Coherent quasiparticle tunneling in *d*-wave superconductor SIS junctions

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(Received 1 March 2002; revised manuscript received 17 July 2002; published 14 October 2002)

Coherent quasiparticle tunneling characteristics are numerically calculated for a superconductor/insulator/superconductor (SIS) tunnel junction with a *d*-wave order parameter. It is found that in coherent tunneling the differential conductance dI/dV exhibits a very sharp peak at the superconducting gap voltage, showing a sharp contrast to the case of incoherent tunneling, where the dI/dV peak is very broad. The sharp dI/dV peak is in good agreement with experimental results for intrinsic Josephson junctions of $Bi_2Sr_2CaCu_2O_{8+\delta}$. It is also found that when a small amount of incoherent tunneling is involved, the tunneling characteristics change abruptly to those of incoherent tunneling. These results imply that the tunneling in the intrinsic Josephson junctions is mostly coherent.

DOI: 10.1103/PhysRevB.66.132507 PACS number(s): 74.50.+r, 74.80.Dm, 74.72.−h, 74.20.Rp

It is widely accepted that the order parameter of high-T_c superconductors is primarily of *d*-wave symmetry, as supported by a large number of experimental results.[1] The symmetry of the superconducting order parameter manifests itself in various superconducting properties. In superconducting tunnel junctions, the difference between *d*- and *s*-wave symmetry leads to a significant change in the current–voltage (I-V) characteristics. It is expected that a tunnel junction made of a *d*-wave supercondutor exhibits a large subgap conductance in its I-V curve due to the line nodes in the *d*-wave order parameter, presenting a sharp contrast to the case of conventional *s*-wave tunnel junctions. Indeed, this is supported by numerical calculations of I-V characteristics for superconductor/insulator/superconductor (SIS) and superconductor/insulator/normal-metal (SIN) junctions with *d*-wave symmetry.[2,3] In SIN junctions, experimental results are basically in good agreement with numerical results.[4,5] On the other hand, in *d*-wave SIS junctions, it is becoming increasingly likely that experimental results do not agree with the numerical calculations.

It is known that in highly anisotropic high-T_c superconductors a layered crystal structure itself makes a stack of almost ideal SIS tunnel junctions, called intrinsic Josephson junctions (IJJ's).[6,7] In crystals of such superconductors, the *c*-axis transport directly represents the tunneling characteristics of a *d*-wave SIS junction. Recently, a very small IJJ mesa was fabricated on a crystal surface and the quasiparticle I-V characteristics of the IJJ's were measured by the short pulse method in an extended range covering the gap voltage $2\Delta/e$.[8] The measurement has revealed that there are significant differences between experimental results and numerical calculations for *d*-wave SIS junctions. A marked difference is that calculated dI/dV-V curves show a very broad conductance peak structure at $V=2\Delta/e$, while experimental results exhibit a sharp peak.[9] This difference is thought to arise from the fact that the former numerical calculations lack an important factor, which we presume to be the coherence of tunneling, or the conservation of the transverse momentum. The coherent tunneling itself is not a novel concept. It was suggested earlier associated with the *c*-axis transport in high-T_c supercondutors.[10] Recently, some experiments support the coherent tunneling based on the low-energy quasiparticle tunneling in IJJ's of $Bi_2Sr_2CaCu_2O_{8+\delta}$.[8,11,12]

This paper presents numerical calculations of the coherent quasiparticle tunneling in *d*-wave SIS junctions to be compared with experimental results. We show numerically that the coherent tunneling constraint causes a drastic change in the I-V characteristics. We further show that when even a small amount of incoherent tunneling is partially involved, the I-V characteristics change drastically to those of incoherent tunneling. This implies in turn that the experimental result on IJJ's is in itself evidence for the coherent quasiparticle tunneling in the *c*-axis direction, if the dI/dV-V curve exhibits a sharp peak at the gap voltage.

