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Systematic study of thermal transport of composite fermions around filling factors $\nu=1\pm\frac{1}{2m}$

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The thermopower is calculated for a two-dimensional electron and hole gas at low temperatures in the fractional quantum Hall-effect regime around filling factors $\nu=1\pm\frac{1}{2m}$, $2m$ being the number of flux quanta attached to each carrier in the composite fermion (CF) picture. The general expressions obtained are applied to the $2m=2$ (i.e., $\nu=\frac{1}{2}$ and $\nu=\frac{3}{2}$) and $2m=4$ (i.e., $\nu=\frac{3}{4}$ and $\nu=\frac{5}{4}$) cases. The analysis leads to the determination, on a firm theoretical base, of the actual number of carriers involved in the expression of the thermopower of the system. We demonstrate the significance of the contribution of the nondiagonal component of the composite fermion thermopower to the diagonal component of the total thermopower of the system around $\nu=\frac{3}{2}$. We investigate the dependence of the CF's effective mass on the magnetic field analyzing the ratios $S_{xx,\nu=3/2}/S_{xx,\nu=1/2}$ and $S_{xx,\nu=3/4}/S_{xx,\nu=1/2}$, together with the available experimental data, for the diffusion case. We also analyze the phonon-drag case. Our theoretical results are in agreement with the experimental data.

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I. INTRODUCTION

During the past decade there has been much interest in understanding the thermoelectric properties of two-dimensional (2D) electron and hole systems in strong magnetic fields and very low temperatures.[1–9] Recent theoretical calculations[6,7,10] based on the composite fermion (CF) picture[11,12] gave a satisfactory interpretation of the experimental data[13] for a wide magnetic-field range, where the fractional quantum Hall effect (FQHE) is observed, for the resistivity[14–16] around filling factors $\nu=\frac{1}{2}$ and $\nu=\frac{3}{2}$ and the diffusion thermopower[17,1] around $\nu=\frac{1}{2}$.

The $\nu=\frac{3}{2}$ case is different than $\nu=\frac{1}{2}$. This is due to the fact that at $\nu=\frac{3}{2}$ the magnetic field is inadequate to transform all the carriers to CF's as in the $\nu=\frac{1}{2}$ case. The $\nu=\frac{3}{2}$ is treated as a system composed of two different gases showing parallel conduction. One is of electrons which fully occupy one of the two spin levels of the lowest Landau level, and a second is of electrons which partially occupy the other spin level and which have been transformed to composite fermions.[7] However, the $\nu=\frac{3}{2}$ case can be treated alternatively as a parallel conduction between two gases, one of electrons fully occupying the lowest Landau level (both spin levels) and a second gas of composite fermions originated from holes occupying the upper half of the upper spin level.[18–23] The $\nu=\frac{1}{2}$ case can also be treated as an electron gas at $\nu=1$ and a composite fermion gas originated from holes, occupying the upper half of the lower spin level of the lowest Landau level, conducting in parallel.

The above analysis can be easily generalized for filling factors $\nu=1\pm\frac{1}{2m}$, where $2m$ is the number of the flux quanta attached to each carrier in the CF picture, to obtain the thermopower around $\nu=1\pm\frac{1}{2m}$, for different values of $2m$, for a wide effective magnetic-field range. Bayot *et al.*[1,2] measured the diagonal diffusion thermopower for a 2D hole gas in a GaAs/AlGaAs heterojunction, for a wide magnetic-field range. Ying *et al.*[3] focused on the diagonal thermopower, S_{xx}, for a 2D hole gas in a GaAs/AlGaAs heterojunction at $\nu=\frac{3}{2}$. According to their data, the diffusion (d) thermopower S_{xx}^{d} is dominant at $T<0.1$ K, while at higher temperatures the phonon-drag contribution becomes important. To interpret their experimental data they used the Mott formula for the diffusion thermopower, attributing the heat transfer to CF's. Thus,

$$S_{xx}^{qp}=-\frac{\pi^2 k_B T(p+1)}{3eE_F}=-\frac{\pi k_B T m^*(p+1)}{6e\hbar^2 n_{CF}}. \qquad (1)$$

Here p is a constant directly connected with the scattering mechanisms. The carrier (CF's) concentration is $n_{CF}=n_h/3$, where n_h is the total hole (h) concentration of the sample and m^* is the quasiparticle (qp) effective mass. However, Cooper *et al.*[8] argued that this was not the case. They ‘‘believe that whoever uses the above equation the quantity n should the total number of carriers, including those in any filled Landau levels.’’

Tieke *et al.*[4] reported measurements of the phonon-drag thermopower of a 2D electron gas, exhibiting $S_{xx,\nu=3/2}/S_{xx,\nu=1/2}=1$ and also $S_{xx,\nu=3/4}/S_{xx,\nu=1/4}=1$. They also reported that at $T=440$ mK and at $\nu=\frac{1}{2}$ the phonon-drag (g) thermopower is proportional to $1/n_e$, where n_e is the electron concentration.

