Magnetization reversal through soliton flip in a biquadratic ferromagnet with varying exchange interactions

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We study the phenomenon of magnetization reversal in the form of a soliton flip in a biquadratic ferromagnetic spin chain induced by varying bilinear and biquadratic exchange interactions. This is carried out by analyzing the evolution of the velocity and amplitude of the soliton using a perturbation analysis.

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I. INTRODUCTION

The magnetization reversal process, or the switching properties obtained through an understanding of the underlying magnetization dynamics in magnetic systems, is an important issue mainly because the dynamic process is nonlinear in nature. The importance of the issue is also based on the fact that the success and development of random access memories depend on the magnetization switching process. The magnetization reversal process is normally based on a coherent rotation of the magnetization and a propagation of domain walls in the presence of the magnetic field, and this has been studied in detail.[1] One cannot rule out the possibility of magnetization reversal without applying external magnetic fields. Among the various approaches, magnetization switching by stress induced anisotropy and thermal activation assume importance.[2,3] In our present study, we propose that site-dependent or inhomogeneous exchange interactions can also be good candidates for activating magnetization or spin reversal processes in a ferromagnet. Most of the available results on the magnetization reversal process is based on experimental studies and numerical simulations, and analytical results are very limited. Recently, it was found that the classical Landau-Lifshitz (LL) equation is a useful model to describe the fast magnetization reversal process.[4] In an entirely different context, the LL equation, corresponding to different magnetic interactions has been proved to be completely integrable, admitting soliton solutions in several cases.[5–7] Thus the soliton has been identified as a very useful object that can describe localized coherent spin or magnetization configurations in classical ferromagnetic systems. Higher order magnetic interactions, inhomogeneous external magnetic fields, etc., introduce perturbations to these solitons.[6,7] Thus it has become increasingly important to investigate the effect of these perturbations on the soliton and, in particular, a study to understand whether any of these perturbations can contribute to magnetization reversal or spin flip in ferromagnetic systems needs immediate attention. In this paper, we try to find answer for the question whether varying (inhomogeneous) exchange interactions can induce magnetization reversal process in a biquadratic ferromagnetic system. After deriving the equation of motion for spins in a classical inhomogeneous biquadratic ferromagnetic system, in Sec. II, we study the effects of quadratic and tangent hyperbolic type nonlinear inhomogeneity on the spin soliton and magnetization reversal in Sec. III. The paper is concluded in Sec. IV.

II. DYNAMICS OF BIQUADRATIC FERROMAGNET WITH VARYING EXCHANGE INTERACTIONS

In some magnetic materials, the biquadratic exchange interaction plays an important role, and therefore in recent years there has been a considerable interest in the study of quantum spin chains with competing bilinear and biquadratic exchange interactions. In particular, the complete integrability of spin chains with a spin magnitude $S>\frac{1}{2}$ has been established if suitable polynomials in $(\mathbf{S}_i\cdot\mathbf{S}_j)$ are added to the original bilinear Heisenberg spin chain $(\mathbf{S}_i\cdot\mathbf{S}_j)$.[8] When the term $(\mathbf{S}_i\cdot\mathbf{S}_j)^2$ alone in the polynomial is added to it, it corresponds to a spin-1 Heisenberg ferromagnet with bilinear and biquadratic exchange interactions. The study of quantum fluctuations in systems with $S>1$ is far from reach. A theoretical explanation for the origin of biquadratic interaction was given by Anderson[9] and Kittel,[10] and further treated by Huang *et al.* and Allan *et al.*[11] Also, much effort has been devoted to studying quantum fluctuations and the low temperature properties of one-dimensional spin Heisenberg ferromagnets since the fascinating conjecture by Haldane was proposed.[12] When the spin value is large, the quantum fluctuation $(1/[S(S+1)])$ ceases and the spin system is considered classical. In this connection, it is also of interest to study the nonlinear spin dynamics of a biquadratic spin system, with varying exchange interactions in the classical limit.