We start with a general expression. Quasiparticle current of a tunnel junction at a finite temperature T is expressed as follows:[13]

$$I(V)=\int\frac{d\mathbf{k}_\mathrm{L}}{(2\pi)^3}\int\frac{d\mathbf{k}_\mathrm{R}}{(2\pi)^3}\int_{-\infty}^{\infty}d\omega A_\mathrm{L}(\mathbf{k}_\mathrm{L},\omega)\times A_\mathrm{R}(\mathbf{k}_\mathrm{R},\omega-eV)\{f(\omega-eV)-f(\omega)\}|t(\mathbf{k}_\mathrm{L},\mathbf{k}_\mathrm{R})|^2, \tag{1}$$

where $A(\mathbf{k},\omega)$ is the spectral function for a quasiparticle with a momentum $\mathbf{k}$ and an energy ω measured from the Fermi level. $t(\mathbf{k}_\mathrm{L},\mathbf{k}_\mathrm{R})$ is the tunneling matrix element across the tunneling barrier, $f(\omega)$ is the Fermi function, and the suffixes L,R denote left and right, respectively. In the coherent tunneling, the momentum perpendicular to the tunnel direction $\mathbf{k}^\perp$ is conserved so that $\mathbf{k}_\mathrm{L}^\perp=\mathbf{k}_\mathrm{R}^\perp$. We assume for simplicity that $t(\mathbf{k}_\mathrm{L},\mathbf{k}_\mathrm{R})$ is independent of $\mathbf{k}^\parallel$, the momentum parallel to the tunnel direction. Let $g_\mathrm{L}(k_\mathrm{L}^\perp,\theta_\mathrm{L},\omega)$ be the $k_\mathrm{L}^\perp$- and θ-dependent quasiparticle density of states expressed as

$$g_\mathrm{L}(k_\mathrm{L}^\perp,\theta_\mathrm{L},\omega)=\int\frac{dk_\mathrm{L}^\parallel}{2\pi}A(k_\mathrm{L}^\perp,k_\mathrm{L}^\parallel,\theta_\mathrm{L},\omega),$$

where $\theta_\mathrm{L}=\tan^{-1}(k_{\mathrm{L}y}^\perp/k_{\mathrm{L}x}^\perp)$. g_R is defined similarly. Then, the constraint arising from the coherent tunneling is simply $k_\mathrm{L}^\perp=k_\mathrm{R}^\perp$ and $\theta_\mathrm{L}=\theta_\mathrm{R}$. In this case, the tunneling matrix element $|t(\mathbf{k}_\mathrm{L},\mathbf{k}_\mathrm{R})|^2$ is rewritten as $(2\pi)^2|t|^2\delta(k_\mathrm{L}-k_\mathrm{R})\delta(\theta_\mathrm{L}^\perp-\theta_\mathrm{R}^\perp)$.

The calculation of Eq. (1) includes the product $g(k_{\mathrm{L}}^{\perp},\theta_{\mathrm{L}},\omega)g(k_{\mathrm{R}}^{\perp},\theta_{\mathrm{R}},\omega-eV)$, which is not generally explicit. To circumvent this complexity, we assume following the conventional way in this problem that E_{F} is sufficiently larger than Δ so that $g(k^{\perp},\theta,\omega)$ is regarded as constant in the nontrivial part of the integration. Then $g(k^{\perp},\theta,\omega)=g_0$ for $T>T_{\mathrm{c}}$ and $g(k^{\perp},\theta,\omega)=g_0N(\theta,\omega)$ for $T<T_{\mathrm{c}}$, where $N(\theta,\omega)$ is the normalized BCS quasiparticle density of states. Using this assumption, the normal tunneling resistance R_{N} is expressed as

$$\frac{1}{eR_{\mathrm{N}}}=\frac{|t|^2}{(2\pi)^2}\int g_0{}^2k_{\mathrm{L}}^{\perp}k_{\mathrm{R}}^{\perp}2\pi\delta(k_{\mathrm{L}}-k_{\mathrm{R}})dk_{\mathrm{L}}^{\perp}dk_{\mathrm{R}}^{\perp}.$$