The ‘‘belief’’ of Cooper *et al.*[8] has been successfully applied by Tsaousidou and Triberis[9] to the phonon-drag thermopower of a 2D electron gas at filling factor $\nu=\frac{3}{2}$, where the Fermi radius of CF's is expressed in terms of $n_{CF}=n_e/3$. Although this seems to be the case there is no explanation, as far as we know, on a concrete theoretical base, justifying the correct number of carriers involved in the expression of the thermopower.

The present paper consists of the following: After the introduction (Sec. I), in Sec. II we present the theory and the general expressions for the thermopower, for the $2m=2$ (i.e., $\nu=\frac{1}{2}$ and $\nu=\frac{3}{2}$) (Sec. II A) and the $2m=4$ (i.e., $\nu=\frac{3}{4}$ and $\nu=\frac{5}{4}$) (Sec. II B) cases. In Sec. III we present results and a discussion for $B_{eff}=0$ (Sec. III A) for the diffusion and phonon-drag cases and $B_{eff}\neq 0$ (Sec. III B) where we investigate the diffusion thermopower around $\nu=\frac{3}{2}$. Finally, in Sec. IV we present our conclusions.

II. THEORY

Recently, the edge states model has been used[10] to evaluate the p value which characterizes the energy dependence of the transport time τ_t, through the relation[10,24] $\tau_t=\tau_0 E^p$, where τ_0 is a constant and p is directly connected with the scattering mechanisms. The p value also determines the diffusion thermopower, S^d, of the system.[10,24,8]

The resistivity of a system at $\nu=\frac{1}{2}$ is given by

$$\rho=\rho^{qp}+\rho^{CS}, \tag{2}$$

where

$$\rho^{qp}=\frac{\nabla\mu^*}{e^2 J_N}\quad(\nabla T=0). \tag{3}$$

ρ^{CS} is the nondiagonal term of the resistivity arising from the statistical potential, and ρ^{qp} is the quasiparticle (CF's) integer quantum Hall-effect (IQHE) resistivity term. Lopez and Fradkin[25] showed that for $\nu=\frac{1}{2}$ and weak external electric fields E_l

$$J_k=\frac{\epsilon_{kl}E_l}{2\rho^{CS}}, \tag{4}$$

where J_k is the induced electric current in the k direction. However, the electric current for the same magnetic field carried by electrons has a different sign than the one carried by holes. Thus

$$\rho^{CS}=\pm\frac{2\pi\hbar 2m}{e^2}, \tag{5}$$

where the positive sign corresponds to electrons and the negative sign to holes. The thermopower of the system is given by

$$S=S^{qp}\quad(J_N=0), \tag{6}$$

where S^{qp} is the quasiparticle component of the thermopower and J_N is the particle current.

2D electron gases in strong magnetic fields and very low temperature where the FQHE appears, at filling factors $\nu=1\pm\frac{1}{2}m$, can be studied as a system of two gases conducted in parallel as described in the introduction. The positive sign in the above hierarchy expression corresponds to electrons transformed to CF's, while the negative sign to holes.

The thermopower of the system is given by

$$\tilde{S}=-\frac{\tilde{L}}{\tilde{\sigma}}. \tag{7}$$

The conductivity of the system is given by

$$\tilde{\sigma}=\tilde{\sigma}_1+\tilde{\sigma}_2, \tag{8}$$

where $\tilde{\sigma}_1$ is the conductivity of the fully occupied (lower) spin level of the lowest Landau level,

$$\tilde{\sigma}_1=\begin{bmatrix} 0 & -\dfrac{e^2}{h} \\ \dfrac{e^2}{h} & 0 \end{bmatrix}, \tag{9}$$

and $\tilde{\sigma}_2$ is the conductivity of the CF gas obtained by inverting the resistivity matrix

$$\tilde{\rho}=\begin{bmatrix} \rho_{xx}^{qp} & -\rho_{xy}^{qp}-\rho^{CS} \\ \rho_{xy}^{qp}+\rho^{CS} & \rho_{xx}^{qp} \end{bmatrix}. \tag{10}$$

ρ_{xy}^{qp} is the nondiagonal quasiparticle CF IQHE conductivity term and ρ_{xx}^{qp} is the quasiparticle diagonal conductivity of the CF's.

The coefficient $\tilde{L}$ is given by

$$\tilde{L}=\tilde{L}_1+\tilde{L}_2, \tag{11}$$

where the $\tilde{L}_1$ corresponds to the fully occupied spin level and $\tilde{L}_2$ to that of the CF's. The matrix elements of transport coefficients $\tilde{L}_i$ $(i=1,2)$ and $\tilde{\sigma}_i$ $(i=1,2)$ are calculated using

$$L_{i,kl}=\frac{1}{eT}\int_{-\infty}^{\infty}\left[-\frac{\partial f(E)}{\partial E}\right](E-E_F)\sigma_{i,kl}^{(0)}(E)dE, \tag{12a}$$

$$\sigma_{i,kl}=\int_{-\infty}^{\infty}\left[-\frac{\partial f(E)}{\partial E}\right]\sigma_{i,kl}^{(0)}(E)dE, \tag{12b}$$

where $\sigma_{i,kl}^{(0)}(E)$ is the zero (0) temperature conductivity for $E=E_F$, for each gas (i) and the subscripts kl denote the diagonal (xx) and nondiagonal (xy) components.

