Electrons in an annealed environment: A special case of the interacting electron problem

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(Received 28 February 2002; revised manuscript received 7 June 2002; published 2 October 2002)

The problem of noninteracting electrons in the presence of annealed magnetic disorder, in addition to nonmagnetic quenched disorder, is considered. It is shown that the proper physical interpretation of this model is one of electrons interacting via a potential that is long ranged in time, and that its technical analysis by means of renormalization-group techniques must also be done in analogy to the interacting problem. As a result, and contrary to previous claims, the model does not simply describe a metal-insulator transition in $d=2+\epsilon(\epsilon \ll 1)$ dimensions. Rather, it describes a transition to a ferromagnetic state that, as a function of the disorder, precedes the metal-insulator transition close to $d=2$. In $d=3$, a transition from a paramagnetic metal to a paramagnetic insulator is possible.

DOI: 10.1103/PhysRevB.66.155101 PACS number(s): 71.30.+h, 75.10.Lp, 64.60.Ak

I. INTRODUCTION

Local magnetic moments are known to play an important role in the behavior of disordered electronic systems, but the precise nature of that role remains incompletely understood.[1] One way to think about such local moments is that, in a disordered environment, the exchange interaction between the electrons may be locally enhanced to the point where the electron spins order magnetically in a finite region in space.[2] The resulting magnetized regions are often referred to as local moments, or droplets, or rare regions. Since they are self-generated by the electron system, they are in thermodynamic equilibrium with the other electronic degrees of freedom. It is therefore intuitively plausible that such local moments can be modeled as annealed magnetic disorder, in addition to the underlying quenched disorder that produces them. In Ref. 4 an explicit derivation has been given that corroborates this argument. There is experimental evidence for such local moments to influence the transport properties of the electron system in important ways, and, in particular they are suspected to influence the critical behavior near the metal-insulator transition (MIT) that is observed in disordered electron systems.[1] However, a theoretical understanding of the coupling between local moments and transport properties has proven to be very hard. Studying and understanding the annealed disorder model mentioned above is expected to shed light on this important problem.

Reference 4 provided such an analysis, and concluded that the annealed disorder leads to a very interesting type of MIT. The most exciting feature was that the transition was driven by the vanishing of the thermodynamic density susceptibility $\partial n/\partial \mu$, and thus resembled more a Mott transition than an Anderson transition.[5] This was even more surprising as the Coulomb interaction between the electrons, which is what usually causes a Mott transition, had not been explicitly taken into account in the model.

Subsequently, Ref. 6 developed a general classification of quantum phase transitions with respect to (1) whether one can describe the transition by means of a local order parameter, and (2) whether the order-parameter susceptibility in the disordered phase is an analytic function of the wave number. The second criterion has an important bearing on which observables can become critical at a MIT: Criticality in $d>2$ ($d=2$ is the lower critical dimension for all known MIT's of disordered interacting electrons) implies a logarithmic dependence on the renormalization-group (RG) length rescaling factor, and hence on the wave number, in perturbation theory in $d=2$. This in turn implies a (weaker) nonanalytic wave number dependence in $d>2$ away from criticality.[7,8] Although the considerations in Ref. 6 do not provide a rigorous mathematical proof, they strongly suggest that $\partial n/\partial \mu$ cannot be critical at a MIT for a large class of models, which includes the model studied in Ref. 4.

In the current paper we provide a thorough reanalysis of the model derived and studied in Ref. 4, and resolve this contradiction. We show that the RG analysis of the model performed in Ref. 4 had an incorrect structure and led to unreliable results. A proper analysis of the model's renormalizability, and the resulting RG flow equations, show that $\partial n/\partial \mu$ is not singularly renormalized and hence not critical, in agreement with Ref. 6. In addition, it reveals that within a controlled ϵ expansion about $d=2$, the model does not simply describe a metal-insulator transition. Rather, it displays a variant of the phase-transition sequence that is known to occur in a related model with both quenched disorder and electron-electron interactions (but no annealed disorder).[10] That is, as the disorder increases, there is first a transition to a ferromagnetic metallic state, and then, with further increasing disorder, a transition to a ferromagnetic insulator state. For $d=3$, a transition directly from a paramagnetic metal to a paramagnetic insulator is possible.

This paper is organized as follows. In the following section, we give intuitive physical arguments that explain our model and our procedure to analyze it, and we summarize our results. In Sec. III we formally define the model and write it in a way that facilitates a renormalization-group

analysis. Section IV performs the renormalization to one-loop order, and Sec. V analyzes the results. Some technical issues regarding the model's renormalization properties are relegated to Appendixes A and B, the flow equations for the interacting and annealed disorder models are compared in Appendix C, and a perturbative analysis of the free energy is given in Appendix D.

II. PHYSICAL ARGUMENTS

Since some of our detailed arguments are quite technical, we start by giving some intuitive physical arguments to explain both our general strategy and our results.

A. Annealed disorder as a model for local moments

We start by recalling the argument for why annealed disorder models local moments.[4] Any field-theoretical treatment of a statistical-mechanics problem starts with a functional integral representation of the partition function,[11]

$$Z = \int D[\phi]\ e^{-S[\phi]}. \tag{2.1a}$$

The form of the action S defines the model under consideration, and the mathematical nature of the field ϕ depends on whether the system is classical or quantum mechanical, consists of fermions or bosons, and whether the model is a microscopic one in terms of fundamental fields or of an effective nature. The usual procedure is to identify a saddle point of S that approximately contains the physics one is interested in, to expand about this saddle point, and to employ perturbation theory and the renormalization-group. In a system with quenched disorder there will be, apart from homogeneous saddle-point solutions, solutions where the field ϕ, or some components of it, are nonzero only in certain regions in space. Such inhomogeneous saddle points have been proposed as a description of rare regions in classical magnets by Dotsenko *et al.*[2] This concept was generalized to quantum magnets,[3] and to the effective-field theories used to describe MIT's in quenched disordered electron systems.[4] In a large system there will be many rare regions that interact only very weakly, and thus exponentially many almost-degenerate saddle points, since the orientation of the field on the rare regions is arbitrary. These saddle points are expected to be separated by large energy barriers, and thus not to be perturbatively accessible from one another. Within the perturbation theory, and denoting the nth saddle-point field configuration by $\Phi^{(n)}$ and the fluctuations by φ, one can therefore write the partition function

$$Z \approx \sum_n D[\varphi] e^{-S[\Phi^{(n)}+\varphi]}. \tag{2.1b}$$

In the thermodynamic limit, the discrete set of saddle points becomes a saddle-point manifold that needs to be integrated over. The saddle-point field configurations Φ thus become degrees of freedom that are governed by some probability distribution $P[\Phi]$ are integrated over at the level of the partition function, and couple to the field φ by means of some coupling S_c that is determined by the action S,

$$Z \approx \int D[\Phi]\ \ P[\Phi] \int D[\varphi] e^{-S[\varphi]+S_c[\Phi,\varphi]}. \tag{2.1c}$$

They therefore act like an annealed disorder. Note that in giving Eq. (2.1c) we implicitly assume that the (annealed) disorder adjusts and comes to equilibrium with the fluctuations φ. If the disorder were fixed on the time scale of the φ fluctuations, then it would be a quenched disorder. In the latter case, for the average over saddle points to be meaningful, $\ln Z$ rather than Z should be averaged over the Φ fields.[12]

In our case, we are interested in rare regions that carry a magnetic moment. According to the arguments recalled above, they can be modeled by annealed magnetic disorder in addition to the quenched disorder that allows for the inhomogeneous saddle-point solutions. In the simplest possible model the annealed disorder has a Gaussian distribution, and is static. The latter means that the coupling constant, or the annealed magnetic disorder strength, will be proportional to the temperature.[4] This is just the Boltzmann weight assigned to these classical degrees of freedom that are in equilibrium with the electrons. We emphasize that this model, and its derivation in Ref. 4, is unaffected by our considerations concerning its analysis and interpretation, which differ from the one given in that reference.

B. Annealed disorder as an effective interaction

The physical effects of the annealed disorder are fundamentally different from those of quenched, or frozen-in, disorder.[12] The former gets integrated over at the level of the partition function, cf. Eq. (2.1c), the latter, at the level of the free energy. Consequently, integrating out the annealed disorder generates an effective physical interaction between the degrees of freedom that couple to it, the electron spin density in our case, which can be understood as resulting from an exchange of annealed disorder fluctuations between the electrons. The effects of the quenched disorder, on the other hand, are more subtle and fundamentally different from those of interactions.

It is therefore plausible that a system of noninteracting electrons in the presence of both the quenched and annealed disorders will behave in many respects as one with the quenched disorder only and an additional electron-electron interaction. As the only difference one would expect that, if the annealed disorder is modeled as static, the resulting effective interaction will be infinitely long ranged in time, a feature that one would not expect to have qualitative effects. This expectation is in contradition with the results of Ref. 4, which found a behavior that was drastically different from that of electrons interacting via an instantaneous interaction. In particular, this reference predicted a MIT of Mott type, where the thermodynamic susceptibility $\partial n/\partial \mu$ vanishes. This is in contradiction to both explicit calculations for quenched disordered, interacting electron systems, which find that $\partial n/\partial \mu$ is not singularly renormalized,[13,1] and very general considerations in Ref. 6.

The analysis that will be presented below removes this contradiction, and illustrates the technical issues behind the above intuitive physical considerations. We will show that the technical treatment of the annealed disorder in analogy to that of the quenched disorder in Ref. 4 was not only in disagreement with the above physical arguments, but led to an unnatural structure of the theory. This in turn led to incorrect assumptions about the behavior under renormalization, and ultimately to physically incorrect results. A treatment of the annealed disorder in analogy to an interaction, on the other hand, does not run into these problems and yields results that are in agreement with all known constraints.

