PHYSICAL REVIEW B **66**, 161320(R) (2002)

Bell inequalities and entanglement in solid-state devices

Nikolai M. Chtchelkatchev,[1,2] Gianni Blatter,[3] Gordey B. Lesovik,[1,3] and Thierry Martin[2]
[1]*L.D. Landau Institute for Theoretical Physics RAS, 117940 Moscow, Russia*
[2]*Centre de Physique Théorique, Université de la Méditerranée, Case 907, 13288 Marseille, France*
[3]*Theoretische Physik, ETH-Hönggerberg, CH-8093 Zürich, Switzerland*

(Received 24 February 2002; revised manuscript received 29 August 2002; published 31 October 2002)

Bell-inequality checks constitute a probe of entanglement—given a source of entangled particles, their violation is a signature of the nonlocal nature of quantum mechanics. Here, we study a solid-state device producing pairs of entangled electrons, a superconductor emitting Cooper pairs properly split into the two arms of a normal-metallic fork with the help of appropriate filters. We formulate Bell-type inequalities in terms of current-current cross correlators, the natural quantities measured in mesoscopic physics; their violation provides evidence that this device indeed is a source of entangled electrons.

DOI: 10.1103/PhysRevB.66.161320 PACS number(s): 03.65.Ta, 03.67.Lx, 85.35.Be, 73.23.Ad

Entanglement[1,2] is a defining feature of quantum-mechanical systems[3] with important new applications in the emerging fields of quantum information theory,[4] quantum computation,[5] quantum cryptography,[6] and quantum teleportation.[7] Many examples of entangled systems can be found in nature, but only in few cases can entanglement be probed and used in applications. So far, much attention has been focused on the preparation and investigation of entangled photons[8,9] and, more recently, of entangled atoms,[10,11] while other studies use elementary particles (kaons)[12] and electrons.[13] Bell inequality (BI)[14] checks have become the accepted method to test entanglement:[15,16] their violation in experiments with particle pairs indicates that there are nonlocal correlations between the particles, as predicted by quantum mechanics, which no local hidden variable theory can explain.[14]

Quasiparticles in solid-state devices are promising candidates as carriers of quantum information. Recent investigations provide strong evidence that electron spins in a semiconductor show unusually long dephasing times approaching microseconds; furthermore, they can be transported phase coherently over distances exceeding 100 μm.[17] Several proposals how to create an Einstein-Podolsky-Rosen[2] (EPR) pair of electrons in solid-state systems have been made recently, e.g., using a superconductor[18,19] or a quantum dot[20] as a source of entangled beams of electrons. At first glance, the possibility of performing BI checks in solid-state systems may seem to be a naive generalization[21–23] of the corresponding tests with photons.[8,9] But in the case of photons, the BIs have been tested using photodetectors measuring coincidence rates (the probability that two photons enter the detectors nearly simultaneously[8,9]). Counting quasiparticles one-by-one (as photodetectors do in quantum optics[9]) is difficult to achieve in solid-state systems where currents and current-current correlators, in particular noise, are the natural observables in a stationary regime.[24] Here, the BIs are reformulated in terms of current-current cross-correlators (noise) and a practical implementation of BIs as a test of quasiparticle entanglement produced via a hybrid superconductor–normal-metal source[18,19] is discussed.[15,25]

Consider a source [Fig. 1(a)] injecting quasi-particles into two arms labeled by indices 1 and 2. The detector includes two filters $F^d_{1(2)}$ selecting electrons by spin; the filter F^d_1 transmits electrons spin-polarized along the direction **a** into lead 5 and deflects electrons with the opposite polarization into lead 3 (and similar for filter F^d_2 with direction **b**). The detector thus measures cross correlations of (spin)currents between the leads; a violation of BIs provides evidence for nonlocal spin correlations between the quasiparticle beams 1 and 2.