The conductivities are calculated using two models. We use the Isihara-Smrčka[26,27] model for low effective magnetic fields and the Englert model[28,29] for higher effective fields. The complete theory has been presented elsewhere.[6,30]

For $i=1$ (i.e., for the fully occupied spin level of the lowest Landau level) our calculation shows that the corresponding $\tilde{L}_{i=1}$ coefficient for the diffusion (d) case is zero. For the phonon-drag (g) case the experimental results presented by Tieke *et al.*[4] showed that for a fully occupied Landau level both the thermopower components are zero.

From

$$S_{xx}=L_{1,xx}\rho_{xx}-L_{1,xy}\rho_{xy}, \tag{13}$$

$$S_{xy}=L_{1,xy}\rho_{xx}-L_{1,xx}\rho_{xy}, \tag{14}$$

and the fact that $\rho_{xx}=0$ and $\rho_{xy}\neq 0$, for the fully occupied Landau levels, we obtain the same result as for the diffusion case, i.e.,

$$\tilde{L}_1=0\quad(\tilde{L}_1^d=\tilde{L}_1^g=0). \tag{15}$$

Then Eq. (7) leads to

$$S_{xx}=\frac{S_{xx}^{qp}\rho_{xx}^{qp^2}-S_{xy}^{qp}\rho_{xx}^{qp}N+M(S_{xy}^{qp}\rho_{xx}^{qp}+S_{xx}^{qp}N)}{M^2+\rho_{xx}^{qp^2}}, \quad (16a)$$

$$S_{xy}=\frac{-S_{xy}^{qp}\rho_{xx}^{qp^2}-S_{xx}^{qp}\rho_{xx}^{qp}N+M(S_{xx}^{qp}\rho_{xx}^{qp}-S_{xy}^{qp}N)}{M^2+\rho_{xx}^{qp^2}}, \quad (16b)$$

where $M=\rho_{xy}^{qp}+\rho^{CS}+e^2/h[\rho_{xx}^{qp^2}+(\rho_{xy}^{qp}+\rho^{CS})^2]$ and $N=(\rho_{xy}^{qp}+\rho^{CS})$.

The expressions given by Eqs. (16a) and (16b) allow us to evaluate the diagonal and nondiagonal components of the thermopower for any filling factor given by $\nu=1\pm\frac{1}{2m}$. We apply them to the cases $2m=2$ and $2m=4$, which are of experimental interest. Our general expressions are applicable on both diffusion and phonon-drag cases.

A. The $2m=2$ case

The $2m=2$ case corresponds to two different filling factors, i.e., $\nu=\frac{1}{2}$ and $\nu=\frac{3}{2}$. The $\nu=\frac{1}{2}$ case could be attributed to a composite fermion gas at $\nu=\frac{1}{2}$, originated from electron or an electron gas $\nu=1$ and a composite fermion gas at $\nu=\frac{1}{2}$ conducting in parallel. In the latter representation the CF gas originates from holes occupying the upper half of the lower spin level of the lowest Landau level. The $\nu=\frac{1}{2}$ state reveals the equivalence of the two approaches.

The hole conductivity at $\nu=\frac{1}{2}$ has an opposite sign to the electronic case.[22,23] Thus, $\rho^{CS}=-2h/e^2$. Equation (16a) for $B_{eff}=0$, i.e., $\rho_{xx}^{qp}\ll 2h/e^2$, $\rho_{xy}^{qp}=0$, and $S_{yx,\nu=1/2}^{qp}(h)=0$ gives

$$S_{xx,\nu=1/2}=-S_{xx,\nu=1/2}^{qp}(h), \quad (17)$$

where $S_{xx,\nu=1/2}^{qp}(h)$ is the quasiparticle diagonal component of the thermopower of CF's originated from holes (which are positive). This is consistent with the alternative picture of electrons occupying the lowest half of the lower spin level of the lowest Landau level for which S_{xx} is negative, i.e.,

$$S_{xx,\nu=1/2}=S_{xx,\nu=1/2}^{qp}(e). \quad (18)$$

For the $\nu=\frac{3}{2}$ case, the two gases conducting in parallel consist of an electron gas fully occupying the lower spin level of the lowest Landau level ($\nu=1$) and another electron gas at $\nu=\frac{1}{2}$ transformed to CF's occupying the lower half of the higher spin level of the lowest Landau level. To each carrier two-flux quanta are attached ($2m=2$).

From Eq. (3) we obtain that $\rho_{CS}=2h/e^2$. Equation (16a) for $B_{eff}=0$, i.e., $\rho_{xx}^{qp}\ll 2h/e^2$, $\rho_{xy}^{qp}=0$, and $S_{yx,\nu=1/2}^{qp}(e)=0$ gives

$$S_{xx,\nu=3/2}=\frac{S_{xx,\nu=1/2}^{qp}(e)}{3}, \quad (19)$$

where $S_{xx,\nu=1/2}^{qp}(e)$ is the quasiparticle diagonal component of the thermopower of CF's, originated from electrons.

