# Spin properties of quantum wells with magnetic barriers. II. Inverted band ordering and spin polarized interface states

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The electronic band-edge spectrum of the interface states in the magnetic semiconductor quantum wells based on narrow-gap semiconductors with mutually inverted band arrangement is studied within the envelope-function formalism. Interface states are shown to appear in these structures in the case of overlapping bulk bands of the constituents. The hybridization between the bare sp-electron states and the d states of the Mn atoms leads to spin splitting. The spin-splitting effect of the interface states as a function of external magnetic field, well width, band offsets, and fraction of the magnetic atoms, is studied. One essential result is that one can design a structure where the states localized at the interfaces only have one spin direction. The results give evidence of the perspective for using the magnetic semiconductor structures in spin electronics.

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## I. INTRODUCTION

In this paper, in the framework of the approach developed in the preceding paper (Ref. 1), we continue to study magnetic semiconductor quantum wells but based on II-VI or III-V narrow-gap semiconductors with mutually inverted band arrangement, in the literature called inverted heterostructures. Among a number of the magnetic semiconductor structures, inverted magnetic heterostructures attract the interest because of specific features of the energy spectrum resulting in the appearance of the interface Tamm states.<sup>2</sup> These states can appear at perfect interfaces and are not related to states due to impurities or other defects appearing because of an attractive electrostatic potential. This property of the inverted magnetic structures may present a promising avenue to add new spin-dependent functionality to commonly used normal magnetic semiconductor heterostructures.

A specific feature of the II-VI or III-V semiconductor with inverted energy structure is the absence of an energy gap between conduction and valence bands which are related to the two branches of the fourfold  $\Gamma_8$  band in the vicinity of the extremum point  $\Gamma$ . Figure 1 shows the relative band arrangement of the II-VI or III-V narrow-gap semiconductors with normal and inverted band spectrum. The  $\Gamma_8$  band is classified by the projection of the total angular momentum J=3/2 onto  $\left|\frac{3}{2},\pm\frac{1}{2}\right\rangle$  and  $\left|\frac{3}{2},\pm\frac{3}{2}\right\rangle$  states. The states with the projection of  $\pm 1/2$  characterize the conduction band-edge, while the states with  $m_J = \pm 3/2$  generate the heavy hole band-edge. The  $\Gamma_6$  band, which in the II-VI or III-V semiconductors with the normal band arrangement corresponds to the conduction band edge, now constitutes the light hole band edge. Then it is usual to consider the band gap of such semiconductors determined by  $E_g = E(\Gamma_6) - E(\Gamma_8)$  as negative.

It is the band arrangement which leads to the most inter-

esting features of the energy spectrum of spatially confinement systems based on semiconductors with mutually inverted band spectrum. It has been shown theoretically<sup>3</sup> that an inverted II-VI semiconductor (HgTe) has specific surface states at the interface between a semiconductor and vacuum. These states, being Tamm type, are a superposition of the electronic states in the valence and conduction bands. In the inverted semiconductor heterojunction where the constituents have opposite band-edge symmetry and overlapping gaps, there has been predicted to be a band of midgap states bound to the interface. The crucial point for these states is that they appear even without an attractive electrostatic potential, being again generated by the conduction and valence



FIG. 1. Principal band diagram of the quantum well  $HgTe/Hg_{1-x}Mn_xTe/HgTe$  considered. The band gaps of the constituent semiconductors are shown by different shading, the region of band gap overlap is marked by cross shading. The symmetries of the different bands are displayed.

band states of the constituent semiconductors. They are referred to as Tamm-like interface states. These states with energies in the range of band gap overlap of the constituents have linear (or near linear) dispersion as a function of the momentum in the interface plane. Volkov and Pankratov<sup>4</sup> seem to be the first to have noticed the midgap states, in a treatment of the simplest two-band approximation (Dirac model) for near-gap states in heterostructures made from IV-VI semiconductors. The interface Tamm states have been predicted by several authors<sup>5,6</sup> to exist in HgTe-CdTe inverted heterostructures. The theoretical analysis in this case is more complex than in IV-VI materials. Because of mixing with heavy-hole levels the theoretical treatment should be based on eight-band Kane or Luttinger models. As was shown in a number of experimental and theoretical articles<sup>7–11</sup> these interface states play a key role in forming the energy spectrum of the two-dimensional states in inverted semiconductor structures.