III. THE MODEL AND ITS RENORMALIZABILITY

In this section we consider the same effective-field theory as in Ref. 4.

A. Effective-field theory

Our starting point, as in Ref. 4, is Wegner's nonlinear sigma model[14] (NLσM) for noninteracting electrons with nonmagnetic quenched disorder. The action reads

$$\mathcal{A}_{\mathrm{NL}\sigma\mathrm{M}}=\frac{-1}{2G}\int d\mathbf{x}\,\mathrm{tr}\,[\boldsymbol{\nabla} Q(\mathbf{x})]^2+2H^{(1)}\int d\mathbf{x}\,\mathrm{tr}\,[\Omega\, Q(\mathbf{x})]. \tag{3.1}$$

Here $Q(\mathbf{x})$ is a matrix field that comprises two fermionic degrees of freedom. Accordingly, Q carries two fermionic Matsubara frequency indices n and m, and two replica indices α and β to deal with the quenched disorder. The matrix elements $Q^{\alpha\beta}_{nm}$ are spin-quaternion valued to allow for particle-hole and spin degrees of freedom. It is convenient to expand them in a basis $\tau_r\otimes s_i$ $(r,i=0,1,3.3)$ where $\tau_0=s_0$ is the 2×2 unit matrix, and $\tau_{1,3.3}=-s_{1,3.3}=-i\sigma_{1,3.3}$, with σ_j the Pauli matrices,[15]

$$Q^{\alpha\beta}_{nm}=\sum_r\sum_i {}^{i}_{r}Q^{\alpha\beta}_{nm}. \tag{3.2a}$$

For simplicity, we will ignore the particle-particle or Cooper channel, which amounts to dropping τ_1 and τ_2 from the spin-quaternion basis.[15,1] The $Q^{\alpha\beta}_{nm}$ are then elements of $\mathcal{C}\times\mathcal{Q}$, with $\mathcal{C}$ and $\mathcal{Q}$ the complex number field and the quaternion field, respectively. The ${}^{i}_{r}Q^{\alpha\beta}_{nm}$ obey the following symmetry properties (for $r=0,3$):[16]

$${}^{0}_{r}Q^{\alpha\beta}_{nm}=(-)^{r0}\,{}^{r0}_{r}Q^{\beta\alpha}_{mn}, \tag{3.2b}$$

$${}^{i}_{r}Q^{\alpha\beta}_{nm}=(-)^{r+1\,i}\,{}^{i}_{r}Q^{\beta\alpha}_{mn}\qquad (i=1,3.3). \tag{3.2c}$$

Alternatively, we can write the spin indices explicitly, and consider matrix elements $Q^{\alpha\beta}_{nm,ij}$ that are complex number valued. Q is subject to the constraints

$$Q^2(\mathbf{x})\equiv 1,\mathrm{tr}\,Q(\mathbf{x})\equiv 0. \tag{3.2d}$$

These constraints are conveniently implemented by parametrizing Q in terms of matrices q whose matrix elements, $q^{\alpha\beta}_{nm}$, are restricted to frequency labels $n>0$, $m<0$. In terms of the q, Q can be written in block matrix form

$$Q=\begin{pmatrix}\sqrt{1-qq^{\dagger}}-1 & q\\ q^{\dagger} & -\sqrt{1-q^{\dagger}q}+1\end{pmatrix}. \tag{3.2e}$$

Here the block matrices, clockwise from the upper left, correspond to frequency labels $n,m>0$; $n>0$, $m<0$; $n,m<0$; and $n<0$, $m>0$, respectively.

$\Omega^{\alpha\beta}_{nm}=\delta_{nm}\delta_{\alpha\beta}\Omega_n(\tau_0\otimes s_0)$ in Eq. (3.1) is a frequency matrix with $\Omega_n=2\pi Tn$ being a bosonic Matsubara frequency and T being the temperature. G is a measure of the disorder that is proportional to the bare resistivity, and the frequency coupling $H^{(1)}$ is proportional to the bare density of states at the Fermi level. tr denotes a trace over all discrete degrees of freedom that are not shown explicitly.

The properties of this model are well known.[14,15,1] The bare action describes diffusive electrons, with $D=1/GH^{(1)}$ the diffusion coefficient. Under renormalization, D decreases with increasing disorder until a MIT is reached at a critical disorder value. The critical behavior is known in an ϵ expansion about the lower critical dimension $d=2$. In the absence of the Cooper channel, the MIT appears only at two-loop order at a critical disorder strength of $O(\sqrt{\epsilon})$. $H^{(1)}$, which determines the specific-heat coefficient, the spin susceptibility, and $\partial n/\partial\mu$ is uncritical, which makes this MIT an Anderson transition.

Now we add magnetic annealed disorder to the model. The motivation for this is the fact that the annealed disorder models certain types of local moments, see Secs. I and II above. A technical derivation of this has been given in Ref. 4, and the main idea has been recapitulated in Sec. II A. The annealed disorder implies that the Q in the resulting terms all carry the same replica index;[12] otherwise, the functional form of the resulting additional term in the action can be taken from Ref. 15, which considered the quenched magnetic disorder. From that reference, we have

$$\mathcal{A}^{(1)}_{\mathrm{ann}}=2TJ^{(1)}\int d\mathbf{x}\sum_{\alpha}\sum_{j=1}^{3}\mathrm{tr}\,[(\tau_3\otimes s_j)\,Q^{\alpha\alpha}(\mathbf{x})]^2. \tag{3.3a}$$

The coupling constant $J^{(1)}$ is a measure of the strength of the magnetic disorder. The temperature prefactor in Eq. (3.3a) is a consequence of the static nature of the local moments considered within this model, as has been explained in Ref. 4 and Sec. II above. Equation (3.3a) is the only annealed magnetic disorder term if fluctuations of the matrix field Q on all length scales are taken into account in calculating the partition function. However, the NLσM is an effective theory for long-wavelength fluctuations, and it is therefore convenient to project the annealed disorder term onto this regime as well. It has been discussed in detail in Ref. 16 that this can be achieved by means of a phase-space decomposition and a relabeling of momenta. Applied to Eq. (3.3a), this procedure generates another contribution to the action,

$$\mathcal{A}_{\rm ann}^{(2)} = 2TJ^{(2)} \int d\mathbf{x} \sum_{\alpha} \sum_{j=1}^{3} [\mathrm{tr}\,(\tau_3 \otimes s_j)\, Q^{\alpha\alpha}(\mathbf{x})]^2. \tag{3.3b}$$

The coupling constant $J^{(2)}$ is, in general, independent of $J^{(1)}$. $\mathcal{A}_{\rm ann}^{(1)}$ and $\mathcal{A}_{\rm ann}^{(2)}$ enter the action additively with the understanding that only long-wavelength fluctuations are integrated over in calculating the partition function. Note that in the case of quenched magnetic disorder, a complete phase-space decomposition leads to a term analogous to Eq. (3.3b), but it is zero in the replica limit because the replica sum is then part of the trace.

As we will see, under renormalization the annealed disorder terms generate another contribution to the action that takes the form

$$\mathcal{A}_{\Omega}^{(2)} = 2H^{(2)} \int d\mathbf{x}\, \mathrm{tr}\,[\mathrm{sgn}\,\Omega\; Q(\mathbf{x})], \tag{3.4}$$

so we add this right away. For a discussion on why this term must be present on physical grounds, see Sec. V B.

$$\mathcal{A} = \mathcal{A}_{\rm NL\sigma M} + \mathcal{A}_{\rm ann}^{(1)} + \mathcal{A}_{\rm ann}^{(2)} + \mathcal{A}_{\Omega}^{(2)} \tag{3.5}$$

is the complete action for our model, and the partition function is obtained as the functional integral

$$Z = \int D[Q]\,\delta[Q^2 - 1]\;\; e^{-\mathcal{A}[Q]}. \tag{3.6}$$

B. Annealed disorder as a long ranged interaction

$\mathcal{A} = \mathcal{A}_{\rm NL\sigma M} + \mathcal{A}_{\rm ann}^{(1)}$ defines the model studied in Ref. 4. $\mathcal{A}_{\rm ann}^{(2)}$ was neglected in that reference, but this term will not be of crucial importance in what follows. Terms that appear under renormalization and indicate the appearance of $\mathcal{A}_{\Omega}^{(2)}$ were interpreted differently in Ref. 4, and we will discuss this point in Sec. V B below. A related point is that we have written $\mathcal{A}_{\rm ann}^{(1)}$ in a form that is different from that in Ref. 4. The latter representation was modeled after the way one would treat the quenched disorder, and it added and subtracted a term where all replica indices of Q are not the same. As we will see, this formulation, which is a matter of taste at this point, is rather unnatural at the stage of a RG analysis, and this led to the incorrect RG treatment of the model in Ref. 4. We therefore write the annealed disorder term in a form that is strictly diagonal in the replica index. This replica structure is common to both the annealed disorder term and any electron-electron interaction term, and one would therefore expect the renormalization properties of the current model and one of interacting electrons to have common features. To underscore this point, we rewrite the annealed disorder part of the action by splitting it into spin-singlet and spin-triplet contributions,

$$\mathcal{A}_{\rm ann} \equiv \mathcal{A}_{\rm ann}^{(1)} + \mathcal{A}_{\rm ann}^{(2)} \equiv \mathcal{A}_{\rm ann}^{(1,s)} + \mathcal{A}_{\rm ann}^{(1,t)} + \mathcal{A}_{\rm ann}^{(2,t)}, \tag{3.7a}$$