B. The $2m=4$ case

The $2m=4$ case corresponds to two different filling factors, i.e., $\nu=\frac{3}{4}$ and $\nu=\frac{5}{4}$. For the $\nu=\frac{3}{4}$ case the system consists of an electron gas fully occupying the lower spin level of the lowest Landau level (at $\nu=1$) and a second of composite fermions (at $\nu=\frac{1}{4}$) conducting in parallel. The CF gas originates from holes, occupying the higher fourth of the lower spin level of the lowest Landau level. To each carrier four-flux quanta are attached ($2m=4$).

The nondiagonal resistivity of the system of the composite fermions, for $B_{eff}=0$, is $\rho^{CS}=-4h/e^2$. Then, Eq. (16a) for $B_{eff}=0$, i.e., $\rho_{xx}^{qp}\ll 4h/e^2$, $\rho_{xy}^{qp}=0$, and $S_{yx,\nu=1/4}^{qp}(h)=0$ gives

$$S_{xx,\nu=3/4}=-\frac{S_{xx,\nu=1/4}^{qp}(h)}{3}. \quad (20)$$

The quasiparticle thermopower, $S_{xx,\nu=1/4}^{qp}(h)$, has a positive sign.

For the $\nu=\frac{5}{4}$ case, the system consists of an electron gas occupying the lower spin level of the lowest Landau level (at $\nu=1$) and a second of composite fermions (at $\nu=\frac{1}{4}$). The CF gas originates from electrons occupying the lowest fourth of the higher spin level of the lowest Landau level. To each carrier four-flux quanta are attached ($2m=4$).

From Eq. (3) we obtain $\rho^{CS}=4h/e^2$. Equation (16a) for $B_{eff}=0$, i.e., $\rho_{xx}^{qp}\ll 4h/e^2$, $\rho_{xy}^{qp}=0$, and $S_{yx,\nu=1/4}^{qp}(e)=0$ gives

$$S_{xx,\nu=5/4}=\frac{S_{xx,\nu=1/4}^{qp}(e)}{5}. \quad (21)$$

Here we have to notice that

$$S_{xx,\nu=1/4}^{qp}(h)=-S_{xx,\nu=1/4}^{qp}(e). \quad (22)$$

Thus

$$S_{xx,\nu=3/4}=-\frac{S_{xx,\nu=1/4}^{qp}(h)}{3}=\frac{S_{xx,\nu=1/4}^{qp}(e)}{3}. \quad (23)$$

The above result could mislead one to assume that the origin of the CF's at $\nu=\frac{3}{4}$ are electrons at their $\nu=\frac{1}{4}$ filling factor, i.e., electrons occupying the lowest fourth of the higher spin level of the lowest Landau level. However, the latter is consistent with the $\nu=\frac{5}{4}$ state[21] which leads to a completely different result for the diagonal component of the system thermopower [Eq. (21)]. This could lead to serious discrepancies in the interpretation of the resistivity data where the true number of carriers transformed to CF's are of crucial importance.

III. RESULTS AND DISCUSSION

A. $B_{eff}=0$

For the *diffusion* case, Eqs. (16a) and (19) give

$$S_{xx,\nu=3/2}=\frac{S^{qp}_{xx,\nu=1/2}}{3}=-\frac{\pi k_B T m^*(p+1)}{3\times(6e\hbar^2 n_{CF})}$$

$$=-\frac{\pi k_B T m^*(p+1)}{6e\hbar^2 n}, \tag{24}$$

where n is the total number of carriers.

For the *phonon-drag* case, the quasiparticle (CF's) thermopower is given by the semiclassical Cantrell-Butcher formula,[9] which for $\nu=\frac{1}{2}$ reads in a compact form

$$S^{qp,g}_{xx,\nu=1/2}=-\frac{\Lambda m_{3/2}}{8\pi^2 k_B T^2 n|e|}\mathcal{F}, \tag{25}$$

where $\mathcal{F}=\sum_{\mathbf{Q},s}\omega_{\mathbf{Q}}(q^2/Q)\int_0^{2\pi}d\theta\times\int d\epsilon_{\mathbf{k}}f^0(\epsilon_{\mathbf{k}})[1-f^0(\epsilon_{\mathbf{k}}+\hbar\omega_{\mathbf{Q}})]\times P^a_{\mathbf{Q}}(\mathbf{k},\mathbf{k}+\mathbf{q})$. Here Λ is the phonon mean free path, $\omega_{\mathbf{Q}}$ is the frequency for the acoustic phonon with wave vector $\mathbf{Q}=(\mathbf{q},q_z)$ and polarization s, $\mathbf{k}=(k_x,k_y)$ is the CF wave vector, $\epsilon_{\mathbf{k}}=\hbar^2k^2/2m_{3/2}$ is the CF energy, f^0 is the Fermi-Dirac function, and θ is the angle between $\mathbf{k}$ and $\mathbf{q}$. In the above equation $P^a_{\mathbf{Q}}(\mathbf{k},\mathbf{k}+\mathbf{q})$ is the rate at which the CF will transfer from $\mathbf{k}$ to $\mathbf{k}+\mathbf{q}$ by absorbing a phonon.[9]