We formulate the BIs in terms of current-current correlators: assuming separability and locality[14,15] (no entanglement, only local correlations are allowed) the density matrix of the source/detector system describing joint events in the leads α, β is given by

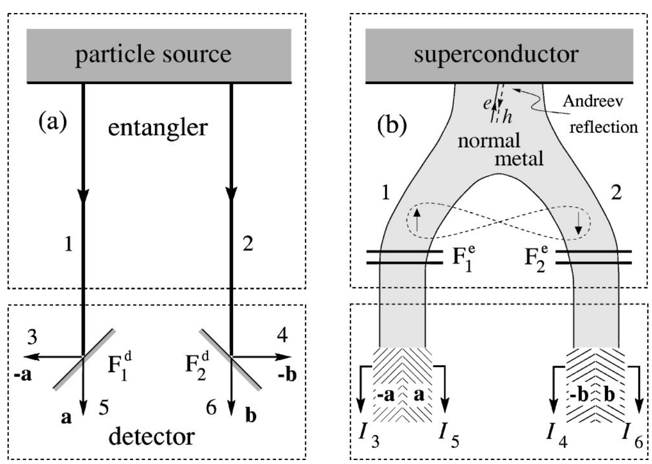

FIG. 1. (a) Schematic setup for the measurement of Bell inequalities: a source emits particles into leads 1 and 2. The detector measures the correlation between beams labeled with odd and even numbers. Filters $F^d_{1(2)}$ select the spin: particles with polarization along the direction $\pm\mathbf{a}(\pm\mathbf{b})$ are transmitted through filter $F^d_{1(2)}$ into lead 5 and 3 (6 and 4). b) Solid state implementation, with superconducting source emitting Cooper pairs into the leads. Filters $F^e_{1,2}$ (e.g., Fabry-Perot double barrier structures or quantum dots) prevent Cooper pairs from entering a single lead. Ferromagnets with orientations $\pm\mathbf{a}, \pm\mathbf{b}$ play the role of the filters $F^d_{1(2)}$ in a); they are transparent for electrons with spin aligned along their magnetization axis.

$$\rho = \int d\lambda f(\lambda) \rho_\alpha(\lambda) \otimes \rho_\beta(\lambda), \tag{1}$$

where the lead index α is even and β is odd (or vice versa); the distribution function $f(\lambda)$ (positive and normalized to unity) describes the "hidden variable" λ. The Hermitian operators $\rho_\alpha(\lambda)$ satisfy the standard axioms of density matrices. For identical particles the assumption (1) implies that Bose and Fermi correlations between leads with odd and even indices are neglected.

Consider the Heisenberg operator of the current $I_\alpha(t)$ in lead $\alpha = 1, \ldots, 6$ (see Fig. 1) and the associated particle number operator $N_\alpha(t,\tau) = \int_t^{t+\tau} dt' \, I_\alpha(t')$ describing the charge going through a cross section of lead α during the time interval $[t, t+\tau]$. For later convenience we introduce the averages $\langle \ldots \rangle_\lambda$, $\langle \ldots \rangle_\rho$ over the density matrices $\rho_\alpha(\lambda)$, ρ, respectively, and over large time periods, e.g.,

$$\langle N_\alpha(\tau) N_\beta(\tau) \rangle \equiv \frac{1}{2T} \int_{-T}^{T} dt \langle N_\alpha(t,\tau) N_\beta(t,\tau) \rangle_\rho , \tag{2}$$

where $T/\tau \to \infty$. Finally, we define the particle number fluctuations $\delta N_\alpha(t,\tau) \equiv N_\alpha(t,\tau) - \langle N_\alpha(\tau) \rangle$.

The derivation of the BI is based on the following lemma: let x, x', y, y', X, Y be real numbers such that $|x/X|$, $|x'/X|$, $|y/Y|$, and $|y'/Y|$ do not exceed unity, then the following inequality holds:[26]

$$-2XY \leqslant xy - xy' + x'y + x'y' \leqslant 2XY. \tag{3}$$

Lemma (3) is applied to our system with

$$x = \langle N_5(t,\tau) \rangle_\lambda - \langle N_3(t,\tau) \rangle_\lambda , \tag{4a}$$

$$x' = \langle N_{5'}(t,\tau) \rangle_\lambda - \langle N_{3'}(t,\tau) \rangle_\lambda , \tag{4b}$$

$$y = \langle N_6(t,\tau) \rangle_\lambda - \langle N_4(t,\tau) \rangle_\lambda , \tag{4c}$$

$$y' = \langle N_{6'}(t,\tau) \rangle_\lambda - \langle N_{4'}(t,\tau) \rangle_\lambda , \tag{4d}$$

