Superconducting states in ferromagnetic metals

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The symmetry of the superconducting states arising directly from the ferromagnetic states in crystals with cubic and orthorhombic symmetries is described. The symmetry nodes in the quasiparticle spectra of such states are pointed out if they exist. The superconducting phase transition in the ferromagnet is accompanied by the formation of superconducting domain structure consisting of complex conjugate states imposed on the ferromagnet domain structure with the opposite direction of the magnetization in the adjacent domains. The interplay between stimulation of a nonunitary superconducting state by the ferromagnetic moment and supression of superconductivity by the diamagnetic orbital currents is established.

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I. INTRODUCTION

A new class of superconducting materials has been revealed very recently where the superconducting state appears from another ordered state of the material—namely, the ferromagnetic state. There are now several metallic compounds demonstrating the coexistence of superconductivity and itinerant ferromagnetism. These are UGe_2,[1,2] $ZrZn_2$,[3] and URhGe.[4] The superconducting states in these materials have to be preferably spin triplet to avoid the large depairing influence of the exchange field. Moreover it seems reasonable[5] that these are the states where only electrons with the spin down direction of the spins are paired, as is the case in the A_1 phase of superfluid He-3.[6] Then the interaction between ferromagnetic and Cooper pair magnetic moments will stimulate the superconducting state. The explanation of the phase diagram of $ZrZn_2$ based on this idea has been proposed.[7] At first sight it seems plausible because $ZrZn_2$ has a cubic crystalline structure allowing multicomponent unconventional superconducting states with spontaneous magnetization. On the contrary the first discovered ferromagnet-superconductor UGe_2 has an orthorhombic structure. The orthorhombic point group obeys only one-dimensional representations that prevents the formation of a superconducting state with spontaneous magnetization in the crystals with strong spin-orbital coupling as a result of a spontaneous phase transition from the normal state.[8] However Fomin has recently[9] shown that the magnetic superconducting phases may arise from the normal ferromagnet state even in the orthorhombic crystal with strong spin-orbital coupling. It means that in this case the stimulation of the superconductivity by the ferromagnetism also takes place.

The goal of this article is to present the detailed analysis of the problem of interaction of triplet pairing superconductivity with magnetization in the ferromagnetic metals. To investigate this problem one must first have the symmetry classification scheme for the superconducting states arising from the ferromagnetic normal state. The point is that the classification of unconventional superconducting states arising from a nonmagnetic normal state, that has been established in Refs. 10–12, does not include the new ferromagnet-superconducting states arising from the normal state with broken time reversal symmetry. So, the discussion of interplay of the stimulation of superconductivity by the ferromagnetism of itinerant electrons and the suppression of superconductivity by diamagnetic currents is forestalled by the symmetry classification of possible triplet superconducting states arising directly from a normal ferromagnet state in crystals with an inversion center. All superconducting magnetic classes in the crystals with orthorhombic (Sec. II) and cubic symmetry (Sec. III) are described and the corresponding superconducting order parameters are presented. The existing symmetry nodes in the spectra of the elementary excitations are pointed out. It is shown that in the superconducting state the ferromagnetic domain structure with opposite direction of magnetization in the adjacent domains causes the appearance of the superconducting domain structure with the complex conjugate order parameters and the opposite directions of the Cooper pair magnetic moments.

II. SUPERCONDUCTIVITY IN THE FERROMAGNET ORTHORHOMBIC METALS

A. Superconducting states

Let us consider first a ferromagnetic orthorhombic crystal with spontaneous magnetization along one of the symmetry axis of the second order chosen as the z direction. The symmetry group

$$G = M \times U(1) \tag{1}$$

consists of the so called magnetic class[13] M and the group of the gauge transformations $U(1)$. Any magnetic superconducting state arising directly from this normal state corresponds to the one of the subgroups of the group G characterized by broken gauge symmetry. In the given case M is equal to $D_2(C_2^z) = (E, C_2^z, RC_2^x, RC_2^y)$, where R is the time reversal operation. Let us look first on the subgroups of G being isomorphic to the initial magnetic group $D_2(C_2^z)$ and constructed by means of combining its elements with phase factor $e^{i\pi}$ being an element of the group of the gauge transformations $U(1)$. The explicit form of these classes are

$$D_2(C_2^z)=(E,C_2^z,RC_2^x,RC_2^y), \tag{2}$$

$$\tilde{D}_2(C_2^z)=(E,C_2^z,RC_2^x e^{i\pi},RC_2^y e^{i\pi}), \tag{3}$$

$$D_2(E)=(E,C_2^z e^{i\pi},RC_2^x e^{i\pi},RC_2^y), \tag{4}$$

$$\tilde{D}_2(E)=(E,C_2^z e^{i\pi},RC_2^x,RC_2^y e^{i\pi}). \tag{5}$$

The superconducting states are characterized by broken gauge symmetry. At the same time the phase transition from the normal paramagnetic state with symmetry $D_2\times U(1)$ to the normal ferromagnetic state with symmetry $D_2(C_2^z)\times U(1)$ obeys this symmetry. That is why the phase transition from a normal paramagnetic state to a normal ferromagnetic state and from a normal ferromagnetic state to a superconducting ferromagnetic state cannot have the same origin contrary to the statement in the paper.[9] As a result the corresponding phase transition lines may intersect each other only accidentally in isolated points in the (P,T) plane. In particular there is no reason for the coincidence of these lines exactly in the quantum critical point at $T=0$. In the existing orthorhombic compound UGe_2 the ferromagnetism and superconductivity at $T=0$ disappear simultaneously above the critical pressure about 17.6 kbar, the low-temperature part of para-ferro phase transition near this pressure is, however, of first order.[2]