The Hamiltonian for a classical Heisenberg ferromagnetic spin chain with varying bilinear and biquadratic exchange interactions can be written as

$$\mathcal{H}=-\sum_i\,[J_e f_i(\mathbf{S}_i\cdot\mathbf{S}_{i+1})+J_b g_i(\mathbf{S}_i\cdot\mathbf{S}_{i+1})^2],$$

$$\mathbf{S}_i=(S_i^x,S_i^y,S_i^z), \tag{1}$$

where J_e and J_b are the bilinear and biquadratic exchange parameters, respectively, and f_i and g_i characterize the variation of the bilinear and biquadratic exchange interactions at the lattice sites along the spin chain. The LL equation of motion[13] corresponding to the spin Hamiltonian [Eq. (1)] in the case of a one-dimensional classical continuum spin chain can be obtained from the lattice equation $d\mathbf{S}_i/dt = \{\mathbf{S}_i,\mathcal{H}\}_{PB}$, and by taking the continuum limit by expanding $\mathbf{S}_{n\pm1}$ and f_{n-1}, g_{n-1} about $\mathbf{S}(x,t)$ and $f(x),g(x)$ in Taylor expansions where x is a continuous variable. The resultant LL equation of motion reads

$$\mathbf{S}_t=\mathbf{S}\wedge\Bigg\{\left[A-\frac{a}{2}A_x+\frac{a^2}{2}A_{xx}+\frac{a^2}{2}J_b g(\mathbf{S}\cdot\mathbf{S}_{xx})\right]\mathbf{S}_{xx} +\left[A_x-\frac{a}{2}A_{xx}+\frac{a^2}{3}J_b g(\mathbf{S}\cdot\mathbf{S}_{xxx})\right]\mathbf{S}_x+\frac{a^2}{12}A\mathbf{S}_{xxxx} +\frac{a^2}{6}A_x\mathbf{S}_{xxx}\Bigg\},\quad \mathbf{S}^2=1, \tag{2}$$

where $A(x)=J_e f(x)+J_b g(x)$. Here the suffices t and x represent partial derivatives with respect to t and x, respectively. As the LL equation (2) is a nontrivial vector nonlinear partial differential equation, it is difficult to solve it in its natural vector form to understand the underlying nonlinear spin dynamics. Hence we intend to map this to one of the well known nonlinear evolution equations that admits solitons. Following Lakshmanan and co-workers' space curve mapping procedure,[14,5] Eq. (2) reduces to the following inhomogeneous generalized higher order nonlinear Schrödinger (NLS) equation,

$$iq_t+(kq)_{xx}+2k|q|^2q+2q\int_{-\infty}^{x}k_{x'}|q|^2dx'+\frac{a^2A}{12}[q_{xxxx} +K_1|q|^2q_{xx}+K_2q^2q^*_{xx}+K_3|q_x|^2q+K_4q^*q_x^2 +3K_2|q|^4q]+\frac{a^2}{12}A_x[q_{xxx}+6|q|^2q_x]=0, \tag{3}$$

where

$$k(x)=\tilde{A}+\frac{a^2}{6}A_{xx},$$

$$K_1=-12\left(1+\frac{4J_bg}{A}\right),$$

$$K_2=-8\left(1+\frac{3J_bg}{A}\right),$$

$$K_3=-36\left(1+\frac{8J_bg}{3A}\right),$$

$$K_4=-14\left(1+\frac{24J_bg}{7A}\right).$$

Thus Eq. (3) describes the dynamics of spins in an inhomogeneous classical continuum biquadratic Heisenberg ferromagnetic spin chain in an equivalent representation where q is related to the energy and current densities of the spin system through the curvature and torsion of the space curve. A Painlevé singularity structure analysis brings out the completely integrable models underlying Eq. (3) at different orders and the integrability properties were studied in detail.[5,6,15] Therefore, we desist from presenting more results about this aspect here.

III. EFFECT OF NONLINEAR INHOMOGENEITY ON THE SPIN SOLITON AND MAGNETIZATION REVERSAL

The constraint on the inhomogeneity in the form of a linear function of x for integrability raises an important question about the effect of nonlinear inhomogeneity on the soliton which forms the major concern of the rest of the paper. We consider the inhomogeneous nonlinear equation from Eq. (3) at the order $O(a^0)$ and put $k(x)=k_0+\lambda k_1(x)$, where λ is a small parameter and $k_1(x)$ is a nonlinear function of x. After a suitable rescaling and a redefinition of λ, the equation reads

$$iq_t+q_{xx}+2|q|^2q+\lambda\left[(k_1q)_{xx}+2k_1|q|^2q +2q\int_{-\infty}^{x}k_{1x'}|q|^2dx'\right]=0. \tag{4}$$