where the "prime" indicates a different direction of spin-selection in the detector's filter [e.g., let $\mathbf{a}$ denote the direction of the electron spins in lead 5 ($-\mathbf{a}$ in lead 3], then the subscript $5'$ in Eq. (4b) refers to electron spins in lead 5 polarized along $\mathbf{a}'$ (along $-\mathbf{a}'$ in the lead 3). The quantities X, Y are defined as

$$X = \langle N_5(t,\tau) \rangle_\lambda + \langle N_3(t,\tau) \rangle_\lambda = \langle N_{5'}(t,\tau) \rangle_\lambda + \langle N_{3'}(t,\tau) \rangle_\lambda = \langle N_1(t,\tau) \rangle_\lambda , \tag{5a}$$

$$Y = \langle N_6(t,\tau) \rangle_\lambda + \langle N_4(t,\tau) \rangle_\lambda = \langle N_{6'}(t,\tau) \rangle_\lambda + \langle N_{4'}(t,\tau) \rangle_\lambda = \langle N_2(t,\tau) \rangle_\lambda ; \tag{5b}$$

the equalities (5a) and (5b) follow from particle number conservation. All terms in (5a) and (5b) have the same sign, hence $|x/X| \leqslant 1$ and $|y/Y| \leqslant 1$.

The BI follows from Eq. (3) after averaging over both time t [see Eq. (2)] and λ,

$$|G(\mathbf{a},\mathbf{b}) - G(\mathbf{a},\mathbf{b}') + G(\mathbf{a}',\mathbf{b}) + G(\mathbf{a}',\mathbf{b}')| \leqslant 2, \tag{6a}$$

$$G(\mathbf{a},\mathbf{b}) = \frac{\langle [N_5(\tau) - N_3(\tau)][N_6(\tau) - N_4(\tau)] \rangle}{\langle [N_5(\tau) + N_3(\tau)][N_6(\tau) + N_4(\tau)] \rangle} \tag{6b}$$

and with $\mathbf{a}, \mathbf{b}$ the polarizations of the filters $\mathrm{F}^{\mathrm{d}}_{1(2)}$.

At this point, the number averages and correlators in (6a) need to be related to measurable quantities, current averages and current noise; this step requires to perform the time averaging introduced in Eq. (2) and implemented in Eq. (6a). The correlator $\langle N_\alpha(\tau) N_\beta(\tau) \rangle$ includes both reducible and irreducible parts. As demonstrated below, the BI (6a) can be violated if the irreducible part of the correlator is of the order of (or larger) than the reducible part. The irreducible correlator $\langle \delta N_\alpha(\tau) \delta N_\beta(\tau) \rangle$ can be expressed through the noise power $S_{\alpha\beta}(\omega) = \int d\tau e^{i\omega\tau} \langle \delta I_\alpha(\tau) \delta I_\beta(0) \rangle$,

$$\langle \delta N_\alpha(\tau) \delta N_\beta(\tau) \rangle = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} S_{\alpha\beta}(\omega) \frac{4 \sin^2(\omega\tau/2)}{\omega^2} . \tag{7}$$

In the limit of large times, $\sin^2(\omega\tau/2)/(\omega/2)^2 \to 2\pi\tau\delta(\omega)$, and therefore

$$\langle N_\alpha(\tau) N_\beta(\tau) \rangle \approx \langle I_\alpha \rangle \langle I_\beta \rangle \tau^2 + \tau S_{\alpha\beta} , \tag{8}$$

where $\langle I_\alpha \rangle$ is the average current in the lead α and $S_{\alpha\beta}$ denotes the shot noise. In reality, the noise power diverges as $1/\omega$ when $\omega \to 0$, but this singular behavior starts from very small $\omega < \omega_{\mathrm{fl}}$;[27] at frequencies $\omega_{\mathrm{fl}} \ll \omega \ll \omega_0$ the noise power is nearly constant.[24] The upper boundary ω_0 of the frequency domain depends on the voltage V of the terminals $3-6$ (the particle source is grounded), on the characteristic time of electron flight τ_{tr} between these terminals, and parameters of the normal leads 1,2. If, for example, a superconductor plays the role of a particle source, the normal leads must include the filters $\mathrm{F}^{\mathrm{e}}_{1,2}$ preventing Cooper pairs to decay into one lead.[18] Consider the case when Fabry-Perot double barrier structures or quantum dots with resonant energies $\pm\varepsilon_0$ and widths $\Gamma_{1(2)} \ll \varepsilon_0$ play the role of the filters $\mathrm{F}^{\mathrm{e}}_{1,2}$. If the leads are equally biased with the respect to the source then $\omega_0 = \min(|V|; \Gamma_{1(2)}; \tau_{\mathrm{tr}}^{-1})$ [carbon nanotubes are promising candidates[28] for the role of the normal wires 1,2 in Fig. 1 as strong electron-electron interactions assume the task of the filters $\mathrm{F}^{\mathrm{e}}_{1,2}$;[29] in this case $\omega_0 = \min(|V|; \tau_{\mathrm{tr}}^{-1})$]. Summarizing, Eq. (7) implies Eq. (8) if

