Hybrid-phonon resonance in a three-dimensional anisotropic quantum well

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This paper reports on a theoretical study of intraband resonances arising in the absorption of electromagnetic radiation by an anisotropic parabolic quantum well. It is shown the scattering of electrons by optical phonons leads to the resonance absorption. The shape of resonance peaks on the absorption curve is studied, and their multiplet nature is demonstrated.

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I. INTRODUCTION

Rapid advances on nanofabrication technology have made it possible to manipulate electron and phonon properties of nanostructures. For example, the size and shape of quantum wells and dots can be controlled in experiments.[1] This opens possibilities to study nanostructures. In particular, the form of confining potentials in quantum wells (dots) depends on growth conditions. Note that a hard-wall potential is a good approximation to describe the lateral confinement of electrons in dots in the case when the dots are obtained from the growth of semiconductor crystallites in a glass or polymer matrix,[2] but is not suitable for dots defined from a two-dimensional electron gas with a subsequent etching processing, ion implantation, or application of electrostatic gates, where one should expect a parabolic confining potential.[3,4] In particular, it was shown that in self-assembled quantum dots generated by the Stranski-Krastanow growth mode the lateral potentials can be treated approximately as being parabolic.[5] The parabolic potential is widely employed in theoretical investigations to study the physical properties of quantum well (dots). We will use the parabolic potential in the present work. Note that theoretical studies[6–8] on the electronic properties of parabolic quantum dots and wells generally in good agreement with the experimental results.[9–11] It is well known that intraband excitations are insensitive to electron-electron interactions in quantum dots (wells) with parabolic potentials as a consequence of the generalized Kohn theorem.[8,13–15] In the case of a parabolic confinement potential the electronic motion can be separated into uncoupled center-of-mass motion and relative motion. However, the far infrared radiation only couples to the center-of-mass motion and we have the same dipole resonance frequencies as a single electron. In particular, positions of resonance peaks are independent of the number of electrons. This is in good agreement with experimental data.[9] Note that deviations from parabolic confinement change the dipole selection rules for the center-of-mass motion, and couple the center-of-mass motion and the relative motion of the electron in the well. In this case electron-electron interaction can also play an important role[16,17] in the case of interband excitations.

The motion of electrons is confined in all directions in our case. Hence optical measurements using resonance techniques are the most preferable means for studying physical properties of quantum wells and dots, because in this case we have no attached contacts which can influence the physical properties of our system.

The hybridization of the size and magnetic quantization can lead to interesting optical properties of these structures placed in a magnetic field.[18–20] Moreover, studies of the intraband absorption by quantum wells are very advanced because semiconductor quantum wells have many potential applications in the electronic devices (for example, the quantum well laser and the quantum well infrared detector), and nonlinear optics. Most of the works on these systems concerned isotropic wells and dots; there have been some recent experimental studies on anisotropic wells.[11,12] It should be noted that if the symmetry of the system is reduced, the degeneracy of the resonance at zero magnetic field is lifted and we can observe two or three resonance peaks at zero magnetic field.[10,11,18,19,21]

The general case of an arbitrary direction of the magnetic field as well as the polarization vector was studied by the authors in the case of a three-dimensional anisotropic parabolic well using the method of linear canonical transformations of the phase space.[21]

Participation of optical phonons in absorption can provide the additional scope for studying physical properties of nanostructures. In view of this, there is a great deal of interest in the phonon properties of the quantum wells and dots.

For example, the hot-electron magnetophonon resonance of quantum wells in tilted magnetic fields was investigated in Ref. 22, optical phonon modes in spherical quantum dots were studied in Refs. 23–25, the influence of spatial dispersion of LO phonons on the energy spectrum of magnetopolarons in quantum wells was investigated in Ref. 26, and multiphonon processes in the quantum dots were studied in Ref. 27. The intraminiband absorption of quantum superlattices was theoretically investigated in Ref. 28. The resonances with the participation of phonons in a quasi-two-dimensional structure were examined in Ref. 29. Note that the generation of coherent confined phonons in nanostructures has been achieved (see, for example, Ref. 30).

Scattering by phonons in a quantum well can lead to a process in which the transition between electronic states occurs under the simultaneous action of two factors, i.e., when the absorption of a quantum $\hbar\omega$ of the high-frequency field is accompanied by the absorption or emission of an optical phonon. Note that, to our knowledge, no experimental work has yet been reported for such a process in quantum wells (dots).

In this paper we present a theoretical study of the hybrid-phonon resonance in an anisotropic parabolic quantum well placed in a magnetic field arbitrarily directed with respect to the potential symmetry axes of the well. Since the spectrum and wave functions of one-electron states have a simple analytic form it is possible to derive the explicit analytic expression for the absorption coefficient of a high-frequency electromagnetic field.

