Spin-orbit scattering effect on critical current in SFIFS tunnel structures

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The spin-orbit scattering effect on the critical current through superconductor/ferromagnet (SF) bilayers separated by an insulator (SFIFS tunnel junction) has been investigated for the case of the absence of superconducting order parameter oscillations (thin F layers). The analysis is based on a microscopic theory for proximity-coupled SF structure with arbitrary values of parameters (boundary transparency, proximity effect strength, relative orientation, and value of the F-layer exchange field). We find that the spin-orbit scattering considerably modifies the dc Josephson current in SFIFS tunnel junctions. In contrast to a simple physical picture, the reduction of the exchange field effects is nonlinear in character, acquiring its maximum in the field's region where the critical current enhancement or transition to the π state takes place. Hence, for understanding the various experimental results on tunnel structures with thin F layers, the coupled effects of the exchange interaction and spin-orbit scattering must be considered.

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I. INTRODUCTION

Nowadays progress in nanotechnology has made it possible to produce nanostructures with new physical phenomena. This has led to renewed attention to hybrid systems consisting of superconductor (S) and ferromagnetic (F) metals, displaying rich and elegant physics, and having potential applications. The transport properties of SF structures with artificial geometry have turned out to be quite unusual. These have been treated by several authors,[1–9] and the results obtained show that in a ferromagnet the Cooper pair potential not only exponentially decays, but also has an oscillatory character; i.e., in a ferromagnet the density of Cooper pairs is spatially inhomogeneous and the superconducting order parameter contains nodes where the phase changes by π. This causes the exchange field dependence of the Josephson coupling energy, and if the exchange energy H_{exc} in a magnetic layer exceeds a certain value, a crossover from 0-phase to π-phase superconductivity takes place. The phenomenon has been theoretically described for SFS weak links with thick F layers.[1–5] The crossover to the π state even in the absence of order parameter oscillations in thin F layers was also predicted for SFIFS tunnel junctions (where I is an insulator) with parallel alignment of the layer magnetization.[6] For an antiparallel orientation, the possibility of critical current enhancement by exchange interaction in SFIFS junctions with thin F layers was discussed both for small[7] and bulk[8,9] S-layer thicknesses. Bergeret *et al.*[7] considered the model when the influence of the F layers on superconductivity is equivalent to inclusion of a homogeneous exchange field with a reduced value and come, actually, to the limit of effective values of the superconducting order parameter and effective magnetic moment. The authors[8,9] consider a more realistic case, particularly taking into account accurate boundary conditions at the S/F interface and spatial variation of the superconducting state in the S layer. Experimentally, π-phase superconductivity has been observed for SFS weak links Ref. 10 and SIFS Ref. 11 and SNFNS Ref. 12 tunnel junctions with thick F layers, while enhancement of the dc Josephson current has not been detected until now.

Some features should be taken into account for the theory to be adjustable for experimental results on SF structures, and spin-flip processes are among the important ones. These processes can be induced, e.g., by the spin-orbit scattering centers presented in the film; another important source of spin-flip processes for nanoscale hybrid structures is a strong electric field arising near metal-metal boundaries.[13] The basic physics behind the spatial oscillations of induced superconductivity in SF sandwiches with spin-orbit scattering has been recently discussed in Refs. 3 and 14. As is known, in the presence of spin-orbit scattering electron spin is no longer a good quantum number and the electron will change its spin state during a characteristic time τ_{SO}, while the Cooper pair will mix with its spin-exchanged counterpartner. This causes a pair to ''feel'' an exchange field which changes sign at a rate proportional to $1/\tau_{SO}$, decreasing the average ferromagnetic field experienced by the pair. That means that the spin-flip processes not only modify the oscillation length, but also lead to an extra decay of the Cooper pair potential and, at a critical strength, these scattering processes can completely suppress the π-phase superconductivity.

The scenario of the 0-π-phase transition, as well as the mechanisms of critical current enhancement, in the limit of thin F layers differs from that for thick F layers. However, the basic physics behind the spin-orbit scattering effect for SF sandwiches where there is no induced order parameter oscillations has not been discussed until now. The purpose of this paper is to clarify the mechanisms of the influence of spin-orbit scattering on the critical current in SFIFS tunnel structures with thin F layers. Our analysis is based on an extension of calculations,[6,8,15] so as to include the effects of the spin-orbit scattering, which allow one to obtain analytical solutions that have not been explored yet. Namely, we study the tunnel junction with leads formed by the proximity-coupled SF bilayers of a massive *s*-wave superconductor and thin F metals, when the spin-orbit scattering processes take place in ferromagnetic layers. The microscopic model of the proximity effect for SF bilayers is employed to discuss the case of arbitrary values of SF boundary transparency, ferromagnetic exchange field, and proximity effect strength (Sec.