We study the effect of nonlinear inhomogeneity on the spin soliton by treating terms proportional to λ in Eq. (4) as a weak perturbation using multiple scale perturbation analysis.[16] It may be noted that while writing Eq. (4), we have dropped terms at $O(a^2)$ because inhomogeneity does not enter at that order. Also, in our earlier studies we have shown that these terms at $O(a^2)$ as a perturbation do not alter the velocity and amplitude of the unperturbed soliton.[6,17] When $\lambda=0$, Eq. (4) reduces to the completely integrable cubic NLS equation which admits envelope one soliton in the form $q=\eta\,\mathrm{sech}\,\eta(\theta-\theta_0)\exp[i\xi(\theta-\theta_0)+i(\sigma-\sigma_0)]$, where $\theta_t=-2\xi$, $\theta_x=1$, $\sigma_t=\eta^2+\xi^2$, and $\sigma_x=0$. Writing η,ξ,θ,θ_0, and σ_0 as functions of a new time scale $T=\lambda t$, and $q=\hat{q}(\theta,T;\lambda)\exp[i\xi(\theta-\theta_0)+i(\sigma-\sigma_0)]$, under the assumption of quasi-stationarity, on expanding $\hat{q}$ in terms of λ as $\hat{q}(\theta,T;\lambda)=\hat{q}_0(\theta,T)+\lambda\hat{q}_1(\theta,T)+\dots$, where $\hat{q}_0=\eta\,\mathrm{sech}\,\eta(\theta-\theta_0)$, at $O(\lambda)$ after substituting $\hat{q}_1=(\phi_1+i\psi_1)$ and (ϕ_1,ψ_1 are real) we obtain $-\eta^2\phi_1+\phi_{1\theta\theta}+6\hat{q}_0^2\phi_1=F_1(\hat{q}_0)$ and $-\eta^2\psi_1+\psi_{1\theta\theta}+2\hat{q}_0^2\psi_1=F_2(\hat{q}_0)$ where $F_1(\hat{q}_0)$ and $F_2(\hat{q}_0)$ are given by $F_1(\hat{q}_0)=-[\xi_T(\theta-\theta_0)-\xi\theta_{0T}]+(k_1\hat{q}_0)_{\theta\theta}-k_1\xi^2\hat{q}_0+2k_1|\hat{q}_0|^2\hat{q}_0+2\hat{q}_0\int_{-\infty}^{\theta}k_{1\theta'}|\hat{q}_0|^2d\theta'$ and $F_2(\hat{q}_0)=[-\hat{q}_{0T}+2(h_1\hat{q}_0)_\theta\xi]$. As $\hat{q}_{0\theta}$ and $\hat{q}_0$ are solutions of the homogeneous parts of the above two equations for ϕ_1 and ψ_1 respectively, the secularity conditions give

$$\int_{-\infty}^{\infty}\hat{q}_{0\theta}F_1d\theta=0,\qquad \int_{-\infty}^{\infty}\hat{q}_0F_2d\theta=0. \tag{5}$$

The effect of inhomogeneity in exchange interactions "k_1" on spin soliton $\hat{q}_0$ can be understood by evaluating the above two integrals for specific forms of "k_1." As it is known that the model supports soliton spin excitations when the inhomogeneity is in the form of a linear function of x,[15] in this paper, we consider (i) a quadratic-type inhomogeneity repre-