$$\omega_0^{-1} \ll \tau \ll \omega_{\mathrm{fl}}^{-1} \tag{9}$$

(we assume a temperature $T < \omega_0$). Using Eqs. (6a) and (8),

$$|F(\mathbf{a},\mathbf{b}) - F(\mathbf{a},\mathbf{b}') + F(\mathbf{a}',\mathbf{b}) + F(\mathbf{a}',\mathbf{b}')| \leqslant 2, \tag{10a}$$

$$F(\mathbf{a},\mathbf{b}) = \frac{S_{56} - S_{54} - S_{36} + S_{34} + \Lambda_-}{S_{56} + S_{54} + S_{36} + S_{34} + \Lambda_+}, \tag{10b}$$

where $\Lambda_\pm = \tau(\langle I_5 \rangle \pm \langle I_3 \rangle)(\langle I_6 \rangle \pm \langle I_4 \rangle)$. The BI (10a) is the expression to be tested in the experiment; as implied by Eq. (10b), its violation requires $|S_{\alpha\beta}| \gtrsim |\Lambda_\pm|$, i.e., the dominance of the irreducible particle-particle correlator over the reducible.

Below, we discuss the violation of BIs in mesoscopic systems. A violation of Eq. (10a) then implies that the assumption (1) does not hold and the correlations are nonclassical; the particles injected by the source into leads 1 and 2 (see Fig. 1) then are entangled (for a system in a pure state, entanglement implies that the wave function cannot be reduced to a product of wave functions of individual particles).

Consider the solid-state analog of the Bell device in Fig. 1(b) where the particle source is a superconductor; two normal leads 1 and 2 are attached in a fork geometry[18,19] and the filters $\mathrm{F}^{\mathrm{e}}_{1,2}$ enforce the splitting of the injected pairs. The filters $\mathrm{F}^{\mathrm{d}}_{1,2}$ play the role of spin-selective beam-splitters in the detector (they can be constructed using ferromagnets,[18] quantum dots,[30] or hybrid superconductor–normal-metal–ferromagnet structures[31]): e.g., quasiparticles injected into lead 1 and spin-polarized along the magnetization **a** enter the ferromagnet 5 and contribute to the current I_5, while quasiparticles with the opposite polarization contribute to the current I_3. The appropriate choice of voltages between the leads and the source fixes the directions of the currents in agreement with Fig. 1(a). Consider, for instance, a biased superconductor with grounded normal leads. To begin with, we assume the filters $\mathrm{F}^{\mathrm{d(e)}}_{1,2}$ to be perfectly efficient. The noise power is calculated using scattering theory.[32,24] In the tunneling limit with a weak coupling between the superconductor and the leads 3–6 (e.g., Cooper pairs decaying from the superconductor into the normal leads through a tunnel barrier) we find the noise

$$S_{\alpha\beta}=S^{(a)}_{\alpha\beta}\sin^2\left(\frac{\theta_{\alpha\beta}}{2}\right), \tag{11}$$

where $\alpha=3,5$, $\beta=4,6$ or vice versa; $\theta_{\alpha\beta}$ denotes the angle between the magnetization of the leads α and β, e.g., $\cos(\theta_{56})=\mathbf{a}\cdot\mathbf{b}$ and $\cos(\theta_{54})=\mathbf{a}\cdot(-\mathbf{b})$, and $S^{(a)}_{\alpha\beta}$ is the noise for *antiparallel* orientations of the ferromagnets α,β [e.g., $S^{(a)}_{56}$ implies $\mathbf{a}\uparrow\downarrow\mathbf{b}$]. Below, we need configurations with different settings **a** and **b** and we define the angle $\theta_{\mathbf{ab}}\equiv\theta_{56}$. In the tunneling limit and for antiparallel polarization of the filters $\mathrm{F}^{\mathrm{d}}_{1,2}$ in the leads α,β the scattering approach[18] produces the expression