II. GENERAL EXPRESSION FOR THE ABSORPTION COEFFICIENT

The energy spectrum of an electron in an anisotropic parabolic quantum well placed in an arbitrarily directed magnetic field $\boldsymbol{B}$ has the form

$$\varepsilon_{nml}=\hbar\omega_1\left(n+\frac{1}{2}\right)+\hbar\omega_2\left(m+\frac{1}{2}\right)+\hbar\omega_3\left(l+\frac{1}{2}\right), \quad (1)$$

where hybrid frequencies $\omega_i (i=1,2,3)$ are obtained from the sixth-order algebraic equation.[31] Let us consider the absorption as the two-stage process in which an electron absorbs a photon, excites in the intermediate state, and, finally, absorbs (emits) a phonon.

Since in our case the electron transitions are between hybrid levels we call a resonance in absorption due to processes of this type a hybrid-phonon resonance in distinction from a cyclotron resonance with transitions between Landau levels. A hybrid-phonon resonance can be observed only if all levels are well-resolved and the photon frequency is sufficiently monochromatic. Hence in what follows we assume that the photon frequency is high ($\omega\tau\gg 1$), the hybrid confinement is sufficiently strong ($\omega_i\tau\gg 1$) and quantizing ($\hbar\omega_i\gg T$). In this case the transitions occur between levels of the discrete spectrum [Eq. (1)]. The possible types of electron transition are depicted in Fig. 1. Let us consider only long-wave phonons. In this case the phonon frequency is quite monochromatic to observe the resonance.

Let us discuss the physical nature of a hybrid-phonon resonance. The absorption peaks in the hybrid phonon resonance are due to the selection rules for the transitions in second-order perturbation theory and the law of energy conservation in such transitions. Because the spectrum of one-electron states is discrete, the first leads, if we ignore optical phonon dispersion, to delta-function singularities in the absorption of high-frequency radiation. The singularities are proportional to $\delta(\varepsilon_{n'm'l'}-\varepsilon_{nml}+\hbar\omega\mp\hbar\omega_q)$, where ω_q is the phonon frequency, and ω is the photon frequency. Note that in the $3D$ case, as was shown in Ref. 32, the absorption coefficient has a logarithmic singularity in the case of cyclotron-phonon resonance. However, if the weak phonon dispersion is taken into account, the singularities become smeared. In particular, the resonance peaks have the complex multiplet structure in our case.

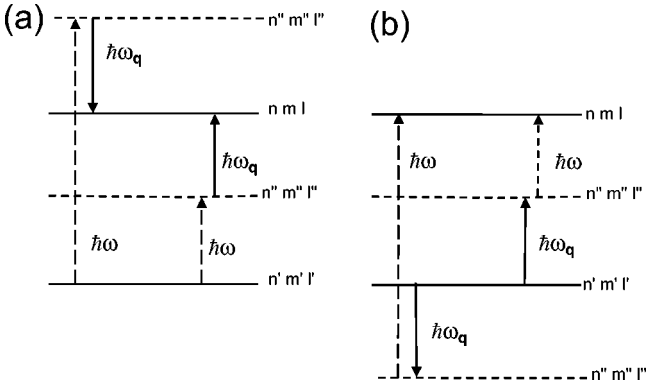

FIG. 1. Transitions leading to resonant absorption.

Using the method suggested in Ref. 32, we find the absorption coefficient by applying ordinary perturbation theory for the interactions of electrons with the high-frequency field H_R and the lattice H_L, which are switched on simultaneously. In this case we use second-order perturbation theory in H_R+H_L to study the electron transitions.

In the case of nondegenerated gas the absorption coefficient is described by the expression

$$\Gamma^{\pm}(\omega)=\frac{2\pi\sqrt{\varepsilon(\omega)}}{c\hbar N_{\boldsymbol{f}}}\left[1-\exp\left(-\frac{\hbar\omega}{T}\right)\right]\sum_{nml}\sum_{n'm'l'}\sum_{\boldsymbol{q}} f_0(\varepsilon_{n'm'l'})|\langle 0,0,nml|V|-\boldsymbol{f},\pm\boldsymbol{q},n'm'l'\rangle|^2$$
$$\times\,\delta(\varepsilon_{n'm'l'}-\varepsilon_{nml}\mp\hbar\omega_{\boldsymbol{q}}+\hbar\omega), \quad (2)$$

where $\varepsilon(\omega)$ is the real part of the dielectric constant, $\boldsymbol{f}$ is the wave vector of photons, $f_0(\varepsilon_{mnp})$ is the electron distribution function normalized to unity, N_f is the number of initial-state photons, and the factor $1-\exp(-\hbar\omega/T)$ takes into account spontaneous transitions.