with

$$\mathcal{A}_{\rm ann}^{(1,s)} = \frac{-\pi T}{4} J^{(1,s)} \sum_{nm} \sum_{\alpha} \sum_{r=0,3} (-)^r \times \mathrm{tr}[(\tau_r \otimes s_0) Q_{nm}^{\alpha\alpha}(\mathbf{x})]\,\mathrm{tr}[(\tau_r \otimes s_0) Q_{mn}^{\alpha\alpha}(\mathbf{x})], \tag{3.7b}$$

$$\mathcal{A}_{\rm ann}^{(1,t)} = \frac{\pi T}{4} J^{(1,t)} \sum_{nm} \sum_{\alpha} \sum_{r=0,3} (-)^r \times \sum_{i=1}^{3} \mathrm{tr}[(\tau_r \otimes s_i) Q_{nm}^{\alpha\alpha}(\mathbf{x})]\,\mathrm{tr}[(\tau_r \otimes s_i) Q_{mn}^{\alpha\alpha}(\mathbf{x})], \tag{3.7c}$$

$$\mathcal{A}_{\rm ann}^{(2,t)} = \frac{-\pi T}{4} J^{(2,t)} \sum_{nm} \sum_{\alpha} \sum_{r=0,3} (-)^r \times \sum_{i=1}^{3} \mathrm{tr}[(\tau_r \otimes s_i) Q_{nn}^{\alpha\alpha}(\mathbf{x})]\,\mathrm{tr}[(\tau_r \otimes s_i) Q_{mm}^{\alpha\alpha}(\mathbf{x})], \tag{3.7d}$$

where we have used Eqs. (3.2a) – (3.2c). Here

$$J^{(2,\mathrm{t})} = 8J^{(2)}/\pi \tag{3.7e}$$

and

$$J^{(1,\mathrm{s})} = -3J^{(1,\mathrm{t})} = -24J^{(1)}/\pi. \tag{3.7f}$$

This relation between the bare values of $J^{(1,\mathrm{s})}$ and $J^{(1,\mathrm{t})}$ will be important later. Notice that $J^{(1,\mathrm{s})} < 0$, while $J^{(1,\mathrm{t})} > 0$, $J^{(2,\mathrm{t})} > 0$.

Comparing these expression to the correspoding ones for an electron-electron interaction,[16] one sees that they have the same structure except for the frequency sector. Transforming from Matsubara frequency space into time space reveals that the annealed disorder corresponds to an interaction that is infinitely long ranged in time. This is physically plausible, as has been explained in Sec. II B.

C. Renormalizability considerations

For reasons explained in Appendixes A and B, we will choose a field-theoretic RG method[9] over a momentum-shell RG.[17] Before we start analyzing our model by means of this method, we need to ask whether the model is renormalizable, and how many renormalization constants are required. Much is known about the renormalization properties of the NLσM, Eq. (3.1), with additional *instantaneous* interaction terms. The pure NLσM is known to be renormalizable with two renormalization constants, one for the coupling constant G and one field renormalization constant.[9] The frequency coupling $H^{(1)}$ turns out not to carry a renormalization constant of its own. In the presence of an instantaneous interaction, the proof of renormalizability for the NLσM breaks down, and the renormalizability of the model has never been proven. However, there is much evidence that the model is still renormalizable, with two additional renormalization constants for the interaction, and with $H^{(1)}$ acquiring a renormalization constant of its own. The two renormalization constants for the interaction terms correspond to symmetric and

antisymmetric combinations of terms bilinear in Q, respectively.[18,19] The same arguments apply to the present model, and are given in Appendix B. From Eqs. (B1), we conclude that we need to write $\mathcal{A}^{(1,s)}_{\rm ann}=\mathcal{A}^{(1,s)}_{+}+\mathcal{A}^{(1,s)}_{-}$, and analogously split $\mathcal{A}^{(1,t)}_{\rm ann}$ and $\mathcal{A}^{(2,t)}_{\rm ann}$, with

$$\mathcal{A}^{(1,s)}_{+}=-2\pi TJ^{(1,s)}_{+}\int d\mathbf{x}\sum_{nm}\sum_{\alpha}\sum_{r=0,3}\left[{}^{0}_{r}Q^{\alpha\alpha}_{nm}(\mathbf{x})\times{}^{0}_{r}Q^{\alpha\alpha}_{nm}(\mathbf{x})+\frac{1}{2}\sum_{i}{}^{i}_{r}Q^{\alpha\alpha}_{nn}(\mathbf{x}){}^{i}_{r}Q^{\alpha\alpha}_{mm}(\mathbf{x})\right], \tag{3.8a}$$

$$\mathcal{A}^{(1,s)}_{-}=-2\pi TJ^{(1,s)}_{-}\int d\mathbf{x}\sum_{nm}\sum_{\alpha}\sum_{r=0,3}\left[{}^{0}_{r}Q^{\alpha\alpha}_{nm}(\mathbf{x})\times{}^{0}_{r}Q^{\alpha\alpha}_{nm}(\mathbf{x})-\frac{1}{2}\sum_{i}{}^{i}_{r}Q^{\alpha\alpha}_{nn}(\mathbf{x}){}^{i}_{r}Q^{\alpha\alpha}_{mm}(\mathbf{x})\right], \tag{3.8b}$$

$$\mathcal{A}^{(1,t)}_{+}=-2\pi TJ^{(1,t)}_{+}\int d\mathbf{x}\sum_{nm}\sum_{\alpha}\sum_{r=0,3}\left[\sum_{i=1}^{3}{}^{i}_{r}Q^{\alpha\alpha}_{nm}(\mathbf{x})\times{}^{i}_{r}Q^{\alpha\alpha}_{nm}(\mathbf{x})+\frac{1}{2}\sum_{i}\begin{pmatrix}3\\-\\-\\-\end{pmatrix}_{i}{}^{i}_{r}Q^{\alpha\alpha}_{nn}(\mathbf{x}){}^{i}_{r}Q^{\alpha\alpha}_{mm}(\mathbf{x})\right], \tag{3.8c}$$

$$\mathcal{A}^{(1,t)}_{-}=-2\pi TJ^{(1,t)}_{-}\int d\mathbf{x}\sum_{nm}\sum_{\alpha}\sum_{r=0,3}\left[\sum_{i=1}^{3}{}^{i}_{r}Q^{\alpha\alpha}_{nm}(\mathbf{x})\times{}^{i}_{r}Q^{\alpha\alpha}_{nm}(\mathbf{x})-\frac{1}{2}\sum_{i}\begin{pmatrix}3\\-\\-\\-\end{pmatrix}_{i}{}^{i}_{r}Q^{\alpha\alpha}_{nn}(\mathbf{x}){}^{i}_{r}Q^{\alpha\alpha}_{mm}(\mathbf{x})\right], \tag{3.8d}$$

$$\mathcal{A}^{(2,t)}_{+}=2\pi TJ^{(2,t)}_{+}\int d\mathbf{x}\sum_{nm}\sum_{\alpha}\sum_{r=0,3}\left[\sum_{i=1}^{3}{}^{i}_{r}Q^{\alpha\alpha}_{nn}(\mathbf{x})\times{}^{i}_{r}Q^{\alpha\alpha}_{mm}(\mathbf{x})+\frac{1}{2}\sum_{i}\begin{pmatrix}3\\-\\-\\-\end{pmatrix}_{i}{}^{i}_{r}Q^{\alpha\alpha}_{nm}(\mathbf{x}){}^{i}_{r}Q^{\alpha\alpha}_{nm}(\mathbf{x})\right], \tag{3.8e}$$

$$\mathcal{A}^{(2,t)}_{-}=2\pi TJ^{(2,t)}_{-}\int d\mathbf{x}\sum_{nm}\sum_{\alpha}\sum_{r=0,3}\left[\sum_{i=1}^{3}{}^{i}_{r}Q^{\alpha\alpha}_{nn}(\mathbf{x})\times{}^{i}_{r}Q^{\alpha\alpha}_{mm}(\mathbf{x})-\frac{1}{2}\sum_{i}\begin{pmatrix}3\\-\\-\\-\end{pmatrix}_{i}{}^{i}_{r}Q^{\alpha\alpha}_{nm}(\mathbf{x}){}^{i}_{r}Q^{\alpha\alpha}_{nm}(\mathbf{x})\right]. \tag{3.8f}$$

In writing Eqs. (3.8) we have made use of Eqs. (3.2a) – (3.2c). The symbol

$$\begin{pmatrix}3\\-\\-\\-\end{pmatrix}_{i}$$

is a shorthand for $3\delta_{i0}-\Sigma_{j=1}^{3}\delta_{ij}$. The $J^{(1,s)}_{\pm}$ are coupling constants whose bare values are equal,

$$J^{(1,s)}_{+}=J^{(1,s)}_{-}=J^{(1,s)}, \tag{3.9a}$$

but in general they renormalize differently. Similarly,

$$J^{(1,t)}_{+}=J^{(1,t)}_{-}=J^{(1,t)}, \tag{3.9b}$$

$$J^{(2,t)}_{+}=J^{(2,t)}_{-}=J^{(2,t)} \tag{3.9c}$$

in the bare theory, but under renormalization these equalities, in general, do not remain valid. All of the J_{+} require only one renormalization constant, which we will denote by Z_{+}, and the J_{-} require another one, Z_{-}. In addition, a renormalization constant for $H^{(2)}$ is needed.

