

Possible weak temperature dependence of electron dephasing

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The first-principle theory of electron dephasing by disorder-induced two state fluctuators is developed. There exist two mechanisms of dephasing. First, dephasing occurs due to direct transitions between the defect levels caused by inelastic electron-defect scattering. The second mechanism is due to violation of the time reversal symmetry caused by time-dependent fluctuations of the scattering potential. These fluctuations originate from an interaction between the dynamic defects and conduction electrons forming a thermal bath. The first contribution to the dephasing rate saturates as temperature decreases. The second contribution does not saturate, although its temperature dependence is rather weak, $\propto T^{1/3}$. The quantitative estimates based on the experimental data show that these mechanisms considered can explain the weak temperature dependence of the dephasing rate in some temperature interval. However, below some temperature dependent on the model of dynamic defects the dephasing rate tends rapidly to zero. The relation to earlier studies of the dephasing caused by the dynamical defects is discussed.

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I. INTRODUCTION

The problem of dephasing of electron states in low-dimensional structures is the focus of interest for many research groups. This is due to novel experiments on the Aharonov-Bohm effect in specially designed mesoscopic circuits^{1,2} and on weak localization magnetoresistance in low-dimensional samples,³ as well as to new theoretical discussions of dephasing.⁴⁻⁹ In particular, dephasing due to defects with internal degrees of freedom as a source of dephasing were recently addressed.^{6,7} According to the model discussed in Ref. 7, a temperature interval can exist in which the dephasing rate is almost temperature independent.

In this work we revisit the dephasing due to dynamic defects which interact with electrons and tunnel between their two states due to interaction with some thermal bath. Examples of such defects are disorder-induced two-state fluctuators^{10,11} present in any disordered material, impurities with noncompensated spin, etc. These defects produce a random time-dependent field and in this way they violate the time-reversal symmetry of the problem. According to a conventional opinion, this property is sufficient to produce dephasing. However, this is true only under the condition that a typical defect relaxation time is shorter than the time during which the electron interference pattern is formed. Indeed, if the defects do not change their state during the pattern formation they act as static ones and can contribute to the interference only in a constructive way.⁷

The purpose of this paper is to develop a systematic theory of weak localization with dephasing due to dynamic defects interacting with electrons which results in a smooth temperature dependence at relatively low temperatures T . The dynamic defects are specified as two-level tunneling states (TLS's) that exist in any crystalline metal.

The main message of this paper is the following. There exist two mechanisms of electron dephasing due to dynamic defects. The first one is due to direct inelastic transitions between the levels of the TLS leading to the possibility of

determining the actual path of the electron, and consequently to loss of interference. The second one is due to relaxation dynamics of dynamic defects which fluctuate due to interaction with the thermal bath. Time dependence of the electron scattering crosssection due to the defects' fluctuations lead to violation of the time-reversal symmetry and, as a consequence, to decoherence. To our knowledge, the theory relevant to the second mechanism has not been developed. However, there exists a temperature interval where this relaxation mechanism is dominating.

The paper is organized as follows. Below we will give physical considerations to describe dephasing by dynamic defects which will be then confirmed by a diagrammatic approach, see Sec. II. In this section the model for electron-TLS interaction will be formulated, Sec. II A; this model will be used to calculate the dephasing rate due to *identical* TLS's, Sec. II B; and, finally, an average procedure over different TLSs will be considered, Sec. II C. Estimates and discussion will be given in Sec. III, while the conclusions will be given in Sec. IV.

Qualitative considerations. Let us start with a toy model which illustrates the essence of the physics involved. Then in Sec. II the results will be confirmed by calculation. Consider the electron motion in a slowly varying potential field $U(\mathbf{r}, t)$. Let us calculate the phase difference $\Delta\varphi$ between the electron waves moving from the same point C along the same closed path \mathcal{P} clockwise and counterclockwise, see Fig. 1.

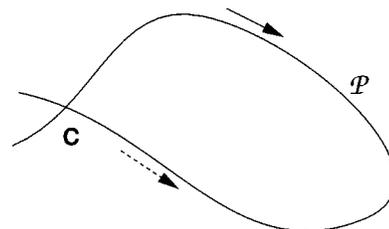


FIG. 1. A closed-loop trajectory.

We begin with evaluation of the variation ΔS of the electron's action S due to the time variation of potential U . We assume that an electron during its motion along the trajectory \mathcal{P} experiences many scattering events against both static and dynamic defects, so that the trajectory can be approximated by a smooth curve. We have

$$S = \int \mathbf{p} d\mathbf{r} = \int \mathbf{p} \cdot \mathbf{n} ds, \quad (1)$$

where $\mathbf{n} = \mathbf{p}/p$ is a unit vector parallel to the tangent to the curve \mathcal{P} and ds is the length element of the curve. This can also be written

$$S = \int ds \sqrt{2m(\mathcal{E} - U)},$$

$\mathcal{E} = p^2/2m$ being the electron kinetic energy while m is the electron effective mass. Expanding this equation in powers of the potential energy U assumed small, one gets

$$\Delta S = - \int \frac{ds}{v} U(s, t) = - \int dt U(s_t, t).$$

Here s_t is the electron's coordinate on the trajectory parametrized by time t . So U depends on time both via the space coordinate s_t and explicitly.

Let now t_0 be the total time of the motion of an electron along the loop \mathcal{P} . Accordingly, the phase variation in the course of a clockwise motion is

$$(\Delta \varphi)_+ = - \frac{1}{\hbar} \int_0^{t_0} dt U(s_t, t), \quad (2)$$

while for the counterclockwise motion one has

$$(\Delta \varphi)_- = - \frac{1}{\hbar} \int_0^{t_0} dt U(s_{t_0-t}, t). \quad (3)$$

The dephasing means a non-vanishing phase difference $\Delta \varphi \equiv (\Delta \varphi)_+ - (\Delta \varphi)_-$. Thus,

$$\overline{(\Delta \varphi)^2} = \sum_{\pm} [\overline{(\Delta \varphi)_{\pm}^2} - \overline{(\Delta \varphi)_{\pm} (\Delta \varphi)_{\mp}}].$$

Using Eqs. (2) and (3) one can express the above expression through $\int_0^{t_0} dt \int_0^{t_0} dt' U(s_{i_t}, t) U(s_{i_{t'}}, t')$ where $i, k = \pm$, $t_+ \equiv t$, $t_- \equiv t_0 - t$. We assume that there is *no spatial correlation* between the scattering centers $U(s_t, t) U(s_{t'}, t') \propto \delta(s_t - s_{t'})$, which implies

$$\overline{U(s_{t_{\pm}}, t) U(s_{t'_{\pm}}, t')} \propto \overline{U^2(s, t)} \delta(t - t'),$$

$$\overline{U(s_{t_{\pm}}, t) U(s_{t'_{\pm}}, t')} \propto \overline{U(s, t) U(s, t_0 - t)} \delta(t + t' - t_0).$$

Using these expressions and introducing the time correlation function of the time-dependent random potential as

$$\overline{U(s, t) U(s, t')} \equiv \overline{U^2} f(t - t'), \quad \overline{U^2} \equiv \overline{U^2(s, t)}, \quad f(0) = 1,$$

one obtains

$$\overline{(\Delta \varphi)^2} \propto \overline{U^2} \int_0^{t_0} dt [1 - f(2t - t_0)].$$

If there are several mechanisms responsible for dephasing characterized by different coupling strengths and different correlation functions the resulting phase variance can be expressed as

$$\overline{(\Delta \varphi)^2} \propto \sum_s \int_0^{t_0} \frac{dt}{\tau_s} [1 - f_s(2t - t_0)]. \quad (4)$$

Here we have absorbed the random scattering potential into the partial relaxation rates τ_s^{-1} . They, as well as the correlation functions, depend on the properties of dynamic defects. We wish to emphasize that Eq. (4) demonstrates the following point indicated above. If the defect has not relaxed during the time $2t - t_0$ between two acts of scattering then in spite of noninvariance of the Hamiltonian respective to the time reversal there is no phase relaxation.

We distinguish two mechanisms of dephasing. The first, which we call the resonant mechanism, is connected to interactions which cause real transitions between different states of the environment. This can be illustrated by the famous double slit experiment. If we send electrons at the double slit, it will pass through both slits and interfere with itself, creating an interference pattern on the screen. Putting detectors to determine which slit the electron really passed through will destroy the interference pattern. If the interaction with the environment in any way allows us to determine the path of the electron, interference is lost. The second mechanism of dephasing is related to a change in the state of the environment due to its own internal dynamics. A dynamic environment leads to a difference in a scattering potential "felt" by an electron state during clockwise and counterclockwise motion. As a result, time-reversal symmetry is broken and the interference pattern decays. We call this mechanism the relaxational mechanism, because it is caused by the relaxation of the environmental states which results when the environment is considered to be in contact with a thermal bath.

At this point we would like to compare our description to the one given in Ref. 12, where it is proved that the dephasing can be described in two equivalent ways. Either you consider the change in the electron phase or you consider the change of state of the environment, where complete dephasing corresponds to the environment being in orthogonal states. The last point of view would imply the existence of only the first mechanism of dephasing that we consider, resonant transitions of environment states. We want to emphasize that our second, relaxational mechanism is not in conflict with this, but is a result of our *description* of the process. In Ref. 12 the environment is considered as a mechanical system evolving according to its own Hamiltonian, whereas we consider the environment to be a statistical system at some temperature. That is, we calculate the action of the environment on the electrons, but do not consider the action of the electrons on the environment. In principle, if one were to follow all the complex dynamics of the environment one would find that it does indeed evolve into orthogonal states

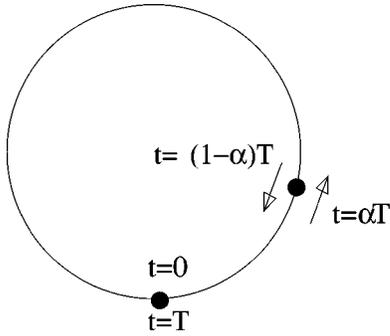


FIG. 2. Closed loop with TLS detector.

as the electron dephases according to the relaxational mechanism, and it would be seen that this is only the resonant mechanism in disguise. However, as the environment consists of a macroscopic number of degrees of freedom it is more natural to treat it statistically as a thermal bath. In other words, the phase of an electron state forming a Cooperon trajectory decays due to *real transitions* in the thermal bath formed by other electrons assisted by *virtual* processes involving dynamic defects. In a perturbative approach these processes occur in the fourth order in the electron-defect coupling constant. In particular, they do not enter the second-order calculation of defect-enhanced electron-electron interaction.¹³ However, it will be shown that they play an important role in dephasing.