We can write the matrix elements of the transition depicted in Fig. 1 as follows:

$$\langle 0,0,nml|V|-\boldsymbol{f},\pm\boldsymbol{q},n'm'l'\rangle=\sum_{n''m''l''}\frac{\langle n'm'l',0|H_R|n''m''l'',-\boldsymbol{f}\rangle\langle n''m''l'',0|H_L|nml,\pm\boldsymbol{q}\rangle}{\varepsilon_{n'm'l'}-\varepsilon_{n''m''l''}+\hbar\omega}$$
$$+\sum_{n''m''l''}\frac{\langle n'm'l',0|H_L|n''m''l'',\pm\boldsymbol{q}\rangle\langle n''m''l'',0|H_R|nml,-\boldsymbol{f}\rangle}{\varepsilon_{nml}-\varepsilon_{n''m''l''}-\hbar\omega}. \quad (3)$$

In the first term transitions pass through the virtual ones $|n''m''l'',-\boldsymbol{f}\rangle$, i.e., at first a number of photons changes and then a number of phonons changes. In the second term transitions pass through the virtual ones $|n''m''l'',\pm\boldsymbol{q}\rangle$, i.e., at first a number of phonons changes and then a number of photons changes. Figure 1(a) shows transitions which are associated with the first term in Eq. (3), and Fig. 1(b) shows transitions which are associated with the second term in Eq. (3).

The interaction with the lattice is represented by the operator H_L,

$$H_L=\sum_{\boldsymbol{q}} h_L(\boldsymbol{q}), \quad h_L(\boldsymbol{q})=D_{\boldsymbol{q}}c_{\boldsymbol{q}}e^{i\boldsymbol{qr}}+\text{c.c.}, \tag{4}$$

here $D_{\boldsymbol{q}}$ is the electron-phonon coupling constant, and the operator H_R of the electron-photon interaction can be expressed as

$$H_R=\frac{e}{m^*}\sqrt{\frac{2\pi\hbar N_f}{\varepsilon(\omega)\omega}}\boldsymbol{e}_f\left(\boldsymbol{p}-\frac{e}{c}\boldsymbol{A}\right) \tag{5}$$

where $\boldsymbol{e}_f$ is the polarization vector, $\boldsymbol{p}$ is the electron momentum, and $\boldsymbol{A}$ is the vector potential of the field $\boldsymbol{B}$.

The matrix elements of H_R have been calculated in Ref. 21 using the method of a linear canonical transformation of the phase space

$$\begin{aligned}\langle n'm'l',0|H_R|n''m''l'',-\boldsymbol{f}\rangle &= ie\hbar\sqrt{\frac{\pi N_f}{m^*\varepsilon(\omega)\omega}}\times\{X_1\sqrt{n'+1}\,\delta_{n',n''-1}\delta_{m',m''}\delta_{l',l''}\\ &+X_2\sqrt{m'+1}\,\delta_{m',m''-1}\delta_{n',n''}\delta_{l',l''}\\ &+X_3\sqrt{l'+1}\,\delta_{l',l''-1}\delta_{m',m''}\delta_{n',n''}\}\end{aligned} \tag{6}$$

here X_i (the coordinates of the vector $\boldsymbol{X}$) were found in Ref. 21. Note that X_i depends both on the hybrid frequencies and magnetic field.

Matrix elements of the operator $h_L(\boldsymbol{q})$ have the form

$$\begin{aligned}&\langle n''m''l'',0|h_L(\boldsymbol{q})|nml,\pm\boldsymbol{q}\rangle\\ &\quad=D_{\boldsymbol{q}}\sqrt{N_{\boldsymbol{q}}+\frac{1}{2}\pm\frac{1}{2}}\langle n''m''l''|e^{\pm i\boldsymbol{qr}}|nml\rangle;\end{aligned} \tag{7}$$

here $N_{\boldsymbol{q}}$ is the number of phonons with the wave vector $\boldsymbol{q}$.

We calculate the matrix elements $h_L(\boldsymbol{q})$ in the new phase coordinates (P_i,Q_i) because in this case the wave function of electrons has the simplest form of the product of the oscillator functions. Let us introduce the matrix $\Lambda=(b_{ij})$ $(i=1,2,3;j=1,\ldots,6)$,

$$\begin{aligned}x&=b_{11}P_1+b_{12}P_2+b_{13}P_3+b_{14}Q_1+b_{15}Q_2+b_{16}Q_3,\\ y&=b_{21}P_1+b_{22}P_2+b_{23}P_3+b_{24}Q_1+b_{25}Q_2+b_{26}Q_3,\\ z&=b_{31}P_1+b_{32}P_2+b_{33}P_3+b_{34}Q_1+b_{35}Q_2+b_{36}Q_3,\end{aligned} \tag{8}$$

where (b_{ij}) were found in Ref. 21.