A theoretical development of a model of interface states for the magnetic semiconductor structures in which one of the constituents is a so-called dilute magnetic (semimagnetic) semiconductor has been presented in Refs. 12 and 13. As model materials, the heterojunctions based on semimagnetic narrow-gap IV-VI semiconductors with mutually inverted bands were considered in the framework of the Dirac model. The interface Tamm states were shown to appear in these structures, being spin split. If the Fermi level lies in one of the interface bands, magnetic ordering appears in the interface plane. The interface magnetization effect has been discussed in Refs. 12 and 13.

In this work we extend the theoretical model of the interface magnetic states to the magnetic quantum structures based on narrow-gap II-VI semiconductors. For simplicity we henceforth denote quantum wells with semimagnetic barriers and spin polarization in the wells as magnetic quantum wells. Quantum wells of the type CdMnTe/HgCdTe/CdMnTe are considered as model materials in which the fractions of the Cd and Mn atoms are chosen such that the signs of the constituent gaps are opposite. Since we focus here on the interface states, we consider only quantum wells with overlapping gaps of the constituent compounds. The interface states are shifted in energy with respect to the usual quantum well states, since they are localized in the range of the overlapping band gaps of the constituents. However, in practice, it is possible to select the constituents in such a way that the usual quantum wells states cannot appear at all. We use here the same approximations as in the previous paper. That is, we study a symmetrical square quantum well in the approximation of flat bands, neglecting the strain effects. Thus any asymmetry effects leading to the bulk<sup>16</sup> or Rashba spin splitting terms<sup>17</sup> do not affect the energy states. The only spin-splitting potential, which will be included in the Hamiltonian, is the spin-dependent hybridization between the *d* and *sp* electrons. When considering the polarized case we neglect the Landau level quantization and the ordinary Zeeman splitting, as well.

A model Hamiltonian for the magnetic semiconductor structures has been constructed starting from a model developed earlier<sup>18–20</sup> and extended to CdMnTe/CdMn quantum wells with the normal band arrangement in the preceding paper (Ref. 1). This approach was found to give a reliable description of the considered magnetic semiconductor structures.<sup>1</sup>

The paper is organized as follows. In Sec. II we describe our theoretical model. The band-edge interface spectrum for the inverted magnetic quantum wells is obtained. The spinsplitting effect is discussed. Spin properties of the electron states are studied. In Sec. III the preceding results are applied to the quantum well  $Hg_{1-x}Mn_xTe/HgTe/Hg_{1-x}Mn_xTe$ . Section IV contains a discussion of the results and Sec. V concludes the paper with a summary of the main results.

## II. SPIN-SPLITTING EFFECT ON THE INTERFACE STATES

The model to be used has been outlined in part I (Ref. 1). We refer to it for the details and give only a sketch of the theoretical treatment here with emphasis on the modifications that occur for inverted band arrangments.

A solution of the problem is found in the framework of perturbation theory. An effective  $8 \times 8 \mathcal{H}_{sp}$  Hamiltonian is obtained by folding the full (20×20) matrix down into the usual Kane *sp* block. It has just the same form as Eq. (15) in part I, but with the only difference that now the bare energies of the  $\Gamma_{6,7,8}$  at k=0 are related to the symmetry of the corresponding bands but not with their conduction or valence character. It can be written as

