Comment on "Ferromagnetic film on a superconducting substrate"

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A superconducting substrate is not able to shrink drastically domains in a ferromagnetic film, contrary to the prediction of Bulaevskii and Chudnovsky. This is shown on the basis of the exact solution for the stripe domain structure.

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In Ref. 1 Bulaevskii and Chudnovsky analyzed the equilibrium stripe domain structure in a ferromagnetic film on a superconducting substrate and predicted a drastic shrinkage of domains. According to them, the domain size is by the factor $(\lambda_L/l)^{1/3}$ smaller than the domain size $l \sim \sqrt{\delta d_M}$ for a film without a superconducting substrate.[2] Here λ_L is the London penetration depth, d_M is the film thickness, and δ is the domain wall thickness ($\delta \ll l$, d_M). In this Comment I shall show that this prediction is incorrect: even in the limit $\lambda_L/l \to 0$, the superconducting substrate can shrink domains only by a numerical factor not more than $\sqrt{1.5}$.

As well in Ref. 1, I consider a ferromagnetic film with its spontaneous magnetic moment $\vec{M}$ normal to the film. The stray magnetic field $\vec{H} = \vec{B} - 4\pi\vec{M}$ must satisfy the equations of magnetostatics:[3]

$$\vec{\nabla} \times \vec{H} = 0, \quad \vec{\nabla} \cdot \vec{H} = 4\pi\rho_M, \tag{1}$$

where $\rho_M = -\vec{\nabla} \cdot \vec{M}$ is the magnetic charge and $\vec{M}$ is the spontaneous magnetization. The second equation in Eqs. (1) follows from the condition that the magnetic induction $\vec{B} = \vec{H} + 4\pi\vec{M}$ is divergence free: $\vec{\nabla} \cdot \vec{B} = 0$. If domain walls are parallel to the magnetization $\vec{M}$, i.e., normal to the film, the magnetic charges appear only on the film surface (Fig. 1).

In the limit $\delta \ll l$, d_M, which was considered in Ref. 1, the distribution of stray fields for the stripe domain structure in a ferromagnetic film can be found exactly using analytical functions on the complex plane.[4] We assume that the film is parallel to the xz plane and is restricted by the planes $y=0$ and $y=d_M$ (Fig. 1). The field components H_x and H_y satisfy Eqs. (1) if they are determined by the real and imaginary parts of an analytical function $\mathcal{H}(w)$ on the complex plane $w = x + iy$. Without a superconducting substrate the solution is

$$\mathcal{H}(w) = -H_x + iH_y = 4M\left[\ln\tan\frac{\pi w}{2l} - \ln\tan\frac{\pi(w - id_M)}{2l}\right]. \tag{2}$$

If the film is put on a superconducting substrate with the London penetration depth much less than the domain size l and the film thickness d_M (the case when Bulaevskii and Chudnovsky predicted a strong effect of the substrate), one can neglect the penetration of the magnetic field into the substrate and obtain the solution of the problem by introducing image charges in the substrate:

$$\mathcal{H}(w) = -H_x + iH_y = 4M\left[2\ln\tan\frac{\pi w}{2l} - \ln\tan\frac{\pi(w - id_M)}{2l} - \ln\tan\frac{\pi(w + id_M)}{2l}\right]. \tag{3}$$

The solutions, Eqs. (2) and (3), are a straightforward generalization of the solutions for a single domain wall obtained in Ref. 4. The single-wall solutions ($l \to \infty$) of Ref. 4 follow from Eqs. (2) and (3) after expansion of the tangent function: $\tan\varphi \approx \varphi$.