To each of the superconducting magnetic classes corresponds an order parameter. All these vector (triplet) order parameters in the crystal with inversion center and strong spin-orbital coupling have the form[8]

$$\mathbf{d}(\mathbf{R},\mathbf{k})=\eta(\mathbf{R})\mathbf{\Psi}(\mathbf{k}), \tag{6}$$

$$\mathbf{\Psi}(\mathbf{k})=\hat{\mathbf{x}}f_x(\mathbf{k})+\hat{\mathbf{y}}f_y(\mathbf{k})+\hat{\mathbf{z}}f_z(\mathbf{k}), \tag{7}$$

where $\hat{x},\hat{y},\hat{z}$ are the unit vectors of the spin coordinate system pinned to the crystal axes and $f_x(\mathbf{k}),\ldots,$ are the odd functions of momentum directions of pairing particles on the Fermi surface. Functions $\mathbf{\Psi}(\mathbf{k})$ for each superconducting state obey a normalization condition

$$\langle\mathbf{\Psi}^*(\mathbf{k})\mathbf{\Psi}(\mathbf{k})\rangle=1, \tag{8}$$

where the angular brackets denote the averaging over $\mathbf{k}$ directions.

The general form of the order parameters for the states (2)–(5) have been pointed out in Ref. 9. We write them here in a somethat different form:

$$\mathbf{\Psi}^{A_1}(\mathbf{k})=\hat{\mathbf{x}}(k_xu_1^{A_1}+ik_yu_2^{A_1})+\hat{\mathbf{y}}(k_yu_3^{A_1}+ik_xu_4^{A_1})+\hat{\mathbf{z}}(k_zu_5^{A_1}+ik_xk_yk_zu_6^{A_1}), \tag{9}$$

$$\mathbf{\Psi}^{A_2}(\mathbf{k})=\hat{\mathbf{x}}(ik_xu_1^{A_2}+k_yu_2^{A_2})+\hat{\mathbf{y}}(ik_yu_3^{A_2}+k_xu_4^{A_2})+\hat{\mathbf{z}}(ik_zu_5^{A_2}+k_xk_yk_zu_6^{A_2}), \tag{10}$$

$$\mathbf{\Psi}^{B_1}(\mathbf{k})=\hat{\mathbf{x}}(k_zu_1^{B_1}+ik_xk_yk_zu_2^{B_1})+\hat{\mathbf{y}}(ik_zu_3^{B_1}+k_xk_yk_zu_4^{B_1})+\hat{\mathbf{z}}(k_xu_5^{B_1}+ik_yu_6^{B_1}), \tag{11}$$

$$\mathbf{\Psi}^{B_2}(\mathbf{k})=\hat{\mathbf{x}}(ik_zu_1^{B_2}+k_xk_yk_zu_2^{B_2})+\hat{\mathbf{y}}(k_zu_3^{B_2}+ik_xk_yk_zu_4^{B_2})+\hat{\mathbf{z}}(ik_xu_5^{B_2}+k_yu_6^{B_2}), \tag{12}$$

where $u_1^A,\ldots,$ are real functions of k_x^2,k_y^2,k_z^2. It is worth noting that the state $\mathbf{\Psi}^{A_2}$ transforms as $i\mathbf{\Psi}^{A_1^*}$ and the state $\mathbf{\Psi}^{B_2}$ transforms as $i\mathbf{\Psi}^{B_1^*}$.

From the expressions for the order parameters (9)–(12) one can conclude that the states A and B have in general no symmetry nodes in the quasiparticle spectrum. Only occasional nodes appear for a particular form of the functions $u_1^A,\ldots$.

The classification of the states in quantum mechanics corresponds to the general statement by Wigner that the different eigenvalues are related to the sets of eigenstates belonging to the different irreducible representations of the group of symmetry of the Hamiltonian. In particular, in absence of the time inversion symmetry violation, the superconducting states relating to the nonequivalent irreducible representations of the point symmetry group of crystal obey the different critical temperatures. Similarly the eigenstates of the particles in the ferromagnetic crystals are classified in accordance with corepresentations Γ of magnetic group M of the crystal.[14] The latter differs from usual representations by the law of multiplication of matrices of representation which is $\Gamma(g_1)\Gamma(g_2)=\Gamma(g_1g_2)$ for elements g_1,g_2 of group M if element g_1 does not include the time inversion operation and $\Gamma(g_1)\Gamma^*(g_2)=\Gamma(g_1g_2)$ if element g_1 does include the time inversion. The matrices of transformation of the order parameters (9)–(12) by the symmetry operations of the group $D_2(C_2^z)=(E,C_2^z,RC_2^x,RC_2^y)$ are just numbers (characters). As usual for one-dimensional representations they are equal ±1. For the state A_1 (9) which is a conventional superconducting state obeying the complete point-magnetic symmetry of initial normal state they are (1,1,1,1). For the order parameter A_2 (10) they are $(1,1,-1,-1)$, where -1 corresponds to the elements of the superconducting symmetry class (3) containing the phase factor $e^{i\pi}$. The same is true for the table of characters of the other states. So all the corepresentations in the present case are real, however, their difference from the usual representations manifests itself in the relationship of equivalence.

