Specific features of a magnetic impurity in a correlated electron model with Ising anisotropy

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(Received 18 January 2002; revised manuscript received 17 September 2002; published 21 November 2002)

We present the Bethe ansatz solution of a magnetic hybridization impurity in a correlated electron host with Ising-like magnetic anisotropy. On the one hand, the strong correlations between host electrons produce a mixed-valence behavior of the localized electrons. On the other hand, the spin-gap of the low-lying unbound electron states caused by the "easy-axis" magnetic anisotropy suppresses the standard Kondo effect. For small enough magnetic fields the magnetic susceptibility of the impurity is zero. The usual Kondo logarithms, characteristic of SU(2)-symmetric systems with gapless spin-carrying excitations, are replaced by square root singularities caused by the closing of the spin gap at a critical field. Similar behavior is predicted for the free edges of an open chain, which again is very different from the isotropic situation. Using the low-lying excitations in the conformal regime we calculate asymptotics of correlation functions.

DOI: 10.1103/PhysRevB.66.184422 PACS number(s): 75.20.Hr, 71.27.+a

I. INTRODUCTION

The Kondo problem[1] describes the effect of a local exchange interaction between the spin of a magnetic impurity and the spins of itinerant electrons. For a free electron host the spins of the conduction electrons screen the impurity spin ($S=\frac{1}{2}$) into a Fermi-liquid fixed point at low energies, while for large enough energies the impurity spin is asymptotically free (with logarithmic corrections). For $S\geqslant\frac{1}{2}$ the impurity spin is undercompensated to an asymptotically free value $S-\frac{1}{2}$ at low energies.[2,3] The crossover energy is the Kondo temperature. Local moment formation and the subsequent screening of the spin is realized within the framework of Anderson's impurity model,[3,4] where localized electrons are hybridized with conduction states. Due to the hybridization the valence of the impurity (expectation value of the number of localized electrons) is in general noninteger, ranging from close to zero (nonmagnetic situation), through the crossover region (mixed-valence regime), to the magnetic or Kondo case (the valence is essentially one).

Magnetic impurities in correlated electron hosts, where the interactions between the itinerant electrons affect the behavior of the impurity, have been studied mostly in one dimension using a wide range of perturbative methods[5] and exact approaches[6–13] (see also Ref. 14). The low-lying excitations of the host in general simultaneously affect the valence of the impurity and screen its magnetic moment. Depending on the host, two situations have to be distinguished. (i) If the excitation spectrum of the correlated electrons is a multicomponent Luttinger liquid (gapless excitations) the screening may be analogous to the ordinary Kondo effect[13] (sometimes the Kondo effect is hidden by interactions[12]). (ii) On the other hand, if the spin excitations of the host are gapped, the Kondo effect is absent as in the case of the Hubbard model with attractive interaction.[8]

All of the abovementioned models assume a local SU(2) spin symmetry. In this study we present exact results for a magnetic impurity in a correlated electron host with magnetic anisotropy. The inclusion of an "easy-axis" magnetic anisotropy (of the Ising type) in the one-dimensional t-J model (with $J=2t$) (Ref. 15) opens a spin-gap in the excitation spectrum without spoiling the integrability of the model,[16,17] i.e., the supersymmetry is preserved. Of course, one-dimensional models have some non-generic features and can only be compared to experimental situations with some caution. However, we expect that the main features of our solution, e.g., the gapped behavior of the magnetic susceptibility for small magnetic fields, are generic to magnetic impurities in spin-gapped hosts.

A magnetic anisotropy appears to be necessary to explain the Kondo screening in some systems.[18] The Kondo effect with a magnetic anisotropy in the local exchange interaction has been studied theoretically[3,19,20] for a free electron host. The magnetic anisotropy is an irrelevant parameter for a $S=\frac{1}{2}$ impurity interacting with a single channel, while for underscreened impurities it can give rise to non-Fermi-liquid effects.[20] The behavior of magnetic impurities in the free electron gas is reminiscent of that of magnetic impurities in spin chains.[21] It is, however, important to keep in mind that correlations between itinerant electrons can affect the properties of a magnetic impurity. Several recent studies derive the Bethe ansatz equations of impurities in spin-gapped correlated electron hosts,[22,23] however, without calculating actual properties of the impurity.

The rest of the paper is organized as follows. The model and the Bethe ansatz equations are introduced in Sec. II. The ground state properties of the system are derived in Sec. III. The finite size corrections to the ground state energy and the mesoscopic properties (conformal dimensions and the Aharonov-Bohm-Casher oscillations) are discussed in Sec. IV. Finally, conclusions follow in Sec. V.