$$\mathcal{H}_{sp} = \begin{pmatrix} \hat{\epsilon}(\Gamma_{6}) & \frac{iP\hbar k_{z}}{m}I & 0 & \frac{iP'\hbar k_{z}}{\sqrt{2}m}I \\ -\frac{iP\hbar k_{z}}{m}I & \hat{\epsilon}_{1}(\Gamma_{8}) & 0 & \hat{\mathcal{V}}_{pd} \\ 0 & 0 & \hat{\epsilon}_{2}(\Gamma_{8}) + \frac{\hbar^{2}k_{z}^{2}}{2m_{hh}}I & k_{z}\hat{\mathcal{P}}_{pd} \\ -\frac{iP'\hbar k_{z}}{\sqrt{2}m}I & \hat{\mathcal{V}}_{pd} & k_{z}\hat{\mathcal{P}}_{pd} & \hat{\epsilon}(\Gamma_{7}) \end{pmatrix}.$$
(1)

The matrix elements of the Hamiltonian are determined by the expressions similar to Eqs. (16)–(20) of paper I with replacement  $\varepsilon_c^0 \rightarrow E(\Gamma_8)$ ,  $\varepsilon_v^0 \rightarrow E(\Gamma_8)$ , and  $\varepsilon_{so}^0 \rightarrow E(\Gamma_7)$ .

Now we treat the problem of finding the interface states of the magnetic semiconductor quantum well in the envelope function approximation by matching the wave functions. It is important to match envelope functions on the both sides of the interface with *identical symmetries* rather than matching states with conduction or valence character. It is not necessary that bands of the same symmetry have similar energies in the two materials for the multiband envelope function approximation to be valid.<sup>14</sup> We start from the bulk Hamiltonian  $\mathcal{H}_{sp}$  with replacement  $k_z \rightarrow -i\partial/\partial z$ . Similar to the previous description in paper I, here we neglect the *k*-dependent polarization-induced block  $\mathcal{P}_{pd}$  reducing the model Hamiltonian to an effective Kane model with renormalized matrix elements and the additional block  $\mathcal{V}_{pd}$ .

The origin of energy is defined as the  $\Gamma_6$  edge of the well material. Since the square quantum structure is considered with the growth direction along the *z* axis, all the matrix elements involved are step functions of *z*. We suppose that z=0 defines the center of the well layer.

The envelope function equation

$$\mathcal{H}_{sp}\mathbf{f} = \mathbf{E} \cdot \mathbf{f} \tag{2}$$

is reduced to two sets of three coupled differential equations for the envelopes  $f_i^{lp}$  of the light particles (electrons, light holes and split-off holes) and to two second order differential equations for the heavy hole envelopes  $f_i^{hh}$ . The envelope function boundary conditions require continuity of the envelopes  $f_i^{lp,hh}$  and  $(1/m_{hh})\partial f_i^{hh}/\partial z$ .

The interface quantum well states have either even or odd parity under  $\mathbf{r} \rightarrow -\mathbf{r}$  within the present symmetrical model. Therefore, all the states can be labeled by the parity. The dispersion relations for the even modes of the interface states associate with the light particles and heavy holes are obtained in the form<sup>21</sup>

$$\sinh k_{lp}L/2 = -\frac{k_{lp}}{\kappa_{lp}^{\pm}} \frac{E - E(\Gamma_6)^b \pm x \langle S_z \rangle (N_0 \alpha)}{E} \cosh k_{lp}L/2,$$
(3)

$$\sinh k_{hh}L/2 = -\frac{k_{hh}}{\kappa_{hh}^{\pm}} \frac{m_{hh}^{b}}{m_{hh}^{a}} \cosh k_{hh}L/2.$$

$$\tag{4}$$

For the odd modes one gets

$$\cosh k_{lp}L/2 = -\frac{k_{lp}}{\kappa_{lp}^{\pm}} \frac{E - E(\Gamma_6)^b \pm x \langle S_z \rangle (N_0 \alpha)}{E} \sinh k_{lp}L/2,$$
(5)

$$\cosh k_{hh}L/2 = -\frac{k_{hh}}{\kappa_{hh}^{\pm}} \frac{m_{hh}^b}{m_{hh}^a} \sinh k_{hh}L/2.$$
(6)

