Phonon drag in ballistic quantum wires in the nonlinear regime

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The acoustic phonon-mediated contribution to the drag current in the ballistic transport regime in two nearby one-dimensional nanowires is calculated. In the nonlinear regime, where the applied bias voltage eV is greater than the temperature T, the threshold of the phonon-mediated drag current with respect to bias or gate voltage is predicted. It is found that the drag current contribution from any two aligned subbands in the drive and drag wires saturates at large bias (driving) voltage.

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I. INTRODUCTION

The purpose of the present paper is to study the phonon contribution to the drag current in the course of *ballistic* (collisionless) electron transport in a nanowire due to a ballistic driving current in an nearby parallel nanowire. Early on the possibility of the Coulomb drag (CD) effect in the ballistic regime in quantum wires was demonstrated by Gurevich, Pevzner, and Fenton[1] and experimentally observed by Debray and co-workers.[2,3] Using the approach of Refs. 4 and 5 we consider two parallel ballistic quantum channels that are connected to two thermal reservoirs, each being in an independent equilibrium state. As was recently shown, although most of the heat from a current through the channel is generated in the reservoirs[6] part of the heat is generated by the current carrying nanostructure itself[7] via emission of phonons. Electrons penetrating into a biased (drive) wire from the leads are characterized by different chemical potentials, the situation is nonequilibrium, and the phonons are generated by the drive wire.

There has been done much experimental[8] and theoretical[9] work on the phonon contribution to the drag current in a two-dimensional electron-gas (2DEG) situation. As for the phonon drag in ballistic quantum wires it has been investigated theoretically[10] only in the linear transport regime $eV \ll T$, where V is the voltage applied along the drive wire. In contrast, the aim of this paper is to investigate the nonlinear regime $eV \gg T$. Our calculations are based on physically transparent semiclassical transport theory (Boltzmann kinetic equations). A fully quantum-mechanical derivation based on the Keldysh diagrammatic approach[11] yields identical results (see the Appendix).

The electrons in the nearby (drag) nanowire being initially in an equilibrium absorb the ballistic phonons emitted by the drive wire and the phonon drag current is created. Similar to the CD situation here we encounter that in the course of backscattering of electrons of the lth subband in the drive wire the phonons with quasimomentum $\hbar q_z/2$ equal to the Fermi momentum $p_n=\sqrt{2m(\mu-\varepsilon_l^{(1)})}$ [$\varepsilon_l^{(1,2)}$ is the bottom of the lth transverse band in the drive (1) and drag (2) wires] of the subband are generated that in their turn are absorbed by the drag wire. For brevity in what follows we omit the superscripts (1,2). The contribution to the current of two subbands having dispersion $\varepsilon_{lp}=\varepsilon_l+p^2/2m$ in the drive wire and $\varepsilon_{np}=\varepsilon_n+p^2/2m$ in the drag wire vanishes unless

$$eV/2>|\varepsilon_l-\varepsilon_n|,$$

again similar to the CD case.[12] However, in the phonon drag we encounter another inequality

$$eV/2>sp_n,$$

where s is the sound velocity. Indeed, the energy-momentum conservation leads to

$$\hbar\omega_{\mathbf{q}}=\varepsilon_{p+\hbar q_z}-\varepsilon_p$$

that can be rewritten, using $q\cos\theta_q=q_z$ as

$$|\cos\theta_q|=\frac{2ms}{|p+\hbar q_z|-p}<1.$$

Now taking into account that both $|p+\hbar q_z|$ and p momenta should be in the vicinity of the Fermi momentum $p_n \pm eV/2v_n$ ($mv_n=p_n$) we obtain the threshold condition that $eV/2$ must be greater than both the Bloch-Grüneisen parameter sp_n and the shift of the band bottoms $|\varepsilon_n-\varepsilon_l|$.

In Sec. II we calculate the spatial dependence of phonons that are generated by a long, uniform drive wire. The current induced by these phonons in the drag wire is considered in Sec. III. In the Appendix we give a brief review how results obtained by our kinetic equation approach can be obtained by the more complicated Keldysh method.[11]

II. PHONON SPATIAL DISTRIBUTION

We assume that the length of the nanowires L is much greater than the transverse dimensions of the wires. Therefore the spatial distribution $N_{\mathbf{q}}(\mathbf{r})$ of the emitted by the *drive* wire ballistic (nonequilibrium) phonons with wave vector $\mathbf{q}$ is given by the stationary Boltzmann equation

$$\mathbf{s}\nabla N_{\mathbf{q}}=\mathcal{R}, \tag{1}$$

where $\mathbf{s}=\partial\omega_{\mathbf{q}}/\partial\mathbf{q}$ is the group sound velocity, $\hbar\omega_{\mathbf{q}}$ is the phonon energy, and $\mathcal{R}$ is the collision operator. The latter can be written as