A different classification can be made by discriminating between the two different regimes of phase dynamics—phase *jumps* and phase *wandering* (or phase diffusion). To understand this, let us consider the resonant mechanism. Consider the case of interest for weak localization, that of one electron traveling around a closed loop both in the clockwise and counterclockwise direction, and interferes with itself after completing a full circuit. If we can determine which direction the electron went, we will not get interference. If we are unable to do so, it will appear. As a detector we use a two level system that is placed at a point on the right hand side of the loop, the distance from the starting (and ending) point being a fraction $\alpha < \frac{1}{2}$ of the total circumference (see Fig. 2).

The energy splitting of the two level system is E , and it starts out in the lower state. When the electron passes, it excites the two level system. We determine the direction of the electron by measuring the time at which this happens. The accuracy with which we can make this measurement is limited by the uncertainty principle $\Delta t \Delta E > \hbar$. If E is large there is no problem, and the interference is destroyed by a single detection event. This case we call a phase jump. In the opposite case where E is small we can not determine with certainty which direction the electron went, and the interference pattern will be smeared out, but not lost entirely. In this case we need the interaction with a number of two level systems along the path, and the combined result of all the detection times can be put together to determine the direction. In this case we speak about phase wandering or phase diffusion, since the random contributions of the difference two level systems makes the electron phase change in a diffusive way.

Let us estimate the dephasing rates in the different cases. Consider first the resonant mechanism where we have inelastic electron scattering due to *direct* transitions between the two TLS states. We emphasize that the inelasticity is not essential to this channel of dephasing. The important point is that there is a real transition between orthogonal states of the TLS. Since we are not considering degenerate states this will mean inelastic scattering in our case. If the energy transfer E is large enough, the phase relaxation time τ_φ is equal to the typical inelastic relaxation time τ_1 , which is a function of the defect parameters. The criterion of “large” E in this case is given as $E\tau_1/\hbar \gg 1$. For smaller E one deals with a phase diffusion or *wandering*. To estimate the dephasing time for this case let us recall that the phase coherence for any two-level system is conserved during the time $t < \hbar/E$. While traversing the trajectory during the time t an electron appears to be coupled with $\bar{N} \sim t/\tau_1$ dynamic defects. The evolution of the electron wave function due to coupling with any of these defects is described by a phase factor $\exp(\pm iEt/\hbar)$ where \pm corresponds to the sign of the energy transfer. If $T \geq E$ the probabilities of the both defect states are almost equal, and the correlation function of the time-dependent random potential is $f(t) = \cos(Et/\hbar)$, see the calculation later. The resulting electron phase shift turns out to be $\bar{N}^{1/2}Et/\hbar$. Consequently, the phase relaxation time can be estimated as $\tau_\varphi \sim \hbar^{2/3}\tau_1^{1/3}/E^{2/3}$. A similar expression for the dephasing time has been introduced in Refs. 14 in connection with decoherence due to quasielastic electron-electron scattering and in Refs. 15,16 in connection with decoherence by low-frequency phonons. In the following we will call this regime the *phase wandering*. Summarizing, we can express the contribution of inelastic processes as

$$\tau_\varphi^{(1)} = \max\{\tau_1, \tau_1^{1/3}(\hbar/E)^{2/3}\}. \quad (5)$$

Moving to the relaxational mechanism, we find that the simplest way to evaluate this contribution is to note that τ_s has a sense of the time at which $\Delta\varphi(t) \approx 1$ provided *all* the involved defects would suffer a transition. It is clear that if the phase shift $\delta\varphi$ due to transition of a single defect is ≥ 1 then a single TLS is enough to produce the dephasing. For $\delta\varphi \ll 1$ the significant phase evolution is possible only with the help of many defects. The actual dephasing time τ_φ is also sensitive to the defect transition rate γ . Indeed, the correlation function $f(t)$ for statistically independent defects is expected to have a form $f(t) = e^{-2\gamma|t|}$ (this form will be supported by the calculations in Sec. II B). If $\gamma\tau_3 \geq 1$, then with a help of Eq. (4) one obtains $(\overline{\Delta\varphi})^2 = t_0/\tau_3$ (for the reasons which will be clear later we ascribe the subscript 3 for the relaxation mechanism). If $\gamma\tau_3 \ll 1$, one has $(\overline{\Delta\varphi})^2 \sim \gamma t_0^2/\tau_3$. We observe that there is a phase wandering regime also for this relaxation mechanism. Again, defining τ_φ as a time at which $(\overline{\Delta\varphi})^2 \sim 1$ one has

$$\tau_\varphi^{(3)} = \max\{\tau_3, (\tau_3/\gamma)^{1/2}\}. \quad (6)$$

One notes that in course of the above considerations we exploit the additions to the electron phase acquired by an electron in course of traversing of the potential induced by

the TLS's. For each TLS the corresponding contribution can be estimated as $\tilde{U}\delta r/v_F$, where \tilde{U} is the potential magnitude, δr is the potential spatial scale, while v_F is the Fermi velocity. An important note should be made in this concern. Since the trajectory in Eq. (1) and in the following ones is considered to be given and the positions of the scatterers are expected to be along the electron trajectory, the phase addition mentioned above is, strictly speaking, beyond the Born approximation for the electron scattering. Indeed, in the Born approximation the scattering amplitude is real at least for symmetric scattering potentials. However it is possible to describe the phase relaxation even within the framework of the Born approximation if one takes into account that the "centers of gravity" of the two TLS states are spatially separated by some vector \mathbf{a} (which is an inherent feature of the model suggested in Ref. 17). In this case the phase variation due to a transition within the i th defect is simply given as $\delta\varphi \sim (\mathbf{p} \cdot \mathbf{a})/\hbar$. Correspondingly, the estimate for the proper rates in Eq. (4) is $\tau_{1,3}^{-1} \approx \tau_{e,d}^{-1} (p_F a/\hbar)^2$ where $\tau_{e,d}^{-1}$ is a typical *elastic* relaxation rate due to dynamic defects.^{11,18}

The previous estimates are relevant to a set of defects having *identical* parameters. However, in real systems the defect parameters are scattered, and one has to perform a proper average. As we will demonstrate, for a realistic model the phase wandering regime turns out to be important. To our knowledge, this fact has not been appreciated in the previous papers dealing with defect-induced decoherence.

In the following sections we will give a more formal derivation of the dephasing rate using the Green function method. It will permit us to consider not only the limiting cases but any relations between various times of relaxation. In the Appendix B we will map the results for the relaxation-dynamics contribution to a simple model of short-range defects hopping between two states separated some distance in real space. This model is often used to interpret results on the so-called random telegraph noise observed in nanostructures.

II. THEORY

A. The model

The dephasing mechanism is based on the assumption that in any crystalline metal there exist dynamic defects of a special type. These defects are tunneling states which are described by the Hamiltonian

$$\tilde{\mathcal{H}}_d = (\Delta\sigma_3 - \Lambda\sigma_1)/2 \quad (7)$$

where Δ is the diagonal level splitting, Λ is the tunneling amplitude, while σ_i are the Pauli matrices. The tunneling amplitude Λ describes the tunneling between the interstitial positions while the spread of Δ is determined by (mesoscopic) disorder around the mobile defect. Consequently, we will assume that the distribution of Λ is narrow and it is centered around some value Λ_0 . As we will see, one can expect smooth temperature dependence of dephasing at $T \gtrsim \Lambda_0$. The above model has been proposed and successfully exploited in Ref. 17 to interpret zero-bias anomalies observed in metallic point contacts. Note that it differs from the well-known TLS model in amorphous metals¹¹ where the

distribution of $\ln \Lambda$ is assumed to be uniform, however resembles the TLS model for crystalline materials suggested by Phillips¹⁹ to describe acoustic experiments in crystalline Si.

Consider now spinless electrons which scatter against tunneling defects with the Hamiltonian (7). The total Hamiltonian then can be expressed in the form

$$\tilde{\mathcal{H}} = \tilde{\mathcal{H}}_d + \sum_{\mathbf{p}} \epsilon_{\mathbf{p}} c_{\mathbf{p}}^{\dagger} c_{\mathbf{p}} + \mathcal{H}_{\text{int}}, \quad (8)$$

where

$$\mathcal{H}_{\text{int}} = \frac{1}{2} \sum_{\mathbf{p}\mathbf{p}'n} (\hat{1}V_{\mathbf{p}\mathbf{p}'}^+ + \sigma_3 V_{\mathbf{p}\mathbf{p}'}^-) c_{\mathbf{p}}^{\dagger} c_{\mathbf{p}'} e^{i(\mathbf{p}-\mathbf{p}') \cdot \mathbf{r}_n/\hbar}. \quad (9)$$

Here $\hat{1}$ is the unit matrix, V^{\pm} represent the components of a short-range defect potential, while \mathbf{r}_n is the coordinate of the n th defect. The Hamiltonian (9) is equivalent to the assumption that the electron scattering amplitudes are $V^+ \pm V^-$ in the "left" and the "right" defect positions, respectively. Estimates for V^+ and V^- are given in Refs. 11,18. After the transform which makes $\tilde{\mathcal{H}}_d$ diagonal we arrive at the Hamiltonian

$$\begin{aligned} \mathcal{H} = & \frac{1}{2} \sum_n E_n \sigma_3 + \sum_{\mathbf{p}} \epsilon_{\mathbf{p}} c_{\mathbf{p}}^{\dagger} c_{\mathbf{p}} \\ & + \frac{1}{2} \sum_{\mathbf{p}\mathbf{p}'n} \left\{ \hat{1}V_{\mathbf{p}\mathbf{p}'}^+ + \left(\frac{\Lambda_n}{E_n} \sigma_1 + \frac{\Delta_n}{E_n} \sigma_3 \right) V_{\mathbf{p}\mathbf{p}'}^- \right\} \\ & \times c_{\mathbf{p}}^{\dagger} c_{\mathbf{p}'} e^{i(\mathbf{p}-\mathbf{p}') \cdot \mathbf{r}_n/\hbar}, \end{aligned} \quad (10)$$

where $E_n = \sqrt{\Delta_n^2 + \Lambda_n^2}$. One observes that there are two processes of electron-defect interaction described by the items proportional to σ_1 and σ_3 , respectively. They correspond to the two mechanisms discussed above and described by Eqs. (5) and (6). Now we proceed to more formal calculations in which the relaxation time τ_1 and τ_3 will be specified.