II. MODEL AND BETHE ANSATZ EQUATIONS

A. Model

The Hamiltonian consists of host and impurity terms $\mathcal{H}=\mathcal{H}_{\rm host}+\mathcal{H}_{\rm imp}$. The host is the one-dimensional supersymmetric t-J model with anisotropic magnetic coupling[17] $\mathcal{H}_{\rm host}=\Sigma_j\mathcal{H}_{j,j+1}$, where

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$$\mathcal{H}_{j,j+1}=-\sum_{\sigma}\mathcal{P}_j(c^{\dagger}_{j,\sigma}c_{j+1,\sigma}+c^{\dagger}_{j+1,\sigma}c_{j,\sigma})\mathcal{P}_j+(c^{\dagger}_{j,\downarrow}c_{j,\uparrow}c^{\dagger}_{j+1,\uparrow}c_{j+1,\downarrow}+c^{\dagger}_{j,\uparrow}c_{j,\downarrow}c^{\dagger}_{j+1,\downarrow}c_{j+1,\uparrow})-[\exp(\eta)n_{j,\uparrow}n_{j+1,\downarrow}+\exp(-\eta)n_{j,\downarrow}n_{j+1,\uparrow}]. \quad (1)$$

Here $c_{j,\sigma}$ ($c^{\dagger}_{j,\sigma}$) destroy (create) an electron at the site j with spin σ, $n_{j,\sigma}=c^{\dagger}_{j,\sigma}c_{j,\sigma}$ is the number operator of electrons with spin σ at the site j, $\mathcal{P}_j=(1-n_{j,-\sigma})(1-n_{j+1,-\sigma})$ is the projection operator which excludes the double occupation at each site, and η is the anisotropy parameter. In spin subspace η leads to an intersite magnetic anisotropy of the "easy-axis" (Ising) type. In the limit $\eta=0$ the isotropic su$(1|2)$-supersymmetric t-J model[15] is recovered. The first term represents the hopping between the neighboring sites (with the hopping matrix element set equal to 1), while the other two terms define the exchange spin-spin interaction between electrons at neighboring sites with the (transversal) exchange constant set equal to 2.

We limit ourselves to the case of an "easy-axis" magnetic anisotropy (of the Ising type), because the integrable variant of the Hamiltonian with an "easy-plane" magnetic anisotropy is non-Hermitian[16,22] and, hence, difficult to justify.

The integrable impurity is located on a link of the chain and interacts with electrons on both sites joined by the link. In the simplest case of an impurity of spin $S=\frac{1}{2}$, situated on the link between sites m and $m+1$ the Hamiltonian $\mathcal{H}_{\text{imp}}$ is given by

$$\mathcal{H}_{\text{imp}}=\frac{\sinh^2\eta}{(\sin^2\theta+\sinh^2\eta)}\left(\hat{B}(\mathcal{H}_{m,\text{imp}}+\mathcal{H}_{\text{imp},m+1})-\mathcal{H}_{m,m+1}-i\frac{\tan\theta}{\sinh\eta}[\mathcal{H}_{m,imp},\mathcal{H}_{imp,m+1}]\right), \quad (2)$$

where the square bracket is a commutator, $\mathcal{H}_{m,\text{imp}}$ is of the same form as Eq. (1) but with one site being the impurity, θ determines the impurity-host coupling, and the operator $\hat{B}$ modifies the hopping and the transverse interaction amplitudes by a factor $\cos\theta$. For $\theta=0$ the impurity reduces to one more site of the host.

The expression of the impurity Hamiltonian for arbitrary spin S is similar to Eq. (2), but with $\mathcal{H}_{m,\text{imp}}$ being much more cumbersome.[11,13] The impurity is still located on a link and interacts with the two sites joined by that link. In general, the Hamiltonian consists of three parts: (a) the hopping and the interaction between the impurity and the two neighboring host sites (with renormalized the hopping constants with respect to the host), (b) the hopping and the coupling between the two host sites adjacent to the impurity are renormalized compared to other host sites, and (c) three-site terms involving the impurity and the two adjacent sites. All the coupling constants of the impurity Hamiltonian depend on two parameters: S, the spin of the impurity, and the off-resonance shift θ, determining the impurity-host coupling, while the integrability restricts η, the anisotropy parameter, to be the same as for the host. The coupling constants of the impurity Hamiltonian, of course, also depend on η. The three-site terms of the impurity Hamiltonian violate the T and P symmetries separately, while their product PT is of course invariant.[7,11] These terms are total time derivatives in the classical sense and are only important in quantum mechanical aspects.[13] The form of the impurity Hamiltonian is independent of the boundary conditions of the chain, i.e., periodic or open (unless the impurity is situated at the edge of the chain).

The three-site terms in $\mathcal{H}_{\text{imp}}$ can be avoided[11,12] by placing the impurity at an open end of the host chain. In this case the impurity has only one neighboring host site and the impurity Hamiltonian simplifies considerably, because parts (b) and (c) are now absent. In the limit $\eta\to 0$ the impurity Hamiltonian reduces to the su$(1|2)$ supersymmetric t-J model with impurity.[13] This is seen by rescaling $\theta\to\eta\theta'$ (with θ' being finite) before taking the limit $\eta\to 0$.