According to our analysis,

$$S^{g}_{xx,\nu=3/2}=\frac{S^{qp,g}_{xx,\nu=1/2}(e)}{3}, \tag{26}$$

given that at the Cantrell-Butcher formula, for our case, $n=n_{CF}=n_e/3$, Eq. (25) reads

$$S^{g}_{xx,\nu=3/2}=\frac{S^{qp,g}_{xx,\nu=1/2}}{3}=-\frac{\Lambda m_{3/2}}{3\times 8\pi^2 k_B T^2 n_{CF}|e|}\mathcal{F}$$

$$=-\frac{\Lambda m_{3/2}}{8\pi^2 k_B T^2 n_e|e|}\mathcal{F}, \tag{27}$$

in absolute agreement with the one used by Tsaousidou and Triberis.[9] The k_F Fermi vector is referred to the CF concentration $n_{CF}=n_e/3$.

The above analysis and discussion justify, on a firm theoretical base, the "belief" expressed by Cooper *et al.*[8] that whoever uses Mott's formula for the diffusion thermopower should use the total number of carriers, including those in any filled Landau levels. Moreover they justify the number of carriers one should use in the expression of the phonon-drag thermopower.

The composite fermion gases are both at their $\nu=\frac{1}{2}$ when the system under study is at filling factors $\nu=\frac{3}{2}$ and $\nu=\frac{1}{2}$ $(2m=2)$. For the diffusion case, assuming that the scattering mechanism is the same for $\nu=\frac{3}{2}$ and $\nu=\frac{1}{2}$, and taking the effective mass to be proportional to $\sqrt{B}$, we obtain from Eqs. (1) and (24) that

$$\frac{S_{xx,\nu=3/2}}{S_{xx,\nu=1/2}}=\frac{m^*_{3/2}}{m^*_{1/2}}=\sqrt{\frac{B_{3/2}}{B_{1/2}}}=\frac{1}{\sqrt{3}}=0.57. \tag{28}$$

TABLE I. Parameters used for the calculation of the thermopower around $\nu=3/2$.

Fitting parameters	
$\Gamma_{N,s}$ (meV)	$0.1\sqrt{B_{eff}/\mu_q}$
$\lambda_{N,s}$ (meV)	$0.025\sqrt{B_{eff}/\mu_q}$
$\Gamma_{N,xx}$	0.9
Physical parameters	
μ (m^2/Vs)	9.0
μ_q (m^2/Vs)	2.10
n_e ($\times 10^{15}$ m^{-2})	2.3
$m^*(m_e)$	0.43
n_{CF} ($B_{\mathrm{eff}}=0$)($\times 10^{15}$ m^{-2})	0.77

Here $m^*_{1/2}$ and $m^*_{3/2}$ are the effective masses of the CF's at the corresponding filling factors of the system under study, i.e., $\nu=\frac{1}{2}$ and $\nu=\frac{3}{2}$, respectively.

Bayot *et al.*[2] measured the diagonal component of the thermopower, S_{xx}, of a sample of hole concentration $p_s=4\times10^{-14}$ m^{-2}, for a wide range of magnetic fields. At $T=100$ mK, where the diffusion thermopower dominates over the phonon drag, they found that $S_{xx,\nu=3/2}/S_{xx,\nu=1/2}=0.75$, a value which is roughly in agreement with Eq. (28).

However, in a later experiment of the same experimental group, Ying *et al.*[3] reported that the ratio $S_{xx,\nu=3/2}/S_{xx,\nu=1/2}=1.4$ for a sample of total hole concentration $p_s=6.5\times10^{-14}$ m^{-2} at $T=60$ mK. These measurements were in qualitative agreement with other reports of the same group (Bayot *et al.*[1]) for a different sample of hole concentration $p_s=14\times10^{-14}$ m^{-2} at $T=169$ mK. For this system they reported for the diffusion thermopower $S_{xx,\nu=3/2}/S_{xx,\nu=1/2}=1.8$ in contradiction with their previous report[2] and Eq. (28) which predicts a ratio lower than 1. The above two measurements show an effective-mass dependence of the magnetic field opposite to what should have been expected.[8]

This inconsistency does not appear in the $S_{xx,\nu=3/4}$ data[2] according to which, $S_{xx,\nu=3/4}/S_{xx,\nu=1/2}=1.3$, in rough agreement with Bayot *et al.*[1] where $S_{xx,\nu=3/4}/S_{xx,\nu=1/2}=1.9$ and Ying *et al.*[3] where $S_{xx,\nu=3/4}/S_{xx,\nu=1/2}=1.5$.