In addition to the relations given by Eqs. (3.9), there is the relation between $J^{(1,s)}$ and $J^{(1,t)}$ given by Eq. (3.7f). It will turn out that these constraints leads to a degeneracy in the RG flow. This is most easily handled by relaxing the condition (3.9a). Instead of Eqs. (3.9) and (3.7f) we therefore write

$$J^{(1,s)}_{\pm}=J^{(1,s)}\pm\Delta, \tag{3.10a}$$

$$J^{(1,t)}_{\pm}=J^{(1,t)}, \tag{3.10b}$$

$$J^{(2,t)}_{\pm}=J^{(2,t)}, \tag{3.10c}$$

and

$$J^{(1,s)}+3J^{(1,t)}=0. \tag{3.10d}$$

Choosing $\Delta\neq 0$ will remove the degeneracy in the RG flow. In the end, we will consider the limit $\Delta\to 0$ to obtain the behavior of our original model.

IV. RENORMALIZATION TO ONE-LOOP ORDER

A. Perturbation theory

1. Gaussian propagators

We now perform a one-loop RG analysis of the model defined in Sec. III. To this end, we expand the action in powers of the matrix q defined by Eq. (3.2e). To Gaussian order we find

$$\mathcal{A}=\frac{-4}{G}\frac{1}{V}\sum_{\mathbf{p}}\sum_{12}\sum_{i,r}{}^{i}_{r}q_{12}(\mathbf{p})\,\Gamma^{(2)}(\mathbf{p},\Omega_{n_1-n_2})\,{}^{i}_{r}q_{12}(-\mathbf{p}), \tag{4.1a}$$

with

$$\Gamma^{(2)}(\mathbf{k},\Omega_n)=\mathbf{k}^2/G+H^{(1)}\Omega_n+H^{(2)}2\pi T + \delta_{\alpha_1\alpha_2}\pi TG\,[\,\delta_{i0}J_{\mathrm{s}}+(1-\delta_{i0})J_{\mathrm{t}}] \tag{4.1b}$$

being the bare two-point vertex. The Gaussian q propagators are obtained by inverting this quadratic form. We find

$$\langle {}^{i}_{r}q_{12}(\mathbf{k})\,{}^{j}_{s}q_{34}(\mathbf{p})\rangle=\delta_{\mathbf{k},-\mathbf{p}}\delta_{13}\,\delta_{24}\,\delta_{rs}\,\delta_{ij}\frac{G}{8}\,{}^{i}\mathcal{D}_{12}(\mathbf{k}). \tag{4.2a}$$

Here $\langle\cdots\rangle$ denotes a Gaussian average, and $1\equiv(n_1,\alpha_1)$, etc., are indices that comprise both the Matsubara frequency index and the replica label. The propagators ${}^{i}\mathcal{D}$ read

$${}^{0}\mathcal{D}_{12}(\mathbf{k})=\mathcal{D}_{n_1-n_2}(\mathbf{k})+\delta_{\alpha_1\alpha_2}\;\Delta\mathcal{D}^{\mathrm{s}}_{n_1-n_2}(\mathbf{k}), \tag{4.2b}$$

$${}^{1,3.3}\mathcal{D}_{12}(\mathbf{k})=\mathcal{D}_{n_1-n_2}(\mathbf{k})+\delta_{\alpha_1\alpha_2}\;\Delta\mathcal{D}^{\mathrm{t}}_{n_1-n_2}(\mathbf{k}), \tag{4.2c}$$

where

$$\mathcal{D}_n(\mathbf{k})=\frac{1}{\mathbf{k}^2+GH^{(1)}\Omega_n+GH^{(2)}2\pi T}, \tag{4.2d}$$

$$\mathcal{D}^{\mathrm{s,t}}_n(\mathbf{k})=\frac{1}{\mathbf{k}^2+GH^{(1)}\Omega_n+GH^{(2)}2\pi T+GJ_{\mathrm{s,t}}2\pi T}, \tag{4.2e}$$

with

$$\Delta\mathcal{D}^{\mathrm{s,t}}_n(\mathbf{k})=\mathcal{D}^{\mathrm{s,t}}_n(\mathbf{k})-\mathcal{D}_n(\mathbf{k}), \tag{4.2f}$$

and

$$J_{\mathrm{s}}=\frac{1}{2}\,(J^{(1,\mathrm{s})}_{+}+J^{(1,\mathrm{s})}_{-})-\frac{3}{4}\,(J^{(2,\mathrm{t})}_{+}-J^{(2,\mathrm{t})}_{-}), \tag{4.2g}$$

$$J_{\mathrm{t}}=\frac{1}{2}\,(J^{(1,\mathrm{t})}_{+}+J^{(1,\mathrm{t})}_{-})+\frac{1}{2}\,(J^{(2,\mathrm{t})}_{+}-J^{(2,\mathrm{t})}_{-}). \tag{4.2h}$$

2. One-loop corrections

By expanding the action to $O(q^4)$ and calculating all diagrams with the topological structure shown in Fig. 1, we obtain the one-loop corrections δG, $\delta H^{(1)}$, etc. to the coupling constants in the Gaussian propagators, Eqs. (4.2), or the two-point vertex, Eq. (4.1b). The explicit calculation is similar to the one for the case of an instantaneous interaction,[1] but substantially simpler due to the absence of cubic terms in the q expansion. We find

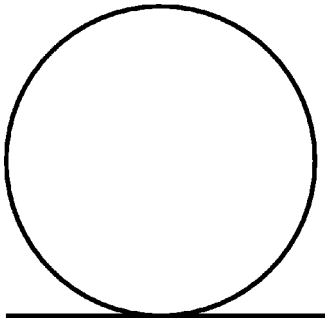

FIG. 1. Structure of diagrams that renormalize the two-point vertex.

$$\delta G=\frac{G^2}{16}\,(K_{+}+K_{-})\,I_2, \tag{4.3a}$$

$$\delta H^{(1)}=\frac{-GH^{(1)}}{16}\,(K_{+}+K_{-})\,I_2, \tag{4.3b}$$

$$\delta H^{(2)}=\frac{-G}{16}\Big[H^{(2)}(K_{+}+K_{-})+\frac{3}{2}\Big(J^{(1,\mathrm{t})}_{+}+J^{(1,\mathrm{t})}_{-}+\frac{1}{2}J^{(2,\mathrm{t})}_{+} -\frac{1}{2}J^{(2,\mathrm{t})}_{-}\Big)(L_{+}+L_{-}-2J^{(2,\mathrm{t})}_{+}+2J^{(2,\mathrm{t})}_{-})\Big]I_2 -\frac{3G}{16}\Big(J^{(2,\mathrm{t})}_{+}+J^{(2,\mathrm{t})}_{-}-\frac{1}{2}L_{+}+\frac{1}{2}L_{-}\Big)\,I_1 +\frac{G}{32}(K_{+}-K_{-})\,I_1, \tag{4.3c}$$

$$\delta J_{\mathrm{s}}=\frac{-G}{8}(J^2_{\mathrm{s}}+3J^2_{\mathrm{t}})\,I_2 +\frac{3G}{16}\Big(J^{(2,\mathrm{t})}_{+}+J^{(2,\mathrm{t})}_{-}-\frac{1}{2}L_{+}+\frac{1}{2}L_{-}\Big)\,I_1, \tag{4.3d}$$

$$\delta J_{\mathrm{t}}=\frac{-G}{16}\Big(J^{(1,\mathrm{t})}_{+}+J^{(1,\mathrm{t})}_{-}+\frac{1}{2}J^{(2,\mathrm{t})}_{+}-\frac{1}{2}J^{(2,\mathrm{t})}_{-}\Big) \times(J^{(1,\mathrm{s})}_{+}+J^{(1,\mathrm{t})}_{+}+J^{(1,\mathrm{s})}_{-}+J^{(1,\mathrm{t})}_{-}-J^{(2,\mathrm{t})}_{+}+J^{(2,\mathrm{t})}_{-})I_2 -\frac{G}{16}\Big(J^{(2,\mathrm{t})}_{+}+J^{(2,\mathrm{t})}_{-}-\frac{1}{2}L_{+}+\frac{1}{2}L_{-}\Big)\,I_1. \tag{4.3e}$$

Here we have defined linear combinations of coupling constants,

$$K_{\pm}=J^{(1,\mathrm{s})}_{\pm}+3J^{(1,\mathrm{t})}_{\pm}=\pm\Delta, \tag{4.4a}$$

$$L_{\pm}=J^{(1,\mathrm{s})}_{\pm}-J^{(1,\mathrm{t})}_{\pm}, \tag{4.4b}$$

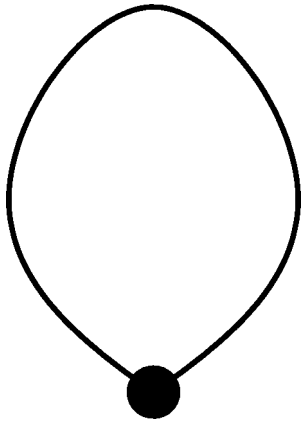

FIG. 2. Structure of diagrams that renormalize the one-point vertex.

where the second equality in Eq. (4.4a) is due to Eqs. (3.10). This will be important later. We have also defined one-loop integrals

$$I_1 = G \int d\mathbf{p}\, \mathcal{D}_n(\mathbf{p}) = -\overline{G}/G\epsilon, \tag{4.5a}$$

$$I_2 = G \int d\mathbf{p}\, 2\pi T \sum_n (\mathcal{D}_n(\mathbf{p}))^2 = -\overline{G}/G H^{(1)} \epsilon. \tag{4.5b}$$

Here $\epsilon = d-2$, and $\overline{G} = G S_d/(2\pi)^d$ with S_d the surface area of the $(d-1)$ sphere. In giving the second equalities in Eqs. (4.5) we have chosen to use dimensional regularization, and in what follows we will use a field-theoretic RG method. At a perturbative level, this is a matter of choice, and we could just as well use the momentum-shell RG method. In that case, the factors of $-1/\epsilon$ in Eqs. (4.5) would be replaced by $\ln b$, with b the RG length rescaling factor. For arguments that go beyond perturbation theory, however, it is advantageous to use the field-theory approach, as is explained in the Appendixes A and B.