Here  $ik_{lp}$  and  $ik_{hh}$  are the bulk wave vectors in the well for the light particles and heavy holes, respectively;  $i\kappa_{lp}^{\pm}$  and  $i\kappa_{hh}^{\pm}$  are the bulk wave vectors in the two spin-split bands of the semimagnetic barriers;  $E(\Gamma_6)^b$  is the energy of the  $\Gamma_6$  band-edge in the magnetic barriers; and *L* is the width of the well. Note that compared to paper I we here have hyperbolic functions instead of trigonometric functions. The bulk wave vectors of the constituents are determined by the requirement  $\det(\mathcal{H}_{sp}-E)=0$  where the coordinate dependent terms should be replaced by the corresponding constant values. So, the governing equations for the bulk wave vectors of the light particles and heavy holes in the well and in the barriers reduce to Eqs. (26)–(29) of paper I with replacement  $k_{lp} \rightarrow -ik_{lp}$  and  $k_{hh} \rightarrow -ik_{hh}$ .

We note the following features of the interface spectrum in comparison with the quantum well states studied in our previous work.<sup>1</sup> First, the interface states are generated from the evanescent modes and can only appear inside the overlapping of the constituent gaps. Therefore the interface states cannot be related to the heavy hole bands. As a result the Eqs. (4) and (6) have no solutions. Second, since the interface states are determined as solutions of the dispersion relations Eqs. (3) and (5) including the nonperiodical functions cosh and sinh, it is possible to find one solution only for the even or odd interface modes. But for the quantum well states<sup>1</sup> in addition to the ground even or odd modes, there appear some excited states, because the dispersion relations include the periodical functions sine and cosine. And third, just as in the case of the quantum well states, the exchange potential breaks time-reversal degeneracy and splits interface modes. The explicit form of the wave functions written in the first order of perturbation theory shows that the average spin vectors  $\langle \Psi^{\pm} | \Sigma | \Psi^{\pm} \rangle$  ( $\Sigma$  is the spin operator) for these split states have opposite direction.

## **III. APPLICATION TO THE QUANTUM WELL**

The preceding results will be applied to the quantum well structure  $Hg_{1-x}Mn_xTe/HgTe/Hg_{1-x}Mn_xTe$ , which for the Mn fraction x > 0.1 gives an example of the inverted band arrangement of the constituent bands. As the input Kane sp parameters of the bulk constituent semiconductors, the values given in Refs. 22 and 23 were used. The concentration dependence of the fundamental gap  $[E_g = E(\Gamma_6) - E(\Gamma_8)]$  in  $Hg_{1-x}Mn_x$ Te was calculated in the virtual crystal approximation by the formula  $E_g(Hg_{1-x}Mn_xTe) = E_g(HgTe) + x3.6$ (eV) presented in Ref. 22. For the parameters related to the sp-d hybridization, we use the values given in Ref. 19. All parameters involved in the numerical calculations are presented in Table I. The well width varies in the limits 20-100 Å. The value of the  $\Gamma_8$  band offset  $\Lambda$  depends on the Mn fraction x and is not accurately known. We have here considered  $\Lambda$  values in the interval 25–250 meV. In the present model calculations it is convenient to vary x and  $\Lambda$  independently to display fundamentally different band arrangements. In practice this can be achieved by extending the treatment to include quaternary alloys like HgCdMnTe. Then x can be used to regulate the strength of the magnetic interactions and afterwards the Cd content can be varied to adjust  $\Lambda$  towards the desired value without changing the magnetic properties. In this paper we for simplicity keep the input parameters for  $Hg_{1-r}Mn_r$ Te and treat  $\Lambda$  as an adjustable parameter. The

TABLE I. Bulk  $\mathbf{k} \cdot \mathbf{p}$  and magnetic exchange parameters.