From Eqs. (23), (19), and (24) we obtained

$$\frac{S_{xx,\nu=3/4}}{S_{xx,\nu=1/2}}=\frac{m^*_{3/4}(p_{3/4}+1)}{m^*_{1/2}(p_{1/2}+1)}. \tag{29}$$

where $p_{1/2}$ and $p_{3/4}$ are constants connected to the scattering mechanisms at the filling factors of the system under study, i.e., $\nu=\frac{1}{2}$ and $\nu=\frac{3}{4}$, respectively, and $m^*_{3/4}$ is the CF's effective mass at $\nu=\frac{3}{4}$. Recently Yeh *et al.*[20] and Onoda *et al.*[31] reported that the effective mass of the four-flux quanta (around $\nu=\frac{1}{4}$) follows the same equation as the two-flux quanta (at $\nu=\frac{1}{2}$) $(m^*=\alpha\sqrt{B})$. However, the constant α of the four-flux quanta is larger by a factor of 2 than the corresponding constant for the two-flux quanta.

Khveshchenko's[24] calculation for the random magnetic-field scattering mechanism gives a $p_{1/2}=0.13$. Following his

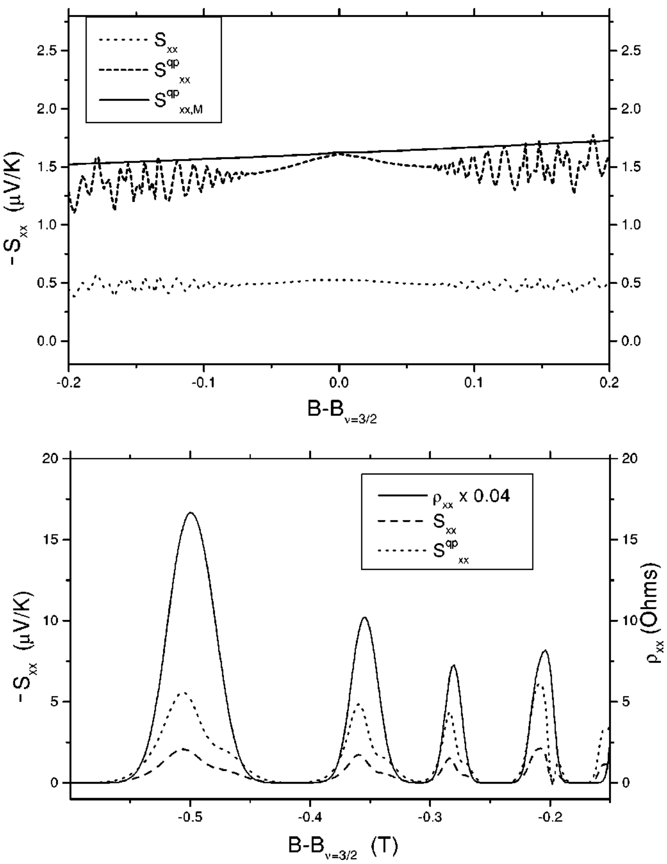

FIG. 1. (a) Calculated S_{xx} at 0.025 K (dotted line), S_{xx}^{qp} (dashed line) using the Isihara-Smrčka model, and the S_{xx} given by the Mott formula for $B_{eff}=0$ around $\nu=\frac{3}{2}$. (b) Calculated S_{xx} (dashed line), S_{xx}^{qp} (dotted line), and ρ_{xx} (scaled by a factor of 0.04 for clarity) (full line) around $\nu=\frac{3}{2}$ using the Englert model at 0.025 K.

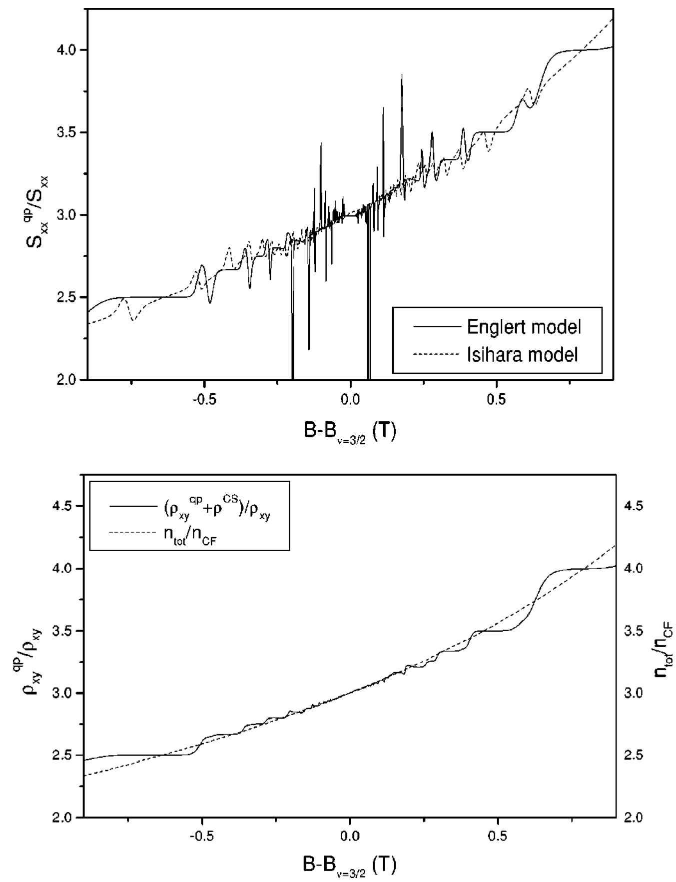

FIG. 2. (a) Calculated S_{xx}^{qp}/S_{xx} at 0.025 K around $\nu=\frac{3}{2}$ for different theoretical models. The dashed line shows the theoretical calculations using the Isihara-Smrčka model and the full line shows the theoretical calculations using the Englert model. b. Calculated ratios n_{tot}/n_{CF} (dashed line) as well with $(\rho_{xy}^{qp}+\rho^{CS})/\rho_{xy}$ (full line) at 0.025 K around $\nu=\frac{3}{2}$.