$$S^{(a)}_{\alpha\beta}=|\langle I_\alpha\rangle|=|\langle I_\beta\rangle|=\frac{1}{h}\int_0^{|V|}d\varepsilon\,\mathrm{Tr}T^{eh}_{\alpha\beta}(\varepsilon), \tag{12}$$

where $T^{eh}_{\alpha\beta}$ is the matrix (in channel space) containing the Andreev transmission probabilities from the lead α into the lead β (the tunneling limit considered here implies $T^{eh}_{\beta\alpha}\ll 1$); V is the bias of the superconductor. Thus the Λ terms in Eq. (10b) can be dropped if

$$\langle I_\alpha\rangle\tau\ll 1, \tag{13}$$

$\alpha=3,\ldots,6$. Then, the BIs (10a)–(10b) neither depend on τ nor on the average current but only on the shot noise, and $F=-\cos(\theta_{\mathbf{ab}})$; the left-hand side of Eq. (10a) has a maximum when $\theta_{\mathbf{ab}}=\theta_{\mathbf{a'b}}=\theta_{\mathbf{a'b'}}=\pi/4$ and $\theta_{\mathbf{ab'}}=3\theta_{\mathbf{ab}}$ (shown as in the photonic case[9] with the substitution $\theta\to\theta/2$). With this choice of angles the BI (10a) is *violated*: it reduces to $1\leq 1/\sqrt{2}$, thus pointing to the nonlocal correlations between electrons in the leads 1,2 [see Fig. 1(b)].

The current in Eq. (13) is of the order of $\Gamma\mathcal{T}_A/h$, where $\Gamma\equiv\min(\Gamma_1,\Gamma_2)$ and $\mathcal{T}_A\equiv\mathrm{Tr}T^{eh}_{\alpha\beta}(\varepsilon_0)/h$. Then the condition (13) becomes $\tau\ll\hbar/\Gamma\mathcal{T}_A$. Equation (10a) becomes the nonlocality criterium if there is no electron exchange between the leads 1 and 2 during the time τ, requiring $\tau\ll\tau_{\mathrm{tr}}/\mathcal{T}_A$.[33] The two conditions can be written as

$$\tau\ll\omega_0^{-1}/\mathcal{T}_A. \tag{14}$$

The condition (14) implies that during the time τ not more than one electron pair can be detected; the noise measurement in this limit then is closely related to the coincidence measurement in quantum optics.[8]

Let us estimate τ. A typical device as sketched in Fig. 1(b) has parameters $\tau_{\mathrm{tr}}^{-1}\lesssim v_F/100\ \mathrm{nm}\sim 1$ GHz, $\Delta\sim 1$ meV, $\Gamma\sim 0.1\Delta\simeq 10$ GHz, $\varepsilon_0<|V|<\Delta$, $\mathcal{T}_A\sim 0.01$; then $\omega_0\sim 100$ GHz. The natural scale for ω_{fl} is 1 kHz.[27] From (9) and (14) follows that $\tau^{-1}\sim 10$ MHz and according to Eq. (13) the current in the leads should not exceed 1 nA.

The BI test described above does not imply that the particles in the leads 1,2 are separated by a spacelike interval; in our approach we try to optimize the parameters of the device to reduce interactions between the particle beams which can destroy the entanglement, e.g., backscattering from one lead to another, but ignore more subtle relativistic effects.

Finally, we probe the robustness of our BI test by allowing the filters $F^e_{1,2}$ to have finite line widths $\Gamma_{1,2}$. If, for instance $\Gamma_{1,2}\sim 2E_0$, the noise correlations will acquire a (small) $S^{(p)}$ contribution (here, the superscript p denotes parallel magnetizations)[34]

$$S_{\alpha\beta}=S^{(a)}_{\alpha\beta}\sin^2\left(\frac{\theta_{\alpha\beta}}{2}\right)+S^{(p)}_{\alpha\beta}\cos^2\left(\frac{\theta_{\alpha\beta}}{2}\right), \tag{15}$$