Parameter	HgTe	$Hg_{1-x}Mn_xTe$
$E(\Gamma_6) - E(\Gamma_8)$ (eV)	-0.3	$-0.3+3.6 \cdot x$
$E(\Gamma_8) - E(\Gamma_7)$ (eV)	1.0	1.0 - 0.1x
$E(\Gamma_8) - \varepsilon_d^0 \text{ (eV)}$		3.4
$m_{hh}/m_0$	0.4	0.6
$2m_0P^2$ (eV)	20	20
U (eV)		7.0
$V_{pd}$ (eV)		0.49
$(N_0 \alpha)$		0.22

absolute temperature is taken to be zero.

In Fig. 2 the spin splitting effect on the interface states of the magnetic quantum well  $Hg_{1-x}Mn_x iTe/HgTe$  with x = 0.2 is shown for three well widths (a) L = 100 Å, (b) 60 Å, and (c) 20 Å. Here the value  $\Lambda = 0.15$  eV was used. The even (odd) modes are labeled by  $E_i^+$  ( $E_i^-$ ). At first, we note that all the levels, being doubly degenerate in the unpolarized case at y=0.5, split in energy when increasing the polarization (or decreasing y). The levels with spin-up and spin-down move in opposite directions. The  $E_i^-$  odd modes are the lowest in energy for a small magnitude of the polarization. However, by increasing the polarization the odd mode with spin down and the even mode with spin up cross each other and change positions. The relative value of the splitting increases with decreasing well width. For example, the magnitude of the shift of the  $E_i^-$  odd modes at y=0.3 is  $\sim$  30 meV in the case of L=100 Å while for L=20 Å it reaches 50 meV.

The most interesting feature of this quantum well is that the even mode with spin up appears in the interface spectrum only. An explanation of this fact follows directly from the band diagram of the  $\Gamma_6$  and  $\Gamma_8$  bands of this quantum structure presented in Fig. 2(d). In the unpolarized case (y =0.5), the diagrams of the  $\Gamma_6$  and  $\Gamma_8$  bands are shown by the solid and dashed lines, respectively. The band gaps of the constituents are shown by different shading, the regions of the band gap overlap being marked by cross shading. In the polarized case (y=0.2), the bulk bands of the magnetic semiconductor split into bands with spin up (dotted lines) and bands with spin down (dash-dotted lines). As a result the overlapping between the gaps of the constituents for the states with spin up increases while the one for the states with spin down decreases. This leads to pushing out the even mode with spin down. The odd modes are found to be allowed for the both spin states.

Another characteristic picture of the spin splitting effect for the interface states is presented in Fig. 3. These are the interface states of the magnetic quantum well  $Hg_{1-x}Mn_xTe/HgTe$  with x=0.1 and  $\Lambda=0.15$  eV shown for (a) L=100 Å, (b) 60 Å and (c) 20 Å. For this magnitude of x the band gap of the magnetic semiconductor is much smaller than that of the nonmagnetic semiconductor which is equal to 0.06 eV. As seen from the band diagram drawn for this quantum well in Fig. 3(d), for any value of the polarization the overlapping between the gaps of the constituents (marked by cross shading) is determined by the value of the gap of the magnetic semiconductor. Both the spin states of the even and odd modes are presented in the interface spectrum of the quantum wells with L = 100 Å and L = 60 Å. But for the thinner quantum well only the interface states with spin down are permitted. This is a result of the larger value of the band gap of the magnetic semiconductor for the states with spin down in comparison with the band gap for the states with spin up. This effect is clearly manifested better for the thin quantum wells. At last we note in Fig. 3(c) that the lowest interface mode  $(E_i^- \text{ odd state})$  appears in the spectrum only in some interval of the nonzero values of the polarization in the case of the thin quantum well. This effect is explained by the band diagram [Fig. 3(d)]. For L=20 Å this odd mode is just pushed out of the allowed energy interval where the interface state can exist. It goes very close to the bulk band of the magnetic semiconductor by turning into a bulk state.