The two corepresentations of the group M are called equivalent[15] if their matrices $\Gamma(g)$ and $\Gamma'(g)$ are transformed to each other by means of the unitary matrix U as $\Gamma'(g)=U^{-1}\Gamma(g)U$ if the element g does not include the time inversion and as $\Gamma'(g)=U^{-1}\Gamma(g)U^*$ if the element g includes the time inversion. The corepresentations for the pair of states A_1 and A_2 are equivalent. In view of one-dimensional character of these corepresentations the matrix of the unitary transformation is simply given by the number $U=i$. The states A_1 and A_2 belong to the same corepresentation and represent two particular forms of the same superconducting state. It will be shown below that if we have state

A_1 in the ferromagnet domains with the magnetization directed up the superconducting state in the domains with down direction of the magnetization corresponds to the superconducting state A_2. The same is true for the pair of states B_1 and B_2.

The critical temperatures of the phase transition from a ferromagnetic normal state to the superconducting states relating to the nonequivalent corepresentations are in general different. The latter is guaranteed by the property of the orthogonality of the order parameters relating to the nonequivalent corepresentations

$$\langle \boldsymbol{\Psi}^{A*}(\mathbf{k}) \boldsymbol{\Psi}^{B}(\mathbf{k}) \rangle = 0. \tag{13}$$

The critical temperatures of equivalent states A_1 and A_2 in the ferromagnetic domains with the opposite orientations of magnetization are equal (see below).

B. Stimulation of superconductivity by ferromagnetism

All the listed above superconducting phases are in principle nonunitary and obey the Cooper pair spin momentum

$$\mathbf{S} = i \langle \boldsymbol{\Psi}^* \times \boldsymbol{\Psi} \rangle = \frac{i}{4} \langle f_-^* f_- - f_+^* f_+ \rangle, \tag{14}$$

where $f_\pm = f_x \pm i f_y$ and Cooper pair angular momentum

$$\mathbf{L} = i \left\langle \Psi_i^* \left(\mathbf{k} \times \frac{\partial}{\partial \mathbf{k}} \right) \Psi_i \right\rangle. \tag{15}$$

The spontaneous Cooper pair magnetic moment as it is clear from Eq. (14) is proportional to the difference in the density of populations of pairs with spin up and spin down. In superfluid He-3 in the A_1 state, only Cooper pairs with spin down are present. Their magnetic moments interact with external field giving rise to an increase of the critical temperature of the phase transition to the superfluid state. In the ferromagnetic metals with strong spin-orbital coupling there are the Cooper pairs with any projection of the total spin. However, the dependence of the critical temperature of the superconducting phase transition from the ferromagnet magnetization also exists. On the microscopic level this dependence originates from the difference of the pairing interaction and the density of states on the Fermi surfaces for the particles with opposite spin projections (see below). Here one needs to note that as usual the word ''spin'' means in fact ''pseudospin'' and it is used to denote Kramers double degeneracy of electron states in a metal with spin-orbital coupling.

On the phenomenological level the shift of the critical temperature determined mostly by the magnetic field action on the electron spins can be described by the following term in the Landau free energy expansion

$$-N_0 f\left(\frac{\mu_B H}{\varepsilon_F} \right) |\eta|^2, \tag{16}$$

where N_0 is the electron density of states on the Fermi surface, μ_B is the Bohr magneton, the function $f(x) \sim x$ at small values of its argument. The magnetic field

$$\mathbf{H} = \mathbf{H}_{\mathrm{ex}} + \mathbf{H}_{\mathrm{ext}} \tag{17}$$

consists of the exchange field $\mathbf{H}_{\mathrm{ex}}$ and the external magnetic field $\mathbf{H}_{\mathrm{ext}}$. Let us stress a very important difference between these fields. $\mathbf{H}_{\mathrm{ex}}$ is frozen into the crystal. It is transformed with any operation of the point symmetry group and completely invariant under magnetic symmetry class $D_2(C_2^z)$ transformations. $\mathbf{H}_{\mathrm{ext}}$ does not relate to these transformations, but as any magnetic field, changes the sign under time inversion.

The exchange field acting on the electron spins stimulates the nonunitary superconducting state. The resulting enhancement of the critical temperature in the absence of an external field can be estimated as

$$\frac{T_c(H_{\mathrm{ex}}) - T_c}{T_c} \approx \frac{\mu_B H_{\mathrm{ex}}}{\varepsilon_F}. \tag{18}$$

The exchange field determines the relative shift of the Fermi surfaces for the spin up and spin down quasiparticles. One can estimate the value of this field for UGe_2 by its Curie temperature. Taking into account that at temperatures lower than ≈ 20 K the phase transition into ferromagnet state starts to be of first order[16] one can say that H_{ex}. is lying in the interval $\approx (20\ \mathrm{T}, 40\ \mathrm{T})$ in the whole interval of the pressures where superconductivity exists.