B. Scattering matrices

Below we will distinguish between the effective low-energy spin S' (screened spin) and the high-energy free spin $S=S'+\frac{1}{2}$. The two-particle scattering matrix for the host is given by

$$\hat{X}(\lambda)=\frac{\hat{I}\sin(\lambda)\cosh(\eta/2)-i\hat{P}\cos(\lambda)\sinh(\eta/2)}{\sin(\lambda-i\eta/2)}, \quad (3)$$

where $\hat{I}$ is the identity and $\hat{P}$ the two-particle permutation operator. The impurity scattering matrix can be written as

$$S^{\sigma,\sigma'}_{M,M'}(\lambda)=\frac{\sin(A\delta_{\sigma,\sigma'}\delta_{M,M'}+B\delta_{\sigma',\sigma}\delta_{M',M+2\sigma})}{\sin[\lambda+i(2S'+1)\eta/2]}, \quad (4)$$

where σ (σ') and M (M') are the electron and impurity spin S' components of the before (after) scattering, and

$$A=\lambda+i(2S'+1)(\eta/2)[1-2(\sigma M|M+\sigma)\times(\sigma' M'|M'+\sigma')],$$

$$B=-i(2S'+1)\eta(\sigma M|M+\sigma)(\sigma' M'|M'+\sigma'). \quad (5)$$

Here $(\sigma M|M+\sigma)$ denotes the Clebsch-Gordan coefficient $(\frac{1}{2}\sigma,S'M|\frac{1}{2}S'SM+\sigma)$ with $S=S'+\frac{1}{2}$.

The condition of integrability requires that the $\hat{X}$ matrices satisfy the Yang-Baxter (triangular) relations among themselves *and* with the impurity $\hat{S}$ matrix for all values of S and θ, i.e.,

$$\hat{X}_{12}(u)\hat{Y}_{13}(u+v)\hat{Y}_{23}(v)=\hat{Y}_{23}(v)\hat{Y}_{13}(u+v)\hat{X}_{12}(u), \quad (6)$$

where u and v are spectral parameters and $\hat{Y}$ is either $\hat{X}$ or $\hat{S}$. For a given host the choice of impurity is then not arbitrary and the anisotropy η must be the same for $\hat{X}$ and $\hat{S}$. For open boundary conditions in addition reflection matrices $\hat{K}(u)$ (for the anisotropic system they also depend on η) are introduced, which satisfy the reflection equations[24]

$$\hat{X}_{12}(u-v)\hat{K}_1(u)\hat{X}_{21}(u+v)\hat{K}_2(v)=\hat{K}_2(v)\hat{X}_{12}(u+v)\hat{K}_1(u)\hat{X}_{21}(u-v). \quad (7)$$

The trace over the auxiliary subspace of the direct product of $\hat{X}$ matrices for each electron and $\hat{S}$ for the impurity determines the transfer matrix of the problem. Transfer matrices for different spectral parameters commute and can be diagonalized simultaneously. It is well known that there are two different (independent) approaches to the algebraic Bethe ansatz for systems of particles with internal degrees of freedom: (i) The one mentioned above, which for impurity models has been extensively reviewed in Refs. 2–4 (for periodic boundary conditions the number of operators in the monodromy is given by the number of electrons and the impurity) and (ii) the graded approach, in which the charge sector is treated as one more degree of freedom[25] (the number of operators in the monodromy is given by the number of sites plus impurity). For magnetic impurities in a correlated electron host this approach was used in Refs. 11–13. Both approaches, however, yield identical results.

C. Bethe ansatz equations

The Bethe equations are derived using the standard quantum inverse scattering method[25,16] and we present here the results for periodic boundary conditions

$$\frac{\sin\left[v_j-\theta+i(2S'+1)\frac{\eta}{2}\right]}{\sin\left[v_j-\theta-i(2S'+1)\frac{\eta}{2}\right]}\left[\frac{\sin\left(v_j+i\frac{\eta}{2}\right)}{\sin\left(v_j-i\frac{\eta}{2}\right)}\right]^{N_a}=\prod_{\alpha=1}^{M}\frac{\sin\left(v_j-\Lambda_\alpha+i\frac{\eta}{2}\right)}{\sin\left(v_j-\Lambda_\alpha-i\frac{\eta}{2}\right)},\quad j=1,\ldots,N,$$

$$\frac{\sin(\Lambda_\alpha-\theta+iS'\eta)}{\sin(\Lambda_\alpha-\theta-iS'\eta)}\prod_{j=1}^{N}\frac{\sin\left(\Lambda_\alpha-v_j+i\frac{\eta}{2}\right)}{\sin\left(\Lambda_\alpha-v_j-i\frac{\eta}{2}\right)}=-\prod_{\beta=1}^{M}\frac{\sin(\Lambda_\alpha-\Lambda_\beta+i\eta)}{\sin(\Lambda_\alpha-\Lambda_\beta-i\eta)},\quad \alpha=1,\ldots,M, \tag{8}$$

where M is the number of down-spin electrons, N_a is the number of host sites, and N is the total number of electrons in the chain. The eigenfunctions and eigenvalues of the total Hamiltonian are parametrized by the charge rapidities v_j, $j=1,\ldots,N$, and the spin rapidities, Λ_α, $\alpha=1,\ldots,M$. The energy of the system is given by

$$E=-2\sum_{j=1}^{N}\frac{1-\cos(2v_j)\cosh(\eta)}{\cosh(\eta)-\cos(2v_j)}. \tag{9}$$

The z projection of the magnetic moment of the system is $S^z=S'+N/2-M$. Only the first factor on the left-hand sides of Eqs. (8) corresponds to the impurity, while the energy, Eq. (9), depends only implicitly on the impurity. The Bethe equations are independent of the position of the impurity on the chain. Recall that $S'=S-1/2$ represents the effective spin of the low-temperature fixed point.