B. Quantum contribution to conductance

The object which we will consider is the weak localization correction to the conductivity $\delta\sigma$, which for the case of a short-range scattering potential can be expressed through the electron Green's functions $G_{R/A}$ in the form¹⁵

$$\begin{aligned} \delta\sigma = & \frac{e^2}{m^2 d} \int (dp)(dq) p^2 \int \left(-\frac{dn}{d\varepsilon} \right) \frac{d\varepsilon}{2\pi} \int \frac{d\omega}{2\pi} \\ & \times G_R(\varepsilon, \mathbf{p}) G_A(\varepsilon, \mathbf{p}) F(\varepsilon, \omega, \mathbf{p}, \mathbf{q} - \mathbf{p}) \\ & \times G_R(\varepsilon + \omega, \mathbf{q} - \mathbf{p}) G_A(\varepsilon + \omega, \mathbf{q} - \mathbf{p}). \end{aligned} \quad (11)$$

Here d is the dimension of the problem ($dp \equiv d^d p / (2\pi\hbar)^d$), $n(\varepsilon)$ is the Fermi function, while $F(\varepsilon, \omega, \mathbf{p}, \mathbf{p}_1)$ is a two-particle Green's function specific for the problem under consideration. The above expression describes the main contribution to the conductivity which arises from the region $\omega\tau, ql \ll 1$, where τ and l are the total

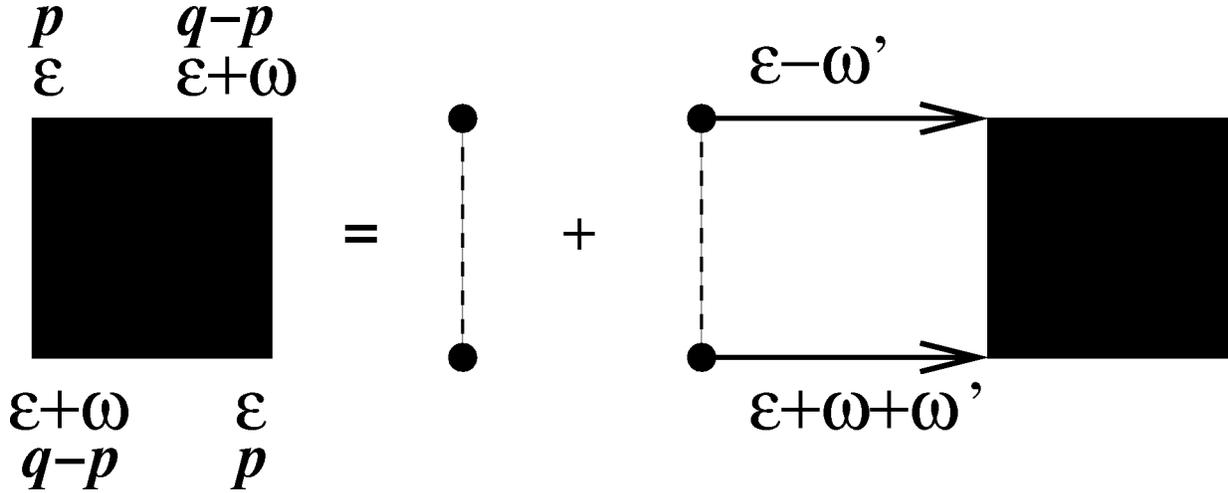


FIG. 3. The equation for the Cooperon.

relaxation time and length, respectively. The function $F(\epsilon, \omega, \mathbf{q}, \mathbf{p})$ can be represented as a sum of the maximally crossed diagrams (the so-called Cooperon) which is a sum of a ladder in the particle-particle channel. This is the same approximation as was used in Ref. 14, and is valid if the inequalities $p_F l / \hbar \gg \ln(\tau_\phi / \tau) \gg 1$ are met. The Cooperon satisfies the Dyson equation shown in Fig. 3.

In this figure, the Cooperon is drawn as a filled square, thick lines with arrows correspond to the Green's functions averaged over the defect position, as well as over the states of the thermal bath, while dotted lines represent propagators for electron scattering against dynamic defects. Since the interaction Hamiltonian (10) contains items of three types, the propagator consists of a sum of three terms. Each propagator can be expressed as a loop graph where dotted lines represent Green's functions for a dynamic defect (see Fig. 4).

To express the propagators in an analytical form we will employ the technique developed by Abrikosov.²⁰ According to this technique, a two-level system describing the dynamic defect is interpreted as a pseudo-Fermion particle with the Green's function

$$g_{\pm}(\epsilon) = (\epsilon \mp E/2 - \lambda + i\delta)^{-1}, \quad (12)$$

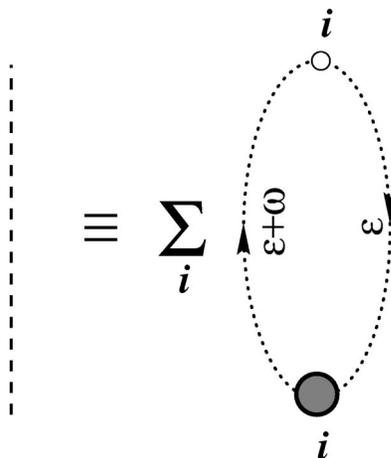


FIG. 4. Schematic representation of the defect propagator.

where λ is an auxiliary ‘‘chemical potential’’ which afterwards will be tended to infinity. This trick allows one to remove extra unphysical states which appear since the Fermi operators have more extended phase space than the spin ones.^{20,21} As a result, the Matsubara technique can be effectively used, and after a proper analytical continuation and the limiting transition $\lambda \rightarrow \infty$ the quantity λ drops out of all the expressions. As a result, the retarded propagator describing the interlevel transitions in the defect can be expressed as

$$D_1^R(\omega) = -\tanh \frac{E}{2T} \left(\frac{1}{\omega - E + i\delta} - \frac{1}{\omega + E + i\delta} \right). \quad (13)$$

Here δ is the adiabatic parameter $\delta \rightarrow +0$. The propagator describing electron-assisted transitions has the form

$$D_3^R(\omega) = \frac{1}{T \cosh^2(E/2T)} \frac{2i\gamma}{\omega + 2i\gamma}. \quad (14)$$

Here

$$\gamma(\Lambda, E) = \left(\frac{\Lambda}{E} \right)^2 \gamma_0(E), \quad \gamma_0(E) = \chi E \coth \frac{E}{2T}, \quad (15)$$

where $\chi = 0.01 - 0.3$ is dimensionless constant dependent on the matrix element $W^{(1)}$ defined below, and where $\gamma_0(E)$ has the meaning of *maximum* hopping rate for the systems with a given interlevel spacing.¹¹

For the elastic component $\propto \hat{I}$ we shall use a trick which will allow us to consider the elastic channel in a unified way with the inelastic ones. Namely, to keep proper analytical properties of the retarded Green's function we define the elastic propagator as

$$D_0^R(\omega) = \frac{\nu}{2T} \left(\frac{1}{\omega + \nu + i\delta} - \frac{1}{\omega - \nu + i\delta} \right). \quad (16)$$

At the final stage, the limiting case $\delta \rightarrow 0, \nu \rightarrow 0$ should be calculated. Note that the factor T^{-1} will be canceled by the Planck function $N_0(\omega)$ which will appear in course of derivation of the equation shown in Fig. 3. The physical reason

of this cancellation is that the elastic impurity scattering is temperature independent. Note that the propagators do not include the electron-defect coupling constant, hence each propagator should be multiplied by $|W^{(i)}|^2$ where $W^{(0)} = V^+$, $W^{(1)} = (\Lambda/2E)V^-$, $W^{(3)} = (\Delta/2E)V^-$. If there are additional static short-range defects their contribution modifies zeroth propagator by the replacement $|W^{(0)}|^2 \rightarrow |W_s|^2 + |W^{(0)}|^2$ where W_s is the contribution of static defects.

