Effective dipolar boundary conditions for dynamic magnetization in thin magnetic stripes

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We show that dynamic magnetization at the lateral edges of a thin, axially magnetized magnetic element with finite in-plane size can be described by effective "pinning" boundary conditions. This effective pinning is of a purely dipolar nature, is not related to the magnetocrystalline surface anisotropy of the magnetic material, and is determined by the inhomogeneity of the dynamic demagnetizing field near the edges of the element. Eigenfunctions and eigenvalues obtained using these effective boundary conditions give quantitative description of the quantized spin wave spectra experimentally observed in long and thin permalloy stripes of a micron-size width.

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Progress in magnetic recording led to the development of patterned recording media where pattern elements (rectangular stripes, prisms, or cylinders) have nonellipsoidal shape and are used to record individual bits of information.[1–4] At the same time, rapid increase of processor speeds in modern computers leads to the necessity to write bits of information during subnanosecond time intervals. To achieve this time scale in writing process, one can take advantage of the so-called precessional remagnetization,[5] where the remagnetizing field is applied for a time interval, which is determined by the resonance frequencies (or eigenfrequencies) of the patterned media. Thus, the theoretical and experimental study of the eigenfrequencies of magnetic excitations in the patterned recording media is of great importance.

The eigenexcitations in the patterned media are actively studied in experiment, and the measurements of spin wave spectra performed by Brillouin light scattering on the simplest pattern of axially magnetized rectangular permalloy stripes have demonstrated quantization of the spin wave frequencies due to the finite width and thickness of the stripe.[1–4] Theoretical calculation of these spin wave eigenfrequencies even in the simplest case of a rectangular stripe (or a rectangular magnetic waveguide) encounters, however, a significant difficulty due to the absence of exact information of the boundary conditions for variable magnetization at the lateral edges of the pattern elements.

It is well known that the usual electrodynamic boundary conditions leave the amplitude of dynamic magnetization at the boundaries of a rectangular magnetic stripe undefined. Thus, in practical calculations it is usually assumed that the boundary conditions at the lateral edges of a stripe (or waveguide) are either of the "free spins" type, or, more often, of the "magnetic wall" type (see, e.g., Chap. 6 in Ref. 6). Both assumptions lead to a simple quantization condition for the component of the spin wave wave vector **k** directed along the particular finite size of the waveguide (e.g., width w) in the form

$$k_r=\frac{r\pi}{w}, \tag{1}$$

where $r=n$ for free spins boundary conditions, $r=n+1$ for magnetic wall boundary conditions, and $n=0,1,2,...$ is a mode number. The eigenfunctions for the dynamic magnetization in these cases are assumed to be cosinusoidal or sinusoidal, respectively, but the fundamental question of the real behavior of the dynamic magnetization at the edges of a finite-size nonellipsoidal magnetic element remains open.

In this Report we calculate profiles and frequencies of the width modes of an axially magnetized magnetic stripe with a rectangular cross section (see Fig. 1) directly from the integral equation describing nonlocal dipole-dipole interaction in the stripe. A similar solution was obtained numerically for the case of an infinitely thin stripe in Ref. 7. We show that our solution, obtained for the stripe having finite thickness L, leads to the boundary conditions for the dynamic magnetization describing effective "pinning" of magnetization at the lateral edges of the stripe. The obtained pinning is of a purely dipolar nature and is a measure of inhomogeneity of the internal dynamic magnetic field along the stripe width. This result allows us not only to accurately predict the spin wave eigenfrequencies of a rectangular magnetic stripe, but also explains why the magnetic wall boundary conditions traditionally used in calculations of eigenmodes of rectangular magnetic waveguides (see, e.g., Chap. 6 in Ref. 6) give approximately correct results.

Spectra of spin wave excitations in magnetic films and other finite magnetic samples can be found using the method of magnetostatic Green's functions.[8,9] In the framework of this method the Landau-Lifshitz equation for the magnetization and the magnetostatic Maxwell equations are reduced to an integrodifferential equation [see, e.g., Eq. (7) in Ref. 8], which is solved by expanding the variable magnetization in a series of eigenfunctions of the exchange differential operator that satisfy the exchange boundary conditions at the film

