Elementary vortex pinning potential in a chiral p-wave superconductor

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The elementary vortex pinning potential is studied in a chiral p-wave superconductor with a pairing $\mathbf{d} = \overline{\mathbf{z}}(\overline{k}_x \pm i\overline{k}_y)$ on the basis of the quasiclassical theory of superconductivity. An analytical investigation and numerical results are presented to show that the vortex pinning potential is dependent on whether the vorticity and chirality are parallel or antiparallel. Mutual cancellation of the vorticity and chirality around a vortex is physically crucial to the effect of the pinning center inside the vortex core.

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Much attention has been focused on the vortex pinning in type-II superconductors. The vortex pinning plays an important role on various vortex-related quantities and phenomena such as the critical current and the hysteresis of the magnetization in superconductors under magnetic fields. The characteristics of the vortex-related phenomena are of particular interest in unconventional superconductors with multiple components of the superconducting order parameter. In such superconductors, multiple states of superconducting order can coexist. Accordingly there appear multiple kinds of vortex structure, where the nature of the vortex pinning can be dependent on the microscopics of the superconducting order.

One of the superconductivities with multiple components of the order parameter is the chiral p-wave one $\mathbf{d} = \overline{\mathbf{z}}(\overline{k}_x \pm i\overline{k}_y)$, which is composed of two degenerate pairing states \overline{k}_x and \overline{k}_y and breaks time-reversal symmetry. This superfluid ³He-A type of chiral p-wave pairing state has been anticipated in a layered ruthenate superconductor $\mathrm{Sr}_2\mathrm{RuO}_4$. ² While identification of the genuine superconducting pairing of this material is still open to further discussion, ³⁻⁶ that chiral p-wave pairing has the simplest and essential form and has attracted a great deal of attention. The vortices for $\mathrm{Sr}_2\mathrm{RuO}_4$ have been investigated intensively. ⁷⁻²⁰ In the context of the vortex pinning, we will see a rich physics contained in that chiral p-wave pairing state.

In this paper, we investigate the elementary vortex pinning potential in the chiral p-wave superconductor with the pairing $\mathbf{d} = \overline{\mathbf{z}}(\overline{k}_x \pm i\overline{k}_y)$. A pointlike pinning center and a single vortex with vorticity perpendicular to conduction layers in a layered superconductor are considered. We show that the vortex pinning potential depends on the sense of the chirality of the Cooper pairs relative to the vorticity of the vortex in the chiral p-wave superconductor. First we analytically discuss the interplay between the chirality and vorticity to explain the mechanism of the chirality dependence of the vortex pinning potential (see also Ref. 21). We then present numerical results for the vortex pinning potential obtained from self-consistent order parameters. Our numerical result confirms the analytical one. The chirality dependence of the vortex pinning would have an influence on the hysteresis of the magnetization and the distribution of the magnetic field in samples, which might be observed with superconducting quantum interference device (SQUID) and magneto-optical imaging techniques.

To investigate the vortex pinning, we use the quasiclassical theory of superconductivity.²² We start with the Eilenberger equation for the quasiclassical Green function in the absence of pinning,

$$\hat{g}_{\text{imt}}(i\omega_n, \mathbf{r}, \overline{\mathbf{k}}) = -i\pi \begin{pmatrix} g_{\text{imt}} & if_{\text{imt}} \\ -if_{\text{imt}}^{\dagger} & -g_{\text{imt}} \end{pmatrix}, \tag{1}$$

namely,

$$i v_{\rm F} \overline{\mathbf{k}} \cdot \nabla \hat{g}_{\rm imt} + [i \omega_n \hat{\tau}_z - \hat{\Delta}, \hat{g}_{\rm imt}] = 0,$$
 (2)