Next we use the relationship

$$\exp\left(\pm i\frac{a}{\hbar}P_i\right)\Phi(x)=\Phi(x\pm a), \quad i=1,2,3 \tag{9}$$

which follows from the fact that the exponential operator is the generator of the translation group. Then

$$\begin{aligned}\langle n''m''l''|e^{\pm i\boldsymbol{qr}}|nml\rangle&=\langle\Phi_{n''}(Q_1)|e^{\pm i\kappa_1Q_1}|\Phi_n(Q_1\pm\lambda_1)\rangle\\ &\times\langle\Phi_{m''}(Q_2)|e^{\pm i\kappa_2Q_2}|\Phi_m(Q_2\pm\lambda_2)\rangle\\ &\times\langle\Phi_{l''}(Q_3)|e^{\pm i\kappa_3Q_3}|\Phi_l(Q_3\pm\lambda_3)\rangle;\end{aligned} \tag{10}$$

here $\Phi_n(x)$ are the oscillator functions, $\lambda_i=\hbar(b_{1i}q_x+b_{2i}q_y+b_{3i}q_z)$ $(i=1,2,3)$, $\kappa_{i-3}=b_{1i}q_x+b_{2i}q_y+b_{3i}q_z$ $(i=4,5,6)$.

After some cumbersome algebra, evaluating the integral in Eq. (10) we obtain

$$\begin{aligned}&\langle n''m''l'',0|h_L(\boldsymbol{q})|nml,\pm\boldsymbol{q}\rangle\\ &\quad=D_{\boldsymbol{q}}\sqrt{N_{\boldsymbol{q}}+\frac{1}{2}\pm\frac{1}{2}}\left(\frac{n''!m''!l''!}{n!m!l!}\right)^{1/2}\\ &\qquad\times(-1)^{n-n''}(-1)^{m-m''}(-1)^{l-l''}g_1^{n-n''}g_2^{m-m''}\\ &\qquad\times g_3^{l-l''}e^{i\varphi_1(n-n'')}e^{i\varphi_2(m-m'')}e^{i\varphi_3(l-l'')}\\ &\qquad\times e^{-g^2/2}e^{-(\kappa_1l_1+\kappa_2l_2+\kappa_3l_3)i/2}\\ &\qquad\times L_{n''}^{n-n''}(g_1^2)L_{m''}^{m-m''}(g_2^2)L_{l''}^{l-l''}(g_3^2);\end{aligned} \tag{11}$$

here $\tan\varphi_i=\kappa_il_i^2/\lambda_i$, $g_i=\sqrt{\lambda_i^2+\kappa_i^2l_i^4}/\sqrt{2}l_i$, $l_i=\sqrt{\hbar/m^*\omega_i}$ $(i=1,2,3)$ are the hybrid lengths, $g^2=g_1^2+g_2^2+g_3^2$, $L_n^{n'}(x)$ are generalized Laguerre polynomials.

Substituting Eqs. (6) and (11) into Eq. (3), we can write the squares of the matrix elements of the perturbation operator

$$\begin{aligned}&|\langle-\boldsymbol{f},\pm\boldsymbol{q},n'm'l'|V|0,0,nml\rangle|^2\\ &\quad=\frac{e^2}{m^*}\frac{N_f(N_{\boldsymbol{q}}+1/2\pm1/2)}{\varepsilon(\omega)\omega}e^{-g^2/2}|D_{\boldsymbol{q}}|^2|A(\omega)|^2\\ &\qquad\times\frac{n'!m'!l'!}{n!m!l!}g_1^{2(n-n')}g_2^{2(m-m')}g_3^{2(l-l')}\\ &\qquad\times[L_{n'}^{n-n'}(g_1^2)L_{m'}^{m-m'}(g_2^2)L_{l'}^{l-l'}(g_3^2)]^2;\end{aligned} \tag{12}$$

here

$$A(\omega)=\frac{X_1g_1e^{-i\varphi_1}}{\omega-\omega_1}+\frac{X_2g_2e^{-i\varphi_2}}{\omega-\omega_2}+\frac{X_3g_3e^{-i\varphi_3}}{\omega-\omega_3}. \tag{13}$$

We introduce partial absorption coefficients by the following formula

$$\overline{\Gamma}^{(\pm)}(\omega)=\sum_{nml,n'm'l'}\overline{\Gamma}^{(\pm)}(n'm'l',nml). \tag{14}$$

Note that we replace $N_{\boldsymbol{q}}$ by the Planck distribution function N_T, which results from thermal averaging over phonon states.