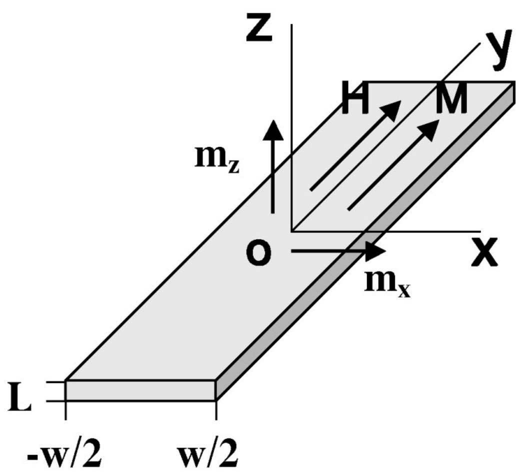

FIG. 1. Geometry of the problem and the system of coordinates.

surfaces.[8] Since the lower width modes in the spin wave spectrum of a finite-width stripe are, in most cases, of a dipolar nature, we, in contrast with Ref. 8, find below a system of eigenfunctions of the *integral operator of nonlocal dipole-dipole interaction*, and then use these eigenfunctions as a basis for expansion of the dynamic magnetization of the stripe.

We consider an infinitely long magnetic stripe of a rectangular cross section with thickness L being much smaller than the width w ($p=L/w\ll 1$). The stripe is uniformly magnetized along the y axis, while the z axis is directed along the stripe thickness (see Fig. 1). Solving magnetostatic Maxwell equations with electrodynamic boundary conditions, we find that the magnetization distribution $\mathbf{m}(\mathbf{r})$ within the stripe leads to the demagnetizing field $\mathbf{H}_d(\mathbf{r})=\int_V\hat{G}(\mathbf{r},\mathbf{r}')\mathbf{m}(\mathbf{r}')d\mathbf{r}'$, where $\hat{G}$ is a tensorial operator (tensorial magnetostatic Green's function). The general expression for $\hat{G}$ is defined in Ref. 10 [see Eq. (5.98) in Ref. 10].

In the case of a thin and infinitely long magnetic stripe the general expression for $\hat{G}$ can be substantially simplified by neglecting dependence on the infinite axial coordinate y and assuming homogeneous distribution of the dynamic magnetization along the coordinate z (stripe thickness). Thus, for the m_z component of the dynamic magnetization the equation for the eigenfunctions of the dipolar integral operator averaged over y and z can be written in the form

$$\lambda_z m_z(x)=\int_{-w/2}^{w/2}dx'\,G_{zz}(x,x')m_z(x'), \tag{2}$$

where the kernel

$$G_{zz}(x,x')=\frac{2}{L}\ln\left[\frac{(x-x')^2}{(x-x')^2+L^2}\right] \tag{3}$$

depends only on the coordinate x (along the stripe width).

The m_x component of the dynamic magnetization also satisfies an integral equation, which is similar to Eq. (2). The eigenvalues λ_x (corresponding to m_x) and λ_z (corresponding to m_z) are connected by the equation $\lambda_x+\lambda_z=-4\pi$. which follows from the definition of the magnetostatic Green's function (2) given in Ref. 10: $G_{xx}(x,x')+G_{zz}(x,x')=-4\pi\delta(x-x')$. In the calculations below we shall omit the indices z and x, defining $\lambda_z=\lambda_n$ and $\lambda_x=-(4\pi+\lambda_n)$. We shall also use dimensionless coordinate $\xi=x/w$ and dimensionless aspect ratio of the stripe $p=L/w$.

If the complete and orthogonal set of eigenfunctions of the stripe is known, then the dynamic magnetization $m(\xi)$ can be expanded in a series of these eigenfunctions $m(\xi)=\Sigma_n B_n m_n(\xi)$, and the eigenvalues λ_n of the problem can be calculated from Eq. (2) as

$$\lambda_n(p)=\frac{1}{\|m_n(\xi)\|^2}\int_{-1/2}^{1/2}\int_{-1/2}^{1/2}d\xi\,d\xi' g \times(\xi-\xi',p)m_n(\xi)m_n(\xi'), \tag{4}$$

where $g(\xi,p)=(2/p)\ln[\xi^2/(\xi^2+p^2)]$.