In addition to these renormalizations of the two-point propagator or vertex function, we will also need the one-point vertex $\Gamma^{(1)}$ to one-loop order. This is given by the diagram shown in Fig. 2, and a simple calculation yields

$$\Gamma^{(1)} \equiv \langle {}^0_0 Q^{\alpha\alpha}_{nn}(\mathbf{x}) \rangle^{-1} = 1 - \frac{G}{16}(K_+ + K_-)\, I_2 . \tag{4.6}$$

For later reference, we notice that the one-loop corrections to G, $H^{(1)}$, and $\Gamma^{(1)}$ vanish in the limit $\Delta \to 0$, and that

$$\delta J_s + 3\,\delta J_t = 0, \tag{4.7}$$

as can be seen by using Eqs. (3.10). Furthermore, the calculation shows that

$$\delta H^{(1)} + \delta H^{(2)} + \delta J_s = \frac{G}{16} \Delta\, I_1 , \tag{4.8}$$

which also vanishes as $\Delta \to 0$.

B. Renormalization

1. Renormalization constants

We now proceed to renormalize the theory, i.e., we absorb the singularities in the $\epsilon \to 0$ limit that are present in the perturbation theory into renormalization constants. We define renormalized coupling constants g, $h^{(1)}$, etc., by

$$\begin{aligned} &\overline{G} = \mu^{-\epsilon} Z_g g, \quad H^{(1)} = Z_h^{(1)} h^{(1)}, \quad H^{(2)} = Z_h^{(2)} h^{(2)}, \\ &J_+^{(1,s)} = Z_+ j_+^{(1,s)}, \quad J_+^{(1,t)} = Z_+ j_+^{(1,t)}, \quad J_+^{(2,t)} = Z_+ j_+^{(2,t)}, \\ &J_-^{(1,s)} = Z_- j_-^{(1,s)}, \quad J_-^{(1,t)} = Z_- j_-^{(1,t)}, \quad J_-^{(2,t)} = Z_- j_-^{(2,t)}, \end{aligned} \tag{4.9}$$

where μ is an arbitrary momentum scale. The renormalization statement is[9]

$$\begin{aligned} &\Gamma_R^{(N)}(\mathbf{p}, \Omega_n; g, h, j_+, j_-; \mu) \\ &\quad = Z^{(N/2)} \Gamma^{(N)}(\mathbf{p}, \Omega_n; G, H, J_+, J_-). \end{aligned} \tag{4.10}$$

Here $\Gamma_R^{(N)}$ is the renormalized N-point vertex function, Z is the field renormalization constant, and H and $J_\pm$ represent the various frequency and annealed disorder coupling constants. The assertion that *all* vertex functions can be made finite to all orders in the loop expansion by the five renormalization constants defined in Eq. (4.2), plus the field renormalization constant, is equivalent to saying that the theory is renormalizable with these renormalization constants. As we have mentioned before, there is strong evidence for this statement to be true, which is recapitulated in Appendix B, but it has not been rigorously proven.

Assuming that the theory is renormalizable, the six equations, Eqs. (4.3) and (4.6), suffice to determine the six renormalization constants to one-loop order. While it is possible to do so for arbitrary bare values of the coupling constants, the results simplify substantially if one uses Eqs. (3.10). Using minimal subtraction,[9] and taking the limit $\Delta \to 0$, we obtain

$$Z = 1 + O(g^2), \tag{4.11a}$$

$$Z_g = 1 + O(g^2), \tag{4.11b}$$

$$Z_h^{(1)} = 1 + O(g^2), \tag{4.11c}$$

$$Z_h^{(2)} = 1 + \frac{g}{\epsilon} \kappa(g, h, j_+, j_-)/h^{(2)}, \tag{4.11d}$$

$$Z_+ = 1 + \frac{g\, 2\phi_s(g, h, j_+, j_-)}{\epsilon \quad j_+^{(1,s)} + j_-^{(1,s)}} + O(g^2), \tag{4.11e}$$

$$Z_- = 1 + \frac{g\, 2\phi_s(g, h, j_+, j_-)}{\epsilon \quad j_+^{(1,s)} + j_-^{(1,s)}} + O(g^2). \tag{4.11f}$$

Here κ and $\phi_{s,t}$ are functions of the renormalized coupling constants that are given by $\delta H^{(2)}$ and $\delta J_{s,t}$ as functions of the bare ones,

$$\kappa(G, H, J) = -\epsilon\, \delta H^{(2)}(G, H, J)/G, \tag{4.12a}$$

$$\phi_{s,t}(G,H,J) = -\epsilon\, \delta J_{s,t}(G,H,J)/G. \tag{4.12b}$$

An inspection shows that, in the limit $\Delta \to 0$,

$$\kappa(G,H,J) = -\phi_{s,t}(G,H,J). \tag{4.12c}$$

Notice that $Z_+ = Z_-$, at least to one-loop order. Since the bare values of the various $J_\pm$ are identical, this means that the renormalized values are also identical, and we can drop the distinction between j_+ and j_-. We will thus write $j_+^{(1,s)} = j_-^{(1,s)} \equiv j^{(1,s)}$, etc. We note that this is a consequence of the relations expressed by Eqs. (3.10), and would not necessarily be true for more general models.

2. Flow equations and their solutions

We are now in a position to determine the RG flow equations for the coupling constants. Defining $l = -\ln \mu$ (or $l = \ln b$ in an alternative momentum-shell approach), and using Eq. (4.12c), we obtain from Eqs. (4.9) and (4.11)

$$\frac{dg}{dl} = -\epsilon g + O(g^3), \tag{4.13a}$$

$$\frac{dh^{(1)}}{dl} = O(g^2), \tag{4.13b}$$

$$\frac{dh^{(2)}}{dl} = -g\, \phi_s(g,h,j) + O(g^2), \tag{4.13c}$$

$$\frac{dj^{(1,t)}}{dl} = \frac{-g}{3}\, \phi_s(g,h,j) + O(g^2). \tag{4.13d}$$

The flow of the remaining coupling constants j can be obtained by relating them to $j^{(1,t)}$. This is a consequence of there being only two renormalization constants for all of the J. We obtain

$$j^{(1,s)} = j^{(1,t)}\, J^{(1,s)}/J^{(1,t)} = -3\, j^{(1,t)}, \tag{4.13e}$$

$$j^{(2,t)} = \frac{J^{(2,t)}}{J^{(1,t)}} j^{(1,t)}. \tag{4.13f}$$

In order to determine the nature of these flows, we calculate ϕ_s from Eqs. (4.3d) and (4.12b). We find

$$\phi_s(g,h,j) = \frac{-3\,(j^{(1,t)})^2}{2\quad h^{(1)}} \left[1 - \frac{j^{(2,t)} h^{(1)}}{4 (j^{(1,t)})^2} \right] + O(g). \tag{4.14}$$

We see that $\phi_s < 0$, unless $J^{(2,t)}$ is larger than $(J^{(1,t)})^2$ in suitable units (note that the J's and H's all have the dimensions of a density of states). This makes physical sense: From Eqs. (3.7c) and (3.7d) we see that $\mathcal{A}^{(1,t)}$ and $\mathcal{A}^{(2,t)}$ are spin-triplet interactions with different signs. $J^{(1,t)} > 0$ promotes ferromagnetism, and $J^{(2,t)} > 0$ weakens that tendency. In two dimensions, for physically sensible values of the coupling constants, we thus have $\phi_s < 0$, and $h^{(2)}$ and $j^{(1,t)}$ both scale to infinity. In $d > 2$, the RG flow equations can be solved explicitly and shown to describe a quantum phase-transition by introducing, as in Ref. 10, a scaling variable $y = g j^{(1,t)}/h^{(1)}$ that obeys

$$\frac{dy}{dl} = -\epsilon y + y^2/2 + O(y^3). \tag{4.15}$$

We see that Eq. (4.15) allows for a fixed-point value $y^* = \epsilon + O(\epsilon^2)$. Denoting the deviation from this fixed-point value by δy, we find

$$\delta y(b) = \delta y(b=1)\, b^{\,\epsilon + O(\epsilon^2)}, \tag{4.16a}$$

and

$$h^{(2)}(b) = h^{(2)}(b=1)\, b^{\,\epsilon + O(\epsilon^2)}, \tag{4.16b}$$

$$j^{(1,t)}(b) = j^{(1,t)}(b=1)\, b^{\,\epsilon + O(\epsilon^2)}, \tag{4.16c}$$

$$h^{(1)}(b) = h^{(1)}(b=1)\, b^{0 + O(\epsilon^2)}, \tag{4.16d}$$

$$g(b) = g(b=1)\, b^{-\epsilon + O(\epsilon^2)}. \tag{4.16e}$$

The behavior of all observables of interest can be deduced from the above flows, see Sec. V A below.