We also introduce the resonance detuning

$$\Delta\omega_q=\omega_1(n-n')+\omega_2(m-m')+\omega_3(l-l')+\omega\mp\omega_q. \tag{15}$$

Then for the partial absorption coefficient we obtained the equation

$$\begin{aligned}\overline{\Gamma}^{(\pm)}&(n'm'l',nml)\\ &=\frac{2\pi e^2}{m^*c\hbar^2\omega\sqrt{\varepsilon(\omega)}}\left[1-\exp\left(\frac{-\hbar\omega}{T}\right)\right]\\ &\times f_0(\varepsilon_{nml})\sum_{\boldsymbol{q}}|D_{\boldsymbol{q}}|^2|A(\omega)|^2e^{-g^2}\frac{n'!m'!l'!}{n!m!l!}g_1^{2(n-n')}\\ &\times g_2^{2(m-m')}g_3^{2(l-l')}[L_{n'}^{n-n'}(g_1^2)L_{m'}^{m-m'}(g_2^2)L_{l'}^{l-l'}(g_3^2)]^2\\ &\times\left(N_{\boldsymbol{q}}+\frac{1}{2}\pm\frac{1}{2}\right)\delta(\Delta\omega_q),\end{aligned} \tag{16}$$

where

$$\begin{aligned}f_0(\varepsilon_{mnl})=&\sinh(\hbar\omega_1/2T)\sinh(\hbar\omega_2/2T)\\ &\times\sinh(\hbar\omega_3/2T)\exp(-\varepsilon_{nml}/T).\end{aligned}$$

Equation (16) clearly shows that if one ignores the optical phonon dispersion, the partial coefficients $\overline{\Gamma}^{(\pm)}(n'm'l',nml)$ have delta-function singularities at the points where $\Delta\omega_q=0$. Replacing the sum by the integral and assuming a parabolic dispersion law for long-wave phonons $\omega_q=\omega_0(1-\omega_0^{-2}v_s^2q^2)$, where ω_0 is the optical-phonon threshold frequency and v_s is the speed of sound, we can easily evaluate the integral with respect to $|\boldsymbol{q}|$ thanks to the presence of a delta function of the form $\delta\omega_q=\delta(\Delta\omega\pm\omega_0^{-1}v_s^2q^2)$, where $\Delta\omega_0=\omega_1(n-n')+\omega_2(m-m')+\omega_3(l-l')+\omega\mp\omega_0$

The electron-phonon coupling constant for deformation potential scattering (DO phonons) and polarization potential scattering (PO phonons) is[32]

$$|D_{\boldsymbol{q}}|^2=\frac{2\pi\hbar^2\alpha_L\omega_0}{m^*}\begin{cases}\sqrt{2m^*\hbar\omega_0}/q^2 & \text{for PO phonons,}\\ 4\hbar^2/\sqrt{2m^*\hbar\omega_0} & \text{for DO phonons.}\end{cases} \tag{17}$$

Here α_L is the dimensionless electron-lattice coupling constant.

Converting to spherical coordinates we obtained the following equation for DO phonons:

$$\begin{aligned}\frac{\overline{\Gamma}_{DO}^{(\pm)}&(n'm'l',nml)}{\Gamma_0}\\ &=\frac{n'!m'!l'!}{n!m!l!}\left(N_0+\frac{1}{2}\pm\frac{1}{2}\right)\\ &\times\left[1-\exp\left(\frac{-\hbar\omega}{T}\right)\right]f_0(\varepsilon_{mnl})\sqrt{\frac{|\Delta\omega_0|}{\omega_0}}\\ &\times\int_0^{2\pi}d\varphi\int_0^{\pi}\sin\theta d\theta y_1^{2(n-n')}y_2^{2(m-m')}\\ &\times y_3^{2(l-l')}|A_0(\omega)|^2e^{-y^2}\\ &\times[L_{n'}^{n-n'}(y_1^2)L_{m'}^{m-m'}(y_2^2)L_{l'}^{l-l'}(y_3^2)]^2,\end{aligned} \tag{18}$$

where $A_0(\omega)=\omega_0A(\omega)/\sqrt{\omega}$, N is the electron concentration, and $\Gamma_0=8\hbar^2\alpha_LNe^2\omega_0^{1/2}/\pi v_s^3c\sqrt{\varepsilon}m^{*2}\sqrt{2m^*\hbar}$, here y_i can be obtained from g_i if we write the vector $\boldsymbol{q}$ in the spherical coordinates.