Similar Bethe equations can be derived for open boundary conditions. *Both*, the Yang-Baxter relations and the reflection equations, have to be satisfied for the integrablity of an open chain. Choosing diagonal reflection matrices (with local boundary potentials $\mu^{\pm}$ acting only on the edges of the open chain), we rewrite the Bethe equations Eqs. (8) for open boundary conditions[11–13]

$$\prod_{\pm}\frac{\sin\left(v_j\pm\theta+i(2S'+1)\frac{\eta}{2}\right)}{\sin\left(v_j\pm\theta-i(2S'+1)\frac{\eta}{2}\right)}\frac{\sin\left(v_j+i\frac{\eta}{2}\xi^{\pm}\right)}{\sin\left(v_j-i\frac{\eta}{2}\xi^{\pm}\right)}\times\left[\frac{\sin\left(v_j+i\frac{\eta}{2}\right)}{\sin\left(v_j-i\frac{\eta}{2}\right)}\right]^{2N_a}=\prod_{\pm}\prod_{\alpha=1}^{M}\frac{\sin\left(v_j\pm\Lambda_\alpha+i\frac{\eta}{2}\right)}{\sin\left(v_j\pm\Lambda_\alpha-i\frac{\eta}{2}\right)},$$

$$\prod_{\pm}\frac{\sin(\Lambda_\alpha\pm\theta+iS'\eta)}{\sin(\Lambda_\alpha\pm\theta-iS'\eta)}\prod_{j=1}^{N}\frac{\sin\left(\Lambda_\alpha\pm v_j+i\frac{\eta}{2}\right)}{\sin\left(\Lambda_\alpha\pm v_j-i\frac{\eta}{2}\right)}=\prod_{\pm}\prod_{\beta=1}^{M}\frac{\sin(\Lambda_\alpha\pm\Lambda_\beta+i\eta)}{\sin(\Lambda_\alpha\pm\Lambda_\beta-i\eta)}. \tag{10}$$

where $j=1,\ldots,N$, $\alpha=1,\ldots,M$, and $\xi^{\pm}$ are related to the boundary potentials $\mu^{\pm}$. We emphasize again that these equations do not depend on the position of the impurity in the chain, i.e., they are the same for the impurity situated at any link in the bulk or at an edge. The choice of one of $\xi^{\pm}$ being equal to zero and the other one equal to $-2i\theta+S'+\frac{1}{2}$ reproduces the Bethe equations of Ge *et al.*[23] The equations formally coincide with the Bethe equations for periodic boundary conditions, Eqs. (8), with only few modifications: The reflection at open edges gives rise to the formal replacement $N_a\to 2N_a$, $N\to 2N$, $M\to 2M$ and *two* impurity factors (instead of one) in each of the Bethe equations (the "period" is now twice as large). Otherwise the Bethe equations for periodic and open boundary conditions are equivalent. Below we perform calculations for periodic boundary conditions and then point out the differences with respect to the open chain case.

III. GROUND STATE PROPERTIES

A. Integral equations

The ground state of the one-dimensional correlated electron system is given by $N-2M$ unbound electron states (with real charge rapidities v_j) and M singlet Cooper-like bound states of for which the charge rapidities are complex conjugated pairs.[17] It follows from Eqs. (8) that the latter are related to spin rapidities Λ_β, such that (to exponential accuracy e^{-N_a})

$$v_\alpha^\pm = \Lambda_\beta \pm i\frac{\eta}{2}. \tag{11}$$

Inserting the real charge rapidities v_j and the pair solutions (characterized by the Λ_α) into Eq. (8) and taking the logarithm of the resulting equations we have

$$\begin{aligned}\Theta[v_j,\eta/2] + \frac{1}{N_a}\Theta[v_j-\theta,(2S'+1)\eta/2] &= \frac{2\pi}{N_a}I_j + \frac{1}{N_a}\sum_{\alpha=1}^{M}\Theta[v_j-\Lambda_\alpha,\eta/2],\\ j&=1,\ldots,N-2M,\end{aligned}$$

$$\begin{aligned}\Theta[\Lambda_\alpha,\eta] + \frac{1}{N_a}\Theta[\Lambda_\alpha-\theta,(S'+1)\eta] &= \frac{2\pi}{N_a}J_\alpha + \frac{1}{N_a}\sum_{j=1}^{N-2M}\Theta[\Lambda_\alpha-v_j,\eta/2]\\ &+\frac{1}{N_a}\sum_{\beta=1}^{M}\Theta[\Lambda_\alpha-\Lambda_\beta,\eta],\\ \alpha&=1,\ldots,M,\end{aligned} \tag{12}$$

where $\Theta[v,\eta]=2\tan^{-1}(\tan v\coth\eta)$, and the quantum numbers I_j and J_α arise because the logarithm is a multivalued function. The quantum numbers completely determine the solutions for the ground state and the elementary excitations. The energy of the system is

$$\begin{aligned}E=-2\sum_{j=1}^{N-2M}\frac{1-\cos(2v_j)\cosh(\eta)}{\cosh(\eta)-\cos(2v_j)}\\ -2\cosh(\eta)\sum_{\alpha=1}^{M}\left(2-\frac{\sinh^2(\eta)}{\sin^2(\Lambda_\alpha)+\sinh^2(\eta)}\right).\end{aligned} \tag{13}$$