Unlike in He-3, in ferromagnetic superconductors the magnetic field acts through the electron charges on the orbital electron motion to suppress the superconducting state. The reduction of the critical temperature due to the orbital effect is

$$\frac{T_c(H_{\mathrm{em}}) - T_c}{T_c} \approx -\frac{\xi_0^2 H_{\mathrm{em}}}{\Phi_0} \approx -\frac{\varepsilon_F \mu_B H_{\mathrm{em}}\ m}{T_c^2\ m^*}, \tag{19}$$

where m and m^* are bare and effective electron mass. The electromagnetic field H_{em} acting on the electron charges is determined by the modulus of the sum of the vectors of the external magnetic field and the dipole field of its own ferromagnet magnetic moment. The latter is much smaller than H_{ex}. In the absence of the external field one can estimate the value of the H_{em} by the value of the magnetic moment density which in UGe_2 is less than 1 kG.[1,2]

The estimations (18) and (19) shows that the stimulation of a nonunitary superconductivity by ferromagnetism takes place at

$$\frac{H_{\mathrm{ex}}}{H_{\mathrm{em}}} > \frac{\varepsilon_F^2\ m}{T_c^2\ m^*}. \tag{20}$$

The interplay between the effects of the stimulation and the suppression of the critical temperature in ferromagnetic superconductors can in principle determine the phase diagrams of these materials as it was suggested in particular for the $ZrZn_2$ in the paper.[7]

It is worth noting however that the estimation (20) is valid far enough from the quantum critical point $T_c(P) = 0$. Near this point the coherence length ξ_0 in Eq. (19) must be substituted by a mean free path l.[17] As result in the quantum critical point vicinity we obtain instead Eq. (20) another criterion for stimulation of superconductivity by ferromagnetism

$$\frac{H_{\rm ex}}{H_{\rm em}} > (k_F l)^2 \frac{m}{m^*}. \tag{21}$$

This criterion is quite general and applies to ferromagnetic superconductors with any crystal symmetry sufficiently close to the pressure where superconductivity disappears. The fulfillment of it is necessary in particular for validity of the explanation of the phase diagram in $ZrZn_2$ proposed in Ref. 7.

One can find the confirmation of these qualitative estimates from the equation for the critical temperature of the superconducting phase transition[8] written (For simplicity we will not use the complete form of the equation for the order parameter taking into account the effect of spontaneous orbital magnetism.[18]) for one of superconducting state (9)–(12) in frame of some particular model of pairing

$$\Delta_{\alpha\beta}(\mathbf{R},\mathbf{r}) = -T\sum_{\omega} \int d\mathbf{r} V_{\beta\alpha,\lambda\mu}(\mathbf{r},\mathbf{r}')G_{\omega}^{\lambda\gamma}(\mathbf{r})G_{-\omega}^{\mu\delta}(\mathbf{r}) \times \exp[i\mathbf{r}\mathbf{D}(\mathbf{R})]\Delta_{\gamma\delta}(\mathbf{R},\mathbf{r}), \tag{22}$$

where

$$\mathbf{D}(\mathbf{R}) = -i\frac{\partial}{\partial \mathbf{R}} + \frac{2e}{c}\mathbf{A}(\mathbf{R}), \quad \nabla\times\mathbf{A} = \mathbf{H}_{\rm em},$$

$$\Delta_{\alpha\beta}(\mathbf{R},\mathbf{r}) = \mathbf{d}(\mathbf{R},\hat{\mathbf{r}})\mathbf{g}_{\alpha\beta},$$

$$V^{\Gamma}_{\beta\alpha,\lambda\mu}(\hat{\mathbf{r}},\hat{\mathbf{r}}') = -\frac{V}{2}[\mathbf{\Phi}^{\Gamma}(\hat{\mathbf{r}})\mathbf{g}_{\beta\alpha}][\mathbf{\Phi}^{\Gamma*}(\hat{\mathbf{r}}')\mathbf{g}^{\dagger}_{\lambda\mu}], \tag{23}$$

$\mathbf{g}_{\alpha\beta} = i(\boldsymbol{\sigma}\sigma_{\rm y})_{\alpha\beta}$, $\boldsymbol{\sigma} = (\sigma_x,\sigma_y,\sigma_z)$ are the Pauli matrices. The functions

$$\mathbf{\Phi}^{\Gamma}(\hat{\mathbf{r}}) = \hat{\mathbf{x}}g_x^{\Gamma}(\hat{\mathbf{r}}) + \hat{\mathbf{y}}g_y^{\Gamma}(\hat{\mathbf{r}}) + \hat{\mathbf{z}}g_z^{\Gamma}(\hat{\mathbf{r}}) \tag{24}$$

are related to the particular corepresentation and in its particular form $\mathbf{\Psi}^{\Gamma}$ (9)–(12). For instance for A_1 case it is

$$\mathbf{\Phi}^{A_1}(\hat{\mathbf{r}}) = \hat{\mathbf{x}}(\hat{r}_x v_1^{A_1} + i\hat{r}_y v_2^{A_1}) + \hat{\mathbf{y}}(\hat{r}_y v_3^{A_1} + i\hat{r}_x v_4^{A_1}) + \hat{\mathbf{z}}(\hat{r}_z v_5^{A_1} + i\hat{r}_x\hat{r}_y\hat{r}_z v_6^{A_1}), \tag{25}$$

where $v_1^A, \ldots,$ are real functions of $\hat{r}_x^2,\hat{r}_y^2,\hat{r}_z^2$. The normal metal electron Green functions are diagonal 2×2 matrices

$$G_{\omega}^{\lambda\gamma}(\mathbf{r}) = \int \frac{d\mathbf{p}}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}}\{[i\omega - \xi(\mathbf{p})]\sigma_0 + 2g(\mathbf{p})\mu_B\sigma_z H_{\rm ex}\}^{-1}_{\lambda\gamma}. \tag{26}$$