From the above analysis we can predict the situation when the one mode with a certain spin direction can appear in the interface spectrum. This will be the case in which the overlapping between the gaps of the constituents disappears for one mode while the gaps for the mode with the opposite spin direction still overlap. This is realized in the quantum well Hg<sub>1-x</sub>Mn<sub>x</sub>Te/HgTe at x=0.2 with the  $\Gamma_8$  band offset  $\Lambda = 0.05$  eV. The result is shown in Fig. 4 for the two well widths: L = 100 Å and 60 Å. In this quantum well only the odd mode appears in the interface spectrum. It is doubly degenerate in the unpolarized case. For nonzero values of the polarization one state with spin up is exhibited in the interface spectrum only, because the overlapping between the gaps for the states with spin down of the constituents is lifted as shown in the band diagram in Fig. 4(b). For the quantum well with  $L \le 60$  Å this last mode is pushed out of the allowed interval of the energy and disappears.

We present in Fig. 5 the characteristic form of the envelope wave functions calculated for the odd mode of the interface states of the quantum well with x = 0.2,  $\Lambda$ =0.05 eV, and L=100 Å. The spin splitting effect for this quantum well was presented in Fig. 4(a). The envelope wave functions for the unpolarized case are shown by solid lines, in the polarized case (y = 0.45 and y = 0.4), the envelopes for the states with spin up and down are shown by dashed and dotted lines, respectively. The odd mode with spin down is allowed only near the point y=0.5, at y<0.45 it disappears, and only the state with spin up is shown at y = 0.4. The form of the envelope wave functions shows that the interface states of the thick quantum wells are localized near the interface boundaries, the dependence of the wave function on the polarization y being rather small. For the thin quantum well, on the other hand, they are extended over a distance of  $\sim 10$  L from the boundaries. We pay attention that the extension of the wave function of the interface states can be regulated by the well width.

#### **IV. DISCUSSION**

In this work the interface states of the magnetic quantum wells have been considered within a model one-electron



FIG. 2. Spin-splitting effect for the interface states of the magnetic quantum well HgTe/Hg<sub>1-x</sub>Mn<sub>x</sub>Te (x=0.2,  $\Lambda$ =0.15 eV) with the well width (a) L=100 Å, (b) 60 Å, and (c) 20 Å. The trend for the states with spin-up (down) is shown by solid (dashed) lines, the calculated points being marked by the crosses (stars). The even (odd) modes are labeled by  $E_i^+$  ( $E_i^-$ ). (d) The corresponding  $\Gamma_6$  and  $\Gamma_8$  bulk band diagram in the cases of y=0.5 (solid and dashed lines, respectively) and y=0.2 (dashed-dotted and dotted lines). |z/L| < 0.5 corresponds to HgMnTe. The band gaps of the constituents are shown by different shading, the regions of band gap overlap are marked by cross shading. The spin-split bulk bands are marked by  $|\mathbf{J}, m_J\rangle$ .



FIG. 3. The same as in Fig. 2, but for the quantum well with x = 0.1,  $\Lambda = 0.15$  eV.

treatment in which, the symmetrical inverted quantum structure has been studied neglecting strain, band bending effects, as well as the effects of Landau level quantization and of Zeeman splitting. We were interested in the band-edge spectrum of the interface states only leaving the problem of their in-plane dispersion out of our consideration. As a result of the used approximations, the problem of the resonant interaction of the interface states with the bulk heavy hole band was beyond this paper. As a matter of fact, the interface states of the inverted zinc blende heterostructures can lie in the energy range of the heavy hole bulk band, being resonant. Therefore, a reliable treatment of the in-plane disper-



FIG. 4. (a) Spin-splitting effect for the interface states of the magnetic quantum well HgTe/Hg<sub>1-x</sub>Mn<sub>x</sub>Te (x=0.2,  $\Lambda$ = 0.05 eV) with the well width L= 100 and 60 Å. (b) The corresponding  $\Gamma_6$  and  $\Gamma_8$  bulk band diagram in the cases of y=0.5(solid and dashed lines, respectively) and y=0.2 (dashed-dotted and dotted lines).The band gaps of the constituent semiconductors are shown by different shading, the region of band gap overlap is marked by cross shading.