II). The critical current of symmetrical SFIFS junctions is discussed in Sec. III where analytical solutions have been obtained for some particular cases. As we shall see, spin-flip scattering plays a major role in the transport properties of superconductor-ferromagnet structures with thin F layers and should be considered for understanding the various experimental results. We close with a Conclusion.

II. SUPERCONDUCTIVITY OF A SF BILAYER WITH SPIN-ORBIT SCATTERING

We consider the case when both metals are in the dirty limit condition $\xi_{S,F} \geqslant l_{S,F}$ and the S layer is thick, $d_S \gg \xi_S$, while the F layer is thin, $d_F \ll \min(\xi_F, \sqrt{D_F/2\pi T_C})$. Here $\xi_S = (D_S/2\pi T_C)^{1/2}$ and $\xi_F = (D_F/2H_{exc})^{1/2}$ are the coherence lengths of the S and F metals; l_S and l_F are the electron mean free paths, $d_{S,F}$ are the thicknesses, and $D_{S,F}$ are the diffusion coefficients of the S and F metals, respectively. These conditions make it possible to neglect the reduction of the critical temperature of the SF bilayer compared to that of the bulk S metal and to imply constant superconducting properties through F-layer thickness. Throughout this work the F layer will be treated as a single-domain film with spin-orbit scattering centers, while there are no such centers in the S layer. Then, the superconducting properties of the SF bilayer are described by the Usadel equations.[16] The latter can be written as (the domain $x \geqslant 0$ is occupied by the S metal and $x < 0$ by the F metal)

$$\Phi_{S\sigma} = \Delta_S + \xi_S{}^2 \frac{\pi T_C}{\omega G_{S\sigma}} [G_{S\sigma}{}^2 \Phi'_{S\sigma}]', \tag{1}$$

$$\Phi_{F\pm} = \xi^2 \frac{\pi T_C}{\tilde{\omega}_\pm G_{F\pm}} [G_{F\pm}^2 \Phi'_{F\pm}]' + \alpha_{SO} G_{F\mp} \left(\frac{\Phi_{F\mp}}{\tilde{\omega}_\mp} - \frac{\Phi_{F\pm}}{\tilde{\omega}_\pm} \right). \tag{2}$$

Here $\tilde{\omega}_\sigma = \omega + i\sigma H_{exc}$, $\sigma = \pm$, and $\omega \equiv \omega_n = \pi T(2n+1)$, $n = \pm 1, \pm 2, \ldots,$ is the Matsubara frequency; $\alpha_{SO} = 2/3\tau_{SO}$ and τ_{SO} is the spin-orbit scattering time; the pair potential Δ_S is determined by the usual self-consistency relations (see, e.g. Ref. 17). We define the x axis as perpendicular to the film surfaces and the prime denotes differentiation with respect to a coordinate x. The modified Usadel functions[18] $\Phi_{S\sigma} = \omega F_{S\sigma}/G_{S\sigma}$, $\Phi_{F\sigma} = \tilde{\omega}_\sigma F_{F\sigma}/G_{F\sigma}$, where $G_{F,S}$ and $F_{F,S}$ are normal and anomalous Green's functions for the F and S material, respectively, are introduced to take into account the normalized confinement of the Green's functions $G^2 + F\tilde{F} = 1$; here $\tilde{F}(\omega, H_{exc}) = F^*(\omega, -H_{exc})$ and $\tilde{\Phi}_{S\sigma} = \omega \tilde{F}_{S\sigma}/G_{S\sigma}$ (see below). We also assume that for a nonsuperconducting F metal the bare value of the order parameter $\Delta_F = 0$; however, the Cooper pair correlation function $F_F \neq 0$ due to the proximity effect. Equation (2) for the F metal is a generalization of the Usadel equations for the case when the spin-orbit scattering is present. At temperatures close to T_C, when $G_{S(F)\sigma} \approx 1$ and $\Phi_{S(F)\sigma}/\tilde{\omega}_\sigma \approx F_{S(F)\sigma}$, Eq. (2) has been simplified, obtaining the form of Eqs. (26) of Ref. 3 If the spin-orbit processes are absent ($\tau_{SO} \to \infty$), the spin-"up" and-"down" subbands do not mix with each other and Eq. (2) obtains the usual form (see, e.g., Refs. 6,15).