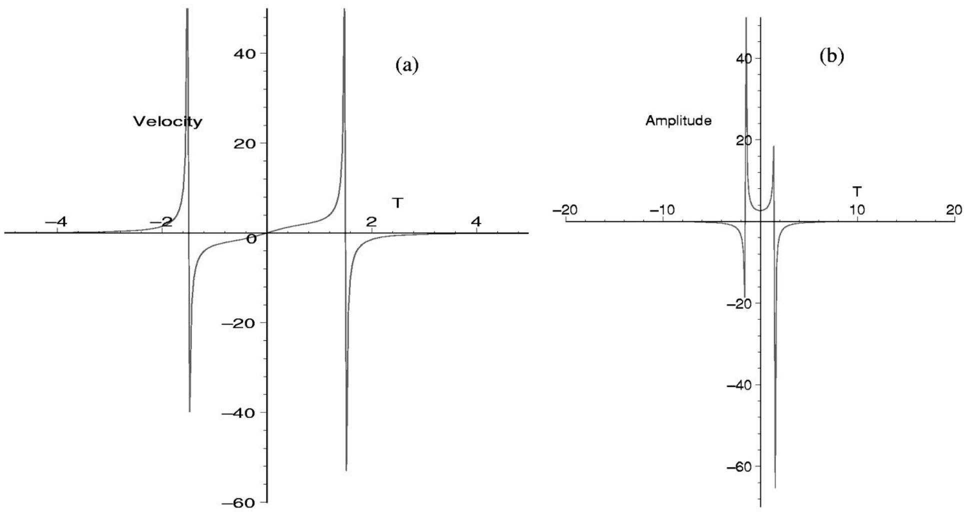

FIG. 1. (a) Time evolution of the velocity (ξ_s) of the soliton and (b) soliton flip in terms of amplitude (η_s), when $C_{s0}=0$ under quadratic inhomogeneity with the initial amplitude $\eta_{s0}=3.0$.

sented by $k_1(x)=\beta_2 x^2+\beta_1 x+\beta_0$, and (ii) a more complicated localized inhomogeneous exchange interaction represented in terms of $k_1(x)=\beta_3 \tanh \eta_p x$ where β_0,β_1,β_2, and β_3 are constants, and η_p replaces the η in the solution.

A. Quadratic inhomogeneity

Substituting the quadratic form of inhomogeneity in secularity conditions (5), we obtain

$$\xi_{sT}=2\hat{\beta}(\eta_s^2-\xi_s^2), \quad \eta_{sT}=2\hat{\beta}\xi_s(\eta_s-2), \tag{6}$$

where ξ_s and η_s are the velocity and amplitude of the soliton in this case (instead of η and ξ), $\hat{\beta}=(\beta_1+\beta_4)$ and $\beta_4=\beta_2\eta_s[(\theta-\theta_0)\tanh\eta_s(\theta-\theta_0)]_{-\infty}^{\infty}$ which is assumed to be finite by choosing θ_0 appropriately. When the inhomogeneity is absent, the velocity and amplitude of the soliton remain constant which is evident from Eq. (6). To understand the nature of evolution of the amplitude and velocity of the soliton under the quadratic form of inhomogeneity, we solve the set of coupled equations (6). We differentiate the second of Eq. (6) and use the first one in the resultant equation to get after integrating once (and suitable rescaling of T) $\eta_{sT}^2-\frac{1}{2}\eta_s^4+\frac{4}{3}\eta_s^3=C_{s0}$, where C_{s0} is the constant of integration. In the above equation C_{s0} is found to be equivalent to the energy of the soliton which oscillates under the potential $(-\frac{1}{2}\eta_s^4+\frac{4}{3}\eta_s^3)$.

When $C_{s0}=0$, the amplitude and velocity of the soliton are found to be respectively $\eta_s=-6Q^{-1}$,

$$\xi_s=\frac{3}{KQ(Q+3)}\left[1-3\left(1-\frac{8}{3\eta_{0s}}\right)^{1/2}-2T\right],$$

where

$$Q=\left[\left(T-\frac{1}{2}\right)^2+3\left(1-\frac{8}{3\eta_{s0}}\right)^{1/2}\left(T-\frac{1}{2}\right)-\frac{6}{\eta_{s0}}\right],$$

and η_{s0} is the initial amplitude of the soliton. Assuming that the soliton with an initial amplitude of $\eta_{s0}=3.0$, starts from rest at $T=0$, we have plotted in Figs. 1(a) and 1(b), the velocity and amplitude of the soliton. From the figures we observe that as time passes on the velocity and amplitude of the soliton increase and when reaching a maximum value suddenly flip leading to magnetization reversal and moves in the opposite direction. Then the soliton dies out slowly due to inhomogeneity of the exchange interactions along the spin chain. As the velocity of the soliton is inversely proportional to the inhomogeneity, the soliton damps very quickly in the case of highly inhomogeneous medium. The soliton would have exploded had it not flipped and reversed when it moved with high speed and fast growing amplitude.