For partial absorption coefficient $\overline{\Gamma}_{PO}^{(\pm)}(n'm'l',nml)$ we have

$$\overline{\Gamma}_{PO}^{(\pm)}(n'm'l',nml)=\frac{mv_s^2}{2\hbar|\Delta\omega|}\overline{\Gamma}_{DO}^{(\pm)}(n'm'l',nml). \tag{19}$$

Transitions from the ground state $n'=m'=l'=0$ will provide the major contribution to the absorption coefficient. In this case the expression for the partial coefficient has the simple form

$$\begin{aligned}\frac{\overline{\Gamma}_{DO}^{(\pm)}(000,nml)}{\Gamma_0}&=n!m!l!\left(N_0+\frac{1}{2}\pm\frac{1}{2}\right)\\ &\times\left[1-\exp\left(\frac{-\hbar\omega}{T}\right)\right]f_0(\varepsilon_{mnl})\\ &\times\int_0^{2\pi}d\varphi\int_0^{\pi}\sin\theta d\theta\\ &\times y_1^{2n}y_2^{2m}y_3^{2l}|A_0(\omega)|^2e^{-y^2}.\end{aligned} \tag{20}$$

Note here that absorption peaks have the amplitude smaller by a factor of $\exp(-\hbar\omega_0/T)$ than emission peaks though their form doesn't vary. If we allow for the smearing of the hybrid-oscillation levels caused by collisions, then $\Gamma(\Delta\omega_q)$ must be replaced by $\mathrm{Re}\Gamma(\Delta\omega_q+i\gamma)$, with the collisional spread $\gamma=\tau^{-1}$.

III. STRUCTURE AND AMPLITUDE OF RESONANCE PEAKS

Let us consider the two most important cases: the polarization vector is perpendicular to the magnetic field and the polarization vector is parallel to the magnetic field.

In the first case $\omega_{cx}=\omega_{cy}=0$ and $\omega_{cz}=\omega_c$ and hybrid frequencies are determined by the formulas

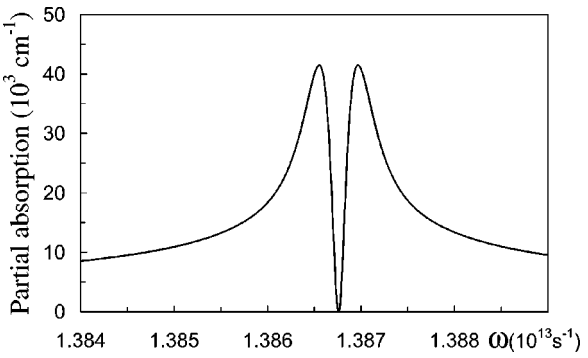

FIG. 2. Partial absorption coefficients in the case of emission of DO phonons in the case of the transitions $|0,0,0\rangle \rightarrow |1,0,0\rangle$. $\varepsilon = 13$, $B = 6.5 \times 10^4$ Oe, $\omega_0 = 8 \times 10^{13}$ s^{-1}, $\Omega_x = 3.7 \times 10^{13}$ s^{-1}, $\Omega_y = 6.2 \times 10^{13}$ s^{-1}, and $\Omega_z = 3.4 \times 10^{13}$ s^{-1}.

$$\omega_{1,2} = \frac{1}{2}[\sqrt{(\Omega_x + \Omega_y)^2 + \omega_c^2} \pm \sqrt{(\Omega_x - \Omega_y)^2 + \omega_c^2}], \quad (21)$$

$$\omega_3 = \Omega_z .$$

The values of y_i have a form

$$y_i = \frac{l_i}{\sqrt{2}} \sqrt{\frac{\omega_0 |\Delta\omega_0|}{v_s^2}} \frac{\sin\theta}{\sqrt{\omega_c^2 \Omega_x^2 + (\Omega_x^2 - \omega_i^2)^2}} \times [(\Omega_x^2 - \omega_i^2)^2 \sin^2\varphi + \omega_c^2 \omega_i^2 \cos^2\varphi]^{1/2}. \quad (22)$$

Let us only examine transitions which originate from the states $|n',0,0\rangle$ into $|n,0,0\rangle$ because the changes of the quantum numbers m l have no influences on the form of absorption peaks.

It should be noted that at the points where the frequency of the electromagnetic radiation satisfies $\Delta\omega_0 = 0$ these coefficients vanish. As seen from Eq. (18) within a small neighborhood of this point, the partial coefficients increase according to a power law, and then fall off exponentially. Hence we can only observe the resonance within a small neighborhood of the point where $\Delta\omega_0 = 0$, namely, in the frequency range order of magnitude 10^9 s^{-1}. This is small compared to the characteristic frequencies $\omega_c, \omega_i, \omega_0$ ($\sim 10^{12}$ s^{-1}) in the system.

Below we consider only transitions $|n',0,0\rangle \rightarrow |n'+1,0,0\rangle$. It is clearly the transitions $|n',0,0\rangle \rightarrow |n'+k,0,0\rangle$, where $k = 2,3, \ldots$ do not differ essentially from $|n',0,0\rangle \rightarrow |n'+1,0,0\rangle$, and partial coefficients are equal in the case of the transitions $|n',0,0\rangle \rightarrow |n'-k,0,0\rangle$ and $|n',0,0\rangle \rightarrow |n'+k,0,0\rangle$.