V. DISCUSSION

A. Physical interpretation and results

For a physical interpretation of our results we first need to relate physical observables to the coupling constants of our theory. Some observables can be identified directly in analogy to the corresponding identification in the case of an instantaneous electron-electron interaction. From the derivation of the NLσM, G is known to be related to the bare conductivity σ via[14,1]

$$\sigma = 8/\pi G. \tag{5.1a}$$

The single-particle or tunneling density of states N at an energy ω from the Fermi level is related to the one-point vertex by[13]

$$N(\epsilon_F + \omega) = \frac{4}{\pi} (\Gamma^{(1)})^{-1} (i\omega_n \to \omega + i0). \tag{5.1b}$$

Equations (4.13a) and (4.11a) show that σ and N are not renormalized, at least to one-loop order,

$$\frac{d\sigma}{dl} = O(g^2), \tag{5.2a}$$

$$\frac{dN}{dl} = O(g^2). \tag{5.2b}$$

The scaling behavior of the relevant operator δy, Eq. (4.16a), determines the correlation length exponent. Denoting the dimensionless distance from the critical point by t, and the correlation length by ξ, one finds for small t

$$\xi \propto |t|^{-\nu}, \tag{5.3a}$$

with a correlation length exponent

$$\nu = 1/\epsilon + O(1). \tag{5.3b}$$

Other quantities of interest are various susceptibilities, in particular the specific heat coefficient $\gamma_V = C_V/T$, the spin susceptibility χ_s, and the density susceptibility $\partial n/\partial\mu$. Their relations to the coupling constants in the field theory are less obvious. We therefore use scaling arguments, in conjunction with the perturbation theory for the free energy, to determine their respective cricital behavior. We start with a homogeneity law for the free energy. From the Gaussian propagators, Eqs. (4.2), we see that, that in principle, there are three different time scales in the theory, given by

$$\tau_1 = \xi^d g h^{(1)} \sim \xi^{z_1}, \tag{5.4a}$$

$$\tau_2 = \xi^d g h^{(2)} \sim \xi^{z_2}, \tag{5.4b}$$

$$\tau_3 = \xi^d g j^{(1,t)} \sim \xi^{z_3}. \tag{5.4c}$$

Here $z_{1,2,3}$ are the dynamical exponents related to these time scales. To one-loop order we have

$$z_1 = d - \epsilon + O(\epsilon^2) = 2 + O(\epsilon^2), \tag{5.5a}$$

$$z_2 = z_3 = d - \epsilon + \epsilon + O(\epsilon^2) = d + O(\epsilon^2) \tag{5.5b}$$

leaving us with two times scales and dynamical exponents. The free energy density f therefore has two different scaling parts, and we can write

$$f(t, T, \ldots) = b^{-(d+z_1)} f_1(t\, b^{1/\nu}, T\, b^{z_1}, T\, b^{z_2}, \ldots) + b^{-(d+z_2)} f_2(t\, b^{1/\nu}, T\, b^{z_1}, T\, b^{z_2}, \ldots). \tag{5.6}$$

Here f_1 and f_2 are scaling functions, and the ellipses denote the dependence of f on external fields that are not shown explicitly.

The specific-heat coefficient is obtained by differenting f twice with respect to T. The leading contribution is obtained by differentiating f_1 with respect to the temperature scale that carries the dynamical exponent z_2. This yields

$$\gamma_V(t) \sim |t|^{-\alpha}, \tag{5.7a}$$

with a critical exponent

$$\alpha = \nu(2z_2 - d - z_1) = 1 + O(\epsilon). \tag{5.7b}$$

To ascertain that this leading contribution has a nonzero prefactor we check against the perturbation theory for the free energy, which is given in Appendix D. From Eqs. (D1b) and (D1c) we see that there is indeed a contribution from differentiating twice with respect to the temperature in the propagators, which carries a dynamical exponent z_2. The temperature prefactor in the expression $f = -(T/V)\ln Z$ for the free energy density has been absorbed into the frequency integration measure. The frequency, however, scales as a wave number squared, and therefore carries an exponent z_1.

A very similar argument applies to the spin susceptibility. A magnetic field B couples to the electrons via a Zeeman term (amongst other coupling mechanisms), and hence can scale as energy or temperature. The spin susceptibility is obtained by differentiating f twice with respect to B, and once therefore expects χ_s to scale as the specific-heat coefficient, viz.

$$\chi_s(t) \sim |t|^{-\gamma}, \tag{5.8a}$$

with a critical exponent

$$\gamma = \alpha = 1 + O(\epsilon). \tag{5.8b}$$

Again, the perturbation theory confirms that the leading contribution obtained in this way is nonzero. This is easily seen from Eqs. (D1b) and (D1c) by taking into account that $B \neq 0$ leads to a mass $\mu_B B$ in two of the spin-triplet propagators that contribute to the Gaussian approximation for the free energy.

Finally, we consider $\partial n/\partial\mu$. Although the chemical potential μ has the dimensionality of energy, it differs fundamentally from either T or $\mu_B B$, since it represents the microscopic energy or inverse time scale. As such, it must have an effective scale dimension of zero. Consequently, we obtain from Eq. (5.6), by differentiating twice with respect to μ,

$$(\partial n/\partial\mu)(t) = \text{const} + O(t^{\nu(d+z_1)}). \tag{5.9}$$

$\partial n/\partial\mu$ thus has only a weak nonanalytic t dependence in addition to a leading noncritical contribution. Again, this is consistent with perturbation theory: The only μ dependence of the free energy, Eq. (D1b), is through the various coupling constants in the propagators. All of these multiply either a frequency or a temperature. Differentiation with respect to μ therefore does not produce a singular integral unless f itself becomes singular. Power counting shows that this happens only for dimensions $d \leqslant -2$, in agreement with Eq. (5.9). This failure of differentiation with respect to a field to produce a singularity is an illustration of a more general argument given in Ref. 6.

The physical interpretation of these results is now clear. The RG flow at one-loop order is qualitatively the same as for electrons interacting via an instantaneous interaction, see the comparison between the two flows given in Appendix C. In the latter case, the runaway flow of the equivalent of $j^{(1,t)}$ (k_t in Appendix C) at one-loop order in $d=2$ suggests a ferromagnetic ground state. In $d = 2 + \epsilon$ there is a phase-transition where the homogeneous magnetic susceptibility diverges. This transition has been identified with a ferromagnetic phase transition where the magnetic susceptibility diverges like $\chi_s \sim |t|^{-\gamma}$, as in Eq. (5.8a).[10,20] The runaway flow thus simply reflects the fact that t is RG relevant at a ferromagnetic transition. The result of this interpretation agrees with a more direct, and more explicit, theory for the ferromagnetic transition.[20] In the current case, the theory describes an infinite-range version of this transition, due to the interaction being infinitely long-ranged in time. These considerations strongly suggest that the physical results we have derived above to one-loop order actually hold to all orders in the loop expansion, as they do in the instantaneous interaction case.[10] In particular, we expect that $\partial n/\partial\mu$ is not renor-

malized to all orders, in agreement with Ref. 6. It also follows that the phase diagram for the present model is qualitatively similar to the one for the interacting case, with a ferromagnetic transition *always* preceding a MIT for $d \gtrsim 2$, while for $d=3$ a direct transition from a paramagnetic metal to a paramagnetic insulator is possible.[10,1] There are, however, differences in the detailed properties of the transition as compared to the one studied in Refs. 20 and 10. For instance, in the latter the specific heat has a much weaker singularity than the spin susceptibility, while here they show the same scaling behavior. In this respect the current case is reminiscient of the Brinkman-Rice theory of the Hubbard MIT.[21]

Although the transition in the present model is classical, in the sense that the order parameter is purely static, it couples to quantum-mechanical degrees of freedom in the form of the diffusive electrons. An explicit description of the transition could be obtained along the lines of Ref. 20. However, given the schematic nature of our model, we will not pursue this here. The same conclusion, namely, that the model under consideration describes a ferromagnetic transition of a classical nature, has recently been reached by Vojta and Narayanan by means of very different arguments.[22] We stress, however, that our goal here has not been to describe a magnetic transition. Rather, it was to resolve the conflict between the results of Refs. 4 and 6, and to check whether or not our model of electrons with both quenched and annealed disorders describes an unusual MIT. As we have seen, the answer to the latter question is negative.

B. Comparison with previous treatments

The crucial difference between the treatment of the annealed disorder model given above and the one in Ref. 4 is related to the occurrence of the coupling constant $H^{(2)}$. In the perturbation theory, i.e., in an expansion in powers of q, the annealed disorder generates terms that have the structure of the last term on the right-hand side of Eq. (4.1b), except that they are not constrained to being diagonal in the replica index. There are two possible interpretations of such terms. (1) They could represent terms quadratic in Q that are not diagonal in replica space. This was the interpretation given in Ref. 4. (2) They could present a new term *linear* in Q, which was not present in the original action. The term with coupling constant $H^{(2)}$ introduced in the present paper serves that purpose. By means of high-order perturbation theory one can, in principle, distinguish between these two possibilities, but this would be extremely cumbersome. Let us instead argue on general structural and physical grounds that the second interpretation is the correct one.

First, we have argued in Sec. II that the annealed disorder, since it gets averaged over at the level of the partition function, should indeed be interpreted as an effective interaction between the electrons. As such, all involved degrees of freedom must occur with the same replica index, and the generation of an interaction term (i.e., one quadratic in Q) for which this is not the case makes no physical sense. Terms quadratic in Q with more than one replica index are characteristic for *quenched* disorder, and indeed the treatment of the annealed disorder in Ref. 4 was modeled after that of quenched magnetic disorder. As we have argued above, this is physically not plausible.