Equations (1) and (2) should be supplemented with the boundary conditions. In the bulk of the S metal the pair potential is equal to the BCS order parameter $\Delta_0(T)$ at the temperature T, $\Phi_S(\infty) = \Delta_S(\infty) = \Delta_0(T)$, while at the free (dielectric) interface of the F metal, $\Phi'_{F\pm}(-d_F) = 0$. Assuming that there are no spin-flip processes at the SF interface, the boundary conditions at this interface (see Ref. 15 for details) can easily be generalized for the case of two fermionic subband in the form

$$\gamma \xi G_{F\pm}^2 \Phi'_{F\pm}/\tilde{\omega}_\pm = \xi_S G_{S\pm}^2 \Phi'_{S\pm}/\omega, \tag{3}$$

$$\gamma_{BF} \xi G_{F\pm} \Phi'_{F\pm} = \tilde{\omega}_\pm G_{S\pm} (\Phi_{S\pm}/\omega - \Phi_{F\pm}/\tilde{\omega}_\pm). \tag{4}$$

The parameters γ and γ_{BF} involved in these relations are given by $\gamma = \rho_S \xi_S/\rho_F \xi$ and $\gamma_{BF} = R_B/\rho_F \xi$, where $\rho_{S,F}$ are the normal-state resistances of the S and F metals, and R_B is the product of SF boundary resistance and its area. In Eqs. (2) and (3) and below, we have used the effective coherence length $\xi = (D_F/2\pi T_C)^{1/2}$, thus providing the regular crossover to both limits $T_C \gg H_{exc} \to 0$ and $H_{exc} \gg T_C$. Relation (3) provides the continuity of the supercurrent flowing through the SF interface at any value of the interface transparency for spin-"up" and -"down" fermionic subbands separately, while condition (4) accounts for the quality of the electric contact. An additional physical approximation for these relations to be valid is the assumption that spin discrimination by the interface is unimportant; i.e., the interface parameters involved are the same for both spin subband. Generalization of Eqs. (3) and (4) to the SF interface with different transmission probabilities for "up" and "down" spin quasiparticles is straightforward; however, the case we consider contains all the new physics we are interested in and is simpler. In such ferromagnetic metals as Ni, Gd, etc., the polarization of the electrons at low temperatures is not more than 10%, and one can expect that the model under consideration reflects the transport properties of these ferromagnets hybrid structures.

Due to the small thickness of the F metal, the proximity effect problem can be reduced to the boundary problem for the S layer and a relation for determining $\Phi_{F\pm}$ at $x=0$. There are three parameters which enter the model: $\gamma_M = \gamma d_F/\xi$, $\gamma_B = \gamma_{BF} d_F/\xi$, and the energy of the exchange field H_{exc}. Using the system of equations (3) and (4), one can obtain the equations determining the unknown value of the functions $\Phi_{F\pm}(0)$ and boundary conditions, for $\Phi_{S\pm}$. Due to these boundary conditions, all the equations for the functions $\Phi_{S\pm}$ and $\Phi_{F\pm}$ are coupled. In the general case, the problem needs self-consistent numerical calculations (similar to those as, e.g., in Refs. 9 and 19). Here, however, we will not discuss the quantitative solution, but pay attention to the qualitative one to consider the new physics we are interested in.

We will hereinafter assume that $\alpha_{SO} \ll 1$ and $d_F/\xi \ll 1$ and solve differential equations by an iteration procedure finding the corrections to $\Phi_{F\pm}(x)$ and $\Phi_{S\pm}(x)$ in small parameters

d_F/ξ and α_{SO}. We also restrict ourselves to the quite realistic experimental case of a bilayer with a weak proximity effect and low transparency of the SF boundary, i.e., $\gamma_M \ll 1$ and $\gamma_B \gtrsim 1$. Then the problem is simplified and reduced to the Usadel Equations (1) for the S layer with boundary conditions (3), while Eq. (4) reduces to (the details will be published elsewhere)

$$\xi_S G_{S\pm}\Phi'_{S\pm}|_{x=0} \approx \overline{\gamma}_M \tilde{\omega}_\pm \frac{\Phi_{S\pm}}{\pi T_C A_\pm}\left\{ 1 \mp 2\alpha_{SO}\frac{iH_{exc}}{\tilde{\omega}_+\tilde{\omega}_-}\frac{G_S}{\omega} \times\left(\frac{\omega}{\overline{\gamma}_B\tilde{\omega}_\pm}\pi T_C + \frac{G_S}{\omega}\frac{\Delta_0^2}{A_\pm^2}\right)\right\}\Bigg|_{x=0}, \qquad (5)$$