The equation shown in Fig. 3 has been analyzed following the procedure of analytical continuation of Matsubara Green's function¹⁵ with making use of analytical properties of two-particle Green's functions.²² The resulting equation for $F(\varepsilon, \omega, \mathbf{p}, \mathbf{q} - \mathbf{p}_1)$ has the form

$$F(\varepsilon, \omega, \mathbf{p}, \mathbf{q} - \mathbf{p}) = \mathcal{D}(\omega) - \int \frac{(dp')d\omega'}{2\pi i} F(\varepsilon, \omega', \mathbf{p}', \mathbf{q} - \mathbf{p}) \\ \times \mathcal{D}(\omega - \omega') G^R(\varepsilon + \omega - \omega', \mathbf{p}') G^A \\ \times (\varepsilon + \omega', \mathbf{q} - \mathbf{p}') [N_0(\omega') - N_0(\omega' - \omega)]. \quad (17)$$

Here $(dp) \equiv 2d^2p/(2\pi\hbar)^2 \equiv \rho d\varepsilon_p d\theta/2\pi$, θ is the angle with the x axis and we write all formulas for the most interesting case of a two-dimensional system

$$\mathcal{D}(\omega) \equiv |W_s|^2 n_s [\mathcal{D}_0^R(\omega) - \mathcal{D}_0^A(\omega)] \\ + \sum_i |W^{(i)}|^2 n_d [\mathcal{D}_i^R(\omega) - \mathcal{D}_i^A(\omega)]. \quad (18)$$

Equation (17) describes the dominant contribution provided the sum of the incoming momenta q is small:

$$ql \ll 1. \quad (19)$$

Here $l = v_F \tau$ is the electron mean free path, while τ is the electron lifetime

$$\tau^{-1} = \tau_e^{-1} + \tau_1^{-1} + \tau_3^{-1}. \quad (20)$$

Here we introduce the elastic relaxation rate as a sum of the contributions of static and dynamic defects $\tau_e^{-1} = \tau_{e,s}^{-1} + \tau_{e,d}^{-1}$ with

$$\tau_{e,s}^{-1} = 2\pi\rho n_s |V_s|^2/\hbar, \quad \tau_{e,d}^{-1} = 2\pi\rho n_d |V_d^+|^2/\hbar,$$

and a typical inelastic relaxation rate as

$$\tau_i^{-1} = 2\pi\rho n_d |V_d^-|^2/\hbar, \quad \tau_{e,d}/\tau_i \approx (p_F a/\hbar)^2 \ll 1. \quad (21)$$

Here n_s is the concentration of static defects while n_d is the concentration of dynamic ones, and ρ is the electron density of states. In principle we now have two sets of relaxation rates. From the interaction vertices we get

$$\tau_1^{-1} = \tau_i^{-1} (\Lambda/2E)^2, \quad \tau_3^{-1} = \tau_i^{-1} (\Delta/2E)^2,$$

while the rates appearing in Eq. (20) arises in the evaluation of the self-energy diagrams as shown in Appendix A, and are given by

$$\tau_1^{-1} = \tau_i^{-1} (\Lambda/2E)^2 \mathcal{G}_1(\varepsilon), \quad \tau_3^{-1} = \tau_i^{-1} (\Delta/2E)^2 \mathcal{G}_3(\varepsilon).$$

The functions $\mathcal{G}_{1,3}$ are discussed in Appendix A. We show that they decay exponentially at $E \gg T$, and only the regions with $\varepsilon \leq T$ are important. Since we only are interested in this region, we will neglect the energy dependence of these functions, and put them to 1 in the following. The two sets of relaxation rates will then be the same.

To analyze Eq. (17) it appears convenient to transform it to the form similar to the Boltzmann equation for an electron diffusion. For this let us take into account that at small q and ω the product of the Green functions in the integrand is a sharp function centered at $\varepsilon = \varepsilon_{\mathbf{p}'} = \varepsilon_F$ where ε_F is the Fermi level. Thus it is natural to assume that $F(\varepsilon, \omega, \mathbf{p}', \mathbf{q} - \mathbf{p}_1)$ depends only on q , ω and the product $\mathbf{q} \cdot \mathbf{v}'$. Having that in mind we first integrate over $\varepsilon_{\mathbf{p}'}$ and make use of the inequalities $p_F l/\hbar \gg 1, \hbar\omega \ll T$ which we assume to be met.

The result can be expressed in terms of a new function

$$\mathcal{F}(\varepsilon, \mathbf{q}, \omega) \equiv \frac{F(\varepsilon, \omega, \mathbf{p}, \mathbf{q} - \mathbf{p})}{\omega(1 - i\tau\mathbf{q} \cdot \mathbf{v})}, \quad (22)$$

where \mathbf{v} is the electron velocity. Here we assume that $\varepsilon \leq T$ and omit the variable ε .

Following the procedure described in Ref. 15 we express the equation for \mathcal{F} in the form:

$$(1 + Dq^2\tau)\mathcal{F}(\varepsilon, \mathbf{q}, \omega) \\ = \frac{\mathcal{D}(\omega)}{4\pi\rho\omega} - T \int \frac{d\omega'}{(2\pi i)(\omega - 2\omega' + i/2\tau)} \\ \times \mathcal{F}(\varepsilon, \mathbf{q}, \omega') \frac{\mathcal{D}(\omega - \omega')}{\omega - \omega'}. \quad (23)$$

Here $D = v_F l/d$ is the diffusion constant. Transforming Eq. (23) to the time representation with respect to ω we obtain

$$(1 + Dq^2\tau)\mathcal{F}(\varepsilon, \mathbf{q}, t) \\ = \frac{\Phi(\varepsilon, t)}{2\tau T\rho} + \int_{-\infty}^t \frac{dt'}{\tau} e^{(t'-t)/\tau} \\ \times \mathcal{F}(\varepsilon, \mathbf{q}, t') \Phi(\varepsilon, 2t - t'). \quad (24)$$

Here we denote

$$\Phi(\varepsilon, t) \equiv \frac{\tau}{\tau_e} + \frac{\tau}{\tau_1} \cos \frac{Et}{\hbar} + \frac{\tau}{\tau_3} e^{-2\gamma t/\hbar}. \quad (25)$$

Here the limiting transition $\nu \rightarrow 0$ has been already done. Note that the function $\Phi(\varepsilon, t)$ depends on the energy variable ε through the relaxation times τ_e , τ_1 , and τ_3 . In the following we omit the variable ε in all the functions keeping in mind that the relaxation rates are energy dependent, see Appendix A. Also in writing the expression for $\Phi(\varepsilon, t)$ we have assumed that $E \ll T$. For $E > T$ it decays to 1 which means that defects with $E > T$ do not contribute to the dephasing, see the discussion below Eq. (33)

Equation (24) can be solved exactly. The solution is based on the relation between the kernel of the integral equation

$$\mathcal{K}(t, t') = (Dq^2\tau - 1)e^{(t'-t)/\tau} [1 - \lambda(2t - t')],$$

$$\lambda(t) \equiv \frac{\tau}{\tau_1} \left(1 - \cos \frac{Et}{\hbar} \right) + \frac{\tau}{\tau_3} (1 - e^{-2\gamma t/\hbar})$$

and its resolvent \mathcal{R} defined by the integral equation

$$\int_{t_1}^t \frac{dt'}{\tau} \mathcal{K}(t_1, t') \mathcal{R}(t', t) = \mathcal{K}(t_1, t) + \mathcal{R}(t_1, t). \quad (26)$$

The relationship has the form²³

$$\mathcal{F}(t, \mathbf{q}) = \Phi(t) + \int_{-\infty}^t \frac{dt'}{\tau} \mathcal{R}(t, t') \Phi(t'). \quad (27)$$

If one can construct a differential operator of the form $\hat{\mathcal{L}}_{t_1} \equiv \sum_j a_j(t_1) (d^j/dt_1^j)$ such that

$$\hat{\mathcal{L}}_{t_1} \mathcal{K}(t_1, t) = 0 \quad (28)$$

for any t , then the integral equation (27) is reduced to the differential equation (28) for a fixed t . That can be directly checked applying the operator $\hat{\mathcal{L}}_{t_1}$ to relation (26). The boundary conditions corresponding for $t_1 \rightarrow t$ can be extracted from the integral relation (26) and its derivatives with respect to t_1 at $t_1 \rightarrow t$.

The results have the simplest form at $\tau_e \ll \tau_1, \tau_3, \hbar/\gamma$. Then one can choose

$$\hat{\mathcal{L}}_{t_1} = \left(\tau \frac{d}{dt_1} + 1 \right)^3.$$

From Eq. (26), the differential equation for the resolvent $\mathcal{R}(t_1, t)$ acquires the form

$$\begin{aligned} & [\tau^3 (d^3/dt^3) + \tau^2 (2 + \lambda_1) (d^2/dt^2) \\ & + \tau (1 + 2\lambda_1) (d/dt) + \lambda_1] \mathcal{R}(t, \theta) = 0. \end{aligned} \quad (29)$$

Here $\lambda_1 = \lambda + Dq^2\tau$. Since the phase relaxation is a slow process with respect to the scale τ the equation (29) has small coefficients at senior derivatives which makes useful the WKB approximation. Consequently, the physical solution can be sought in the form $\mathcal{R}(t, \theta) \propto \exp[\varphi(t)/\tau]$ where $\varphi(t)$ satisfies the equation, $(\dot{\varphi} + 1)^2 [\dot{\varphi} + \lambda_1(t)] = 0$. Since $\lambda(t) \ll 1$, the WKB solution corresponds to the equation

$$\dot{\varphi} + \lambda_1(t) = 0. \quad (30)$$

The boundary condition to Eq. (30) can be extracted from the relation $\mathcal{R}(t, t) = 0$. In this way, we obtain the quasiclassical solution in the form

$$\mathcal{R}(t, t_1) = \exp \left(- \int_{t_1}^t \frac{dt'}{\tau} \lambda_1(t') \right). \quad (31)$$

Now we substitute Eq. (31) in the expression (27) to obtain the final expression for the Cooperon \mathcal{F} . The first item in Eq. (27) is the contribution of lowest order scattering and it should be neglected in the diffusion approximation. Here we analyze the quantum contribution to the static conductance, so only $\mathcal{F}(0)$ is important. As a result, we obtain

$$\mathcal{F}(0, \mathbf{q}) = \int_{-\infty}^0 \frac{dt'}{\tau} \Phi(t') e^{\vartheta(t')}, \quad (32)$$

$$\begin{aligned} \vartheta(t) = & Dq^2t + \left[\frac{t}{\tau_1} - \frac{\sin(Et/\hbar)}{E\tau_1/\hbar} \right] \\ & + \left[\frac{t}{\tau_3} - \frac{\hbar}{2\gamma\tau_3} (e^{2\gamma t/\hbar} - 1) \right], \quad t < 0. \end{aligned} \quad (33)$$

An important feature of Eq. (33) is that if one neglects the processes in which the defect changes its state then the *dephasing is absent*. Indeed, putting $\tau_1 = \tau_3 = \infty$ we get $\Phi(t) = 1$ and $\mathcal{F}(\mathbf{q}) = (Dq^2)^{-1}$. This results in logarithmic divergence of the conductance in the 2D case. Another important feature is that at small time t , which has a physical meaning of the time difference for the collision act for clockwise and counterclockwise partial waves, no linear in t term is originated by inelastic processes. Physically it means that no dephasing takes place if the scattering defect had no enough time to change its state. Therefore, the dephasing appears proportional to the probability for the defect to escape the state in which it has been registered by one partial wave.