calculation and using the same assumptions for $\nu=\frac{1}{4}$ we obtain $p_{1/4}=0.088$. Thus, Eq. (29) gives

$$\frac{S_{xx,\nu=3/4}}{S_{xx,\nu=1/2}}=1.56 \tag{30}$$

in reasonable agreement with the above experimental data.[2,1,3]

However, in a previous work of ours we have proposed a method of evaluating the p value from the resistivity experimental data.[10] This was based on the assumption that the scattering mechanism is the same for both $\nu=\frac{1}{2}$ and $\nu=\frac{3}{2}$. The p value obtained for electrons was in very good agreement with Khveshchenko's reported value,[24] i.e., $p=0.13$ and the available experimental data. For holes the p value obtained was different ($p\sim-0.8$), in qualitative agreement with experimental data for zero magnetic field,[32] indicating probably a different scattering mechanism. We could apply our method to $\nu=\frac{1}{4}$ and $\nu=\frac{3}{4}$, if we had the ρ_{xx} measurements of each sample at the above filling factors. We believe that $m^*\sim\sqrt{B}$ holds and the reason for the large deviations of the experimental data by a factor of more than 20% from the above theoretical result is the incorrect value of p used.

According to Tieke *et al.*[4] measurements of the phonon-drag thermopower lead to $S_{xx,\nu=3/2}/S_{xx,\nu=1/2}=1$. They also reported that at $T=440$ mK and at $\nu=\frac{1}{2}$ the phonon-drag thermopower is proportional to $1/n$, where n is the total carrier concentration of the sample. This means that the $\mathcal{F}$ term of Eq. (27) is independent of the carrier concentration at least at this temperature. Also, the prefactor in Eq. (27) is independent of the actual composite fermion concentration and depends only on the temperature and the total carrier concentration of the sample and consequently the total thermopower at $\nu=\frac{3}{2}$ equals to that at $\nu=\frac{1}{2}$. The composite fermion gases at filling factors of the system under study, $\nu=\frac{3}{4}$ and $\nu=\frac{1}{4}$ $(2m=4)$, are both at their $\nu=\frac{1}{4}$.

Tieke *et al.*[4] also reported measurements of the *phonon-drag* thermopower according to which $S_{xx,\nu=3/4}/S_{xx,\nu=1/4}=1$. The $\nu=\frac{1}{4}$ behavior of the system is similar to the $\nu=\frac{1}{2}$, and the $\nu=\frac{3}{4}$ thermopower can be calculated from an identical expression such as Eq. (27). Thus, the equivalence of the two results is expected.

B. $B_{eff}\neq 0$ $(\nu=3/2)$

We calculate the diffusion thermopower around the filling factor $\nu=\frac{3}{2}$ for a wide range of magnetic field using

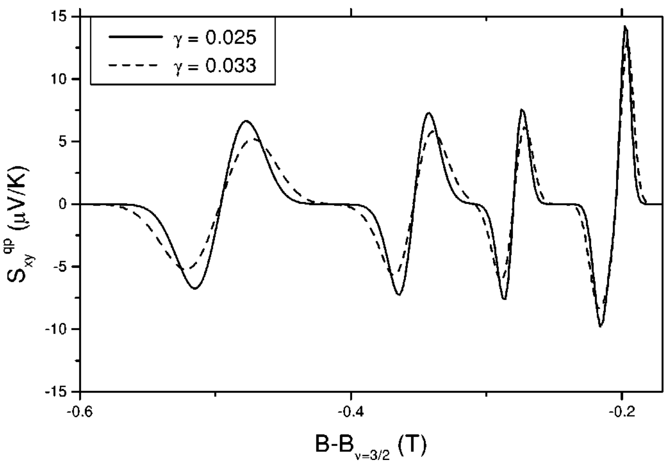

FIG. 3. Calculated S_{xy} using the Englert model around $\nu=\frac{3}{2}$ for two different Landau-level broadenings $\lambda=\gamma\sqrt{B_{eff}/\mu_q}$. The full line is for $\gamma=0.025$ and the dashed line is for $\gamma=0.033$.

Englert[28] and Isihara-Smrčka[26] (IS) models. We consider the lower spin level of the lowest Landau level fully occupied by electrons and a half filled higher spin level of the lowest Landau level occupied by electrons which are transformed to CF's. The parameters used in our calculations are presented in Table I. They are the same we used in our previous work[7] for the interpretation of the Eisenstein *et al.*[16] resistivity experimental data.