where we abbreviated $A_\pm = [1 + 2\overline{\gamma}_B G_S \tilde{\omega}_\pm + \overline{\gamma}_B^2 \tilde{\omega}_\pm^2]^{1/2}$, $G_S = \omega/(\omega^2+\Delta_0^2)^{1/2}$, and $\overline{\gamma}_{B(M)} \equiv \gamma_{B(M)}/\pi T_C$ (functions that are multiplied by α_{SO} have been used in the limit $\alpha_{SO} \to 0$). Now, the equation for F layer function $\Phi_{F\pm}(0)$ has the form

$$\Phi_{F\pm}(0) \approx G_{S\pm}\Phi_{S\pm}\left\{ 1 \pm 2\alpha_{SO}\frac{iH_{exc}}{\tilde{\omega}_+\tilde{\omega}_-}\right\} \Big/ \times [\omega(\overline{\gamma}_B + G_{S\pm}/\tilde{\omega}_\pm]|_{x=0}. \qquad (6)$$

In zeroth approximation in γ_M, $\Phi'_{S\pm}(0)=0$. So we can neglect the suppression of superconductivity in the S layer assuming that $\Phi_{S\pm}(x)$ is spatially homogeneous: $\Phi_{S\pm}(x) = \Delta_S(x) = \Delta_0(T)$. In the next order in γ_M, by linearizing the Usadel equations for the $\Phi_{S\pm}(x)$ and making use of the relation (3), the general solution of the linearized equation (1) is given by

$$\Phi_{S\pm}(x) = \Delta_0\{1 - C_\pm \exp(-\beta x/\xi_S)\}, \qquad (7)$$

where $\beta^2 = (\omega^2+\Delta_0^2)^{1/2}/\pi T_C$. Substituting this solutions into the boundary relations (5), we get for $C_\pm$

$$C_\pm \approx \frac{\overline{\gamma}_M \beta \tilde{\omega}_\pm}{\overline{\gamma}_M \beta\tilde{\omega}_\pm + \omega A_\pm}\left\{ 1 \mp 2\alpha_{SO}\frac{iH_{exc}}{\tilde{\omega}_+\tilde{\omega}_-} \times\left(\frac{G_S}{\overline{\gamma}_B\tilde{\omega}_\pm} + \frac{G_S^2}{\omega^2}\frac{\Delta_0^2}{A_\pm^2}\right)\right\} \qquad (8)$$

We see that due to the proximity with the F layer magnetic correlations spread into the S layer, the fermionic symmetry of the subbands has been lost and the Cooper pair mixes with its spin-exchanged counterpartner. Using Eq. (7) for $x=0$ and relation (6), one can find expressions for $\Phi_{F\pm}(\omega,0)$. However, we will not show here these expressions because of their cumbersome structure. For $H_{exc} \to 0$ the quantity $\tilde{\omega}_\pm \to \omega$, and solution (7), (8) reproduces the results obtained in Ref. 17 for an *SN* bilayer. If $\tau_{SO} \to \infty$, the expressions restore earlier results for *SF* bilayer (see Refs. 6,9,15).

Let us make some comments on the results obtained. The $\Phi_S(x)$ dependence, Eq. (7), shows the *S*-layer order parameter suppression only qualitatively. In order to obtain quantitative accuracy in the small-γ_M limit, it is necessary to take into account self-consistently the corrections to the pair potential, $\Delta_S(x)$, as has been done in Ref. 20. This fact, however, does not influence the main qualitative results of the paper, but may lead to quantitative corrections for curves plotted in Figs. 2 and 4 (see below). As a rule, the value of

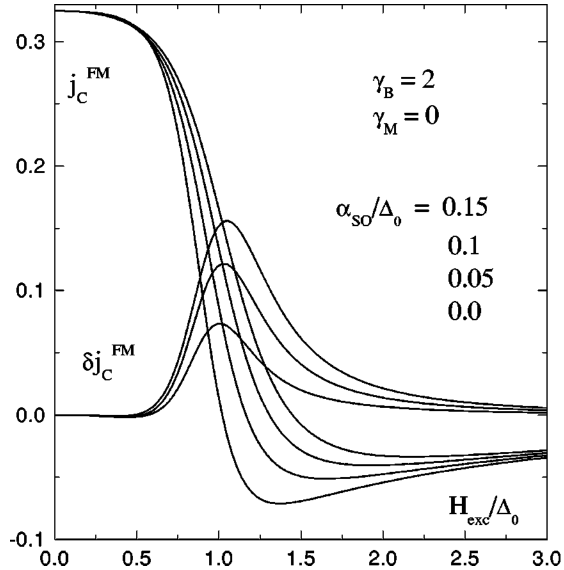

FIG. 1. Critical current of SFIFS tunnel junctions with parallel orientation of the F-layer magnetization j_C^{FM} vs exchange energy for various $\alpha_{SO}/\Delta_0 = 0$, 0.05 , 0.1, and 0.15; the SF interface transparency is low $\gamma_B = 2$ and the proximity effect is weak $\gamma_M = 0$. The additive part of critical current due to spin-orbit scattering, δj_C^{FM}, is also shown. $T=0.1T_C$.