Second, the appearance of a term with the structure of $\mathcal{A}_{\Omega}^{(2)}$, Eq. (3.4), is plausible on physical grounds. The term with coupling constant $H^{(1)}$ in the NLσM represents a frequency coupling with a microscopic time scale, on the order of an inverse Fermi energy (in units where $\hbar=1$). An interaction that is short ranged in time does not add a new time scale to the problem. It therefore renormalizes $H^{(1)}$, but does not generate a new frequency coupling. An interaction that is long ranged in time, on the other hand, does introduce a new time scale and hence a new frequency coupling. In the general case of a frequency-dependent interaction with a continuum of time scales, one would expect a frequency-dependent coupling constant H whose scaling properties would have to be studied by means of a functional RG. In our simple model where the annealed disorder is static, which means that the resulting effective interaction has an infinite range in time, one additional frequency coupling suffices, which is $H^{(2)}$. The infinite-time scale corresponds to a vanishing frequency scale, in accord with the discontinuous frequency dependence $\mathrm{sgn}\,\Omega$ in Eq. (3.4). In this context, we note that the $H^{(2)}$ term does not represent an inelastic lifetime. Rather, it is a true mass in the two-point propagators that is produced by the interaction long ranged in time. This is analogous to the mass corresponding to the plasmon pole that is produced by an interaction that is long ranged in space.

Third, the structure of the renormalization scheme used in Ref. 4 did not reflect the constraints discussed in Appendix B. This is only of minor concern if one neglects the coupling constant $J^{(2,t)}$ and uses one renormalization constant each for J_s and J_t, as was done in Ref. 4. It becomes crucial, however, in the presence of $J^{(2,t)}$, which forces the issue of how many renormalization constants are needed.

Finally, the treatment of Ref. 4 led to results that were not consistent with independent, very general, considerations. In particular, its prediction that $\partial n/\partial\mu$ is singularly renormalized, and critical at a MIT, contradicted one of the results of Ref. 6. This point requires some explanation. The critical behavior predicted implies a nonanalytic dependence on the RG length scale, and hence a nonanalytic dependence on the wave number $|\mathbf{q}|$ in perturbation theory. In two dimensions, this takes the form of a $\ln|\mathbf{q}|$ term in perturbation theory that is caused by the diffusive electron dynamics. In $d>2$, these same integrals over diffusion poles lead to a $|\mathbf{q}|^{d-2}$ dependence.[8] The predicted critical behavior of $\partial n/\partial\mu$ at the MIT, and the mechanism that causes it, therefore implies a nonanalytic wave number dependence of this susceptibility in the metallic phase. However, it was shown on general grounds in Ref. 6 that $\partial n/\partial\mu$ is an analytic function of the wave number for a large class of models, which includes the one under consideration here. This discrepancy prompted the current investigation, see the discussion in Sec. I above.

C. Conclusion and outlook

In conclusion, we have found that the treatment in Ref. 4 of the electron problem in the presence of annealed disorder,

in addition to quenched one, was not correct. The perturbation theory was correct, but the assumptions made about the RG structure of the theory were not. This was the reason for the discrepancy between the explicit results found in Ref. 4 and later, more general considerations.[6] The current procedure, which considers the annealed disorder as an effective electron-electron interaction that is long ranged in time, is physically and technically more convincing. It yields results that are consistent with all the available information, and in particular with Ref. 6. Physically, the model of static, annealed magnetic disorder representing a type of local moments thus turns out to be less interesting than Ref. 4 had given reason to believe. Instead of describing an unusual MIT, the model describes a variant of the ferromagnetic transition of itinerant electrons that has been studied before. It is important to note that the same model with quenched instead of annealed magnetic disorder is well known to contain a MIT in $d=2+\epsilon$.[23] This serves to underscore the fundamental physical difference between the quenched and annealed disorders that we have stressed several times in this paper.

We finally mention a possible consequence of our observation, discussed in Sec. V B, that an electron-electron interaction with more than one time scale produces more than one frequency coupling in the NLσM, which in turn requires additional renormalization constants. (In the present case, there was one additional time scale, infinity, one additional coupling, $H^{(2)}$, and one additional renormalization constant.) At a MIT, the coupling constant that was denoted above by $H^{(1)}$ acquires a power-law frequency dependence. This is equivalent to saying that there are infinitely many time scales in the problem, and this raises doubts about the validity of renormalizing the action with just one renormalization constant for the frequency coupling. It is therefore possible that a complete description of the dynamics near a MIT would require a functional RG. A complete understanding of this problem would also require a solution of the renormalizability problem for models of interacting electrons, which is explained in Appendix B.

ACKNOWLEDGMENTS

This work was initiated at the Aspen Center for Physics, and it is a pleasure to thank the Center for its hospitality. This work was supported by the NSF under Grant Nos. DMR-98-70597, DMR-99-75259, DMR-01-32555, and DMR-01-32726.

APPENDIX A: MOMENTUM-SHELL RENORMALIZATION VERSUS FIELD-THEORETIC RENORMALIZATION

In this appendix we explain our choice of a field-theoretic formulation of the renormalization procedure.

There exist two basic formulations of the RG, the field-theoretic one that originated in high-energy physics,[9] and Wilson's momentum-shell method,[17] which was invented for the study of critical points. After Wilson's breakthrough, it was shown that the field-theoretic method can also be applied to critical phenomena.[9] The relation between these two formulations of the RG is complicated,[24] but for our purposes we can restrict ourselves to a few basic features.

In the Wilsonian method one renormalizes the Hamiltonian or action itself, generating new interactions as one goes along, and checking all newly generated terms for their scale dimensions, and hence for their being RG relevant, irrelevant, or marginal. Irrelevant ones can be dropped, while relevant or marginal ones must be added to the model and included in a repetition of the renormalization process. In the field-theoretic method, one renormalizes specific propagators or vertex functions, and one needs to know from the outset how many renormalization constants are needed in order to make all the vertex functions finite to all orders.

For many models (e.g., for ϕ^4 theory) there is only a small number of relevant or marginal terms. In these cases, the momentum-shell method is often preferred since it is physically more intuitive, and since it provides an explicit check for the generation of additional terms that must be kept. However, the NLσM does not belong to this class, as it has an infinite number of marginal terms in $d=2$: In an expansion of Eq. (3.1) in powers of q, *all* terms are marginal. It is *a priori* unclear how the infinitely many coupling constants multiplying these terms renormalize, although their bare values all coincide. The field-theoretic RG method proves that these coupling constants all renormalize the same way.[25,9] This fixes the structure of the renormalized theory, and it then suffices to consider a small number of vertex functions in order to determine the renormalized theory explicitly. In the momentum-shell method, on the other hand, one needs to explicitly consider a large number of vertex functions or propagators (in principle, infinitely many in the case of the NLσM) in order to do the same.

The same considerations apply to the terms in addition to the NLσM. Equations (3.7) add six coupling constants to the model. Within a momentum-shell RG, one would have to consider q^4 vertices in order to determine how they renormalize. The field-theoretic method, on the other hand, allows us to argue that all of the J split into two pieces that pairwise renormalize in the same way, see Sec. III C and Appendix B. As a result, we need to explicitly renormalize q^2 vertices only. This is the reason why in this paper we choose the field-theoretic method over the momentum-shell one.

APPENDIX B: INVARIANT DECOMPOSITION OF ANNEALED DISORDER TERMS

In this appendix we recall the answer to the following question: Consider the NLσM, Eq. (3.1), which is known to be renormalizable in two-dimensions with two renormalization constants.[25,9] Now add to this action symmetry-breaking operators. How does this affect the renormalizability, and how many additional renormalization constants are needed?

For the case of operators that give some components of the basic field, $Q(\mathbf{x})$ in our case, a mass (massive insertions), this question has been studied in detail.[26] If the NLσM is invariant under transformations that form a symmetry group $\mathcal{G}$, then the operators in question must be expanded in a basis of irreducible representations of $\mathcal{G}$. All operators that belong to the same irreducible representation renormalize the same way, i.e., for each irreducible representation one additional

renormalization constant is needed.

In our case, it is most convenient to write the spin degrees of freedom explicitly, and consider the complex numbers $Q^{\alpha\beta}_{nm,ij}$ as the matrix elements of Q. The NLσM action is then invariant under unitary transformations. We are interested in symmetry-breaking operators that are quadratic in Q. This case was first considered by Pruisken.[18] There are two irreducible representations that correspond to symmetrized and antisymmetrized products of Q. Any operator

$$O=\int d\mathbf{x}\sum_{1234} v_{12,34}\, Q_{12}(\mathbf{x})\, Q_{34}(\mathbf{x}) \tag{B1a}$$

should thus be written as

$$O=O_{+}+O_{-}, \tag{B1b}$$

with

$$O_{\pm}=\frac{1}{2}\int d\mathbf{x}\sum_{12,34} v_{12,34}[Q_{12}(\mathbf{x})\, Q_{34}(\mathbf{x})\pm Q_{32}(\mathbf{x})\, Q_{14}(\mathbf{x})]. \tag{B1c}$$

Here $1\equiv(n_1,\alpha_1,i_1)$, etc. O_{+} and O_{-} require one renormalization constant each, so two additional renormalization constants are needed to renormalize the NLσM with arbitrary massive insertions of order Q^2.

A complication lies in the fact that in the present model, the coupling constants $H^{(1)}$ and $H^{(2)}$ multiply frequency-dependent terms, and the frequency gets integrated over in perturbation theory. As a result, ratios of the J and H appear in perturbation theory, and the proof given in Refs. 26 does not apply. This is true *a fortiori* in the case of an instantaneous electron-electron interaction, where the additional operators are not even massive insertions. Nevertheless, while no actual proof of renormalizability exists in this case, Ref. 19 has presented substantial evidence from perturbation theory that the model is still renormalizable with two additional renormalization constants for the interaction. The same conclusion is expected to hold in the annealed disorder case.

APPENDIX C: COMPARISON WITH THE CASE OF AN INSTANTANEOUS INTERACTION

In this appendix we compare the flow equations derived in Sec. IV B with those for the case of an instantaneous electron-electron interaction.