If the dipolar eigenvalues λ_n are found from Eq. (4), then, neglecting anisotropy and exchange, the frequencies of the quantized spin wave modes (or eigenmodes) of the stripe in ω_M units can be found from the solution[8,9] of the Landau-Lifshitz equation as

$$\left(\frac{\omega_n}{\omega_M}\right)^2=\left(\frac{\omega_H}{\omega_M}+1+\frac{\lambda_n}{4\pi}\right)\left(\frac{\omega_H}{\omega_M}-\frac{\lambda_n}{4\pi}\right), \tag{5}$$

where $\omega_H=\gamma H$, $\omega_M=\gamma 4\pi M$, H is the bias magnetic field applied along the stripe axis, and M_s is the saturation magnetization of the stripe. Obviously, Eq. (5) where exchange interaction is neglected will give best results for the lowest spin wave modes $n=0,1$, while for the higher modes the full dipole-exchange spin wave dispersion equation should be calculated using the formalism of Refs. 8 and 9.

To find the eigenvalues and eigenfunctions of the integral equation (2) we can replace this equation with an infinite-order differential equation of the form

$$\lambda_n m_n(\xi)=\sum_{\nu=0} I_\nu(\xi,p)\frac{\partial^\nu m_n(\xi)}{\partial\xi^\nu}, \tag{6}$$

where

$$I_\nu(\xi,p)=\frac{1}{\Gamma(\nu+1)}\int_{-1/2-\xi}^{1/2-\xi}dt\,g(t,p)t^\nu. \tag{7}$$

The nonanalytic behavior of the functions $I_0(\xi,p)$ and $I_1(\xi,p)$ near the stripe borders $|\xi|=\pm\frac{1}{2}$ does not allow one to get a solution of Eq. (6) in a reasonable closed form. Fortunately, for $p\ll 1$ the first two terms ($\nu=0,1$) in Eq. (6), containing functions $I_0(\xi,p)$ and $I_1(\xi,p)$, give a dominant contribution to the decomposition (6) near the lateral boundaries of the stripe. Direct calculation shows that $I_0(-\frac{1}{2},p)=-2\pi+2p$, $I_1(-\frac{1}{2},p)=-p+2p\ln p$, and all the others terms ($\nu\geqslant 2$) in the decomposition (6) containing $I_\nu(-\frac{1}{2},p)=-2p/(\nu-1)\Gamma(\nu+1)$ are substantially smaller than the first two. A similar situation exists at the other boundary $\xi=\frac{1}{2}$. This leads to the relations between the amplitude of the magnetization mode m_n and its first derivative at the stripe boundaries that can be interpreted as effective boundary conditions of the form

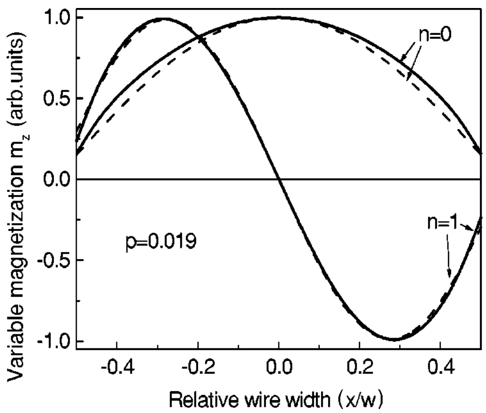

FIG. 2. Profiles of z component of the dynamic magnetization $m_z(\xi)$ in the two lowest ($n=0$ and $n=1$) width modes of an axially magnetized stripe for $p=L/w=0.019$: broken lines, results of the numerical solution of Eq. (2); solid lines, graphs of approximate eigenfunctions Eq. (10).

$$\frac{\partial m_n(\xi)}{\partial \xi} \pm d(p) m_n(\xi)|_{\xi=\pm 1/2}=0, \tag{8}$$

where for $p \ll 1$ the effective "pinning" parameter $d(p)>0$ depends only on the stripe aspect ratio p and is the same for all mode numbers n:

$$d(p)=\frac{2\pi}{p[1+2\ln(1/p)]}. \tag{9}$$

We would like to stress, that, although the boundary conditions (8) look formally analogous to the exchange boundary conditions in a perpendicularly magnetized film [see, e.g., Eq. (7.45) in Ref. 6], the obtained effective pinning is of a purely dipolar nature. In contrast to the usual "exchange" pinning, this dipolar pinning parameter (9) is not related to the magnetocrystalline surface anisotropy of the magnetic material. The physical role of this pinning is to minimize the surface magnetostatic energy which results from the induced surface charges $\boldsymbol{\sigma}=(\mathbf{m}\cdot\mathbf{n})_S$ at the edges of a nonellipsoidal magnetic stripe. It is interesting to note that for $p\rightarrow 0$ the effective pinning parameter $d(p)$, Eq. (9), is rather large, and the boundary conditions (8) are close to the magnetic wall boundary conditions that were traditionally used at the lateral edges of a thin magnetic waveguide (see, e.g., paragraph 6.2 in Ref. 6).