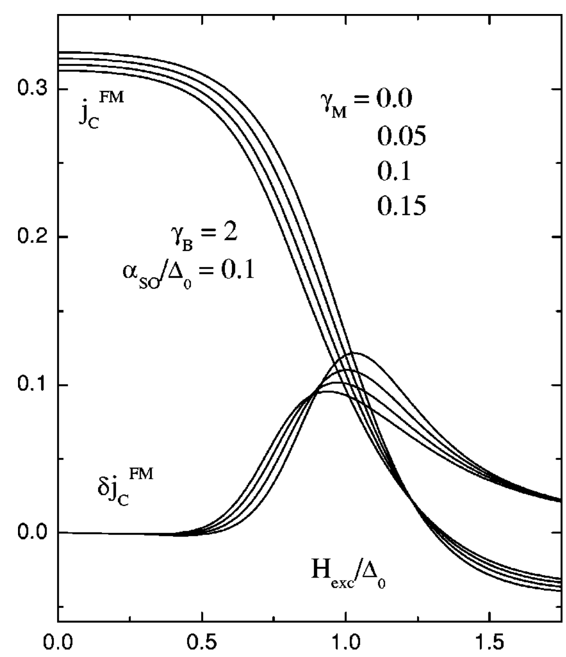

FIG. 2. Critical current of SFIFS tunnel junctions with parallel orientation of the F-layer magnetization j_C^{FM} vs exchange energy for various $\gamma_M = 0$, 0.05, 0.1, 0.15; $\gamma_B = 2$ and $\alpha_{SO}/\Delta_0 = 0.1$. The additive part of critical current due to spin-orbit scattering, δj_C^{FM}, is also shown. $T=0.1T_C$.

these corrections is no more than 5%.[20] For the structures under consideration, one can also expect a kind of induced magnetic properties for the S layer, which are the result of phenomena similar to the superconducting proximity effect:[21] equilibrium leakage of magnetism into the S metal results, for example, in a spatially dependent magnetization of the S layer, local bands that appear inside the energy gap in the S layer, and others. These induced magnetic properties of the S metal are quite important for SF nanostructures with thin S layers ($d_S < \xi_S$) and should be also taken into account to obtain self-consistent numerical results.

III. CRITICAL CURRENT OF SFIFS TUNNEL JUNCTION

We assume that both banks of the Josephson SFIFS tunnel junctions are formed by equivalent SF bilayers, and the transparency of the insulating layer is small enough to neglect the effect of the tunnel current on the superconducting state of the electrons. The plane SF boundary can have arbitrary finite transparency, but it is large compared to the transparency of the junction barrier. The transverse dimensions of the junction are supposed to be much less than the Josephson penetration depth, so all quantities depend only on a single coordinate x normal to the interface surface of the materials. Using the above- obtained results we investigate the influence of spin-flip scattering on the critical Josephson current in SFIFS tunnel junctions. The critical current I_C of the $(SF)_L I (FS)_R$ tunnel contact can be written in the form (see, e.g., Ref. 6):

$$j_C = (eR_N/2\pi T_C) I_C = \frac{T}{T_C} \mathrm{Re} \sum_{\omega>0,\sigma=\pm} \frac{G_{F\sigma}\Phi_{F\sigma}}{\tilde{\omega}_\sigma} \times \left| {}_L \frac{G_{F\sigma}\Phi_{F\sigma}}{\tilde{\omega}_\sigma} \right|_R \tag{9}$$

where R_N is the resistance of the contact in the normal state; the subscript L (R) labels quantities referring to the left (right) bank and the sign of the exchange field depends on mutual orientation of the bank magnetizations.

A. Parallel orientation of the layer magnetizations

For parallel alignment of the layer magnetizations, i.e., with $\tilde{\omega}_L = \tilde{\omega}_R$, the expression for the critical current reads

$$j_C^{FM} = \frac{T}{T_C} \mathrm{Re} \sum_{\omega>0,\sigma=\pm} \frac{G_{S\sigma}^2 \Phi_{S\sigma}^2}{\omega^2} \left\{ 1 + 4\sigma\alpha_{SO}\gamma_B \frac{iH_{exc}}{\tilde{\omega}_+ \tilde{\omega}_-} \right\} \times \left\{ 1 + 2\overline{\gamma}_B G_{S\sigma}\tilde{\omega}_\sigma + \overline{\gamma}_B^2 \tilde{\omega}_\sigma^2 + \sigma 4\alpha_{SO}\overline{\gamma}_B \frac{iH_{exc}}{\tilde{\omega}_+\tilde{\omega}_-} \frac{G_{S\sigma}^2 \Phi_{S\sigma}\tilde{\Phi}_{S\sigma}}{\omega^2} \right\}^{-1}. \tag{10}$$