With substitution of Eq. (32) into Eq. (22) and then into Eq. (11) we obtain for 2D case the expression for the quantum contribution to the conductance. Since only $|\varepsilon| \leq T \ll \varepsilon_F$ are important one can neglect ε dependence of the relaxation rates and put $\varepsilon = 0$ in the expressions for the these quantities. As a result, one arrives at the well-known expression

$$\delta\sigma = - \frac{e^2}{2\pi^2\hbar} \ln \frac{\tau_\varphi}{\tau},$$

where τ_φ is defined according to the equation

$$\ln \frac{\tau_\varphi}{\tau} \equiv \int_1^\infty \frac{d\eta}{\eta} e^{-\Gamma_1(\eta, E, \Lambda) - \Gamma_3(\eta, E, \Lambda)}, \quad (34)$$

$$\Gamma_1(\eta, E, \Lambda) = \frac{\tau}{\tau_1} \left[\eta - \frac{\sin(\eta E \tau/\hbar)}{E\tau/\hbar} \right],$$

$$\Gamma_3(\eta, E, \Lambda) = \frac{\tau}{\tau_3} \left[\eta - \frac{\hbar}{2\gamma\tau} (1 - e^{-2\eta\gamma\tau/\hbar}) \right],$$

where $\eta = t/\tau$. This equation is obtained by the integration over \mathbf{q} .

We can now recover our estimates (5) and (6) by the approximate value of the integral

$$I = \int_1^\infty \frac{d\eta}{\eta} e^{-\alpha\eta^n}$$

in the case where $\alpha \ll 1$. We split the integral at the point η^* :

$$I = \int_1^{\eta^*} \frac{d\eta}{\eta} e^{-\alpha\eta^n} + \int_{\eta^*}^\infty \frac{d\eta}{\eta} e^{-\alpha\eta^n}.$$

Defining η^* such that $\alpha\eta^{*n}=1$, we have $\eta^*\gg 1$. The integrand will be very small in the last integral, and we neglect this. In the first integral we put the exponent equal to 0, and get

$$I \approx \ln \eta^* = \ln \alpha^{-1/n}. \quad (35)$$

Expression (34) depends upon two dimensionless quantities. The first one is $E t/\hbar$. As follows from Eq. (35), the typical value of t is τ_φ . This parameter determines the efficiency of the resonant mechanism of dephasing, that of direct transitions of the TLS states. If $E\tau_\varphi/\hbar \ll 1$ we are in the regime of phase wandering, and we can expand Γ_1 in Eq. (34) in powers of this parameter up to the lowest order. However, if $E\tau_\varphi/\hbar \geq 1$ we have the case of phase jumps, and we can neglect the sine term in Γ_1 . These expansions are given in Eqs. (38) and (39) below. In both cases we can use the formula (35) to arrive at the estimate (5).

The second dimensionless parameter is $\gamma\tau_3/\hbar$. It describes the effect of the relaxational mechanism of dephasing arising from the σ_3 vertex. The physical explanation is that the dephasing occurs only if the partial waves meet the scatterer in different states. Expanding in small [phase wandering, Eq. (45)] and large [phase jumps, Eq. (46)] values of this parameter we arrive at the estimates (6).

In addition there is also the dimensionless parameter $\gamma\tau_1/\hbar$ which will control the effect of the relaxational mechanism acting through the σ_1 vertex. This has been neglected in the above calculations since the inequality $\gamma \ll E/\hbar$ is met.

If the estimates (5) and (6) have different orders of magnitude then the shortest one is effective. However,

$$1/\tau_\varphi \neq 1/\tau_\varphi^{(1)} + 1/\tau_\varphi^{(3)}$$

since Γ_1 and Γ_3 depend on time in different ways. The most clear manifestation of this fact is seen in the magnetic field dependence of the quantum contribution.

C. Average over different dynamic defects

To calculate the quantum contribution to conductance one has to sum over different dynamic defects. In the previous considerations we have assumed that all dynamic defect have the same interlevel distance E and the same tunneling amplitude Λ . Consequently, the summation over different defects has been allowed for by the factor n_d in the expressions for the relaxation times τ_i . However, in realistic systems both E and Λ can be distributed over a significant range. Since the number of dynamic defects at a typical electron trajectory is assumed to be large the summation over different defects can be replaced by a proper average. To calculate the latter it is necessary to specify the distribution function $\mathcal{P}(E, \Lambda)$ which we assume to be normalized to 1. To specify this function, let us come back to the effective Hamiltonian (7). Since Δ is determined by the defect's neighborhood while Λ is determined by the distance between two metastable states it is natural to assume Δ and Λ to be *uncorrelated*, $\mathcal{P}(\Delta, \Lambda) = \mathcal{P}_\Delta(\Delta)\mathcal{P}_\Lambda(\Lambda)$.

Below we will discriminate between two model distributions. The first one will be referred to as the ‘‘glass model’’ (GM).^{10,24} According to this model the distribution of Δ is assumed to be smooth, $\mathcal{P}_\Delta = \mathcal{P}_0$. Since the tunneling integral Λ is an exponential function of the distance between the potential minima and the latter is smoothly distributed, it is assumed that $\mathcal{P}_\Lambda \propto \Lambda^{-1}$. Within this model it is natural to choose the interlevel splitting E and the quantity $p \equiv (\Lambda/E)^2$ as independent parameters. Since $\gamma \propto p$, Eq. (15) can be rewritten as $\gamma = p\gamma_0(E)$. Consequently, the GM results in the *exponentially broad* distribution of relaxation rates. Furthermore, to keep the distribution normalized, we introduce cutoff $p_{\min}(E) = \gamma_{\min}(E)/\gamma_0(E) \ll 1$ and assume $\mathcal{L} \equiv \ln(1/p_{\min}) = \ln(\gamma_0/\gamma_{\min})$ to be energy independent. A cutoff energy in the smooth distribution of E at some E^* is also assumed. As a result, we get the distribution

$$\mathcal{P}_{\text{GM}}(E, p) = \frac{\Theta(E^* - E)}{E^* \mathcal{L}} \frac{1}{p \sqrt{1-p}}. \quad (36)$$

Another model which we will call the ‘‘tunneling-states model’’ (TM) is more appropriate for crystalline materials. There the tunneling integrals Λ is determined by the crystalline structure and are almost the same for all dynamical defects. On the other hand, the parameter Δ is determined by long-range interactions, and it is assumed distributed smoothly within some band, cf. Ref. 17. Then

$$\mathcal{P}_{\text{TM}}(E, \Lambda) = \frac{\Theta(E^* - E)}{E^*} \frac{E}{\sqrt{E^2 - \Lambda_0^2}} \delta(\Lambda - \Lambda_0). \quad (37)$$

In the following we will assume that the dynamical defects are characterized by $\Lambda_0 \ll T$. To calculate the total contribution of the dynamical defects in the case when their parameters are random one has to replace Γ_i in Eq. (34) by the averages $\bar{\Gamma}_i(\eta) = \int dE d\Lambda \Gamma_i(\eta, E, \Lambda)$. Below we will discuss in detail only the tunneling-states model which seems to be more appropriate for crystalline materials. Let us discuss the contribution of direct transitions and relaxation separately.

(a) *Contribution of direct (resonant) transitions.* The item Γ_1 , responsible for the direct transitions, is proportional to $|W_1|^2 = (\Lambda/E)^2 |V^{(1)}|^2$. Since $\mathcal{P}_{\text{TM}}(E, \Lambda) \propto \delta(\Lambda - \Lambda_0)$ the integral over Λ yields the factor $\Lambda_0^2/E \sqrt{E^2 - \Lambda_0^2}$. Using Eq. (35), the quantity $\tau_\varphi^{(1)}$ is estimated from the expression $\bar{\Gamma}_1(\tau_\varphi^{(1)}/\tau) = 1$. The following calculation depends on the relationship between E and τ_φ . At $E\tau_\varphi/\hbar \ll 1$ one can expand the expression for Γ_1 as

$$\Gamma_1(\eta, E, \Lambda) \sim (\Lambda/\hbar)^2 (\eta\tau)^3 / \tau_i, \quad (38)$$

while at $E\tau_\varphi/\hbar \gg 1$

$$\Gamma_1(\eta, E, \Lambda) \sim (\Lambda/E)^2 \eta\tau / \tau_i. \quad (39)$$

To estimate $\bar{\Gamma}_1(\eta)$ let us introduce the energy splitting T_Λ at which $E\tau_\varphi/\hbar = 1$. The meaning of this is that a TLS with an energy splitting less than T_Λ will not by itself cause com-

plete phase loss, i.e., we are in the regime of phase wandering. A TLS with $E > T_\Lambda$ causes a phase jump. Let us first assume that

$$\Lambda_0 \ll T_\Lambda \ll T. \quad (40)$$

Using the distribution (37) and returning to dimensional time one obtains

$$\bar{\Gamma}_1(t) \approx \frac{t}{\tau_i} \frac{\Lambda_0^2}{T_\Lambda E^*} + \frac{t}{\tau_i} \left(\frac{\Lambda_0 t}{\hbar} \right)^2 \frac{T_\Lambda}{E^*}. \quad (41)$$

Now, let us define $\tau_\varphi^{(1)}$ and T_Λ to make both contributions to $\bar{\Gamma}_1(\tau_\varphi^{(1)})$ equal to 1. This definition of T_Λ is consistent with that given above within the accuracy of the approximation because the two terms are the expansions in large and small values of $E\tau_\varphi/\hbar$. The point where both terms becomes of the order 1 should then correspond to the crossover point $E\tau_\varphi/\hbar = 1$. This is easily checked from the formulas below. In this way we get

$$\tau_\varphi^{(1)} = \tau_\Lambda (T_\Lambda / \Lambda_0), \quad (42)$$

where

$$T_\Lambda = (\hbar \Lambda_0 / \tau_\Lambda)^{1/2}, \quad \tau_\Lambda = \tau_i (E^* / \Lambda_0). \quad (43)$$

The time τ_Λ is due to the dynamic defects with symmetric potentials and energy splitting equal to Λ_0 . At $T \leq T_\Lambda$ for all energies the inequality $E\tau_\varphi/\hbar \ll 1$ is met, and only the second item in Eq. (41) is important. One should replace T_Λ by T in this expression to obtain $\tau_\varphi^{(1)} = \tau_\Lambda (T_\Lambda / \Lambda_0) (T_\Lambda / T)^{1/3}$. In contrast, if $T_\Lambda \leq \Lambda_0$ then only the first item in Eq. (41) is important. In this case $\tau_\varphi^{(1)} = \tau_\Lambda$.