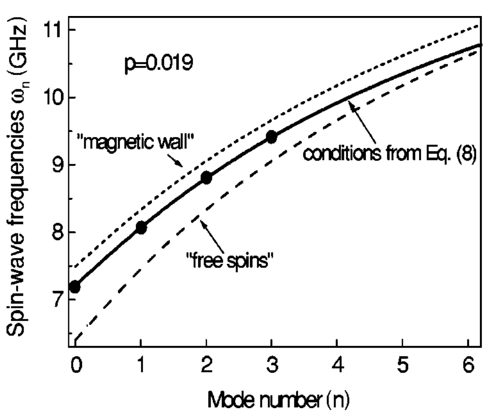

FIG. 3. Calculated and measured frequencies of quantized spin wave modes of an axially magnetized magnetic stripe of finite width w: lower broken line, free boundary conditions; upper broken line, magnetic wall boundary conditions; solid line, frequencies calculated using the boundary conditions, Eq. (8). Circles show the experimental data from Ref. 2 for a FeNi stripe of the thickness $L=29$ nm and width the $w=1.5\ \mu$m ($p=0.019$).

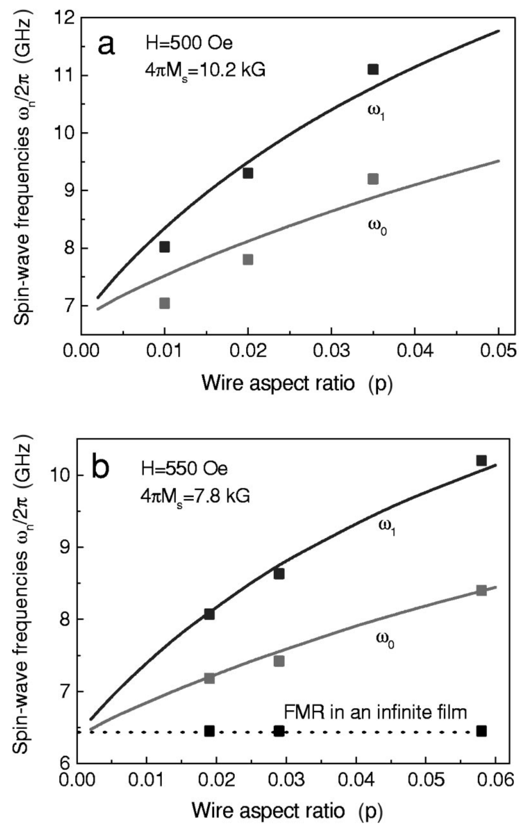

FIG. 4. Comparison of frequencies of two lowest ($n=0$ and $n=1$) quantized spin wave modes calculated using Eqs. (4), (5), and (10) for different values of stripe aspect ratio p (solid lines) to the experimental frequencies measured by Brillouin light scattering in permalloy stripes (symbols): (a) comparison with the experimental results from Ref. 1 for $H=500$ Oe, $4\pi M_s=10.2$ kG, (b) comparison with the experimental results from Ref. 2 for $H=550$ Oe, $4\pi M_s=7.8$ kG.

Taking into account the rectangular geometry of the problem and using the above-mentioned analogy with exchange pinning in a perpendicularly magnetized film, it is natural to assume that the dipolar eigenfunctions of the stripe would have a simple sinusoidal form, analogous to the form of spin wave resonance modes in a perpendicularly magnetized film. For symmetric $m_n^s(x)$ and antisymmetric $m_n^a(x)$ modes these functions have the form

$$m_n^s(x)=A_n^s\cos(\kappa_n^s x), \quad m_n^a(x)=A_n^a\sin(\kappa_n^a x), \quad n=0,1,2\ldots. \tag{10}$$