In the thermodynamic limit (i.e., $N_a,N,M\to\infty$ with the ratios N/N_a and M/N_a remaining fixed) we introduce densities for the rapidities $\rho(v)$ and $\sigma(\Lambda)$, and their "holes" $\rho_h(v)$ and $\sigma_h(\Lambda)$. The Bethe equations satisfied by the densities are

$$\begin{aligned}\Theta'[v,\eta/2]+\frac{1}{N_a}X(v) &= \int d\Lambda\,\Theta'[v-\Lambda,\eta/2]\sigma(\Lambda)\\ &+2\pi[\rho(v)+\rho_h(v)],\end{aligned}$$

$$\begin{aligned}\Theta'[\Lambda,\eta]+\frac{1}{N_a}Y(\Lambda) &= \int dv\,\Theta'[\Lambda-v,\eta/2]\rho(v)\\ &+\int dz\,\Theta'[\Lambda-z,\eta]\sigma(z)\\ &+2\pi[\sigma(\Lambda)+\sigma_h(\Lambda)],\end{aligned} \tag{14}$$

where the prime denotes derivative with respect to the first argument. The driving terms for periodic boundary conditions are

$$\begin{aligned}X(v)&=\Theta'[v-\theta,(2S'+1)\eta/2],\\ Y(\Lambda)&=\Theta'[\Lambda-\theta,(S'+1)\eta]\end{aligned} \tag{15}$$

and for open boundary conditions

$$\begin{aligned}X(v)&=\frac{1}{2}\left\{\Theta'[v,\eta/2]+\sum_{\pm}\Theta'[v\pm\theta,(2S'+1)\eta/2]\right\},\\ Y(\Lambda)&=\frac{1}{2}\left\{\Theta'[\Lambda,\eta/2]-\Theta'[\Lambda,\eta]\right.\\ &\left.+\sum_{\pm}\Theta'[\Lambda\pm\theta,(S'+1)\eta]\right\}.\end{aligned} \tag{16}$$

In the thermodynamic limit the energy of the system is

$$\begin{aligned}E=-2\int\rho(v)\left[\frac{1-\cos(2v)\cosh(\eta)}{\cosh(\eta)-\cos(2v)}\right]dv\\ -2\cosh(\eta)\int\sigma(\Lambda)\left[2-\frac{\sinh^2(\eta)}{\sin^2(\Lambda)+\sinh^2(\eta)}\right]d\Lambda.\end{aligned} \tag{17}$$

The second terms on the left-hand side of Eqs. (12) and (14) are the driving terms due to the impurity. The energy only depends implicitly on the parameters of the impurity. The number of electrons and the z projection of the magnetization per site are given by $N/N_a=2\int d\Lambda\,\sigma(\Lambda)+\int dv\,\rho(v)$ and $M^z=(S'/N_a)+(1/2)\int dv\,\rho(v)$, respectively.

We introduce the usual "dressed" energies, $\varepsilon(v)$ for the unbound electron states and $\Psi(\Lambda)$ for the singlet pairs, which satisfy the following integral equations:

$$\begin{aligned}\Theta'[v,\eta/2]-\mu-\frac{H}{2} &= \frac{1}{2\pi}\int d\Lambda\,\Theta'[v-\Lambda,\eta/2]\Psi(\Lambda)\\ &+\varepsilon(v),\\ \Theta'[\Lambda,\eta]-2\mu &= \frac{1}{2\pi}\int dv\,\Theta'[\Lambda-v,\eta/2]\varepsilon(v)\\ &+\frac{1}{2\pi}\int dz\,\Theta'[\Lambda-z,\eta]\Psi(z)+\Psi(\Lambda),\end{aligned} \tag{18}$$

where H is the external magnetic field and μ is the chemical potential of the electrons. H and μ are introduced as the Lagrange multipliers for the conservation of the magnetization and the total number of electrons, respectively. All states with negative (positive) dressed energy are populated (empty). The bands $\varepsilon(v)$ and $\Psi(\Lambda)$ can form Dirac seas with the filling beginning at the edges of the interval $[-\pi,\pi]$, where the dressed energies have their minimum. The integral equations for ''dressed energies'' do not depend on the impurity.