where the order parameter is $\hat{\Delta}(\mathbf{r}, \overline{\mathbf{k}}) = [(\hat{\tau}_x + i\hat{\tau}_y)\Delta(\mathbf{r}, \overline{\mathbf{k}})] - (\hat{\tau}_x - i\hat{\tau}_y)\Delta^*(\mathbf{r}, \overline{\mathbf{k}})]/2$ and $\hat{\tau}_i$ the Pauli matrices. The Eilenberger equation (2) is supplemented by the normalization condition $\hat{g}_{\text{imt}}(i\omega_n, \mathbf{r}, \overline{\mathbf{k}})^2 = -\pi^2\hat{1}$, and the commutator is $[\hat{a}, \hat{b}] = \hat{a}\hat{b} - \hat{b}\hat{a}$. The vector $\mathbf{r} = (r\cos\phi, r\sin\phi)$ is the center-of-mass coordinate, and the unit vector $\overline{\mathbf{k}} = (\cos\theta, \sin\theta)$ represents the relative coordinate of the Cooper pair. A cylindrical Fermi surface is assumed. We use units in which $\hbar = k_{\rm B} = 1$.

Following Thuneberg *et al.*, $^{23-25}$ the effect of pinning is introduced into the quasiclassical theory of superconductivity as follows. The quasiclassical Green function \hat{g} in the presence of a pointlike nonmagnetic defect situated at $\mathbf{r} = \mathbf{R}$ is obtained from the Eilenberger equation

$$iv_{F}\mathbf{\bar{k}}\cdot\nabla\hat{g} + [i\omega_{n}\hat{\tau}_{z} - \hat{\Delta},\hat{g}] = [\hat{t},\hat{g}_{imt}]\delta(\mathbf{r}'),$$
 (3)

and the t matrix due to the defect

$$\hat{t}(i\omega_n, \mathbf{r}') = \frac{v}{D} [\hat{1} + N_0 v \langle \hat{g}_{imt}(i\omega_n, \mathbf{r}', \overline{\mathbf{k}}) \rangle_{\theta}], \qquad (4)$$

where $\mathbf{r}' = \mathbf{r} - \mathbf{R}$, the denominator $D = 1 + (\pi N_0 v)^2 [\langle g_{\text{imt}} \rangle_{\theta}^2]$ + $\langle f_{\text{imt}} \rangle_{\theta} \langle f_{\text{imt}}^{\dagger} \rangle_{\theta}]$, the average over the Fermi surface $\langle \cdots \rangle_{\theta} = \int \cdots d\theta / 2\pi$, the normal-state density of states on the Fermi surface N_0 , and we assume *s*-wave scattering v when obtaining Eq. (4). We define a parameter $\sigma = (\pi N_0 v)^2 / [1 + (\pi N_0 v)^2]$, which measures how strong the scattering potential of the defect is.

The free energy in the presence of the defect is, at the temperature T, given as $^{23-26}$

$$\delta\Omega(\mathbf{R}) = N_0 T \int_0^1 d\lambda \sum_{\omega_n} \int d\overline{\mathbf{k}} \int d\mathbf{r} \text{Tr}[\delta \hat{g}_{\lambda} \hat{\Delta}_b], \quad (5)$$

where $\delta \hat{g}_{\lambda} = \hat{g} - \hat{g}_{\rm imt}$ is evaluated at $\hat{\Delta} = \lambda \hat{\Delta}_b$, and $\hat{\Delta}_b$ is the order parameter in the absence of the defect. Equation (5) represents the difference in the free energy between the states with and without the defect, and then gives the vortex pinning potential $\delta \Omega(\mathbf{R})$.

For the chiral *p*-wave pairing state $\mathbf{d} = \overline{\mathbf{z}}(\overline{k}_x + i\overline{k}_y)$ = $\overline{\mathbf{z}} \exp(i\theta)$, it is known that the order parameter around a single vortex, $\Delta_b(\mathbf{r}, \overline{\mathbf{k}})$ [= $\Delta_b(r, \phi; \theta)$], has two possible forms depending on whether the chirality and vorticity are parallel or antiparallel each other. 9.17,18 One form is

$$\Delta_{h}^{+-}(r,\phi;\theta) = \Delta_{+}(r)e^{i(\theta-\phi)} + \Delta_{-}(r)e^{i(-\theta+\phi)},$$
 (6)