The result can be summarized as

$$\frac{1}{\tau_\varphi^{(1)}} \approx \frac{1}{\tau_\Lambda} \frac{\Lambda_0}{T_\Lambda} \begin{cases} \min\{(T/T_\Lambda)^{1/3}, 1\}, & T_\Lambda \gg \Lambda_0, \\ T_\Lambda / \Lambda_0, & T_\Lambda \ll \Lambda_0. \end{cases} \quad (44)$$

(b) *Contribution of relaxation processes.* Since only $E \leq T$ are important, for estimates one can assume $E \coth E/2T \approx 2T$. Thus, $\gamma_0 \approx 2\chi T$ becomes E -independent. For the same reason τ_3^{-1} can be approximated as $\tau_i^{-1}(\Delta/E)$. The following calculation depends on the relationship between γ and τ_φ . At $\gamma\tau_\varphi \ll 1$ one can expand the expression for Γ_3 as

$$\Gamma_3(\eta, E, \Lambda) \approx \eta^2 \gamma \tau^2 / \tau_3 = \gamma_0 (\eta \tau)^2 \tau_i^{-1} \Delta^2 \Lambda^2 / E^4, \quad (45)$$

while at $\gamma\tau_\varphi \gg 1$ one has

$$\Gamma_3 = \eta (\tau / \tau_3) \propto (\Delta/E)^2. \quad (46)$$

To estimate $\Gamma_3(\eta)$ let us introduce the energy splitting E_χ at which $\gamma\tau_\varphi = 1$. The meaning of this is that a TLS with $E < E_\chi$ will probably jump during the trajectory traversal time τ_φ (it is ‘‘fast moving’’), whereas a TLS with $E > E_\chi$ will have a low probability to jump in the same time (it is ‘‘slow moving’’). First we assume that

$$\Lambda_0 \ll E_\chi \ll T. \quad (47)$$

Using the distribution (37) and returning to dimensional time one obtains

$$\bar{\Gamma}_3(t) \approx \frac{t}{\tau_i} \frac{E_\chi}{E^*} + \frac{\chi T t^2}{\hbar \tau_i} \frac{\Lambda_0^2}{E_\chi E^*}. \quad (48)$$

Now, let us define τ_φ and E_χ to make both contributions to $\bar{\Gamma}_3$ equal to 1. This procedure indicates that the defects with $E = E_\chi$ are those which experience a hop during the typical Cooperon trajectory traversal time. One obtains

$$1/\tau_\varphi^{(3)} = \tau_i^{-1} (E_\chi / E^*), \quad E_\chi = \Lambda_0 (\chi T \tau_\Lambda / \hbar)^{1/3}. \quad (49)$$

Introducing the characteristic temperature $T_\alpha = \hbar / \chi \tau_\Lambda$ at which $E_\chi = \Lambda_0$ one can express the dephasing rate as

$$1/\tau_\varphi^{(3)} = \tau_\Lambda^{-1} (T / T_\alpha)^{1/3}. \quad (50)$$

Another important characteristic energy is the temperature T_β at which $E_\chi = T$, $T_\beta = \Lambda_0^{3/2} / T_\alpha^{1/2}$. The ratio $T_\alpha / T_\beta = (\chi \Lambda_0 \tau_\Lambda / \hbar)^{3/2}$ can be arbitrary.

The meaning of T_α and T_β can be understood as follows. Imagine starting at some large temperature where $T \gg E_\chi \gg \Lambda_0$. As we lower the temperature E_χ is also decreasing, but it decreases at a slower rate than T ($E_\chi \sim T^{1/3}$). At the temperature T_α , $E_\chi = \Lambda_0$. Since Λ_0 is the lower cutoff for E , if $T < T_\alpha$ all defects are slow moving because no defects exist with sufficiently low splitting. Alternatively, since T is decreasing faster than E_χ , T will overtake E_χ at the temperature T_β . For $T < T_\beta$ all defects are fast, because the slow ones are frozen out. Which temperature is reached first depends on the specific values of the parameters (since the ratio T_α / T_β is arbitrary). Also, if $T_\alpha > T_\beta$ then $E_\chi < \Lambda_0$ for all $T < T_\alpha$, so in particular $T_\beta < \Lambda_0$ and is thus unimportant. Similarly, if $T_\alpha < T_\beta$ then $T_\alpha < \Lambda_0$.

The result (49) is valid if $T \gg E_\chi \gg \Lambda_0$, or at $T \gg T_\alpha, T_\beta$. At $E_\chi \gg T \gg \Lambda_0$, or $T_\beta \gg T \gg \Lambda_0$, only the first item in Eq. (48) exists and the quantity E_χ should be replaced by T . As a result,

$$1/\tau_\varphi^{(3)} \approx \tau_\Lambda^{-1} (T / \Lambda_0). \quad (51)$$

Since $\Lambda_0 = T_\beta^{2/3} T_\alpha^{1/3}$ the results (50) and (51) match at $T = T_\beta$. This temperature region exists only if $T_\beta > T_\alpha$. At $T \leq \Lambda_0$ the dephasing rate strongly decreases with the temperature decrease.

If $T_\alpha \gg T \gg T_\beta$ the relaxation is slow for all the dynamic defects and the second item in Eq. (48) is important. However, in this case one has to replace $E_\chi \rightarrow \Lambda_0$ in its estimate. As a result,

$$1/\tau_\varphi^{(3)} = \tau_\Lambda^{-1} (T / T_\alpha)^{1/2}. \quad (52)$$

The temperature dependence of τ_φ is sketched in Fig. 5 for $T_\alpha < T_\beta$ and in Fig. 6 for $T_\alpha > T_\beta$.

(c) *Comparison.* Now we are in a position to compare the contributions to the dephasing rate. Both contributions to $1/\tau_\varphi$ are parametrized by the quantity $1/\tau_\Lambda$, the relative contributions being dependent on the temperature. The relative resonant contribution crosses over from $(\Lambda_0 / T_\Lambda) (T / T_\Lambda)^{1/3}$ to Λ_0 / T_Λ at $T = T_\Lambda$. The relative relaxation contribution

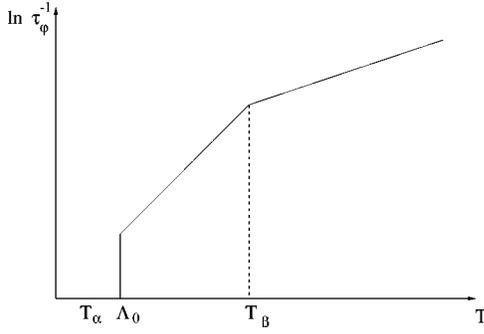


FIG. 5. Schematic picture of $\ln \tau_\phi^{-1}$ as function of temperature for $T_\alpha < T_\beta$.

crosses over from T/Λ_0 to $(T/T_\alpha)^{1/3}$ at $T=T_\beta$ if $T_\beta \geq T_\alpha$. In the opposite case it crosses over to $(T/T_\alpha)^{1/2}$ at $T=\Lambda_0$ and then to $(T/T_\alpha)^{1/3}$ at $T=T_\alpha$.

We conclude that at $T \geq T_\alpha, T_\beta$ the relaxation contribution dominates, and the dephasing rate is

$$\tau_\phi^{-1} = \tau_\Lambda^{-1} [(T/T_\alpha)^{1/3} + \zeta], \quad (53)$$

where ζ is a constant of the order 1 originating from the resonant contribution.

At low temperatures both contributions can be important, their interplay depending on the relationship between the temperature T and the characteristic energies Λ_0 , T_Λ , T_α , and T_β . For both mechanisms the dephasing rate vanishes as $T \rightarrow 0$ and there is a region in which the dephasing rate is proportional to $T^{1/3}$.

As it is seen, can the suggested mechanism indeed exhibit saturationlike behavior in some temperature region. Unfortunately the existing experimental data are obtained in a relatively narrow temperature interval (typically no more than 2 decades) which does not allow one to determine the temperature dependence with an accuracy that is sufficient to prove or disprove Eq. (53).