The driving terms in Eqs. (14), i.e. the terms that do not explicitly depend on ρ and σ, are either of order 1 or of order $1/N_a$. The terms of order 1 determine the behavior of the host, while the ones of order $1/N_a$ drive the impurity. Eqs. (14) are linear integral equations, such that we may write $\rho=\rho_{\mathrm{host}}+(1/N_a)\rho_{\mathrm{imp}}$ and $\sigma=\sigma_{\mathrm{host}}+(1/N_a)\sigma_{\mathrm{imp}}$, and obtain separate integral equations for the rapidity densities for the host and the impurity.

B. Properties

The energy of the unbound electron states are gapped for an external magnetic field less than a critical value H_c, given by

$$H_c=-2\mu+2\Theta'[\pi,\eta/2]-\frac{1}{\pi}\int d\Lambda\,\Theta'[\pi-\Lambda,\eta/2]\Psi(\Lambda). \tag{19}$$

In other words, H_c is one half of the minimal external magnetic field necessary to depair a singlet bound state. If the value of the external magnetic field is larger than H_s, given by

$$H_s=-2\mu+2\Theta'[\pi,\eta/2], \tag{20}$$

the magnetization is maximal, i.e., saturated. At this saturation field the system undergoes a second order phase transition into the ferromagnetic spin-polarized state, in which there are no pairs because the ''dressed'' energy of bound electrons is gapped. This behavior is similar to a type-II superconductor in a magnetic field: For $H\leqslant H_c$ there are only Cooper-pairs, while for $H_c\leqslant H\leqslant H_s$ pairs and unbound electrons coexist, which is reminiscent of the Meissner effect. Note, however, that in an one-dimensional electron gas there is no true superconducting order with off-diagonal long range order, but the correlation functions of the singlet pairs and/or unbound electrons fall off with power-laws for long times and/or distances. For $H\geqslant H_s$ it is straightforward to obtain the ground state energy. In the intermediate phase, $H_c\leqslant H\leqslant H_s$, however, the ground state energy depends on the filling of both Dirac seas.

We first study the case $H<H_c$, where the ground state consists only of singlet pairs $(2M=N)$.[17] In this case the Bethe equations reduce to only one set of equations

$$\Theta[\Lambda_\alpha,\eta]+\frac{1}{N_a}\Theta[\Lambda_\alpha-\theta,(S'+1)\eta]$$
$$=\frac{2\pi}{N_a}J_\alpha+\frac{1}{N_a}\sum_{\beta=1}^{M}\Theta[\Lambda_\alpha-\Lambda_\beta,\eta],$$
$$\alpha=1,\ldots,M, \tag{21}$$

where the J_α are integers (half-integers) for odd (even) $M+1$, limited by $J_{\max}$, i.e.,

$$|J_\alpha|\leqslant(N_a-M-1)/2=J_{\max}. \tag{22}$$

The ground state corresponds to two sequences of quantum numbers J_α belonging to the interval $[-J_{\max},J_{\max}]$, beginning at the edges of the interval, i.e., $-J_{\max},-J_{\max}+1,\ldots$ and $\ldots,J_{\max}-1,J_{\max}$ with both sequences being equally long. In the thermodynamic limit we have

$$\Theta'[\Lambda,\eta]+\frac{1}{N_a}Y(\Lambda)$$
$$=2\pi[\sigma(\Lambda)+\sigma_h(\Lambda)]$$
$$+\left[\int_{-\pi}^{-\Lambda_0}+\int_{\Lambda_0}^{\pi}\right]dz\,\Theta'[\Lambda-z,\eta]\sigma(z). \tag{23}$$

Here Λ_0 plays the role of a Fermi point, because in the ground state only states with $\Lambda\in[-\pi,-\Lambda_0]\cup[\Lambda_0,\pi]$ are filled. The wave functions of pairs are symmetric (pairs of electrons form bosons), but these bosons are hard-core ones, satisfying an anyonlike exclusion statistics, as a consequence of the interactions among the pairs. Because they are hard-core bosons the Cooper-pairs form a Dirac sea. The parameter Λ_0 is related to the chemical potential μ via $\Psi(\pm\Lambda_0)=0$. The energy and the total number of electrons are now given by Eq. (17) with only the σ density term integrated over the occupied states.

The magnetization of the impurity is exactly S' for $H\leqslant H_c$ for both, open or periodic, boundary conditions. This implies that the Kondo effect is absent in the present model, due to the spin-gap induced by the Ising-like (''easy-axis'') magnetic anisotropy. The low-lying excitations do not carry spin, and, consequently, cannot couple to the spin S' to form a magnetic moment of spin S for $H\leqslant H_c$. The valence, i.e., the number of localized electrons at the impurity, varies with Λ_0 (or μ), from one at $\Lambda_0=0$ (i.e., for a filled band or the maximal number of conduction electrons) to zero for $\Lambda_0=\pi$, i.e. for an empty band of conduction electrons.