where the chirality and vorticity are antiparallel (case I). The other is

$$\Delta_{h}^{++}(r,\phi;\theta) = \Delta_{+}(r)e^{i(\theta+\phi)} + \Delta_{-}(r)e^{i(-\theta+3\phi)},$$
 (7)

where the chirality and vorticity are parallel (case II). Here, the vortex center is situated at $\mathbf{r}=0$, the dominant component $\Delta_+(r\to\infty)=\Delta_{\rm BCS}(T)$, and the induced one $\Delta_-(r\to\infty)=0$. Because of the axisymmetry of the system, we can take $\Delta_+(r)$ to be real.

First we analytically investigate the vortex pinning potential. We discuss the quantity $\delta\Omega(R=0)$, where both the defect and vortex center are situated just at the origin $\mathbf{r}=0$ $(R\equiv|\mathbf{R}|)$. From the quasiclassical viewpoint, the quasiparticles inside the vortex core, subject to Andreev reflection, run along straight lines called quasiparticle paths. ^{27–29} We consider the quasiparticle paths which go through the origin $\mathbf{r}=0$. On those paths with zero impact parameter, the position vector is parallel to the direction of the quasiparticle path (i.e., $\mathbf{r}||\mathbf{\bar{k}}|$), and therefore $\phi=\theta,\theta+\pi$. In this situation ($\phi=\theta$), from Eqs. (6) and (7), the order parameter on the path is

$$\Delta_b^{+-}(r,\phi=\theta;\theta) = \Delta_+(r) + \Delta_-(r) \tag{8}$$

in case I and

$$\Delta_h^{++}(r,\phi=\theta;\theta) = [\Delta_+(r) + \Delta_-(r)]e^{2i\theta}$$
 (9)

in case II. The cancellation between the chirality and vorticity occurs in Eq. (8) and not in Eq. (9). Of importance is the resultant difference in the phase factor of these order parameters

On the basis of an analysis of the so-called zero-core vortex model in Ref. 24, the matrix elements of \hat{g}_{imt} at the vortex center are approximately obtained as³⁰

$$g_{\rm imt} = \sqrt{\omega_n^2 + |\widetilde{\Delta}|^2} \omega_n^{-1}, \quad f_{\rm imt} = -\widetilde{\Delta} \omega_n^{-1}, \quad f_{\rm imt}^{\dagger} = \widetilde{\Delta}^* \omega_n^{-1}, \quad (10)$$

where $\widetilde{\Delta} = \Delta_b^{+\pm}(r \rightarrow \infty, \phi = \theta; \theta)$. Here, Eq. (10) is obtained assuming that the amplitude of the order parameter is constant (i.e., zero core) around the vortex, which is the only approximation in this analysis. Inserting the order parameter

of Eq. (8) into Eq. (10), we obtain the anomalous Green functions integrated over the Fermi surface as, in case I,

$$\langle f_{\rm imt} \rangle_{\theta} = f_{\rm imt}, \ \langle f_{\rm imt}^{\dagger} \rangle_{\theta} = f_{\rm imt}^{\dagger}$$
 (11)

because of the absence of any phase factors in Eq. (8), i.e., because of the cancellation between the chirality factor $\exp(i\theta)$ and the vorticity factor $\exp(-i\phi)$ in Eq. (6). On the other hand, in case II,

$$\langle f_{\rm imt} \rangle_{\theta} = 0, \quad \langle f_{\rm imt}^{\dagger} \rangle_{\theta} = 0$$
 (12)

because of the phase factor $\exp(2i\theta)$ contained in Eq. (9). The diagonal component of $\langle \hat{g}_{\rm imt} \rangle_{\theta}$ is $\langle g_{\rm imt} \rangle_{\theta} = g_{\rm imt}$ both in case I and II. Consequently, in case I, $\langle \hat{g}_{\rm imt} \rangle_{\theta} = \hat{g}_{\rm imt}$ and we obtain $[\hat{t}, \hat{g}_{\rm imt}] = 0$ from Eq. (4). In case II, $\langle \hat{g}_{\rm imt} \rangle_{\theta} \neq \hat{g}_{\rm imt}$ and $[\hat{t}, \hat{g}_{\rm imt}] \neq 0$ generally.