Later on we restrict ourselves to the case when the film thickness d_M essentially exceeds the domain structure period l. Then the stray fields on two film boundaries ($y=0$ and $y=d_M$) do not overlap and can be calculated separately. Near the boundary $y=0$ in absence of a superconducting substrate

$$\mathcal{H}(w) = 4M\left(\ln\tan\frac{\pi w}{2l} - i\frac{\pi}{2}\right). \tag{4}$$

In the presence of a superconducting substrate Eq. (3) yields by a factor 2 larger values of $\mathcal{H}$ at $y > 0$, but $\mathcal{H} = 0$ at y

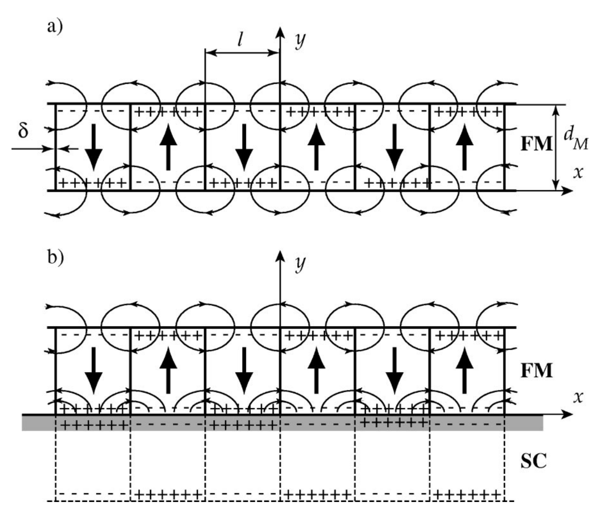

FIG. 1. Magnetic charges (+ and −) and magnetic flux (thin lines with arrows) in a ferromagnetic film (FM) without (a) and with (b) a superconducting substrate (SC). The magnetization vectors in domains are shown by thick arrows.

<0. However, one should remember that we are solving the problem in the limit $\lambda_L \to 0$. For finite λ_L the jump of the tangential component H_x at the plane $y=0$ transforms into an exponential decrease of H_x at $y<0$ down to zero at the distance λ_L, and H_x is continuous in accordance with the laws of electrodynamics (see below).

Especially important for us is the magnetic field at the ferromagnetic film boundary $y=0^+$. Without a superconducting substrate,

$$H_x(x) = -\mathrm{Re}\mathcal{H} = -4M \ln\left|\tan\frac{\pi x}{2l}\right|, \tag{5}$$

$$H_y(x) = \mathrm{Im}\mathcal{H} = \mp 2\pi M \,\mathrm{sgn}\left(\tan\frac{\pi x}{2l}\right) \quad \text{at} \quad y \to \pm 0. \tag{6}$$

The field pattern is periodic with the period $2l$ along the axis x. The magnetic charge on the film boundary $y=0$ is

$$\rho_M = \frac{1}{4\pi}[H(x+i0) - H(x-i0)]\delta(y)$$
$$= -M\delta(y)\,\mathrm{sgn}\left(\tan\frac{\pi x}{2l}\right). \tag{7}$$

So in the limit of $\lambda_L \to 0$ the method of complex variables provides the exact solution of the problem in terms of elementary functions without using the Fourier expansion. For finite λ_L the exact solution in the form of the infinite Fourier series is also known and agrees with our $\lambda_L \to 0$ solution. One may check it, comparing the magnetic energy of the two solutions. The magnetic energy can be calculated using the potential ϕ for the magnetic field ($\vec{H} = \vec{\nabla}\phi$) and integration by parts:

$$\mathcal{E}_m = \int dV \frac{H^2}{8\pi} = \frac{1}{2}\int dS \rho_M \phi, \tag{8}$$

where the surface integral should be taken over all planes which confine the magnetic charge ρ_M. Without a superconducting substrate in the limit $d_M \gg l$ the energy of the stray fields near the plane $y=0$ per unit area in the plane xz is

$$E_m = \frac{M}{2l}\int_0^l dx \int_0^x dx' \, 4M \ln\tan\frac{\pi x'}{2l} = \frac{8M^2 l}{\pi^2}\int_0^{\pi/2} d\varphi \varphi \ln\tan\varphi$$
$$= \frac{7\zeta(3)}{\pi^2} M^2 l \approx 0.852 M^2 l, \tag{9}$$

where $\zeta(z)$ is the zeta function. The same value of energy was obtained with the Fourier-expansion method in the problem after Sec. 44 in the book by Landau and Lifshitz.[2] The Fourier-series solution for the magnetostatic problem of a ferromagnetic film on a superconducting substrate for arbitrary λ_L was found by Stankiewicz *et al.*,[5] and their solution also agrees with our $\lambda_L \to 0$ solution. This is checked in detail in Ref. 6.