Finally we obtain the expression for the coherent quasiparticle current,

$$I(V)=\frac{1}{2\pi eR_{\mathrm{N}}}\int_0^{2\pi}d\theta\int_{-\infty}^{\infty}d\omega N(\theta,\omega)\times N(\theta,\omega-eV)\{f(\omega-eV)-f(\omega)\}. \tag{2}$$

In d-wave superconductors, $N(\theta,\omega)$ is expressed as

$$N(\theta,\omega)=\mathrm{Re}\left[\frac{\omega}{\sqrt{\omega^2-\Delta^2\cos^2 2\theta}}\right]. \tag{3}$$

Using Eqs. (2) and (3), we first calculate the simplest case where t is constant. The result is compared with earlier numerical calculations,[2,3] where the tunneling is tacitly assumed to be incoherent. Then we extend our calculation to the case where the coherent tunneling occurs with a finite broadness in the θ dependence of the tunneling probability. In this case, the δ function in the tunneling matrix element is replaced by a Gaussian distribution function, representing smeared coherent tunneling. This turns out to be particularly important to understand the nature of quasiparticle tunneling from the actual dI/dV-V characteristics.

Figures 1(a) and (b) show two sets of I-V or dI/dV-V curves for coherent tunneling calculated using Eqs. (2) and (3). In this calculation and hereafter, eR_{N} is set unity and values for Δ and T are chosen tentatively from Ref. 3 for the purpose of comparison. These values themselves have no particular physical implication except that they are similar to experimental values for IJJ's in $Bi_2Sr_2CaCu_2O_{8+\delta}$. For comparison, we show in Fig. 2 the case of incoherent tunneling for a d-wave superconductor SIS tunnel junction, which is basically the same as the previous results.[3] At a glance, it is clear that the present results provide a sharp contrast to Fig. 2. The most significant difference is seen in the conductance peak at $V=2\Delta/e$. In the incoherent tunneling case (Fig. 2), the conductance peak is very broad with a width of about Δ/e, and the peak height is less significant, while in the coherent tunneling case the conductance peak is pronouncedly sharp, presenting a sharp contrast to the incoherent case. This is the most dramatic change in the tunneling characteristics brought about by the incorporation of the coherent tunneling constraint.

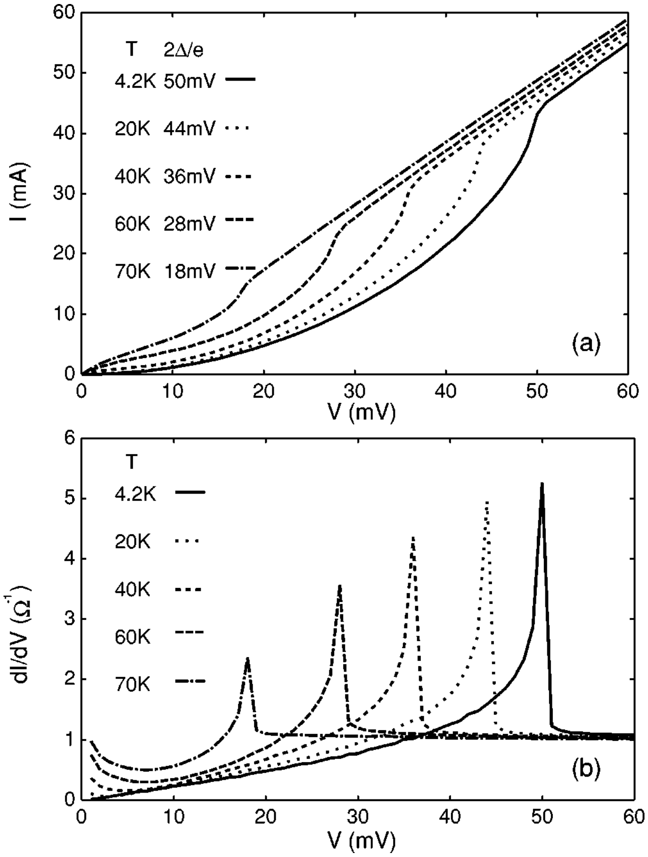

FIG. 1. (a) A set of I-V curves calculated using Eqs. (3) and (4) at various temperatures and (b) corresponding dI/dV-V curves, showing very sharp conductance peaks.