When $[\hat{t},\hat{g}_{imt}]=0$, the Eilenberger equation (3) in the presence of the defect is identical to Eq. (2) (the equation in the absence of the defect); namely, the defect has no influence on the Green function and the free energy. From this and the above results of the analysis of the factor $[\hat{t},\hat{g}_{imt}]$, we find that $\delta\Omega(0)=0$ in case I when the chirality is antiparallel to the vorticity and $\delta\Omega(0)\neq 0$ in case II when the sense of the chirality is the same as that of the vorticity. It means that the vortex pinning depends on the chirality in the chiral p-wave superconductor.

The above analytical result is based on the zero-core vortex model, i.e., on the non-self-consistent (constant) amplitude of the order parameter. We next investigate the vortex pinning potential $\delta\Omega(R)$ numerically with the self-consistent order parameters around the vortex which have the forms of Eqs. (6) and (7). As the self-consistent amplitude $\Delta_{\pm}(r)$ in Eqs. (6) and (7), we adopt numerical data which we have obtained in Ref. 18 by solving self-consistently the Eilenberger equation.

In Fig. 1, we show the numerical results for $\delta\Omega(R)$ in the Born limit ($\sigma \leq 1$) and the unitary limit ($\sigma \rightarrow 1$). We present those results for chiral p-wave pairing and the isotropic s-wave one. As noted in Fig. 1, in the case of the s-wave pairing (dot-dashed lines), the difference in the free energy between the states with and without the defect, $\delta\Omega(R)$, is equal to zero at $R \rightarrow \infty$ (R is the distance between the vortex center and the defect). This is because the Anderson's theorem³¹ is valid far away from the vortex core. On the other hand, in the chiral p-wave pairing cases (solid and dashed lines), $\delta\Omega(R\rightarrow\infty)$ is finite and positive as seen in Fig. 1. The quantity $\delta\Omega(R\to\infty)$ is equal to the loss of condensation energy due to the pair breaking effect of the defect far away from the vortex core (i.e., the breakdown of Anderson's theorem). As noted in Figs. 1(a) and 1(b), at T $=0.8T_c$ (high temperature), the condensation energy loss in bulk $\delta\Omega(R\to\infty)$ dominantly contributes to the depth of the vortex pinning potential $\delta\Omega(R)$, i.e., to the vortex pinning energy. From Figs. 1(a) and 1(b) it is noticed that the vortex pinning energies of the chiral p-wave pairing cases at a high temperature are about 10 times larger than those of the s-wave pairing case. This enhancement of the pinning effect is due to the breakdown of the Anderson's theorem, and then

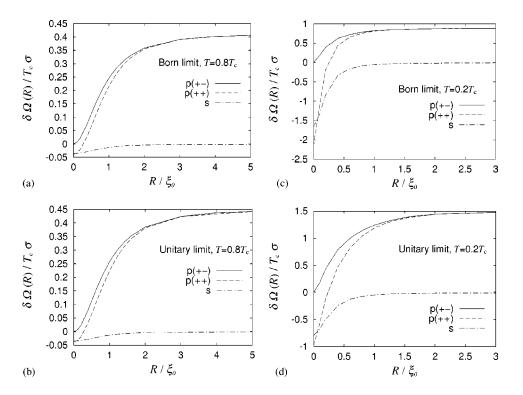


FIG. 1. The vortex pinning potential as a function of the distance R between the vortex center and the defect. Solid lines p(+-)] correspond to the case of the p-wave pairing with the chirality antiparallel to the vorticity (case I). Dashed lines [p(++)] correspond to the case of the p-wave pairing with the chirality parallel to the vorticity (case II). Dotdashed lines correspond to the case of the isotropic s-wave pairing. T_c is the superconducting critical temperature. The distance R is normalized with the coherence length $\xi_0 = v_F/\Delta_{BCS}(T=0)$.

it must be a common feature of unconventional superconductors. 25,32,33,21 For example, in the case of high- $T_{\rm c}$ cuprates, this may be one of the reasons why small point defects such as Zn atoms 34 and oxygen vacancies 35 are efficient pinning centers.