Here $\xi(\mathbf{p})$ and $g(\mathbf{p})$ are correspondingly the momentum dependent quasiparticle spectrum and g factor. It is convenient to work with the Green functions by introducing the following notations:

$$\hat{G}_{\omega}(\mathbf{r}) = \frac{1}{2}\{[G_{\omega}^{\uparrow}(\mathbf{r}) + G_{\omega}^{\downarrow}(\mathbf{r})]\sigma_0 + [G_{\omega}^{\uparrow}(\mathbf{r}) - G_{\omega}^{\downarrow}(\mathbf{r})]\sigma_z\}.$$

Using the general form of self-consistency equation (22) one can easily obtain the following system of equations:

$$\langle g_-^*(\hat{\mathbf{r}})f_-(\hat{\mathbf{r}})\rangle\eta(\mathbf{R}) = \langle g_-^*(\hat{\mathbf{r}})g_-(\hat{\mathbf{r}})\rangle\hat{L}\eta(\mathbf{R}), \tag{27}$$

$$\langle g_z^*(\hat{\mathbf{r}})f_z(\hat{\mathbf{r}})\rangle\eta(\mathbf{R}) = \langle g_z^*(\hat{\mathbf{r}})g_z(\hat{\mathbf{r}})\rangle\hat{L}\eta(\mathbf{R}), \tag{28}$$

$$\langle g_+^*(\hat{\mathbf{r}})f_+(\hat{\mathbf{r}})\rangle\eta(\mathbf{R}) = \langle g_+^*(\hat{\mathbf{r}})g_+(\hat{\mathbf{r}})\rangle\hat{L}\eta(\mathbf{R}), \tag{29}$$

where the combinations $g_{\pm} = g_x \pm i g_y$, g_z, and $f_{\pm} = f_x \pm i f_y$, f_z correspond to the pairing interaction and the order parameter amplitudes with spins up-up, down-down, and zero projection of the pair spin on the z direction. The angular brackets denote the averaging over the directions of unit vector $\hat{\mathbf{r}}$ and the integral operator in the right-hand side is

$$\hat{L}\eta(\mathbf{R}) = \frac{1}{2}VT\sum_{\omega}\int d\mathbf{r}\{g_-^*(\hat{\mathbf{r}})f_-(\hat{\mathbf{r}})G_{\omega}^{\uparrow}(\mathbf{r})G_{-\omega}^{\uparrow}(\mathbf{r}) + g_z^*(\hat{\mathbf{r}})f_z(\hat{\mathbf{r}})[G_{\omega}^{\uparrow}(\mathbf{r})G_{-\omega}^{\downarrow}(\mathbf{r}) + G_{\omega}^{\downarrow}(\mathbf{r})G_{-\omega}^{\uparrow}(\mathbf{r})] + g_+^*(\hat{\mathbf{r}})f_+(\hat{\mathbf{r}})G_{\omega}^{\downarrow}(\mathbf{r})G_{-\omega}^{\downarrow}(\mathbf{r})\}\exp[i\mathbf{r}\mathbf{D}(\mathbf{R})]\eta(\mathbf{R}). \tag{30}$$

One must to add to this equation the normalization condition (8)

$$\left\langle \frac{1}{2}[f_+^*(\hat{\mathbf{r}})f_+(\hat{\mathbf{r}}) + f_-^*(\hat{\mathbf{r}})f_-(\hat{\mathbf{r}})] + f_z^*(\hat{\mathbf{r}})f_z(\hat{\mathbf{r}})\right\rangle = 1. \tag{31}$$

After finding of the eigen function $\eta(R)$ of the operator $\hat{L}$ one must find the critical temperature from the condition of zero value of the determinant of the linear system of equations (27)–(29) for the amplitudes $\langle g^* f\rangle$. It is worth noting that, for the interaction in the form of A_1 state, the order parameter can be chosen correspondingly as belonging to one of A_1 or A_2 states. As a result all the amplitudes $\langle g^* f\rangle$ will be correspondingly real or imaginary and we deal with the system of equations of the third order determining critical temperature common for the both states. The same is correct for the B type corepresentation.

Then the appearance of the linear shift of the critical temperature due to exchange field (18) follows trivially from the linear shifts of the amplitudes

$$\langle g_-^* g_-\rangle - \langle g_-^* g_-\rangle_{H_{\rm ex}=0} \sim -\frac{\mu_B H_{\rm ex}}{\varepsilon_F},$$

$$\langle g_+^* g_+\rangle - \langle g_+^* g_+\rangle_{H_{\rm ex}=0} \sim \frac{\mu_B H_{\rm ex}}{\varepsilon_F},$$

$$\int d\mathbf{r}[G^{\uparrow}_{\omega}G^{\uparrow}_{-\omega}-G^{\uparrow}_{\omega}(H_{\text{ex}}=0)G^{\uparrow}_{-\omega}(H_{\text{ex}}=0)]\sim-\frac{\mu_B H_{\text{ex}}}{\varepsilon_F},$$

$$\int d\mathbf{r}[G^{\downarrow}_{\omega}G^{\downarrow}_{-\omega}-G^{\downarrow}_{\omega}(H_{\text{ex}}=0)G^{\downarrow}_{-\omega}(H_{\text{ex}}=0)]\sim\frac{\mu_B H_{\text{ex}}}{\varepsilon_F}.$$