sion dependence of the interface states should be developed in the frame of a self-consistent approach including the coupling effects. The resonant interaction will result in broadening of the interface bands. In the nonmagnetic inverted II-VI heterojunctions this problem has been studied in Ref. 10 where it has been shown that the broadening of the interface bands due to resonance with the heavy holes is essential only over a narrow range of the in-plane momentum. The value of



FIG. 5. The envelope wave functions calculated for the odd mode of the interface states of the quantum well with x=0.2,  $\Lambda = 0.05$  eV, and L=100 Å. The envelope wave functions for the unpolarized case are shown by solid lines, in the polarized case (y=0.45 and y=0.4) the envelopes for the states with spin up and down are shown by dashed and dotted lines, respectively.

this range is obviously related to the magnitude of the half width of the probability density function of the interface states. As mentioned above, the region of the extension of the interface wave function increases with decreasing well width. So, the resonant effect is larger the narrower the well is. It is worth mentioning that in the case of magnetic heterostructures the resonant nature of the interface states can give rise to nontrivial effects resulting from the combination of their confinement and magnetic properties.

We have stated that, since the interface states are generated from the evanescent modes, and that is why, they appear inside the overlapping of the constituent gaps, the interface states cannot be related to the heavy hole bands. But it is important to note that this conclusion is restricted to the used Hamiltonian  $\mathcal{H}_{sp}$  in which the *k*-dependent off diagonal blocks  $\mathcal{P}_{pd}$  were left out. In the case of the full Hamiltonian in the form Eq. (1), the heavy hole and light particle states cannot be decoupled and the interface spectrum will be generated by all the coupled conduction and valence band states.

At last we note that in this work we did not aim at getting an exact quantitative description of the spin-splitting effect for the considered magnetic inverted structures. Our aim was, mainly, to understand the nature of the effect and to follow its generation. Quantitative calculations demand, first, self-consistent treatment of the problem in the frame of the full Hamiltonian (1); and secondly, careful selection of the input parameters for the actual quantum structure.

#### V. SUMMARY

In the framework of the developed model the spinsplitting effect on the interface states in the inverted magnetic quantum well structure  $Hg_{1-x}Mn_xTe/HgTe$  has been studied in some detail. The following features of the interface states in comparison with the quantum well states studied in our previous work<sup>1</sup> have been emphasized. First, since the interface states are the evanescent modes of the constituents they can only appear inside the overlapping region of the constituent gaps. Second, in contrast to the quantum well states, it is possible to find either the even or odd mode in the interface spectrum. And third, just as in the case of the quantum well states the interface modes are split by the exchange potential showing a visible spin splitting effect. The states with opposite average spin directions are shifted in energy in opposite directions. The splitting clearly increases with increasing y and with decreasing well width L. The generation of the energy shifts of the interface states follows directly from the analysis of the bulk energy spectrum of the constituents. If for some values of y the polarization effect is larger than the overlapping energy between the bulk band gaps of the constituents then the level with a certain spin direction is not permitted and disappears. As a result, the situation can be predicted in the magnetic quantum structure when only the interface states with one spin direction can exist. It is worth stressing that a partial spin polarization in the semimagnetic barriers can yield a complete spin polarization of the interfaces states or ordinary bound states in the nonmagnetic quantum well material. This situation can be easily controlled by the choice of the constituent semiconductors and by the magnetic field.<sup>24</sup>

We conclude that the magnetic semiconductor heterostructure based on narrow-gap semiconductors with mutually inverted bands give an example of resonant tunneling structures relevant for spin-dependent tunneling phenomenon discussed extensively in the literature.<sup>25–27</sup> We emphasize that a possibility of tuning the interface spectrum spin-splitting in the studied structures points out that they have spin filtering properties. This conclusion is not limited to the quantum wells  $Hg_{1-x}Mn_xTe/HgTe$  for which the numerical calculations have been presented here. The model can also be applied to other inverted magnetic quantum wells type HgMnTe/HgCdTe. When varying the fractions of the Cd and Mn atoms it is possible to shift the bulk bands of the constituents in these quantum structures and in this way change the interface spectrum.

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