Another difference in the tunneling characteristics is seen in the subgap conductance $\sigma(V)\equiv dI/dV$ for $V<2\Delta/e$. In the coherent tunneling case, $\sigma(V)$ is almost linear in V, while in the incoherent tunneling case, $\sigma(V)$ is nearly proportional to V^2 for $V<\Delta/e$. This difference in the V dependence comes from the difference in the evaluation of the joint density of states in Eq. (2). In the coherent tunneling, the inte-

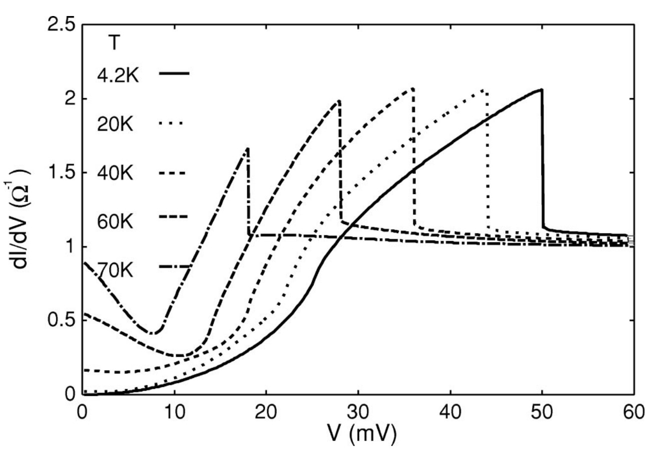

FIG. 2. A set of dI/dV-V curves for incoherent tunneling calculated under otherwise the same condition as that of Fig. 1, showing very broad conductance peaks.

gral of $N(\theta,\omega)N(\theta,\omega-eV)$ with respect to θ is evaluated, while in the incoherent tunneling, that of $N(\theta_L,\omega)N(\theta_R,\omega-eV)$ is evaluated with respect to θ_L and θ_R independently. In the former case, dI/dV for small V is close to $\int N(\theta,\omega)d\theta$, which is the quasiparticle density of states of a d-wave superconductor at low temperatures.[14] Then dI/dV is linear for lower V as is $N(\omega)$ for a d-wave superconductor. In the incoherent tunneling, on the other hand, the joint density of states to be evaluated increases quadratically with respect to N, from which it follows that dI/dV is nearly proportional to V^2 for smaller V. The experimental result is nearly expressed in terms of V^2 dependence, indicating that the tunneling partially includes incoherent process or some other processes described in a later section.

These two differences represent the most pronounced changes brought about by the incorporation of coherence in tunneling. As will be mentioned later, actual characteristics for d-wave SIS junctions compare much well with the coherent tunneling case.

Artemenko[15–17] also calculated the coherent tunneling characteristics for the interlayer Josephson effect in a d-wave layered superconductor. He found that $\sigma(V)$ for small V decreases with increasing V, which is similar to the present result. However, his result implies that I decreases for $V>2\Delta/e$, which is at variance both with the present result and the experimental result. It should be noted that the Artemenko's result is based on a multistacked junctions, in which interplay of Josephson current, scattering and charging effect plays an important role. Their model takes into account the suppressed dispersion in the tunneling direction, which is more realistic than in the present case for a single junction.