Next we consider the situation $H_c\leqslant H\leqslant H_s$, where both, unbound electrons and singlet pairs, have gapless low-lying excitations, i.e., form Dirac seas. The valence of the impurity again depends on the density of electrons, and, interestingly, also on the external magnetic field. Due to the van Hove singularity of the empty band of unpaired electron states, the magnetization of the host is proportional to $\sqrt{H-H_c}$ for fields H slightly larger than H_c. This feature is characteristic of a Pokrovskii-Talapov level-crossing transition, which is the analog of a second order phase transitions in one-dimension. The magnetization of the impurity is driven by the host,

$$M_{\mathrm{imp}}^z=S'+f_S(\theta,\eta)\sqrt{H-H_c}, \tag{24}$$

where $f_S(\theta,\eta)$ is also a function of the band filling. The magnetic susceptibility of the impurity has a square root singularity as H_c is approached from above. This is also very different from the standard Kondo effect, where for spin $\frac{1}{2}$ the magnetic susceptibility of the impurity is finite for small magnetic fields.[7]

For open boundary conditions the magnetic susceptibility of the impurity diverges as strongly as the magnetic susceptibility of the open edges themselves (all inversely proportional to $\sqrt{H-H_c}$). This is also very different from the usual behavior of a magnetic impurity in an open correlated electron chain with SU(2) spin symmetry and gapless excitations, where the magnetic susceptibility of the edges diverges (though logarithmically), while the one for an impurity of spin $S=\frac{1}{2}$ remains finite.[11,13]

With increasing magnetic field the population of the Dirac sea of singlet pairs gradually decreases until H_s is reached, which is the field at which the band is empty. For fields larger than the saturation field H_s the magnetization of the impurity is equal to $M^z_{\rm imp}=S'+(n_{\rm imp}/2)$, where $n_{\rm imp}$ is the valence of the impurity.

For imaginary θ, the Hamiltonian of the impurity if placed in the bulk, i.e., not at the edge, is non-Hermitian (the energy eigenvalues are real, though). This is independent of the boundary conditions (open or periodic). In this case the incoming and reflecting waves of the electrons "see" two different effective spins of the impurity corresponding to $S'\pm|\theta|$.[12] However, the Ising magnetic anisotropy of the model again suppresses any manifestation of the Kondo effect, since only the spin-singlet pairs are gapless for $H\leqslant H_c$, but cannot screen the effective spins of the impurity. For fields slightly larger than H_c the van Hove singularity of the empty band of unbound electron states manifests itself, rather than the weaker logarithmic Kondo singularities.

At finite but low temperatures the magnetic susceptibility of the impurity (as well as the susceptibility of the edges of an open chain) is exponentially small for $H<H_c$ and $H>H_s$. At $H=H_c$ or H_s the magnetic susceptibility and the Sommerfeld coefficient of the specific heat display the $T^{1/2}$ features corresponding to the van Hove singularities of the empty bands. For $H_c<H<H_s$, on the other hand, the magnetic susceptibility is finite for $S=\frac{1}{2}$ as $T\to 0$ and Curie-like for $S\geqslant\frac{1}{2}$. The specific heat is proportional to the temperature everywhere away from the van Hove singularities.

IV. MESOSCOPIC PROPERTIES

The mesoscopic (finite size) effects reveal the difference between periodic and open boundaries conditions. In the gapless region, the correlation functions of operators asymptotically decay algebraically as a consequence of the low-lying excitations. For periodic boundary conditions they are proportional to[26,27]

$$(x-iv_\rho t)^{-2\Delta^+_\rho}(x+iv_\rho t)^{-2\Delta^-_\rho}(x-iv_\sigma t)^{-2\Delta^+_\sigma} \times(x+iv_\sigma t)^{-2\Delta^-_\sigma}, \tag{25}$$

where $v_\rho=\varepsilon'/2\pi\rho|_{v=v_0}$ [v_0 is the Fermi point for unbound electron states, defined by $\varepsilon(\pm v_0)=0$ for $H\geqslant H_c$], and $v_\sigma=\Psi'/2\pi\sigma|_{\Lambda=\Lambda_0}$ are the Fermi velocities of unbound electrons and spin-singlet pairs, respectively. Here ε' and Ψ' are the derivatives of the dressed energies with respect to the rapidity at the Fermi point. The conformal dimensions $\Delta^\pm_i$ of primary operators are

$$2\Delta^\pm_i=2n^\pm_i+\left[\frac{z_{\rho,i}(\Delta N_\sigma-\delta_\sigma)-z_{\sigma,i}(\Delta N_\rho-\delta_\rho)}{\det\hat z}\pm[z^T_{i,\sigma}(\Delta D_\sigma-d_\sigma)+z^T_{i,\rho}(\Delta D_\rho-d_\rho)]\right]^2, \tag{26}$$

where $i=\rho,\sigma$ and $n^\pm_i$ are the number of particle-hole excitations at the right and left Fermi points of each of the Dirac seas. Here ΔN_i denotes the change in the number of quasiparticles in the Dirac sea and ΔD_i represents the number of backscattering excitations (particle transfer from the left to the right Fermi points).