When $C_{s0}\neq 0$, the equation can be integrated and the solution and hence the amplitude η_s and velocity ξ_s of the soliton can be expressed in terms of Jacobian elliptic functions.[18] The amplitude has the form

$$\eta_s(T)=\frac{(\epsilon_- - C_2\epsilon_+)-\epsilon_+ cn\left(\frac{T}{g}\right)}{(\delta_- - C_2\delta_+)-\delta_+ cn\left(\frac{T}{g}\right)},$$

where $\epsilon_\pm=\eta_2 a_2\pm\eta_1 b_2$, $\delta_\pm=a_2\pm b_2$,

$$C_2=\frac{2(\eta_1-\eta_{s0})b_2}{\eta_{s0}\delta_+-\epsilon_+},$$

$$g=\frac{1}{\sqrt{a_2 b_2}},$$

$$a_1^2=\frac{-(\eta_3-\eta_3^*)^2}{4},$$

$$b_1=\frac{(\eta_3+\eta_3^*)}{2},$$

$a_2^2=(\eta_1-b_1)^2+a_1^2$, and $b_2^2=(\eta_2-b_1)^2+a_1^2$. Here η_{s0} is the initial amplitude $\eta_s(0)$ of the soliton at $T=0$ and η_1 and η_2 are the two real roots of the polynomial $\eta_s^4-\frac{8}{3}\eta_s^3+\frac{7}{81}=0$, η_3 is the complex root of the same and η_3^* its complex conjugate. The velocity ξ_s of the soliton is of the form

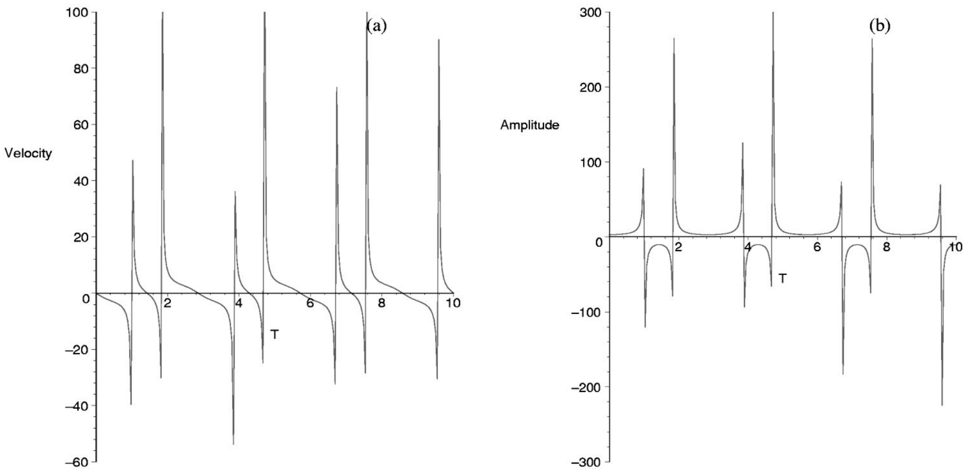

FIG. 2. (a) Time evolution of the velocity(ξ_s) of the soliton and (b) soliton flip in terms of amplitude(η_s), when $C_{s0}=\frac{7}{81}$ under quadratic inhomogeneity with the initial amplitude $\eta_{s0}=3.0$.

$$\xi_s=\frac{(\epsilon_+\delta_- - \delta_+\epsilon_-)\mathrm{sn}\left(\frac{T}{g}\right)dn\left(\frac{T}{g}\right)}{Kg\left\{\left[(\delta_- - C_2\delta_+)-\delta_+ cn\left(\frac{T}{g}\right)\right]\left[(\epsilon_- - C_2\epsilon_+)-\epsilon_+ cn\left(\frac{T}{g}\right)\right]-2\left[(\delta_- - C_2\delta_+)-\delta_+ cn\left(\frac{T}{g}\right)\right]^2\right\}}.$$

Here $\mathrm{sn}(T/g),\mathrm{cn}(T/g)$ and $\mathrm{dn}(T/g)$ are Jacobian elliptic functions. While evaluating the above expressions for amplitude and velocity C_{s0} is chosen as $\frac{7}{81}$ for convenience.