(d) *Averaging over the tunneling matrix element.* In our considerations we have assumed that $\mathcal{P}_{\text{TM}} \propto \delta(\Lambda - \Lambda_0)$, i.e., that the tunneling matrix element Λ is given. Note, however, that due to disorder the barrier parameters are also scattered. To discuss role of such a scatter let us assume that the overlap integral Λ is given by the expression $\Lambda = (\hbar \omega_0 / \pi) e^{-\lambda}$ where ω_0 is some attempt frequency while the barrier strength λ is distributed according to Gaussian law around some central value $\lambda_0 = \ln \hbar \omega_0 / \pi \Lambda_0 \gg 1$,

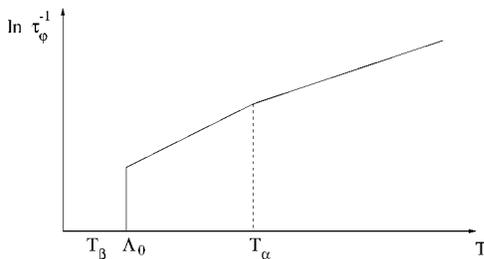


FIG. 6. Schematic picture of $\ln \tau_\phi^{-1}$ as function of temperature for $T_\alpha > T_\beta$.

$$\mathcal{P}_\lambda = \mathcal{N} e^{-(\lambda - \lambda_0)^2 / 2\bar{\lambda}^2}. \quad (54)$$

Here \mathcal{N} is a proper normalization factor which at $\bar{\lambda} \ll \lambda_0$ is equal to $(2\pi\bar{\lambda}^2)^{-1/2}$. As we have seen, for most interesting regimes it is the quantity Λ^2 that has to be averaged. The results then can be expressed as

$$\frac{\overline{\Lambda^2}}{\Lambda_0^2} = \mathcal{N} \int_{-\lambda_0}^{\infty} d\xi e^{-2\xi - \xi^2 / 2\bar{\lambda}^2} \approx e^{2\bar{\lambda}^2} \quad \text{at } \lambda_0 \gg \bar{\lambda}.$$

As it is seen, the only effect of the scatter in λ corresponds to renormalization of the tunneling matrix element by a constant factor $e^{\bar{\lambda}^2}$. Since this factor is temperature independent, it does not affect qualitatively the picture obtained with an assumption of a fixed value of Λ . One also notes that the averaging procedure practically cuts out a contribution of the region $\lambda > \lambda_0$ since at this region the tunneling matrix element exponentially decays with λ . Thus the picture is not sensitive to this region, and any distribution of λ with a lower cutoff (even allowing a Gaussian smearing of this cutoff) does not change the considerations made in an assumption of a fixed value of $\Lambda = \Lambda_0$.

III. ESTIMATES AND DISCUSSION

To make estimates we rewrite the expression (21) for τ_i in the form

$$\tau_i^{-1} = \sigma_{\text{in}} v_F n_d, \quad \sigma_{\text{in}} \equiv \sigma_e^d |V^- / V^+|^2, \quad (55)$$

where σ_e^d is the cross section of elastic electron scattering by a dynamic defect. Correspondingly, the key parameter of our theory τ_Λ is given as

$$\tau_\Lambda^{-1} = \Lambda_0 P_d \sigma_{\text{in}} v_F, \quad (56)$$

where $P_d = n_d / E^*$ is the density of states of the dynamic defects.

The density of states P_d can be, in principle, estimated for a given material on the basis of point contact measurements. Namely, metallic point contacts are known to exhibit, first, telegraph resistance noise²⁵ and, second, zero-bias anomalies;²⁶ both effects are expected to be associated with the dynamic defects.^{17,25,26}

Although we appreciate that the material preparation procedure can significantly affect the defect system, we believe that such experiments can provide more or less reasonable estimates for P_d . The telegraph noise studies²⁵ for a Co nanoconstriction with a size of ~ 10 nm revealed the presence of about several dynamic defects at energies less than 10 mV. This would give us the value $P_d \sim (3-5) \times 10^{32} \text{ erg}^{-1} \text{ cm}^{-3}$. However, the telegraph noise is related to TLS with rather slow relaxation rates ($\leq 10^3 \text{ s}^{-1}$) while we are interested in the defects with switching times of the order of 10^{-9} s. Consequently, these estimates most probably significantly *underestimate* P_d . What is more instructive, the magnitude of the resistance noise revealed rather large defect asymmetry corresponding to the estimate $\sigma_{\text{in}} \sim \sigma_e^d \sim 10^{-15} \text{ cm}^2$.

We believe that the zero bias anomalies can give more reliable information concerning P_d . The magnitude of these anomalies for Co nanoconstrictions²⁶ of the same type as mentioned above corresponds to a presence of several tens of TLS at the energy region about 1 meV.^{17,26} Correspondingly, one obtains $P_d \sim (3-5) \times 10^{34} \text{ erg}^{-1} \text{ cm}^{-3}$.

Based on these estimates and taking $P_d \approx 10^{34} \text{ erg}^{-1} \text{ cm}^{-3}$, $\sigma_{\text{in}} \approx 10^{-15} \text{ cm}^2$, $v_F \approx 10^8 \text{ cm/s}$, and $\Lambda_0 \approx 10 \text{ mK}$ we obtain $\tau_\Lambda \approx 10^{-9} \text{ s}$. Equations (55) and (56) yield $T_\Lambda \approx \Lambda_0$. Thus at temperatures larger than $T_\Lambda \approx \Lambda_0 \approx 10 \text{ mK}$ one expects, according to Eq. (44), temperature-independent contribution of resonant processes.

For the relaxation channel, one obtains $T_\alpha \approx T_\beta \approx 10 \text{ mK}$. Consequently, at $T \gtrsim T_\alpha \approx T_\Lambda \approx 10 \text{ mK}$ one expects that dephasing rate obeys Eq. (53) with $\tau_\Lambda \approx 10^{-9} \text{ s}$. Now let us check if our assumption $\Lambda_0 \approx 10 \text{ mK}$ is realistic. We will exploit a crude estimate

$$\Lambda_0 \approx \frac{\hbar \omega_0}{\pi} \exp\left(-\frac{2}{\hbar} \int_0^a dr \sqrt{2MU(r)}\right), \quad (57)$$

where $U(r)$ is a potential relief between the two stable defect positions separated by a distance a and M is the defect mass. Taking as an example $U(r) = (U_0/2)[1 - \cos(2\pi r/a)]$ one obtains for the exponent $(2a/\pi\hbar)\sqrt{2U_0M}$. Taking for a light defect $\omega_0 \approx 10^{14} \text{ s}^{-1}$ and assuming $a \approx 10^{-8} \text{ cm}$, $U_0 \approx 0.2 \text{ eV}$ one estimates that the value $\Lambda = 10 \text{ mK}$ is achievable for $M \approx 2 \times 10^{-23} \text{ g}$ which corresponds to atomic weight ≈ 10 .

Summarizing our estimates, we can conclude that for realistic parameters of the dynamic defects one can indeed expect a slow temperature dependence of the dephasing rate given by Eq. (53) crossing over to a rapid decrease at low temperatures. The crossover temperature, as well as the behavior below that temperature, depends on the distribution of Λ . For a deltalike distribution (37) the TLS spectrum has a gap of Λ_0 . Thus the TLS contribution to dephasing rate is exponentially frozen out at for $T < \Lambda_0$, and we are left with the ‘‘standard’’ mechanisms such as electron-electron scattering. However for the Gaussian distribution (54) with $\bar{\lambda} \gg 1$ the situation is different. In this case the cutoff temperature is given by the renormalized tunneling coupling $\Lambda_0 e^{\bar{\lambda}^2}$ while for lower temperatures one deals with rather flat distribution of λ within the region $\lambda \leq \lambda_0 + \bar{\lambda}$. Correspondingly, at these temperatures one deals with a glasslike TLS distribution for which $\tau_\varphi \propto T^{-1}$.

Although some papers, e.g., Refs. 9,27, stated that to explain the dephasing saturation by a TLS contribution one would need an unreasonably large concentration of the TLS, this conclusion was mainly based on the ‘‘glassy’’ model of the TLS while we exploited the tunneling state model of Refs. 17,19. In general, to obtain independent information concerning the TLS concentration based on ‘‘bulk’’ measurements such as acoustic measurements or heat capacity measurements in conductors is rather difficult due to a presence of electronic contributions. In particular, the value $P_d \sim 10^{34} \text{ erg}^{-1} \text{ cm}^{-3}$ exploited above is still less than the electron density of states ($\sim 10^{35} \text{ erg}^{-1} \text{ cm}^{-3}$ for Co) and so the

TLS are not expected to affect significantly the properties of the material, such as heat capacity.

Now we would like to compare our results with the calculations given in Refs. 7–9 where a similar problem was considered. The authors of Ref. 7 gave a semiphenomenological treatment of the problem. They exploited the TLS distribution typical for the standard glassy TLS model, but with the upper cutoff $\Lambda_{0,\text{max}}$ for the tunneling matrix element. For the resulting dephasing time they reported a proportionality of τ_φ^{-1} to $\Lambda_{0,\text{max}}/E^*$ (in our notations) in the limit $\tau_\varphi \Lambda_{0,\text{max}} > \hbar$. One notes that such a proportionality is in agreement with the second line of our Eq. (44) although the total expression for τ_φ was some different from ours. Furthermore, the estimate for the opposite limiting case was completely different from our Eq. (44). Since Eq. (44) corresponds to the ‘‘resonant’’ or ‘‘inelastic’’ channel we conclude that the authors of Ref. 7 accounted for only these inelastic processes of electron dephasing. The ‘‘elastic,’’ or σ_3 , channel (which, as we have seen, can dominate with respect to the ‘‘inelastic’’ one) seems to stay beyond the quantitative results of Ref. 7.

In Ref. 8 the dephasing due to dynamical defects was treated within the framework of the two-channel Kondo model. We believe that this model is not relevant to the metallic samples we are interested in, see Ref. 28.

Then, the dephasing by TLS was also considered in the recent paper Ref. 9 where the saturation behavior of τ_φ in quantum dots for the TLS distribution with fixed Λ_0 was claimed. As follows from the derivation,⁹ only the transitions between TLS states due to interaction with the electron forming the interference loop are taken into account. At the same time, the transitions due to other electrons forming a thermal bath (second mechanism of dephasing) are ignored. Hence, only the σ_1 channel is taken into account, and the result is similar to the second item in our Eq. (49). However, the main contribution arising from the σ_3 channel is omitted.