We begin with an analytical consideration of the case of a vanishing effective pair breaking parameter near the SF boundary, $\gamma_M = 0$; i.e., the influence of the F layer on the superconducting properties of the S metal can be neglected and the order parameter in the S region is spatially homogeneous: $\Phi_{S\pm}(x) = \Delta_0(T)$ [see Eqs. (7) and (8)]. For the amplitude of the Josephson current we than obtained

$$j_C^{FM} \approx 2\frac{T}{T_C} \sum_{\omega>0} \frac{\Delta_0^2}{\Delta_0^2 + \omega^2} \left\{ 1 + 2\overline{\gamma}_B \omega G_S + \overline{\gamma}_B^2(\omega^2 - H_{exc}^2) + 8\alpha_{SO}\overline{\gamma}_B^2 \frac{\omega H_{exc}^2}{\omega^2 + H_{exc}^2} \right\} \{[1 + 2\overline{\gamma}_B \omega G_S + \overline{\gamma}_B^2(\omega^2 - H_{exc}^2)]^2 + 4H_{exc}^2 \overline{\gamma}_B^2 (G_S + \overline{\gamma}_B\omega)^2\}^{-1}. \tag{11}$$

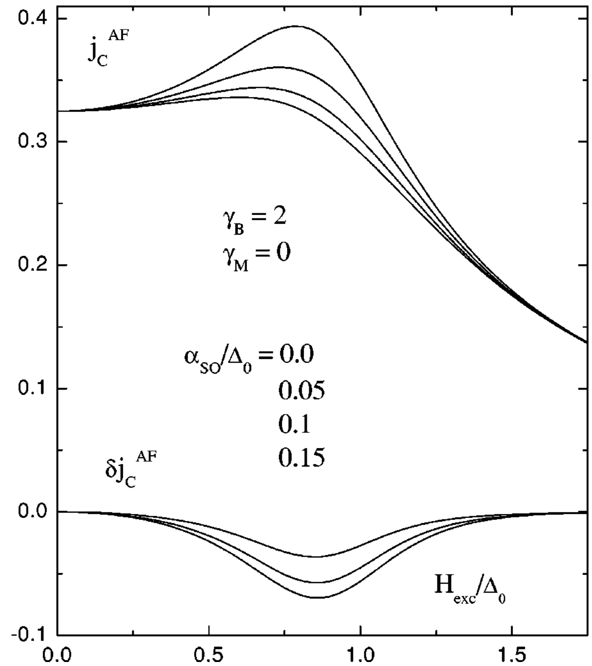

FIG. 3. Same as in Fig. 1 but for antiparallel orientation of the F-layer magnetization. The additive part of the critical current due to spin-orbit scattering, δj_C^{AF}, is also shown.

If $H_{exc} \to 0$, the expression (11) restores the result for SNINS junctions [see, e.g., Eq. (28a) of Ref. 18 for the parameters values under consideration]. As is seen from this expression, for large enough H_{exc}, the supercurrent changes its sign; i.e., with increasing magnetic energy the junction crosses over from 0-phase-type to π-phase-type superconductivity. However, the spin-flip processes exert influence upon junction's tendency to set in the π-phase state.

In Fig. 1 we plot a family of the Josephson current amplitude (11) as a function of H_{exc} when $\gamma_B = 2$ and $\gamma_M = 0$ for various values of the spin-orbit scattering intensity α_{SO}. Here we also show the function $\delta j_C^{FM} = j_C^{FM}(\alpha_{SO}) - j_C^{FM}(\alpha_{SO} = 0)$. The main feature here is that the increase of exchange energy pulls the SFIFS junction to the π state. The new result of this figure is that, as the intensity of spin-orbit scattering processes increases, the critical current amplitude $j_C^{FM}(\alpha_{SO})$ for the π state decreases. The spin-orbit scattering reduces the effect of exchange field, and this reduction has a nonlinear character with its maximum in the region where the transition to the π state takes place.

A more realistic case is shown in Fig. 2, where we plot the results of numerical calculations of the critical current, Eq. (10), for the tunnel junction when suppression of the S layer superconductivity occurs due to the proximity effect ($\gamma_M \neq 0$). The function δj_C^{FM} has also been shown. Again, the main result of these calculations is that spin-flip processes can sizably reduce the SFIFS junction tendency to a π-phase state. In contrast to a simple physical picture, the suppression has nonlinear behavior, acquiring its maximum near the H_{exc} region of crossover from 0-type to π-type superconductivity.