The above expressions for the velocity (ξ_s) and amplitude (η_s) of the soliton are plotted in Figs. 2(a) and 2(b). The figures show that unlike the case corresponding to $C_{s0}=0$, here the flipping of the soliton amplitude or magnetization reversal and the reversal of the velocity of the soliton occur doubly periodically and continues indefinitely. The soliton in this case does not die out. This is because as mentioned earlier, C_{s0}, now acts as a source of energy for soliton flip or for magnetic reversal, sustaining the soliton to oscillate doubly indefinitely periodically under the potential $[(-\eta_s^4/2)+\frac{4}{3}\eta_s^3]$ (forced oscillations).

B. Localized (kink) inhomogeneity

As a second example, we consider the inhomogeneity of the exchange interactions in the form $k_1(x)=\beta_5\tanh\eta_p x$. Substituting the above form of $k_1(x)$ in the secularity conditions and on evaluating the integrals and after rescaling $T\to(-4/3)T$, $\xi_p\to -4\xi_p$, we obtain

$$\eta_{pT}=\beta_5\eta_p^2\xi_p, \quad \xi_{pT}=-\beta_5\eta_p\left(\frac{3}{8}\eta_p^2+2\xi_p^2\right). \tag{7}$$

The above equations can be combined together to give $\eta_{pTT}+(3\beta_5^2/8)\eta_p^5=0$, which on integrating once becomes $(\eta_{pT})^2-(\beta_5^2/8)\eta_p^6=C_3$, where C_3 is an arbitrary constant of integration. In order to understand the effect of the present inhomogeneity on the velocity and amplitude of the soliton more transparently, we choose $C_3=\pm\beta_5^2/8$ for convenience. For instance, when $C_3=-\beta_5^2/8$, the solution after integrating once can be written in terms of the complete elliptic integral of first kind, $F(\hat{\psi},\hat{k})=3^{1/4}[2C_4-(\beta_5^2/4)T]$, where

$$\hat{\psi}=\cos^{-1}\left[\frac{1+(1-\sqrt{3})\eta_p^2}{1+(1+\sqrt{3})\eta_p^2}\right]$$

and the period is given by

$$\hat{k}=\frac{\sqrt{(2+\sqrt{3})}}{2}.$$

When $C_3=\beta_5^2/8$, using the transformation $\eta_p^2=1/(1+z^2)$, the equation after integrating once can be written as

$$\int_y^\infty\frac{dz}{\sqrt{z^4+3z^2+3}}=\frac{-\beta_5^2}{8}T+C_5,$$

where C_5 is another constant of integration. The above equation can be integrated and z can be expressed in terms of Jacobian elliptic functions. Thus, the final form of the amplitude η_p and velocity ξ_p read as

$$\eta_p=\frac{1}{\sqrt{(1+\sqrt{3})}}\left[\frac{1-\chi cnT}{1-\frac{1-\sqrt{3}}{1+\sqrt{3}\ cnT}}\right]^{1/2},$$

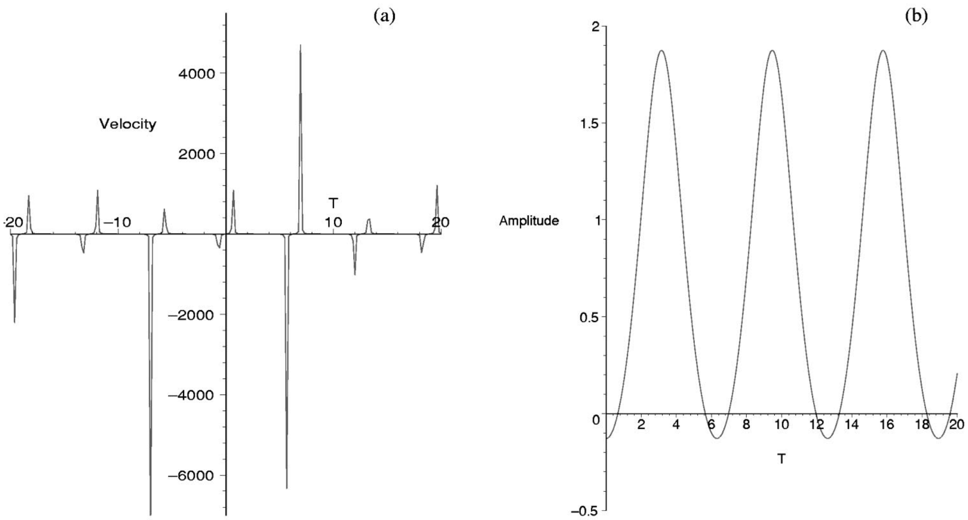

FIG. 3. Time evolution of (a) the velocity (ξ_p) and (b) the amplitude (η_p) of the soliton under tangent hyperbolic inhomogeneity when the initial amplitude is $\eta_{p0}=(1-\sqrt{3})/4\sqrt{3}(1+\sqrt{3})$.