Next we consider the case of smeared coherent tunneling where a small shift in θ is involved on the occasion of tunneling. This is likely because, to a greater or lesser degree, imperfections at the junction interfaces scatter quasiparticles, giving a finite shift in θ. We assume that the tunneling probability is proportional to the Gaussian distribution, i.e.,

$$|t(\theta_L,\theta_R)|^2=2\pi\frac{|t|^2}{\sqrt{2\pi}S}\exp\left\{-\frac{\phi^2}{2S^2}\right\}\equiv 2\pi h(\phi), \quad (4)$$

where $\phi=\theta_L-\theta_R$, and S is the standard deviation of the shift ϕ. It is implied that a large S reflects a large amount of scattering centers at the interfaces. Then we obtain

$$I(V)=\frac{1}{2\pi e R_N}\int_0^{2\pi}d\theta\int_{-\infty}^{\infty}d\phi h(\phi)\int_{-\infty}^{\infty}d\omega N(\theta,\omega) \times N(\theta+\phi,\omega-eV)\{f(\omega-eV)-f(\omega)\}. \quad (5)$$

Figures 3(a) and (b) show, respectively, two sets of I-V or dI/dV-V curves calculated using Eqs. (4) and (5) for various S values at 4.2 K. We see from Figs. 2 and 3 that I-V and dI/dV-V curves smoothly change with increasing S from the coherent tunneling curve to the incoherent curve, as expected. When S is increased to a value of 20°, the conductance peak height decreases to almost half the value in the case of $S=0$. For $S=30°$ and larger, the tunneling characteristics are almost identical to those of incoherent tunneling.

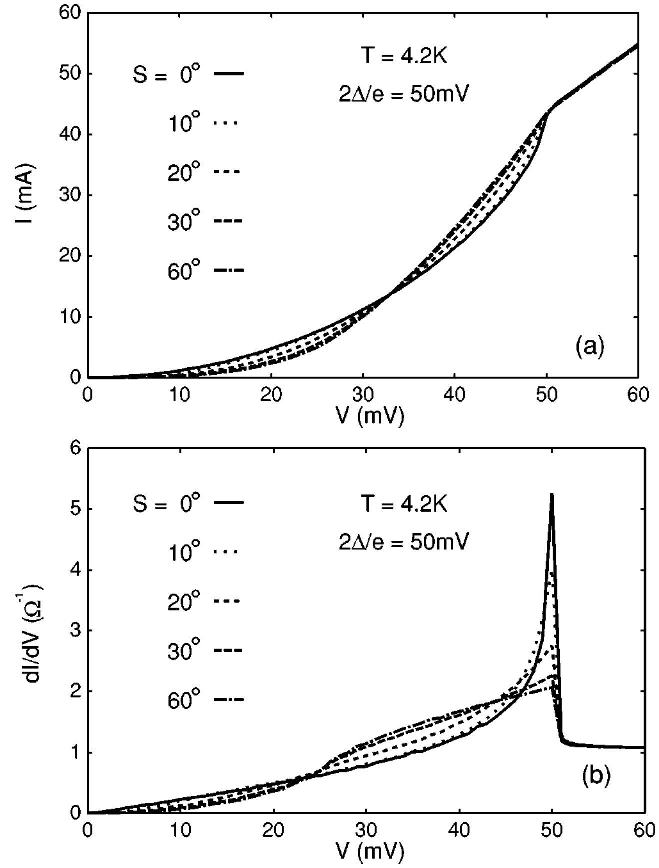

FIG. 3. (a) A set of I-V curves for $2\Delta/e=50$ mV at $T=10$ K. (b) Corresponding dI/dV-V curves for various S values. It is seen that the conductance peak becomes broader very rapidly as S increases.

It is striking that the I-V curve almost completely changes to that of the incoherent tunneling characteristics at a small value of $S=30°$. This implies that the tunneling characteristics are very sensitive to the fraction of incoherent tunneling. This result further implies that when the tunneling characteristics similar to those in Fig. 1 are observed, the tunneling is thought to be coherent for a sizable fraction of tunneling quasiparticles.