In view of the complexity of the analytic formulas for the partial coefficient, a detailed description of the absorption peaks requires numerical studies. In the case of transitions from the ground state ($n = m = l = 0$) the absorption coefficients have two symmetrically positioned sharp peaks (to the left and right of the point) within a small neighborhood of the point where $\Delta\omega_0 = 0$, i.e., the partial absorption peaks have a doublet structure (Fig. 2).

In the case of transitions from the first state ($n = 1, m, l = 0$) each of the doublet peak splits up into two ones (Fig. 3), in the case of transitions from the second state ($n = 2, m, l = 0$) each of the doublet peak splits up into three ones (Fig. 4), in the generic case of transitions from the state ($n', m, l = 0$) each of the doublet peak splits up into $n'+1$ ones. Thus we can determine the number of the resonance level if we know the number of maxima in the neighborhood of a point $\Delta\omega_0 = 0$. The number of peaks is determined by the the low number of the Laguerre polynomials. In particular, if the low number of the Laguerre polynomials is equal to k then the number of maxima at the right (left) of the point $\Delta\omega_0 = 0$ is equal to $k+1$. It should be pointed that the exponential character of the dependence of the partial coefficient on the number n', m', and l' guarantees a sharp decrease in partial absorption as the level number grows.

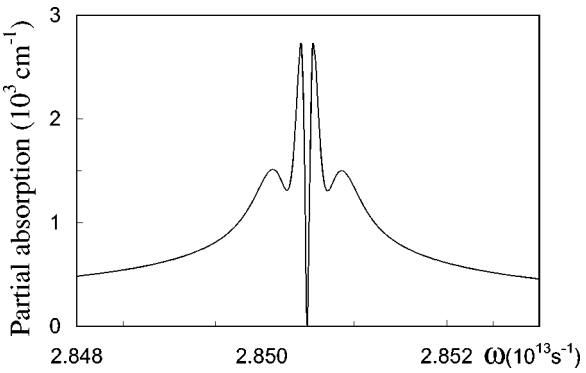

FIG. 3. Partial absorption coefficients in the case of emission of DO phonons in the case of the transitions $|1,0,0\rangle \rightarrow |2,0,0\rangle$. $\varepsilon = 13$, $B = 10^5$ Oe, $\omega_0 = 9 \times 10^{13}$ s^{-1}, $\Omega_x = 5.1 \times 10^{13}$ s^{-1}, $\Omega_y = 3.2 \times 10^{13}$ s^{-1}, and $\Omega_z = 1.3 \times 10^{13}$ s^{-1}.

Let us consider the case when the polarization vector is parallel to the magnetic field. In this case $\omega_{cx} = \omega_{cz} = 0$, $\omega_{cy} = \omega_c$, and hybrid frequencies are determined by the formulas

$$\omega_{1,3} = \frac{1}{2}[\sqrt{(\Omega_x + \Omega_z)^2 + \omega_c^2} \pm \sqrt{(\Omega_x - \Omega_z)^2 + \omega_c^2}],$$

$$\omega_2 = \Omega_y . \quad (23)$$

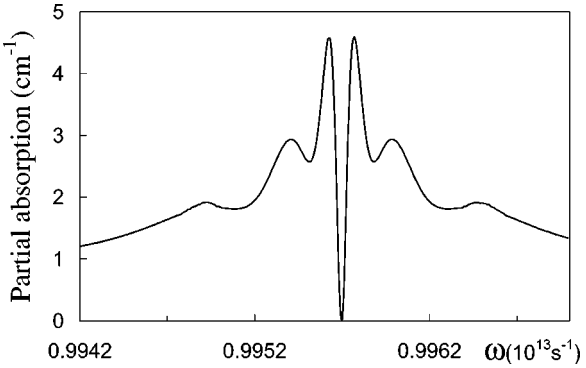

FIG. 4. Partial absorption coefficients in the case of emission of DO phonons in the case of the transitions $|2,0,0\rangle \rightarrow |3,0,0\rangle$. $\varepsilon = 13$, $B = 9 \times 10^4$ Oe, $\omega_0 = 7.5 \times 10^{13}$ s^{-1}, $\Omega_x = 5.7 \times 10^{13}$ s^{-1}, $\Omega_y = 3.5 \times 10^{13}$ s^{-1}, and $\Omega_z = 2.3 \times 10^{13}$ s^{-1}.