The superconducting substrate increases the magnetic energy density in the film by 4 times, but contracts the area occupied by the magnetic field by 2 times. Thus the magnetic energy at the boundary $y=0$ in the presence of the substrate is 2 times larger than without it. On the other hand, in our limit $d_M \gg l$ the substrate has no effect on the magnetic energy at the other boundary $y=d_M$. Eventually the substrate increases the total magnetic energy by 1.5 times. The energy of the domain walls per unit length along the axis x is inversely proportional to the period l and the energy of the stray fields is proportional to l. The period l is determined by minimization of the total energy per unit length, and the growth of the magnetic energy by two times decreases the domain width l only by $\sqrt{1.5}$ times.

In any domain the surface charges on the film boundary $y=0$ generate the magnetic flux $\Phi = \pm 4\pi M l$. Without a superconducting substrate, half of this flux enters the film itself, and another half exits from the film [Fig. 1(a)]. The superconducting substrate does not allow for the magnetic flux to exit from the film, and the whole flux enters the film [Fig. 1(b)]. Let us consider now the effect of a small, but finite London penetration depth. The magnetic field inside the superconductor is determined by the boundary value of the tangential field H_x in the ferromagnetic film at $y=0$, which is of the order of M. Then the magnetic flux, which enters the superconductor, is $\sim M\lambda_L$, i.e., about λ_L/l times smaller than the total flux $4\pi M l$. This provides a correction of the relative order λ_L/l to the magnetic flux of the stray magnetic fields inside the film. The energy $\sim M^2\lambda_L$ inside the superconductor is also a small correction of the same relative order.

The latter discussion helps to understand the source of an error in Ref. 1. Looking for the magnetic field distribution, Bulaevskii and Chudnovsky[1] assumed that the magnetic field component normal to the film boundary is the same inside the film and inside the superconducting substrate [see their Eq. (7)]. So according to their solution half of the total stray magnetic flux enters the superconductor even in the limit $\lambda_L \to 0$. Meanwhile, only a small part $\propto \lambda_L/l$ of the total flux is able to penetrate to the superconducting substrate. It is worth stressing that the solution of the problem does not need any *a priori* assumption on distribution of the magnetic flux between the ferromagnet and the superconductor at all. One must simply use correct electrodynamic boundary conditions[2] at the interface $y=0$: continuity of the normal component of the magnetic induction $\vec{B}$ and continuity of the tangential component of the magnetic field $\vec{H}$. The solution by Bulaevskii and Chudnovsky satisfies the first condition but violates the second one. Indeed, the values of the Fourier components of H_x inside and outside the superconductor, which are given by Bulaevskii and Chudnovsky after their Eq. (8), differ by a large factor $1/q\lambda_L$, where $q \sim 1/l$ is the wave number in the Fourier expansion used by Bulaevskii and Chudnovsky. Because of this error, they essentially overestimated the energy inside the superconductor, and as a result of it, predicted a strong shrinkage of the domains.

In summary, the result of Bulaevskii and Chudnovsky[1] on domain structure in a ferromagnetic film on a superconducting substrate is incorrect because they ignored the electrodynamic boundary condition that the tangential component of the magnetic field must be continuous at the ferromagnet-superconductor interface. Instead of it they used the incorrect assumption that the magnetic flux produced by magnetic charges at the interface is equally distributed between the ferromagnet and superconductor. The correct solution of the problem in terms of elementary analytic functions on the complex plane is given, which is exact in the limit of large ratio of the domain size to the London penetration depth.

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