$$\xi_p=\frac{3^{1/8}(1+\sqrt{3})}{\sqrt{16[(1+\sqrt{3})-(1-\sqrt{3})\mathrm{cn}T]}}\left[\frac{(1-\sqrt{3})}{(1+\sqrt{3})}\{1-\chi\mathrm{cn}T\}^{-1/2}+\chi\left[1-\frac{(1-\sqrt{3})}{(1+\sqrt{3})}\mathrm{cn}T\right]\{1-\chi\mathrm{cn}T\}^{-3/2}\right]\mathrm{sn}T\mathrm{dn}T,$$

where $\chi=[1-2\sqrt{3}(1+\sqrt{3})\eta_{p0}^2]$; sn, cn, and dn are Jacobian elliptic functions with the period $\frac{1}{2}\sqrt{(2-\sqrt{3})}$, and η_{p0} is the initial amplitude of the soliton in this case. In Figs. 3(a) and 3(b), we plot the velocity and amplitude of the soliton by choosing the initial amplitude of the soliton as

$$\eta_{p0}=\frac{(1-\sqrt{3})}{4\sqrt{3}(1+\sqrt{3})}$$

for convenience. Unlike the case of quadratic inhomogeneity, in this case, due to the high nonlinear nature of the inhomogeneity the soliton flip and hence the magnetization reversal does not occur very dominantly. The amplitude of the soliton in fact oscillates doubly periodically smoothly with a marginal reversal in the amplitude. However, the velocity of the soliton shows dramatic turns at the points when it reverses or switches. It is observed that when the soliton amplitude changes from positive (negative) to negative (positive), it suddenly moves either forward or backward and on all other occasions it moves very slowly and the soliton is almost at rest. Thus, in this case, due to the high nonlinear nature of inhomogeneity the soliton is almost arrested and jumps suddenly forward or backward when the amplitude of the soliton reverses. The perturbed soliton can be constructed by solving ϕ_1 and ψ_1 equations after using the velocity and amplitude evolution equations corresponding to quadratic and tangent hyperbolic inhomogeneities. The resultant solutions contain secular terms which make the solutions unbounded. We remove these secular terms by choosing the arbitrary constants in the solutions appropriately. Also the boundary conditions determine the more arbitrary constants available in the solutions. Construction of the solutions involve an evaluation of several complicated integrals and very lengthy calculations. The details of calculations and the perturbed solutions will be published elsewhere.

IV. CONCLUSIONS

In summary we studied the effect of nonlinear inhomogeneity in bilinear and biquadratic exchange interactions on the spin soliton of a classical continuum Heisenberg ferromagnetic spin chain, the dynamics of which is governed by an inhomogeneous generalized higher order NLS equation upon mapping the spin chain onto a moving space curve. The effect of inhomogeneity was understood by carrying out a multiple scale perturbation analysis on an inhomogeneous NLS equation and by analyzing the evolution of the velocity and amplitude of the soliton. As examples, we considered quadratic and tangent hyperbolic-type inhomogeneities. The results of the perturbation analysis show an interesting phenomenon of soliton flipping leading to magnetization(spin) reversal in the ferromagnetic medium. When the soliton moves along the spin chain with quadratic-type inhomogeneity, starting from rest with a finite amplitude, the soliton amplitude grows and is accelerated and, when the velocity reaches a maximum value, it suddenly flips (magnetization reversal), then slows down, and once again flips; and this phenomenon is found to occur doubly periodically for an indefinite time. However, when the inhomogeneity is in the form of a tangent hyperbolic function, the soliton jumps back and forth and the amplitude in this case changes smoothly, and doubly periodically, though only with a marginal amount of negative amplitude. Finally, we also constructed perturbed solitons in both cases of inhomogeneity. The above spin soliton flipping phenomenon which leads to magnetization reversal in a ferromangetic medium is expected to have potential applications in magnetic memories and recording.

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