The physical mechanism of the 0-π transition for junctions with thin F layers differs from those studied before, where this transition was due to spatial oscillation of the order parameter in thick F layers. For the case of thin ferromagnetic layers a large enough H_{exc} prompts phase jumps by $\pi/2$ at each SF interface, providing a total π shift across the contact.[6,9] However, if there is spin-orbit scattering, this causes a pair to ''see'' a ''smeared'' exchange field which changes sign at a rate $\sim 1/\tau_{SO}$. This scenario can be clearly illustrated in the limit of large γ_B and H_{exc}, and $\gamma_M = 0$, where Eqs. (6) and (7) yield

$$F_{F\pm}(0) \sim -i\Delta_0 \frac{H_{exc} \pm i2\alpha_{SO}}{\overline{\gamma}_B H_{exc}} \mathrm{sgn}(H_{exc}).$$

Due to the decrease of the effective exchange field experienced by the pair the π-phase state has been shifted to high value of the exchange fields (compare in Fig. 1 the cases $\alpha_{SO}=0$ and $\alpha_{SO}\neq 0$).

While our results have been obtained for SF bilayers with a weak proximity effect and low boundary transparency, qualitatively the conclusions should be valid for arbitrary γ_B and γ_M values, too.

B. Antiparallel orientation of the layer magnetizations

To be definite, we took $\tilde{\omega}_L = \omega + iH_{exc}$, $\tilde{\omega}_R = \omega - iH_{exc}$. Now the critical current of the $(\mathrm{SF})_L\mathrm{I}(\mathrm{FS})_R$ tunnel contact can be given in the form

$$\begin{aligned} j_C^{AF} = {} & 2\frac{T}{T_C}\mathrm{Re}\sum_{\omega>0} \frac{G_{S+}\Phi_{S+}G_{S-}\Phi_{S-}}{\omega^2}\Bigg\{ (G_{S+} + \overline{\gamma}_B\tilde{\omega}_+)^2 \\ & + \frac{G_{S+}^2\Phi_{S+}\tilde{\Phi}_{S+}}{\omega^2}\left(1 + 4\alpha_{SO}\frac{iH_{exc}}{\tilde{\omega}_+\tilde{\omega}_-}\right)\Bigg\}^{-1} \\ & \times \Bigg\{ (G_{S-} + \overline{\gamma}_B\tilde{\omega}_-)^2 + \frac{G_{S-}^2\Phi_{S-}\tilde{\Phi}_{S-}}{\omega^2} \\ & \times \left(1 - 4\alpha_{SO}\frac{iH_{exc}}{\tilde{\omega}_+\tilde{\omega}_-}\right)\Bigg\}^{-1} \end{aligned} \tag{12}$$

First we assume that one can neglect the suppression of the superconductivity in the S layer ($\gamma_M=0$). Then, after simple but cumbersome algebra we have for the critical current

$$\begin{aligned} j_C^{AF} \approx {} & 2\frac{T}{T_C}\sum_{\omega>0} \frac{\Delta_0^2}{\Delta^2+\omega_0^2}\Bigg\{ 1 - 8\alpha_{SO}\frac{\Delta_0^2}{\Delta_0^2+\omega^2} \\ & \times \frac{\omega H_{exc}^2}{\overline{\gamma}_B^2(\omega^2+H_{exc}^2)}\Bigg\}\{[1+2\overline{\gamma}_B\omega G_S + \overline{\gamma}_B^2(\omega^2 - H_{exc}^2)]^2 \\ & + 4H_{exc}^2\overline{\gamma}_B^2(G_S+\overline{\gamma}_B\omega)^2\}^{-1/2}. \end{aligned} \tag{13}$$

For $H_{exc}\to 0$ expression (13) restores the result for SNINS junctions. If $\tau_{SO}\to\infty$ and $H_{exc}\neq 0$, the expression reproduces the main results of Refs. 7–9. Namely, the SFIFS junction is always in the 0-phase state, but in some interval of the exchange field energy the enhancement of the dc Josephson current occurs. The physical mechanism of the enhancement becomes transparent if we use the real energy ε representation and a formal analogy between the energy shifting due to an electric potential V and the energy shifting due to the exchange field (this analogy for SF structures have been noted in Refs. 22 and 23). One can show[9] that the maximum of $I_C^{AF}(H_{exc})$ is achieved due to the overlap of two $\varepsilon^{-1/2}$ singularities in the quasiparticle density of states. In the limit $T\to 0$ this leads formally to logarithmic divergency of the critical current, similar to the Riedel singularity of nonstationary supercurrent in SIS tunnel junctions at voltage $eV=2\Delta_0$. If there is a spin-orbit scattering process in the F layer, then, as has been noted above, the ''smeared'' exchange field has an effect on Cooper pairs, reducing the current enhancement [see Eq. (13)]. To continue our analogy, one can say that the influence of the spin-orbit processes on I_C^{AF} is similar to temperature: finite temperature smears out the Riedel singularity.