In Fig. 1(a) we plot the diagonal term of the system diffusion thermopower around $\nu=\frac{3}{2}$. We present the results obtained from Eq. (16a) as well as the Mott formula value and the quasiparticle component of the S_{xx}, calculated from the IS model for $B-B_{\nu=3/2}$ in the range of $[-0.2\text{–}0.2]$ T. This model is valid for low effective fields. The value of the thermopower obtained using the Mott formula $S_{xx,M}^{qp}$ is different for each value of the magnetic field because the number of the electrons transformed to composite fermions changes with the magnetic field as the number of the electrons occupying the filled lower spin level changes. With increasing magnetic field the number of the electrons transformed to CF's decreases and thus the "Mott thermopower" increases. This thermopower would have been the thermopower of the composite fermion gas if at that field the CF's were at their $\nu=\frac{1}{2}$. The slope of the S_{xx}^{qp} curve follows the $S_{xx,M}^{qp}$ behavior.

In Fig. 1(b) we present results obtained from Eq. (16a) for the diagonal term of the system diffusion thermopower for a wide range of magnetic fields (where the composite fermion Landau levels are well separated) around $\nu=\frac{3}{2}$ and also the resistivity, scaled by a factor of 0.04, and the quasiparticle component of the S_{xx}, calculated from the Englert model. The diffusion thermopower follows the same behavior as the resistivity of the system.

In Fig. 2(a) we plot the ratio S_{xx}^{qp}/S_{xx} for both the IS and Englert models while in Fig. 2(b) the ratio $(\rho_{xy}^{qp}+\rho^{CS})/\rho_{xy}$ and n/n_{CF} around $\nu=3/2$ are plotted. At zero effective magnetic field the values of all plotted quantities equals 3. This ratio, in contrast with the n/n_{CF} ratio, does not increase continuously but shows characteristic plateaus.

However, our theory predicts, in the magnetic-field range where the Landau levels are well separated, large peaks in the ratio S_{xx}^{qp}/S_{xx} in contrast with the ratio $(\rho_{xy}^{qp}+\rho^{CS})/\rho_{xy}^{qp}$. These peaks appear due to the contribution of the nondiagonal component of the quasiparticle thermopower, S_{yx}. With decreasing magnetic effective fields, the S_{yx} becomes larger and the contribution of the peaks increases. At low fields, where the Landau levels are not well separated, these peaks vanish. Thus, it is evident that near $\nu=\frac{3}{2}$ we can calculate even at nonzero magnetic effective fields the thermopower from the following equation:

$$S_{xx,\nu=3/2}=\frac{S_{xx,\nu=1/2}^{qp}}{(\rho_{xy}^{qp}+\rho^{CS})/\rho_{xy}^{qp}}=\frac{S_{xx,\nu=1/2}^{qp}}{n/n_{CF}}. \tag{31}$$

On the other hand, every attempt to calculate the diagonal component of the thermopower, at magnetic fields high enough to separate the composite fermion Landau levels, these corrections should be taken into account, in order to obtain quantitative agreement with experiment.

In Fig. 3 we plot the S_{xy}^{qp} using the Englert model for two Landau-level broadenings ($\lambda=0.025\sqrt{B_{eff}/\mu_q}$ and $\lambda=0.033\sqrt{B_{eff}/\mu_q}$). Here μ_q is the quantum mobility calculated from the single-particle relaxation τ_q time through the relation $\mu_q=e\tau_q/m^*$. Comparing Figs. 2(a) and 3 we conclude that the large peaks appearing in the ratio of S_{xx}^{qp}/S_{xx} are caused by the contribution of the nondiagonal term of the quasiparticle thermopower in the total thermopower of the system. This deviations from the $(\rho_{xy}^{qp}+\rho^{CS})/\rho_{xy}^{qp}$ can also be observed in the system thermopower. The quasiparticle nondiagonal thermopower component depends strongly on the Landau-level broadening. The dashed line in Fig. 3 refers to large broadening. The peaks are significantly lower. Thus, we expect that the corrections in the diagonal thermopower, due to the nondiagonal term, to vanish with increasing Landau level broadening. A full analysis of the effect of the Landau-level broadening on the thermopower will be presented on the near future.

IV. CONCLUSIONS

We have presented a systematic study of thermal transport of composite fermions around filling factors $\nu=1\pm\dfrac{1}{2m}$. The general expressions obtained for the thermopower have been applied to the $2m=2$ (i.e., $\nu=\frac{1}{2}$ and $\nu=\frac{3}{2}$) and $2m=4$ (i.e., $\nu=\frac{3}{4}$ and $\nu=\frac{5}{4}$) cases. The analysis demonstrates on a firm theoretical base that the actual number of carriers involved in the expression of the thermopower of the system is that of the CF's. It is concluded that if $m^*\sim\sqrt{B}$ holds it is the p value, which is connected with the scattering mechanisms, that determines the $S_{xx,\nu=3/2}/S_{xx,\nu=1/2}$ and $S_{xx,\nu=3/4}/S_{xx,\nu=1/2}$ ratios.

For the $\nu=\frac{3}{2}$ case the calculation of the diffusion thermopower shows that in order to reproduce the behavior of the diagonal component of the thermopower of the system we have to include, at least for low Landau-level broadening, the corrections imposed by the nondiagonal term.

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