To demonstrate the validity of two latter relationships it is enough to look at the expression for electron Green function in a normal metal with isotropic spectrum $\xi(\mathbf{p})=\xi$ and isotropic g factor $g(\mathbf{p})=1/2$ (see Ref. 18):

$$G^{\lambda}_{\omega}(\mathbf{r})=-\frac{\pi N_0}{p_0 r}\exp\left(ip_0^{\lambda}r\ \text{sgn}\omega-\frac{r|\omega|}{v_0^{\lambda}}\right), \qquad (32)$$

here $p_0^{\lambda}=[2m(\varepsilon_F-\lambda\mu_B H_{\text{ex}})]^{1/2}$, $p_0=p_0^{\lambda}(H_{\text{ex}}=0)$ is the Fermi momentum, $\lambda=\uparrow,\downarrow$ or $+1,-1$, $v_0^{\lambda}=p_0^{\lambda}/m$, is the Fermi velocity on the corresponding sheet of the Fermi surface, $v_0=p_0/m$, $N_0 m p_0/2\pi^2$ is the density of states on the one spin projection.

As for the second term in the right-hand side of Eq. (30) due to the difference in the Fermi momenta with spin up and spin down it contains the fast oscillating products of two Green functions and starts to be negligibly small. The smallness of this term, however, does not result in the disappearance of the amplitude f_z of the Cooper pair state with zero projection of spin because all three amplitudes f_+,f_-,f_z obey coupled linear equations (27)–(29). This fact is the direct consequence of the strong spin-orbital coupling. Unlike this, in the superfluid He-3, all three amplitudes f_+,f_-,f_z obey independent equations characterized by different critical temperatures[18] such that the amplitudes f_+ and f_z are equal to zero at the critical temperature where the amplitude f_- appears.

Generally speaking the second term in Eq. (30) promotes the appearance of the oscillating solution

$$\eta(\mathbf{R})=\eta(x,y)e^{iQz}. \qquad (33)$$

On the other hand, the first and the third terms in Eq. (30) make these oscillations nonprofitable (oscillations in the order parameter decrease the critical temperature). In superconductors with *s* pairing the appearance of a solution with nonvanishing Q or so called Fulde-Ferrell-Larkin-Ovchinnikov state[19,20] is possible for large enough values of H_{c2}^{orb} in comparison with the paramagnetic limiting field.[21] In superconductors with triplet pairing when $H_{\text{ex}}\gg H_{c2}^{\text{orb}}$ one would not expect the appearance of FFLO state. This question, however, demands a special investigation in the frame of some particular model of pairing interaction. One can find a quick look on the problem in the paper.[22] It should be mentioned also that inhomogeneous superconducting states break the space parity. This demands in principle a generalization of the classification of the superconducting states proposed in the present paper.

C. Domain structures

To discuss the domain structures in the ferromagnetic superconductors in the absence of the external magnetic field let us assume that we have interactions (23), (24) in the form corresponding to A_1 and A_2 states such that the corresponding functions $v_i^{A_1}$ and $v_i^{A_2}$ are equal. Let us fix the solutions of the two corresponding sets of equations (27)–(29) as relating to A_1 and A_2 states. (As it has been mentioned above one can discuss conversely the pair of solutions relating correspondingly to A_2 and A_1 states. This choice does not change the conclusions of this subsection.) Such a pair of states possess equal and opposite direction Cooper pair magnetic moments. It is easy to see that the following equalities are obeyed:

$$\langle g_-^{A_1^*}g_-^{A_1}\rangle=\langle g_+^{A_2^*}g_+^{A_2}\rangle,\quad \langle g_+^{A_1^*}g_+^{A_1}\rangle=\langle g_-^{A_2^*}g_-^{A_2}\rangle,$$

$$\langle g_z^{A_1^*}g_z^{A_1}\rangle=\langle g_z^{A_2^*}g_z^{A_2}\rangle.$$

Hence, if the state A_1 is the solution of the system of equations (27)–(29) with critical temperature T_c, the state A_2 is the solution of the same system with opposite direction of the H_{ex} and the same critical temperature. This means, if the pairing interaction in the ferromagnet with up direction of H_{ex} corresponds to the pure A_1 state, the superconducting states in ferromagnetic domains with opposite orientation of magnetization will be A_2. One can say that this is the consequence of the above mentioned property of conjugacy between the states A_1 and A_2. The same is true for another pair of conjugate states B_1 and B_2.

The ferromagnet domain structure with alternating up-down direction of the magnetization is always accompanied by the superconducting domain structure with alternating properties of the complex conjugacy of the order parameter and alternating up-down direction of the Cooper pair magnetic moment. The superconducting order parameter distribution in the vicinity of the domain wall between of two adjacent domains demands special investigation.

It is quite natural that the Abrikosov vortices having in the A_1 state some fixed direction of the current and flux will have opposite orientations of the current and flux in the adjacent ferromagnet domain with the opposite direction of magnetization. We stress once again that these conclusions have been obtained in the absence of the external magnetic field. The Abrikosov state in the ferromagnetic superconductors in the presence of the external field has been discussed in frame of phenomenological approach by Sonin and Felner.[23]

III. SUPERCONDUCTIVITY IN FERROMAGNET METALS WITH CUBIC SYMMETRY

Symmetric orientations of magnetic moments in cubic crystals along the symmetry axes of the fourth or of the third order give rise to a decreasing of the initial cubic symmetry of the normal state to the magnetic classes $D_4(C_4)=(E,C_4,C_2,C_4^3,RU_x,RU_y,RU',RU'')$ and $D_3(C_3)=(E,C_3,C_3^2,RU_1,RU_2,RU_3)$, correspondingly.[13] Let as look first at the tetragonal magnetic class.