It is interesting to compare the numerical calculation with experimental results. The inset in Fig. 4 shows a dI/dV-V curve for IJJ's of $Bi_2Sr_2CaCu_2O_{8+\delta}$ measured by short pulse tunneling spectroscopy using a small mesa of IJJ's.[18] Since the sample providing this dI/dV-V curve is in the overdoped region, the influence of the pseudogap is much less significant.[18] On the other hand, Fig. 4 shows the numerical result for the coherent tunneling with almost identical values for Δ, T, and R_N. It is clearly seen that the numerical result compares with the experimental result fairly well in that the tunneling conductance peak at $V=45$ mV is sharp and that $\sigma(V)$ for $\Delta/e<V<2\Delta/e$ is suppressed compared with the characteristics in Fig. 2. This agreement strongly supports that the tunneling of quasiparticles in IJJ's of $Bi_2Sr_2CaCu_2O_{8+\delta}$ is likely to be coherent.

It is also seen in Fig. 4 that there are still some differences between the numerical calculations and the experimental re-

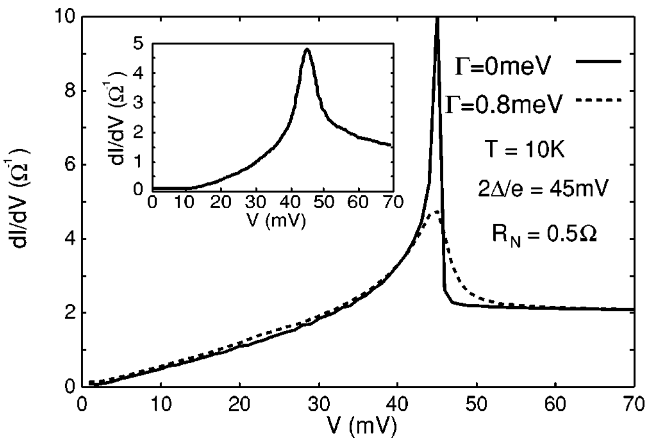

FIG. 4. Numerical calculation of coherent tunneling dI/dV-V curve for a d-wave SIS tunnel junction with values of $T=10$ K, $2\Delta/e=45$ mV and $R_N=0.5\ \Omega$. The dashed line is calculated using the quasiparticle relaxation model with $\Gamma=0.8$ meV. The inset shows an experimental dI/dV-V curve for a small mesa of IJJ's of $Bi_2Sr_2CaCu_2O_{8+\delta}$ measured by the short pulse method at 10 K.

sults. For example, in the region of smaller V, experimental results behave like $\sigma\propto V^2$, while the calculation for the coherent tunneling behaves like $\sigma\propto V$. This difference is probably reduced to a great extent if we introduce θ-dependent tunneling matrix elements, which does not reach the general consensus.[19–21] If we adopt the quasiparticle relaxation model by Dynes[22] or a finite value for ϕ or both, the agreement is much improved as depicted by dashed line in Fig. 4. It is also noteworthy that calculated dI/dV-V curves at higher temperatures exhibit a cusp at $V=0$, which is missing in experiments on IJJ's of $Bi_2Sr_2CaCu_2O_{8+\delta}$.[8,18,23] However, this cusp readily vanishes if we introduce a finite quasiparticle relaxation time Γ, which is very likely.

In conclusion, we have calculated numerically the coherent quasiparticle tunneling characteristics for an SIS tunnel junction with a superconducting order parameter of d-wave symmetry. It is shown that introducing a factor of coherent tunneling makes the conductance peak in the dI/dV-V curve significantly sharper, which better agrees with interlayer tunneling experiments. It is also shown that a small fraction of incoherent tunneling causes the tunneling characteristics to change drastically to those of incoherent tunneling. Good agreement with the experimental results suggests that the quasiparticle tunneling in IJJ's of $Bi_2Sr_2CaCu_2O_{8+\delta}$ is mostly coherent.

Note added. After the completion of this manuscript, we noted a paper by Krasnov,[24] in which a similar result is described in passing.

We thank T. Shibauchi and K. Anagawa for useful discussions. This work was partially supported by a Grant-In-Aid for Scientific Research from Ministry of Education, Culture, Sports, Science and Technology of Japan, and The Mitsubishi Foundation.

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