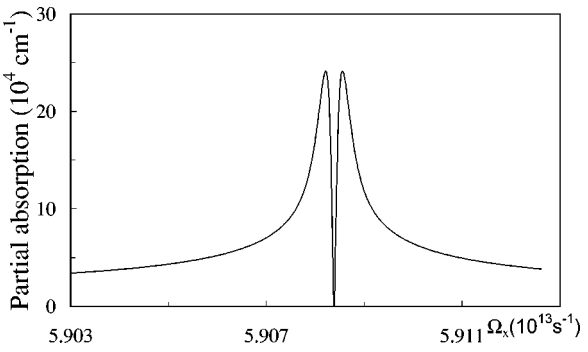

FIG. 5. Partial absorption coefficients in the case of emission of DO phonons in the case of the transitions $|0,0,0\rangle \rightarrow |1,0,0\rangle$. $\varepsilon = 13$, $B = 5\times 10^4$ Oe, $\omega_0 = 9\times 10^{13}$ s^{-1}, $\omega = 2.847\times 10^{13}$ s^{-1}, $\Omega_y = 3.2\times 10^{13}$ s^{-1}, and $\Omega_z = 1.3\times 10^{13}$ s^{-1}.

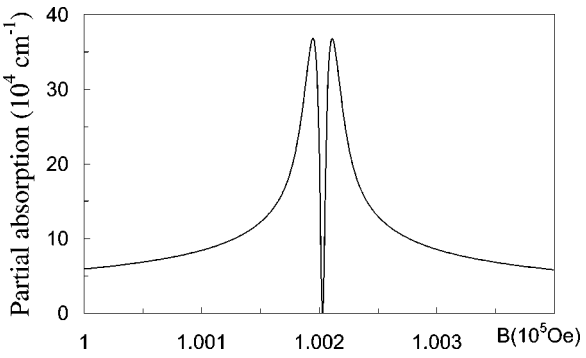

FIG. 6. Partial absorption coefficients in the case of emission of DO phonons in the case of the transitions $|0,0,0\rangle \rightarrow |1,0,0\rangle$. $\varepsilon = 13$, $\omega_0 = 9\times 10^{13}$ s^{-1}, $\omega = 2.847\times 10^{13}$, $\omega_x = 5.1\times 10^{13}$ s^{-1}, $\Omega_y = 3.2\times 10^{13}$ s^{-1}, and $\Omega_z = 1.3\times 10^{13}$ s^{-1}

It is easy to show that transitions between the level with the different n and l are forbidden, and the coordinates $X_i (i = 1,3)$ are equal to zero. The function y_2 has the form

$$y_2 = \frac{l_2}{\sqrt{2}} \sqrt{\frac{\omega_0 |\Delta\omega_0|}{v_s^2}} \sin\theta \sin\varphi. \tag{24}$$

In this case we have the same multiplet structure of the resonance peaks as in the first one. It is interesting to note that in the case of the pure hybrid resonance (without phonons) the transitions between the levels with the different $n\ l$ are also forbidden and we have only one resonance peak but not three.[21]

In conclusion, we have investigated theoretically the hybrid-phonon resonance of a three-dimensional anisotropic quantum well in the presence of an arbitrarily directed magnetic field and polarization vector. If we ignore optical phonon dispersion, the partial absorption peaks have a delta-function singularity at the points where $\Delta\omega_0 = 0$. Note that the singularity of the partial absorption peaks is connected with singularities in the density of the initial and final states.

The interesting multiplate structure of peaks arises if one takes into account the dispersion of optical phonons. That is, at the points where the frequency of the electromagnetic radiation satisfy conditions $\Delta\omega_0 = 0$ the partial coeficients vanish. Within a small neighborhood of this point $\Gamma^{(\pm)}(n'm'l', nml)$ has $n'+1$ peaks symmetrically positioned (to the left and right) to the points where $\Delta\omega_0 = 0$. Note that the number of transitions from the state n' is equal to $n'+1$. The absorption peaks in the hybrid-phonon resonance are due to the selection rules for the transitions in second-order perturbation theory and the law of energy conservation in such transitions. The width and the position of the resonance peaks depend strongly on the magnetic field and the characteristic frequencies of the parabolic confinement (Figs. 3–6) but on the whole the general nature of the $\Gamma^{(\pm)}$ vs the magnetic field or the characteristic frequencies is the same as for $\Gamma^{\pm}(\omega)$ (Figs. 5 and 6)

The frequency factor $A(\omega)$ in the absorption coefficient has singularities at the points $\omega_i (i = 1,2,3)$. These singularities, however, are unimportant in studies of the hybrid-phonon resonance, since they are shifted far in relation to the points where $\Delta\omega_0 = 0$ by the optical phonon frequency, with the result they lie in the distant region of the wings of the hybrid-phonon resonance lines. This is an analog of a cyclotron resonance in the system which was studied by the authors in Ref. 21.

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