A. Tetragonal magnetic class $D_4(C_4)$

As before we construct first the groups being isomorphic to the initial magnetic group $D_4(C_4)$ by means of combining its elements with $e^{i\pi}$ and $e^{\pm i\pi/2}$ phase factors from the group of the gauge transformations $U(1)$. The explicit form of these superconducting magnetic classes are

$$D_4(C_4)=(E,C_4,C_2,C_4^3,RU_x,RU_y,RU',RU''), \quad (34)$$

$$\tilde{D}_4(C_4)=(E,C_4,C_2,C_4^3,e^{i\pi}RU_x,e^{i\pi}RU_y,e^{i\pi}RU',e^{i\pi}RU''), \quad (35)$$

$$\tilde{D}_4(D_2)=(E,e^{i\pi}C_4,C_2,e^{i\pi}C_4^3,RU_x,RU_y,e^{i\pi}RU',e^{i\pi}RU''), \quad (36)$$

$$\tilde{D}_4(D_2')=(E,e^{i\pi}C_4,C_2,e^{i\pi}C_4^3,e^{i\pi}RU_x,e^{i\pi}RU_y,RU',RU''), \quad (37)$$

$$D_4(E)=(E,e^{i\pi/2}C_4,e^{i\pi}C_2,e^{3i\pi/2}C_4^3,e^{i\pi}RU_x,RU_y,e^{3i\pi/2}RU',e^{i\pi/2}RU''), \quad (38)$$

$$D_4(E)=(E,e^{3i\pi/2}C_4,e^{i\pi}C_2,e^{i\pi/2}C_4^3,e^{i\pi}RU_x,RU_y,e^{i\pi/2}RU',e^{3i\pi/2}RU''). \quad (39)$$

The order parameters of the superconducting states corresponding to these classes are

$$\mathbf{\Psi}^{A_1}(\mathbf{k})=\hat{\mathbf{x}}(k_xu_1^{A_1}-ik_yu_2^{A_1})+\hat{\mathbf{y}}(k_yu_1^{A_1}+ik_xu_2^{A_1})+\hat{\mathbf{z}}[k_zu_3^{A_1}+ik_xk_yk_z(k_x^2-k_y^2)u_4^{A_1}], \quad (40)$$

$$\mathbf{\Psi}^{A_2}(\mathbf{k})=\hat{\mathbf{x}}(ik_xu_1^{A_2}-k_yu_2^{A_2})+\hat{\mathbf{y}}(ik_yu_1^{A_2}+k_xu_2^{A_2})+\hat{\mathbf{z}}[ik_zu_3^{A_2}+k_xk_yk_z(k_x^2-k_y^2)u_4^{A_2}], \quad (41)$$

$$\mathbf{\Psi}^{B_1}(\mathbf{k})=\hat{\mathbf{x}}(k_xu_1^{B_1}+ik_yu_2^{B_1})+\hat{\mathbf{y}}(-k_yu_1^{B_1}+ik_xu_2^{B_1})+\hat{\mathbf{z}}[k_z(k_x^2-k_y^2)u_3^{B_1}+ik_xk_yk_zu_4^{B_1}], \quad (42)$$

$$\mathbf{\Psi}^{B_2}(\mathbf{k})=\hat{\mathbf{x}}(ik_xu_1^{B_2}+k_yu_2^{B_2})+\hat{\mathbf{y}}(-ik_yu_1^{B_2}+k_xu_2^{B_2})+\hat{\mathbf{z}}[ik_z(k_x^2-k_y^2)u_3^{B_2}+k_xk_yk_zu_4^{B_2}], \quad (43)$$

$$\mathbf{\Psi}^{E_+}(\mathbf{k})=(k_x+ik_y)[\hat{z}u_1^{E_+}+ik_z(k_x\hat{y}-k_y\hat{x})u_2^{E_+}]+(\hat{x}+i\hat{y})[k_zu_3^{E_+}+ik_xk_yk_z(k_x^2-k_y^2)u_4^{E_+}], \quad (44)$$

$$\mathbf{\Psi}^{E_-}(\mathbf{k})=(k_x-ik_y)[\hat{z}u_1^{E_-}+ik_z(k_x\hat{y}-k_y\hat{x})u_2^{E_-}]+(\hat{x}-i\hat{y})[k_zu_3^{E_-}+ik_xk_yk_z(k_x^2-k_y^2)u_4^{E_-}], \quad (45)$$

where $u_1^{A_1}, \ldots,$ are real functions of $k_x^2+k_y^2, k_z^2$.

As for the orthorhombic case the states A_1,A_2 and B_1,B_2 represent the pairs of equivalent corepresentations. Another two superconducting states E_+ and E_- are related to nonequivalent corepresentations. In total there are four different superconducting states. The states A and $E_\pm$ have no symmetry zeros in the quasiparticle spectra. Only the states of B type have symmetry points of zeros lying on the northern and southern poles of the Fermi surface $k_x=k_y=0$. This is easy to see directly from the expressions (40)–(45).

Again in alternating ferromagnet domains with opposite directions of the magnetization there is alternating sequence of A_1 and A_2, or B_1 and B_2, or E_+ and E_- states. As for the latter pair of states one can check this statement directly from the system of equations (27)–(29).