and with the same choice of angles as considered above the BI (10a) reduces to

$$\left|\frac{S^{(a)}_{\alpha\beta}-S^{(p)}_{\alpha\beta}}{S^{(a)}_{\alpha\beta}+S^{(p)}_{\alpha\beta}}\right|\leq\frac{1}{\sqrt{2}}; \tag{16}$$

the BIs (16) still can be violated, though not maximally. Alternatively, Eq. (16) can be used to estimate the quality of the filters $\mathrm{F}^{\mathrm{e}}_{1,2}$. Here, we have discussed the violation of BIs in an idealized situation ignoring paramagnetic impurities, spin-orbit interaction, etc. Imperfect $\mathrm{F}^{\mathrm{d}}_{1,2}$ filters should be considered in a similar way as in the quantum-optics literature;[9] it can be shown in a similar way as in Ref. 21 that 90% spin polarization of the filters is enough for the BI-violation test under consideration; appropriate spin filters can be realized with current technology in spin electronics.[35] Note that there are other inequalities which test entanglement for two-[36] and many-[15] particle systems; tests of such inequalities can be implemented in a similar manner as discussed above. Moreover, while electron-electron interactions were neglected here, it has been suggested[37,38] that they do not destroy entanglement.

The BI violation discussed above applied to a superconducting source of entangled particles. In other cases, e.g., when quantum dots play the role of entangler,[13,20] a violation of BIs can be demonstrated in a similar way.

Finally, consider the same device but with a normal metal source rather than a superconducting one and without $\mathrm{F}^{e}_{1,2}$ filters. Using the scattering approach[32,24] in the tunneling limit between the leads and the source we find that $\Lambda_{\pm} \sim \tau g_1 g_2(\omega_0)^2$ and $S_{\alpha\beta} \sim g_1 g_2 \omega_0$, where $g_{1(2)} \ll 1$ is the conductance between the lead $1(2)$ and the particle source and $\omega_0 = V/\hbar$. The $\Lambda_{\pm}$ terms in Eq. (10b) then cannot be dropped as otherwise we arrive at a contradiction with the condition (9). In this case $F \simeq \Lambda_-/\Lambda_+$ and the BI (10a) assumes the form

$$|\Lambda_-/\Lambda_+| \leqslant 1, \tag{17}$$

which *cannot* be violated, a strong indication that a normal source itself cannot produce entangled particles. However, if the electrons strongly interact with each other on their way from the normal source into the leads then, according to Ref. 20, a finite entanglement can be produced by the interactions; in this case the violation of BIs can be demonstrated as well.

In conclusion, we propose a general form of BI tests in solid-state systems formulated in terms of current-current cross correlators (noise), the natural observables in the stationary transport regime of a solid state device. For a superconducting source injecting correlated pairs into a normal-metal fork completed with appropriate filters,[18,19] the analysis of such BIs shows that this device is a source of entangled electrons when the fork is weakly coupled to a superconductor. BI-checks can thus be applied to test electronic devices with applications in quantum communication and quantum computation where entangled states are basic to their functionality.

We thank Yu.V. Nazarov and F. Marquardt for stimulating discussions. The research of N.M.C. and of G.B.L. was supported by the RFBR (Projects No. 00-02-16617, 02-02-16622, and 02-02-06509), by Forschungszentrum Jülich (Landau Scholarship), by the Netherlands Organization for Scientific Research (NWO), by the Swiss NSF, and by the Russian Ministry of Science (projects *Mesoscopic Systems*, and *Quantum Computations*).