As noted in Figs. 1(c) and 1(d), at $T = 0.2T_c$ (low temperature), the contribution of the vortex core $(R \approx 0)$ to the depth of $\delta\Omega(R)$ is nonzero in case II (dashed lines). Here, the contribution of the vortex core means the energy gain due to the presence of the scattering center in the vortex core. In contrast, the depth of $\delta\Omega(R)$, i.e., the vortex pinning energy, is determined in case I (solid lines) only by the loss of condensation energy far away from the vortex core. It is noticeable that certainly $\delta\Omega(R=0)$ equals zero in case I. This numerical result confirms the analytical one discussed above. The vortex pinning energy depends on whether the chirality and vorticity are antiparallel (solid lines) or parallel (dashed lines). Especially in the Born limit, the difference in the vortex pinning energy is eminent as noticed in Fig. 1(c), because in this limit the loss of condensation energy in the bulk is relatively small compared to the contribution of the vortex core to the depth of $\delta\Omega(R)$.

In general, the two chiral states of cases I and II can coexist as domain structures in samples under magnetic fields. The spatial gradient of the magnetic field in a sample is proportional to the local strength of the vortex pinning in the critical state. In terms of the present chirality-dependent vortex pinning, the gradient inside the domain of the case-II state is predicted to be steeper than that inside the domain of the case-I state. This may be experimentally observed as a signature of the chiral state. Also the domain structure of the two chiral states depends on the hysteresis of the applied magnetic field, and therefore the present chirality-dependent

vortex pinning may affect the hysteresis curve during multiple cycles of the magnetization as observed¹ in UPt₃.

In the case of the usual winding-1 vortex $\Delta \propto \exp(i\phi)$, the chiral "p-wave" pairing $\bar{k}_x \pm i\bar{k}_y = \exp(\pm i\theta)$ is essential for the cancellation between the chirality and vorticity. If winding-2 vortices $\Delta \propto \exp(2i\phi)$ are realized in a chiral d-wave state $\bar{k}_x^2 - \bar{k}_y^2 \pm i\bar{k}_x\bar{k}_y = \exp(\pm 2i\theta)$, the same kind of cancellation occurs.

We comment on the relation of the present vortex pinning to the superconducting gap structure in Sr₂RuO₄. In this material, it has been pointed out from experiments that the gap had line nodes³⁻⁵ and little in-plane anisotropy.⁴ Models for the gap structure consistent with those experimental facts were proposed in which there existed horizontal line nodes perpendicular to the axis of the cylindrical Fermi surface.^{4,5} Now, for the present theory of chirality-dependent vortex pinning, what is important is that the Fermi surface averages of the anomalous Green functions (i.e., the average of the order parameter except for the chiral part) are finite as in Eq. (11). The present chirality dependence of the vortex pinning does not occur if the order parameters have sign changes on all Fermi surfaces relevant to superconductivity as $\Delta(\mathbf{k})$ $\sim \exp(\pm i\theta)\cos(ck_z)$. It occurs if there are no sign changes as $\Delta(\mathbf{k}) \sim \exp(\pm i\theta) |\cos(ck_z)|$. In another case,⁵ the chirality dependence is expected to occur when the order parameter is nodeless on the major Fermi surface with a dominant density of states, even if there are gap nodes and sign changes on the other minor Fermi surfaces.

In conclusion, we investigated the elementary vortex pinning potential $\delta\Omega(R)$ on the basis of the quasiclassical theory of superconductivity. In the chiral p-wave pairing state, $\delta\Omega(R)$ was dependent on the sense of the chirality relative to the vorticity at a low temperature. In terms of the

present chirality-dependent vortex pinning, a theoretical analysis for anomalies in the hysteresis of the magnetization observed experimentally in $\rm Sr_2RuO_4$ (Ref. 36) would be interesting and is left for future work.

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