B. Trigonal magnetic class $D_3(C_3)$

The groups being isomorphic to the initial magnetic group $D_3(C_3)$ are constructed by the combinations of its elements with elements $e^{i\pi}$ and $e^{\pm 2i\pi/3}$ of the gauge group $U(1)$. That yields the superconducting magnetic classes of symmetry

$$D_3(C_3)=(E,C_3,C_3^2,RU,RU_2,RU_3), \quad (46)$$

$$\tilde{D}_3(C_3)=(E,C_3,C_3^2,e^{i\pi}RU_1,e^{i\pi}RU_2,e^{i\pi}RU_3), \quad (47)$$

$$D_3(E)=(E,e^{2i\pi/3}C_3,e^{-2i\pi/3}C_3^2,RU,e^{-2i\pi/3}RU_2,e^{2i\pi/3}RU_3), \quad (48)$$

$$\tilde{D}_3(E)=(E,e^{-2i\pi/3}C_3,e^{2i\pi/3}C_3^2,RU,e^{2i\pi/3}RU_2,e^{-2i\pi/3}RU_3), \quad (49)$$

where the elements U_1,U_2,U_3 are the rotations on the angle π around axes

$$\hat{\phi}_1=\hat{\mathbf{x}}, \quad \hat{\phi}_2=\frac{1}{2}(-\hat{\mathbf{x}}+\sqrt{3}\hat{\mathbf{y}}), \quad \hat{\phi}_3=\frac{1}{2}(-\hat{\mathbf{x}}-\sqrt{3}\hat{\mathbf{y}}).$$

The corresponding order parameters are

$$\mathbf{\Psi}^{A_1}(\mathbf{k})=i(k_x\hat{\mathbf{y}}-k_y\hat{\mathbf{x}})u_1^{A_1}+k_z\hat{\mathbf{z}}u_2^{A_1}+(k_y\hat{\mathbf{y}}-k_x\hat{\mathbf{x}})u_3^{A_1}, \quad (50)$$

$$\mathbf{\Psi}^{A_2}(\mathbf{k})=(k_x\hat{\mathbf{y}}-k_y\hat{\mathbf{x}})u_1^{A_2}+ik_z\hat{\mathbf{z}}u_2^{A_2}+i(k_y\hat{\mathbf{y}}-k_x\hat{\mathbf{x}})u_3^{A_2}, \tag{51}$$

$$\mathbf{\Psi}^{E_+}(\mathbf{k})=(\phi_1+e^{2i\pi/3}\phi_2+e^{-2i\pi/3}\phi_3)[i\hat{\mathbf{z}}u_1^{E_+}+k_z(k_x\hat{\mathbf{y}}-k_y\hat{\mathbf{y}})u_2^{E_+}]+(\hat{\phi}_1+e^{2i\pi/3}\hat{\phi}_2+e^{-2i\pi/3}\hat{\phi}_3)[ik_zu_3^{E_+}+\phi_1\phi_2\phi_3u_4^{E_+}], \tag{52}$$

$$\mathbf{\Psi}^{E_-}(\mathbf{k})=(\phi_1+e^{-2i\pi/3}\phi_2+e^{2i\pi/3}\phi_3)[i\hat{\mathbf{z}}u_1^{E_-}+k_z(k_x\hat{\mathbf{y}}-k_y\hat{\mathbf{y}})u_2^{E_-}]+(\hat{\phi}_1+e^{-2i\pi/3}\hat{\phi}_2+e^{2i\pi/3}\hat{\phi}_3)[ik_zu_3^{E_-}+\phi_1\phi_2\phi_3u_4^{E_-}], \tag{53}$$

where

$$\phi_1=k_x,\quad \phi_2=\tfrac{1}{2}(-k_x+\sqrt{3}k_y),\quad \phi_3=\tfrac{1}{2}(-k_x-\sqrt{3}k_y),$$

and $u_1^{A_1},\ldots,$ are the real functions invariant under the transformations D_3 group. The z-axes for both spin and orbital coordinate systems are chosen along the symmetry axis of the third order.

As before the A_1 and A_2 states correspond to the equivalent corepresentations. The states $E_\pm$ are related to nonequivalent representations.

None of these states have the symmetry nodes in the quasiparticle spectra.

IV. CONCLUSION

The symmetry classifications of the superconducting states with triplet pairing in the orthorhombic and cubic ferromagnet crystals with strong spin-orbital coupling is presented. It is found that unlike the case of weak spin-orbital interaction where the nonunitary magnetic superconducting states are possible only in the case of multicomponent superconductivity[8] any superconducting state in the ferromagnet metals with strong spin-orbital coupling is in general nonunitary. It is demonstrated that in general none of states (besides B states in the cubic crystals) obey the symmetry nodes in the quasiparticle spectra. The situation with zeros is changed however if due to some reason the pairing amplitude with zero projection of the Cooper pair spin is absent.[22,24]

The ferromagnetism stimulates in general the triplet superconductivity even with a one-component order parameter. The mechanism of this stimulation is due to the difference of the pairing interaction and the density of states for electrons with opposite directions of spin. However, the competitive mechanism supressing superconductivity due to the orbital diamagnetic currents is always present. The comparison of these two influences of ferromagnetism on superconductivity near the quantum critical point leads to the criterion given by formula (21).

The presence of the ferromagnet domain structure in the superconducting state is always accompanied by the corresponding superconducting domain structure of the complex conjugate states. The adjacent domains in the absence of the external field contain the quantized vortices with opposite directions of currents and fluxes.

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