In Fig. 3 we present a family of the Josephson current amplitude (13) as a function of H_{exc} when $\gamma_B=2$ and $\gamma_M=0$ for various values of the spin-orbit scattering intensity α_{SO}. Here we also plot the function $\delta j_C^{AF} = j_C^{AF}(\alpha_{SO}) - j_C^{AF}(\alpha_{SO}=0)$. One can see that in the range of $H_{exc}<\Delta_S$ there is the effect of enhancement of the SFIFS junction critical current. The new result in this figure is that, as the intensity of spin-orbit scattering processes increases, the tendency of the dc Josephson current $j_C^{AF}(\alpha_{SO})$ to enhance decreases. The spin-orbit scattering reduces the effect of exchange field, and this reduction is nonlinear in character with its maximum in the region most interesting for experimentalists. More general cases are shown in Fig. 4, where we illustrate the results of numerical calculations of the critical current, Eq. (12), taking into account a suppression of the S-layer superconductivity by the proximity effect ($\gamma_M\neq 0$). The function δj_C^{AF} has also been shown. One can see, again, that spin-orbit suppression has nonlinear behavior, obtaining its maximum in the region of H_{exc}^x most interesting for experimental investigation. Qualitatively, these results should be valid for arbitrary values of γ_B and γ_M, too.

IV. CONCLUSION

We do not consider here the experimental situation, which is now unclear and controversial even for SF hybrid struc-

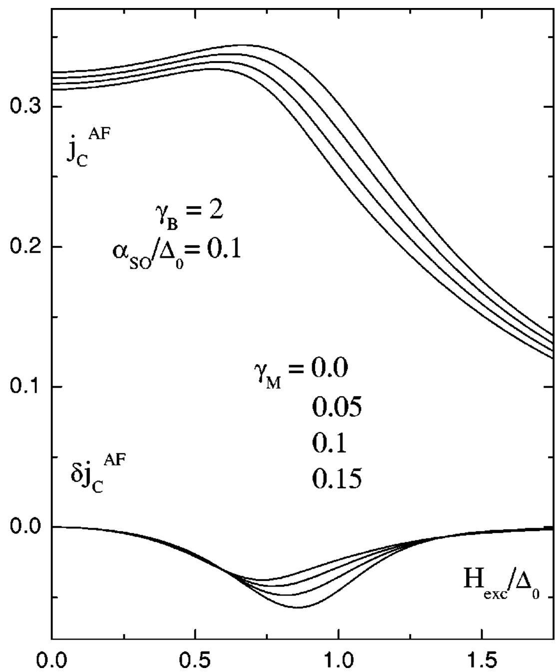

FIG. 4. Same as in Fig. 2 but for parallel orientation of the F-layer magnetization. The additive part of the critical current due to spin-orbit scattering, δj_C^{FM}, is also shown.

tures with thick ferromagnetic layers. Let us only note that spin-orbit scattering is relevant for ferromagnetic conductors containing large Z numbers. The magnetic inhomogeneity of the materials, such as the F-layer multidomain structure, domain walls, and the inhomogeneous ''cryptoferromagnetic'' state imposed by superconductors also gives a nonzero probability amplitude for the spin-flip scattering. For nanoscale hybrid structures a strong electric field arising near metal-metal boundaries is also an important source of the spin-flip processes. In the absence of any precise information about the magnetic structure of the samples used in experiments on SF sandwiches, we restrict ourselves to making only qualitative analytical calculations.

In conclusion, we investigate the spin-orbit scattering effect on critical current of SFIFS tunnel junctions for the case of thin F layers, when the superconducting order parameter oscillation is absent. Instead, the parameter phase jumps at the SF interfaces. The analysis is based on a microscopic theory for proximity-coupled S and F layers. The main result of our calculations is that the spin-flip processes can sizably modify the behavior of the dc Josephson current versus the exchange field for SFIFS tunnel junctions. We find that reduction of the exchange field effects is nonlinear in character, acquiring its maximum in the ferromagnetic field region where the critical current enhancement or transition to the π state occurs. Hence, for understanding the various experimental results on tunnel structures with thin F layers, the coupled effects of the exchange interaction and spin-orbit scattering must be considered.

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