[1] E. Schrödinger, Naturwissenschaften **23**, 807 (1935); **23**, 823 (1935); **23**, 844 (1935).
[2] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. Lett. **47**, 777 (1935).
[3] M.B. Menskii, Phys. Usp. **44**, 438 (2001).
[4] A. Zeilinger, Phys. World **11**, 35 (1998).
[5] A. Steane, Rep. Prog. Phys. **61**, 117 (1998).
[6] A.K. Ekert, Phys. Rev. Lett. **67**, 661 (1991).
[7] C.H. Bennett *et al.*, Phys. Rev. Lett. **70**, 1895 (1993).
[8] A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982); Z.Y. Ou and L. Mandel, *ibid.* **61**, 50 (1988); Y.H. Shih and C.O. Alley, *ibid.* **61**, 2921 (1988); G. Weihs *et al.*, *ibid.* **81**, 5039 (1998); A. Aspect, Nature (London) **398**, 189 (1999).
[9] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge, 1995).
[10] J. Cirac, Nature (London) **413**, 375 (2001).
[11] M. Rowe *et al.*, Nature (London) **409**, 791 (2001).
[12] R.A. Bertlmann *et al.*, Phys. Rev. A **63**, 062112 (2001).
[13] D. P. DiVincenzo *et al.*, in *Mesoscopic Phenomena and Mesoscopic Devices in Microelectronics*, edited by I. O. Kulik and R. Ellialtioglu (Kluwer, Dordrecht, 2000), Vol. 559.
[14] J.S. Bell, Physics (Long Island City, NY) **1**, 195 (1965); J.S. Bell, Rev. Mod. Phys. **38**, 447 (1966); J.F. Clauser *et al.*, Phys. Rev. Lett. **23**, 880 (1969).
[15] R. Werner and M. Wolf, quant-ph/0107093 (unpublished).
[16] N. Mermin, Rev. Mod. Phys. **65**, 803 (1993); A. Grib, Phys. Usp. **27**, 284 (1984).
[17] J.M. Kikkawa and D.D. Awschalom, Phys. Rev. Lett. **80**, 4313 (1998); Nature (London) **397**, 139 (1999).
[18] G.B. Lesovik, T. Martin, and G. Blatter, Eur. Phys. J. B **24**, 287 (2001).
[19] P. Recher *et al.*, Phys. Rev. B **63**, 165314 (2001).
[20] W.D. Oliver *et al.*, Phys. Rev. Lett. **88**, 037901 (2002).
[21] S. Kawabata, J. Phys. Soc. Jpn. **70**, 1210 (2001).
[22] X. Maître *et al.*, Physica E **6**, 301 (2000).
[23] R. Ionicioiu *et al.*, Phys. Rev. A **63**, 050101(R) (2001).
[24] Ya. Blanter, M. Büttiker, Phys. Rep. **336**, 1 (2000).
[25] S. Popescu, Phys. Rev. Lett. **74**, 2619 (1995); N. Gizin, Phys. Lett. A **210**, 151 (1996).
[26] If $|X| = |Y| = 1$ then $xy - xy' = xy(1 \pm x'y') - xy'(1 \pm x'y)$. So $|xy - xy'| \leqslant |xy(1 \pm x'y')| + |xy'(1 \pm x'y)| \leqslant (1 \pm x'y') + (1 \pm x'y) = 2 \pm (x'y' + x'y)$. Thus $-(2 + (x'y' + x'y)) \leqslant xy - xy' \leqslant 2 - (x'y' + x'y)$; the last inequality is Eq. (3).
[27] Y. Imry, *Introduction to Mesoscopic Physics* (Oxford University Press, Oxford, 1997).
[28] V. Bouchiat *et al.*, cond-mat/0206005 (unpublished).
[29] P. Recher and D. Loss, Phys. Rev. B **65**, 165327 (2002); C. Bena *et al.*, Phys. Rev. Lett. **89**, 037901 (2002).
[30] P. Recher *et al.*, Phys. Rev. Lett. **85**, 1962 (2000).
[31] D. Huertas-Hernando, Yu.V. Nazarov, and W. Belzig, Phys. Rev. Lett. **88**, 047003 (2002).
[32] G.B. Lesovik, JETP Lett. **49**, 592 (1989).
[33] This condition excludes processes where, e.g., an electron quasiparticle in lead 1 is not absorbed by the terminals 3,5, but is reflected back to the superconductor and finally transformed into a hole propagating through lead 2.
[34] Spin-orbit interactions and spin-flip processes (e.g., due to paramagnetic impurities) in the leads are neglected and we assume that rotation of the magnetizations **a**,**b** does not change the conductances of the contacts between the lead 1 and the terminals 3,5 (lead 2 and terminals 4,6).
[35] J.S. Moodera *et al.*, Phys. Rev. Lett. **70**, 853 (1993).
[36] J.F. Clauser and M.A. Horne, Phys. Rev. D **10**, 526 (1974).
[37] G. Burkard *et al.*, Phys. Rev. B **61**, R16 303 (2000).
[38] M.P. Anantram *et al.*, Phys. Rev. B **53**, 16 390 (1996).