Substituting functions (10) for m_n in the boundary conditions (8) we get the well-known equations for the quantized wave numbers κ_n [see, e.g., Eqs. (7.48) and (7.49) in Ref. 6]. In the limit of small p [and large $d(p)$] it is possible to obtain a simple explicit approximate expression for κ_n:

$$\kappa_n = \frac{(n+1)\pi}{w}\left[1 - \frac{2}{d(p)}\right], \quad n = 0,1,2...\,, \tag{11}$$

which is a new quantization condition for the spin wave wave vector along the stripe width, replacing the traditional formula (1). This condition does not have the form of the quantization condition $\kappa_n = (n-1/3)\pi/w$ empirically found for this case in Ref. 2 [see Eq. (11) in Ref. 2], although it gives very similar numerical values of κ_n for the first four modes. It should be also mentioned that condition (11) can be rewritten as $\kappa_n = (n+1)\pi/w_{\mathrm{eff}}$, where the effective width of the stripe $w_{\mathrm{eff}} = w[d/(d-2)]$ approaches the real stripe width w when the aspect ratio $p = L/w$ of the stripe is decreasing.

Substituting functions (10) for $m_n(x)$ in Eq. (4) one can find dipolar eigenvalues λ_n of the problem (2), and using the dispersion equation (5) it is then easy to calculate quantized spin wave eigenfrequencies ω_n.

To obtain an approximate explicit expression for λ_n one may neglect the dependence of eigenfunctions $m_n(\xi)$ on p, and retain this dependence only in the kernel (3) of Eq. (2) and in the eigenvalues [assuming that $\lambda_n = \lambda_n(p)$]. This assumption allows us to regularize the kernel of Eq. (2), and, using the Fredholm's theory,[11] to derive an approximate analytic expression for the eigenvalues of the problem (2) in the form

$$\lambda_n(p) = -4\pi + \pi^2(2n+1)p, \quad n = 0,1,2,\ldots\,. \tag{12}$$

Figure 2 shows the result of comparison of the two lowest eigenfunctions $m_0^s(x)$ and $m_1^a(x)$ calculated numerically from Eq. (2) (solid lines), and obtained analytically using Eqs. (10) and (11) (broken lines). It can be seen from Fig. 2 that explicit expressions (10) obtained from the approximate boundary conditions (8) give a very good approximation of the eigenfunctions of the original problem, Eq. (2).

The calculation of eigenvalues λ_n of the stripe for $p \ll 1$ done by direct numerical solution of the integral equation (2) and using the explicit expressions (10) for the eigenfunctions $m_n(x)$ of the stripe in Eq. (4) give very similar results. The approximate explicit formula for eigenvalues (12) also gives reasonably accurate results when $n > 1$.

It is interesting to compare our results for dipolar spin wave eigenfrequencies ω_n calculated from Eq. (5) [using Eq. (4) and approximate eigenfinctions (10)] to the results obtained from the simple quantization of the dipolar spin wave dispersion equation (e.g., Damon-Eshbach equation for magnetostatic surface wave[12]) using the quantization condition (1). The results of this comparison are presented in Fig. 3, where the lower broken line connects the points obtained from the quantization of the Damon-Eshbach dispersion equation using a "free" boundary, the upper broken line corresponds to the magnetic wall boundary conditions, while the solid line in the middle connects the points obtained from Eq. (5) for the approximate boundary conditions (8). The symbols show the experimental data obtained in Ref. 2 for the FeNi stripe of the thickness $L = 29$ nm and the width $w = 1.5\ \mu$m $(p = 0.019)$. It is clear from Fig. 3 that for small mode numbers $n < 2$ our theoretical result (solid line) is closer to the result obtained from the magnetic wall condition, while for lager mode numbers our result tends to come closer to the result obtained for free spins boundary conditions. It is also clear that our calculations performed using the dipolar boundary conditions (8) and (9) give a quantitative description of the experiment.[2]

The results of comparison of dipolar spin wave eigenfrequencies ω_n calculated from Eq. (5) for the two lowest modes ($n = 0$ and $n = 1$ shown in Fig. 2) with the experimental data for magnetic (permalloy) stripes of micron-sized width published in two papers[1,2] are presented in Figs. 4(a) and 4(b), respectively. Figure 4 shows that the above developed approximate theory of dipolar eigenfunctions and eigenvalues gives good quantitative description of experiments performed by different groups, on magnetic stripes with different saturation magnetization, and in a reasonably wide range of magnitudes of the stripe aspect ratio p. We believe that although our calculations were done in a rectangular geometry, similar effective boundary conditions could be obtained in other geometries, and, in particular, in the case of a perpendicularly magnetized thin magnetic dot of a circular or elliptic shape.

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