The dressed charge matrix $\hat z=\hat\xi|_{v=v_0,\Lambda=\Lambda_0}$ measures correlations between the bands. The quantities $\xi_{i,k}$ are defined as $\xi_{i,k}=-\partial\epsilon_k/\partial\mu_i$, where $\epsilon_\rho=\varepsilon$, $\epsilon_\sigma=\Psi$, $\mu_\rho=\mu+(H/2)$ and $\mu_\sigma=2\mu$. The integral equations satisfied by the components of $\hat\xi$ are obtained by differentiating the integral equations for the dressed energies, Eqs. (18), with respect to μ_i.[17,27]

The quantities δ_i and d_i renormalize ΔN_i and ΔD_i because of the impurity and the free edges of an open chain (in the case of periodic boundary conditions external fluxes through the ring also contribute to these quantities). The impurity contributions are related to the valence and magnetization of the impurity and of the free edges of the chain (in the case of open boundary conditions) via the Friedel's sum rule. Neither the Fermi velocities nor the matrix of dressed charges depend on the boundary conditions. Since the unbound electrons are gapped for $H\leqslant H_c$, the Fermi velocity v_ρ and the elements of the dressed charge matrix related to the unbound electrons vanish.

Clearly, backscattering processes are absent for open boundary conditions and hence $\Delta D_i=d_i=0$, such that only chiral excitations determine the physics in this case. For periodic boundary conditions the phase shifts caused by external magnetic and electric fluxes (Aharonov-Bohm and Aharonov-Casher effect)[28] are taken into account with $\Delta D_i\to\Delta D_i+\phi_i$ ($\phi_\sigma=\{\{2\Phi/\Phi_0\}\}$, and $\phi_\rho=\{\{\Phi/\Phi_0\}\}+\{\{F/F_0\}\}$, where Φ is the magnetic flux, $\Phi_0=hc/e$ is the unit magnetic flux, $F=4\pi\tau$ is the electric flux generated by a string passing through the center of the ring with linear charge density τ, $F_0=hc/\mu_B$ is the unit electric flux, μ_B is the Bohr magneton, and $\{\{a\}\}$ denotes the fractional part of a to the nearest integer). The fluxes give rise to charge and spin persistent currents of the Aharonov-Bohm-Casher type in a closed ring configuration. For $H\leqslant H_c$ there is no Aharonov-Casher effect, but only oscillations of the charge persistent current of period $\Phi_0/2$ due to the spin-singlet pairs (charge $-2e$). For $H_c\leqslant H\leqslant H_s$, on the other hand, there is an interference of Aharonov-Bohm oscillations in the charge persistent current with period Φ_0 and $\Phi_0/2$ and the spin persistent current displays Aharonov-Casher oscillations of period F_0.

V. CONCLUSIONS

In summary, we have studied the behavior of a magnetic hybridization impurity in a strongly correlated electron host with a magnetic anisotropy of the ''easy-axis'' type (i.e., Ising-like). The magnetic anisotropy of the Ising type gives rise to a gap for low-energy unbound electron states at small external magnetic fields.

The behavior of the magnetic impurity is determined by the properties of the host. The valence of the impurity varies with the electron density from zero to one (for the half-filled case), similar to the situation of an impurity in the t-J model with SU(2) spin symmetry. Also, similarly to the isotropic-exchange case, the effective spin of the impurity at very high fields dynamically yields $S'+(n_{\mathrm{imp}}/2)$ (cf. Refs. 7,11–13), i.e., the interaction changes the effective spin of the impurity.

However, due to the spin-gap of the host the impurity has no Kondo effect in the usual sense. For fields smaller than the field necessary to close the spin-gap, the spin of the impurity is S', independent of the field and the valence. For $H>H_c$ the impurity magnetization increases with field, indicating that there is partial screening of the impurity spin, however, without the usual Kondo logarithms, characteristic of asymptotic freedom in the SU(2)-symmetric gapless situation. In the present case, the van Hove singularity of the empty band of unbound electrons gives rise to a square root discontinuity in the properties and the weaker Kondo logarithms are suppressed. In other words, the impurity behavior is dominated by the gap.

For open boundary conditions the edges in general also contribute to the magnetic susceptibility, e.g., for SU(2) spin symmetry this contribution diverges, i.e., in zero field it is larger than the susceptibility of a magnetic impurity of spin $S=\frac{1}{2}$ (which is finite due to the Kondo effect). However, the spin gap caused by the magnetic anisotropy suppresses the magnetic susceptibility of edges for $H<H_c$. The van Hove singularity of the empty band of unpaired electrons gives rise to a square root-divergent susceptibility of the edges and the impurity for H slightly above H_c, without revealing Kondo logarithms.

Of course, some of the features of this one-dimensional model are not realistic to an experimental situation. For example, the square root van Hove singularity is a one-dimensional property, but also appears in BCS-like density of states. However, we expect that the main properties of our solution, namely, the behavior of the magnetic susceptibility of the impurity in a host with spin gap at small external magnetic fields, are generic features for magnetic impurities in correlated electron hosts with the Ising-like (''easy-axis'') magnetic anisotropy.

ACKNOWLEDGMENTS

P.S. acknowledges the support by the National Science Foundation under Grant No. DMR01-05431 and the Department of Energy under Grant No. DE-FG02-98ER45797.

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