In the instantaneous interaction case one has spin-singlet and spin-triplet interactions amplitudes K_{s} and K_{t}, which are analogous to $J^{(1,\mathrm{s})}$ and $J^{(1,\mathrm{t})}$, respectively. The analog of $J^{(2,\mathrm{t})}$ does not exist. Instead of the two frequency couplings $H^{(1)}$ and $H^{(2)}$ there is only one coupling constant H, which is proportional to the specific-heat coefficient. $\partial n/\partial\mu$ and χ_{s} are proportional to $H+K_{\mathrm{s}}$ and $H+K_{\mathrm{t}}$, respectively.[1]

In the absence of magnetic impurities, a magnetic field, or spin-orbit scattering, K_{t} flows towards large values, and after some transient behavior the one-loop flow equations take the form

$$\frac{dg}{dl}=-\epsilon g+O(g^3), \tag{C1a}$$

$$\frac{dh}{dl}=\frac{3}{8}gk_{\mathrm{t}}+O(g^2), \tag{C1b}$$

$$\frac{dk_{\mathrm{s}}}{dl}=\frac{-3}{8}gk_{\mathrm{t}}+O(g^2), \tag{C1c}$$

$$\frac{dk_{\mathrm{t}}}{dl}=\frac{1}{2}gk_{\mathrm{t}}^2/h. \tag{C1d}$$

A comparison with Eqs. (4.13) shows that the two behaviors are very similar, except that in the instantaneous interaction case k_{t} flows to infinity much faster than $-k_{\mathrm{s}}$. In particular, the conductivity and $\partial n/\partial\mu$ are not renormalized in either case (and neither is the density of states), while the magnetic susceptibility and the specific-heat coefficient both diverge, albeit the latter only logarithmically in the instantaneous interaction case.[10] Strictly at one-loop order, the physical interpretation of the RG flow was long considered not obvious, as has been stressed in the literature many times. However, the analysis given in Ref. 10, combined with the detailed discussion of the ferromagnetic transition in Ref. 20, has shown that the proper interpretation is in terms of a ferromagnetic transition in $d=2+\epsilon$, as has been discussed in Sec. V A.

APPENDIX D: PERTURBATION THEORY FOR THE FREE ENERGY

Here we calculate the free energy in perturbation theory. This serves as a check on our scaling arguments for various observables in Sec. V A.

To zeroth order in a loop expansion, the free energy density f is given by the saddle-point action. This yields free-electron values for all thermodynamic quantities. The first correction, Δf, is obtained by integrating over the fields q in Gaussian approximation. From Eqs. (4.1) we find

$$\Delta f=\Delta f_{\mathrm{s}}+3\Delta f_{\mathrm{t}}. \tag{D1a}$$

Here

$$\Delta f_{\mathrm{s,t}}=\frac{iG}{H^{(1)}}J_{\mathrm{s,t}}\int_0^1 d\eta\,(H^{(2)}+\eta J_{\mathrm{s,t}}) \times\frac{1}{V}\sum_{\mathrm{k}}\int_0^\infty d\omega\, n(\omega/T)\,\mathcal{D}^{\mathrm{s,t}}(\mathbf{k},\omega,T), \tag{D1b}$$

with [cf. Eq. (4.2e)]

$$\mathcal{D}^{\mathrm{s,t}}(\mathbf{k},\omega,T)=\frac{1}{\mathbf{k}^2-iGH^{(1)}\omega+G(H^{(2)}+\eta J_{\mathrm{s,t}})2\pi T}. \tag{D1c}$$

The function

$$n(x) = \frac{1}{2}\coth\left(\frac{x}{2}\right) - \frac{1}{x}, \tag{D1d}$$

serves as a convenient means for transforming the sum over Matsubara frequencies into a real-frequency integral, and we have used the familiar ''charging formula'' trick of integrating over the interaction constants in order to improve convergence.

[1] See, e.g., D. Belitz and T.R. Kirkpatrick, Rev. Mod. Phys. **66**, 261 (1994).

[2] This is the local moment concept that was considered in a classical context by Viktor Dotsenko, A.B. Harris, D. Sherrington, and R.B. Stinchcombe, J. Phys. A **28**, 3093 (1995), and that was generalized to quantum-field theories in Refs. 3 and 4. Other, more spatially localized, local moment concepts have also frequently been discussed in the literature; see R.N. Bhatt and P.A. Lee, Phys. Rev. Lett. **48**, 344 (1982); R.N. Bhatt and D.S. Fisher, *ibid.* **68**, 3072 (1992); In the context of the disordered interacting electron problem that we will frequently refer to, local moments have been discussed by C. Castellani, C. DiCastro, P.A. Lee, M. Ma, S. Sorella, and E. Tabet, Phys. Rev. B **30**, 1596 (1984); A.M. Finkelstein, Pisma Zh. Éksp. Teor. Fiz. **40**, 63 (1984) [JETP Lett. **40**, 796 (1984)]; and more recently by A.V. Andreev and A. Kamenev, Phys. Rev. Lett. **81**, 3199 (1998); and by B.N. Narozhny, I.L. Aleiner, and A.I. Larkin, Phys. Rev. B **62**, 14 898 (2000).

[3] R. Narayanan, T. Vojta, D. Belitz, and T.R. Kirkpatrick, Phys. Rev. B **60**, 10 150 (1999).

[4] D. Belitz, T.R. Kirkpatrick, and Thomas Vojta, Phys. Rev. Lett. **84**, 5176 (2000).

[5] For a general discussion and classification of metal-insulator transitions, see, N.F. Mott, *Metal-Insulator Transitions* (Taylor & Francis, London 1990).

[6] D. Belitz, T.R. Kirkpatrick, and Thomas Vojta, Phys. Rev. B **65**, 165112 (2002).

[7] D. Belitz, T.R. Kirkpatrick, and Thomas Vojta, Phys. Rev. B **55**, 9452 (1997).

[8] This is true within the NLσM description of the MIT, where the Goldstone modes in the ordered (i.e., metallic) phase are also the soft modes that drive the transition. This results in the somewhat surprising conclusion that, for any given observable, there is a relation between its critical behavior at the critical fixed point and its corrections to scaling at the stable fixed point that describes the ordered phase. This is true for any NLσM, including the one for the classical Heisenberg ferromagnet (Ref. 9). Related considerations have at times led to a debate as to whether the ferromagnetic transition found within the NLσM in $d=2+\epsilon$ and the one found within ϕ^4 theory in $d=4-\epsilon$ are really physically identical; see E. Brézin and S. Hikami, cond-mat/9612016 (unpublished), and references therein.

[9] See, J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena* (Clarendon Press, Oxford, 1989), and references therein.

[10] T.R. Kirkpatrick and D. Belitz, J. Phys.: Condens. Matter **2**, 5259 (1990); Phys. Rev. B **45**, 3187 (1992). In these papers, the precise nature of the first phase-transition was unclear. The identification as a ferromagnetic transition was provided by T.R. Kirkpatrick and D. Belitz, *ibid.* **53**, 14 364 (1996), and in Ref. 20.

[11] See, e.g., C. Itzykson and J.-M. Drouffe, *Statistical Field Theory* (Cambridge University Press, Cambridge, 1989).

[12] See, e.g., G. Grinstein, in *Fundamental Problems in Statistical Mechanics VI*, edited by Cohen E.G.D. (North Holland, Amsterdam, 1985).

[13] A.M. Finkel'stein, Zh. Éksp. Teor. Fiz. **84**, 168 (1983) [Sov. Phys. JETP **57**, 97 (1983)].

[14] F. Wegner, Z. Phys. B **35**, 207 (1979). We use the fermionic formulation of the model given in Ref. 15. See also Ref. 16.

[15] K.B. Efetov, A.I. Larkin, and D.E. Khmelnitskii, Zh. Éksp. Teor. Fiz. **79**, 1120 (1980) [Sov. Phys. JETP **52**, 568 (1980)].

[16] D. Belitz and T.R. Kirkpatrick, Phys. Rev. B **56**, 6513 (1997); D. Belitz, T.R. Kirkpatrick, and F. Evers, *ibid.* **58**, 9710 (1998).

[17] K.G. Wilson and J. Kogut, Phys. Rep. **12**, 75 (1974).

[18] A.M.M. Pruisken, Phys. Rev. B **31**, 416 (1985).

[19] D. Belitz and T.R. Kirkpatrick, Nucl. Phys. B **316**, 509 (1989).

[20] D. Belitz, T.R. Kirkpatrick, Maria Teresa Mercaldo, and Sharon L. Sessions, Phys. Rev. B **63**, 174427 (2001); **63**, 174428 (2001).

[21] W.F. Brinkman and T.M. Rice, Phys. Rev. B **2**, 4302 (1970).

[22] Thomas Vojta and R. Narayanan (private communication).

[23] A.M. Finkel'stein, Zh. Éksp. Teor. Fiz. **86**, 367 (1984) [Sov. Phys. JETP **59**, 212 (1984)].

[24] S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, Cambridge, 1995) Vol. 1, Sec. 12.4, and references therein.

[25] E. Brézin, J. Zinn-Justin, and J.C. Le Guillou, Phys. Rev. D **14**, 2615 (1976).

[26] E. Brézin, J. Zinn-Justin, and J.C. Le Guillou, Phys. Rev. B **14**, 4976 (1976); D. Höf and F. Wegner, Nucl. Phys. B: Field Theory Stat. Syst. **275 [FS17]**, 561 (1986); F. Wegner, *ibid.* **280 [FS18]**, 193 (1987); **280 [FS18]**, 210 (1987). The first of these papers uses the field-theory terminology ''soft insertions'' for the terms in question.