It is worthwhile to mention that similar ideas were used to explain the magnetoresistance of polymers.²⁹ Polymer systems exhibiting rather large fraction of free volume are expected to form readily mobile and metastable defects of different types.

IV. CONCLUSIONS

To conclude, we have shown that the dynamic defects can be responsible for the slowing down of the temperature dependence of the dephasing rate at low temperatures. There are two mechanisms of dephasing. The first one corresponds to direct inelastic scattering of electrons by the defects, while the second one is due to violation of the time reversal symmetry caused by fluctuations of the scattering potential. The first mechanism can indeed lead to the saturation, while the second one still contains a temperature dependence although a weak one. However, when $T \lesssim \Lambda_0$ the dephasing rate rapidly tends to 0.

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APPENDIX A: CALCULATION OF THE RELAXATION RATES

The relaxation rate is determined as an imaginary part of the analytically continued Matsubara self-energy. In general, we have three contribution to the self-energy due to three different types of the electron-TLS interaction. Since for a short-range scattering potential the interaction vertexes do not have an internal structure, for each bosonic propagator \mathcal{D}_i one obtains

$$\Sigma_i^M(\varepsilon_k) = \rho g_i^2 \int d\xi_p F_i^M(\varepsilon_k, \xi_p),$$

$$F_i^M(\varepsilon_k, \xi_p) = T \sum_s \mathcal{D}_i^M(\omega_s) G(\varepsilon_k - \omega_s, \xi_p). \quad (\text{A1})$$

Here ρ is the electron density of states, $\omega_s = 2\pi sT$, $\varepsilon_k = 2\pi(k+1)T$, $\xi_p = p^2/2m - \mu$ and g_i is the proper coupling constant determined by the Hamiltonian (10). The analytical continuation is performed in a usual way. Since there are two cuts in the complex ω plane, at $\text{Im } \omega = 0$ and $\text{Re } \text{Im}(\varepsilon - \omega) = 0$ for each i we get $F^R(\varepsilon_k, \xi_p) = F_1 + F_2$ where

$$F_1 = \int_{-\infty + \varepsilon_k}^{\infty + \varepsilon_k} \frac{N(\omega) \mathcal{D}^R(\omega) d\omega}{2\pi i} [G_A(\varepsilon_k - \omega) - G_R(\varepsilon_k - \omega)],$$

$$F_2 = \int_{-\infty}^{\infty} \frac{N(\omega) G_R(\omega) d\omega}{2\pi i} [\mathcal{D}^R(\omega) - \mathcal{D}^A(\omega)]. \quad (\text{A2})$$

For brevity we omit the arguments ξ_p of the electron Green's functions. Now we replace the integration variable in the expression for Σ_1 as $\omega \rightarrow \omega - \varepsilon_k$ and then combine two integrals. Taking in account that $N(\omega + i\pi T) = -n(\omega)$ where $n(\omega) = (e^{\omega/T} + 1)^{-1}$, $\text{Im } G_R(\varepsilon, \xi_p) = \pi \delta(\varepsilon - \xi_p)$ and making a straightforward algebra we obtain

$$\text{Im } F^R(\varepsilon, \xi_p) = - \int_{-\infty}^{\infty} d\omega \left[\coth\left(\frac{\omega}{2T}\right) + \tanh\left(\frac{\varepsilon - \omega}{2T}\right) \right] \\ \times \text{Im}[\mathcal{D}^R(\omega)] \delta(\varepsilon - \omega - \xi_p).$$

Performing trivial integration over ξ_p and taking into account the $1/2\tau = -\text{Im}\Sigma_R$ we finally obtain

$$\tau_i^{-1}(\varepsilon) = 2\pi\rho g_i^2 \mathcal{G}_i(\varepsilon), \quad (\text{A3})$$

$$\mathcal{G}_i(\varepsilon) = \int_{-\infty}^{\infty} d\omega N(\omega) n(\varepsilon - \omega) n^{-1}(\varepsilon) \text{Im}[\mathcal{D}_i^R(\omega)]. \quad (\text{A4})$$

Using Eq. (13) we obtain

$$\mathcal{G}_1 = n(\varepsilon + E) n(\varepsilon - E) n^{-2}(\varepsilon). \quad (\text{A5})$$

At $E \gg T$ it is proportional to $e^{-E/T}$ at any finite ε .

While calculating \mathcal{G}_3 one can expand $N(\omega) \approx T/\omega$, $n(\varepsilon - \omega)/n(\varepsilon) \approx 1$. After that

$$\mathcal{G}_3 = \cosh^{-2}(E/2T). \quad (\text{A6})$$

It can be easily shown that $\mathcal{G}_0 = 1$.

APPENDIX B: MAPPING TO A RANDOM-TELEGRAPH-NOISE MODEL

Consider an electron trajectory with the total traversal time t_0 which contains N dynamic defects able to hop between two sites. They are rather rare, so a typical neighbor of any active dynamic defect is a *static* one. The total length of the trajectory is

$$\mathcal{L}_0 = t_0/v_F = \sum_{s=1}^M |\mathbf{R}_{s+1}^{(0)} - \mathbf{R}_s^{(0)}|,$$

where M is the total number of defects $M \gg N$.

Let us parameterize the electron motion along the trajectory by time t and allow some of the defects (labeled by j) to make transitions between their states. For those defects,

$$\mathbf{R}_j(t) = \mathbf{R}_j^{(0)} + \mathbf{u}_j(t).$$

The length of distorted trajectory \mathcal{L}^+ traversed in the *positive* direction is

$$\mathcal{L}^+ = \sum_{s=1}^M |\mathbf{R}_{s+1}^{(0)} + \mathbf{u}_{s+1}(t_{s+1}) - \mathbf{R}_s^{(0)} - \mathbf{u}_s(t_s)| \\ = \mathcal{L}^{(0)} + v^{-1} \sum_{j=1}^N \tilde{\mathbf{v}}_j \cdot \mathbf{u}_j(t_j).$$

Here $\tilde{\mathbf{v}}_j \equiv (\mathbf{v}_{j-1} - \mathbf{v}_j)$ is the change in the electron velocity due to scattering by j th EF.

Now we can specify the displacement of j th EF as

$$\mathbf{u}_j(t) \equiv \mathbf{a}_j \xi_j(t),$$

where $\xi_j(t)$ is a random telegraph process (RTP), i.e., a function switching between the values ± 1 at random times and having the correlation function

$$\langle \xi_j(t) \xi_k(t') \rangle = \delta_{jk} e^{-2\gamma_j |t-t'|}.$$

Then the time-dependent contribution to the length is

$$(\delta\mathcal{L})_j(t) = l_j \xi_j(t), \quad l_j \equiv (\mathbf{v}_j \cdot \mathbf{a}_j)/v.$$

For a given defect j , the phase difference is just

$$(\delta\Phi)_j(t_0) = (p_F l_j / \hbar) [\xi(t_j) - \xi(t_0 - t_j)].$$

Let us split the calculation of the average $\langle e^{i\delta\Phi} \rangle$ in two steps. First let us calculate the average over different realizations of a given RTP

$$k(t_0) = \langle e^{iJ[\xi(t) - \xi(t_0 - t)]} \rangle_{\text{RTP}}, \quad J \equiv p_F l / \hbar.$$

This sum can be calculated using the generation function (for $t_\beta > t_\alpha$)

$$K(x, y) = \langle e^{-ix\xi(t_\alpha) - iy\xi(t_\beta)} \rangle = e^{-\gamma(t_\beta - t_\alpha)} [\cos(x+y) \cosh \gamma \\ \times (t_\beta - t_\alpha) + \cos(x-y) \sinh \gamma (t_\beta - t_\alpha)].$$

Substituting $x = -y = J$ and considering arbitrary times we obtain

$$k(t, t_0) = 2 \cos^2 J + 2 \sin^2 J e^{-2\gamma|t_0 - 2t|}.$$

We observe that the function depends explicitly on the position of the scatterer along the trajectory, that is natural. It does not contain complete destruction of the interference—it appears only after averaging over different dynamic defects.

The average over different dynamic defects will be performed using the Holtmark procedure (see, e.g., Ref. 30 for a review) according to which

$$\langle e^{i(\Delta\Phi)} \rangle_d = e^{-W(t_0)}, \quad W(t_0) \equiv n_{\text{eff}} V_c \kappa(t_0).$$

Here n_{eff} is the concentration of “active” defects, V_c is the “contact volume,” while

$$W(t_0) = \langle 1 - k(t, t_0) \rangle_d = \langle 2 \sin^2 J \eta(t_0 - 2t) \rangle_d,$$

where $\eta(t) \equiv 1 - e^{-2\gamma|t|}$. The contact volume is estimated as $V_c = \sigma v_F t_0$, where σ is the scattering cross section.

Let us for simplicity assume that the hopping distances a_j of the defects are the same. We have also to assume that the

phase changes due to individual hops to be small to keep the treatment consistent. Assuming $J \ll 1$ we easily average over the directions of hops to get $\overline{J^2} = (4\pi^2/3)(p_F a/\hbar)^2$. Now let us average over the positions of the defects along the trajectories. This is done as

$$\kappa(t_0) = 2\overline{J^2} \int_0^{t_0/2} \eta(t_0 - t) \frac{dt}{t_0} = \frac{\overline{J^2}}{2\gamma} (2\gamma - 1 + e^{-2\gamma t_0}).$$

We observe that $\kappa(t_0) \sim \overline{J^2} \min\{\gamma t_0, 1\}$. Collecting the factors, we obtain

$$W(t_0) \approx (t/\tau_3) \min\{\gamma t_0, 1\},$$

where

$$\tau_3^{-1} \approx (4\pi^2 n_{\text{eff}}/3)(p_F a/\hbar)^2 \sigma v_F.$$

The concentration n_{eff} depends on the distribution of the TLS parameters. After evaluating it in a proper way one recovers the results of Eq. (41).

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