$$\mathcal{R}=\frac{1}{\mathcal{V}}\sum_{ll'}\int\frac{2Ldk}{2\pi\hbar}W_{\mathbf{q}}|C_{ll'}(\mathbf{q}_\perp)|^2 \times[f_{l,k+\hbar q_z}(1-f_{l'k})(N_{\mathbf{q}}+1)-f_{l',k}(1-f_{l,k+\hbar q_z})N_{\mathbf{q}}] \times\delta(\varepsilon_{l,k+\hbar q_z}-\varepsilon_{l',k}-\hbar\omega_{\mathbf{q}}), \quad (2)$$

where $C_{ll'}(\mathbf{q})=\langle \boldsymbol{l}|\mathbf{e}^{\mathbf{iqr}_\perp}|\boldsymbol{l}'\rangle$ is the matrix element for phonon induced transitions, $\mathcal{V}$ is the volume of the channel, and $W_{\mathbf{q}}$ is the electron–phonon coupling constant, that for the deformation potential interaction is $W_{\mathbf{q}}=\pi\Lambda^2q^2/\rho\omega_{\mathbf{q}}$, where Λ is the deformation potential constant, and ρ is the mass density. We rewrite the Boltzmann equation in the form

$$(\mathbf{s}\nabla+\Pi_{\mathbf{q}})N_{\mathbf{q}}=\mathcal{R}_s, \quad (3)$$

where

$$\Pi_{\mathbf{q}}=\frac{1}{\mathcal{V}}\sum_{ll'}\int\frac{2Ldk}{2\pi\hbar}W_{\mathbf{q}}|C_{ll'}(\mathbf{q}_\perp)|^2[f_{l,k+\hbar q_z}-f_{l',k}] \times\delta(\varepsilon_{l,k+\hbar q_z}-\varepsilon_{l',k}-\hbar\omega_{\mathbf{q}}) \quad (4)$$

is the polarization operator describing renormalization (screening) of the electron-phonon interaction (see the Appendix for details), and $\mathcal{R}_s$ is the source of the phonons:

$$\mathcal{R}_s=\frac{1}{\mathcal{V}}\sum_{ll'}\int\frac{2Ldk}{2\pi\hbar}W_{\mathbf{q}}|C_{ll'}(\mathbf{q}_\perp)|^2 f_{l,k+\hbar q_z}(1-f_{l'k}) \times\delta(\varepsilon_{l,k+\hbar q_z}-\varepsilon_{l',k}-\hbar\omega_{\mathbf{q}}). \quad (5)$$

We restrict ourselves by the low temperatures $T\ll sp_l\sim eV$. Therefore, seeking the solution of Eq. (3) in the form $N_{\mathbf{q}}=N_{\mathbf{q}}^{eq}+\Delta N_{\mathbf{q}}$ [$N_{\mathbf{q}}^{eq}$ is the equilibrium phonon distribution (Bose) function] we omit the term $\Pi_{\mathbf{q}}N_{\mathbf{q}}^{eq}$. For the nonequilibrium part $\Delta N_{\mathbf{q}}$ we do not take into account the electron-phonon interaction renormalization and therefore omit in Eq. (3) the polarization operator entirely. The Boltzmann equation takes the form

$$(\partial_x+\alpha\partial_y)\Delta N_{\mathbf{q}}(x,y)=\frac{\mathcal{R}_s}{s_x}, \quad \alpha\equiv\frac{s_y}{s_x}=\tan\varphi_q. \quad (6)$$

We assume that the spatial distribution of the generated phonons depends only on the transverse coordinates x,y.

According to the approach in Refs. 4 and 5, we express the distribution functions in Eq. (5) as the Fermi functions $f^F(\varepsilon_{lp}-\mu^{(R,L)})$ with shifted chemical potentials $\mu^{(R,L)}=\mu\pm eV/2$; μ is the quasi-Fermi level that depends on the gate voltage and V is the bias voltage.

Concerning the collision operator $\mathcal{R}_s$ spatial dependence we assume that it has nonzero values only within the nanowire. Although this quantity is discussed in detail in Ref. 7, we briefly discuss how phonon emission can arise. We consider only intraband transitions. Consider an electron having negative initial momentum $k+\hbar q_z$ and propagating to the left. This electron is described by $f^F(\varepsilon_{lk+\hbar q_z}-\mu-eV/2)$. After phonon generation the final state must be propagating to the right ($k>0$), and is described by $f^F(\varepsilon_{lk}-\mu+eV/2)$. Since the initial state must be occupied and the final state must be empty one obtains the following product of distribution functions in Eq. (5):

$$f^F(\varepsilon_{lk+\hbar q_z}-\mu-eV/2)[1-f^F(\varepsilon_{lk}-\mu+eV/2)].$$

Since at low temperatures the distribution functions are unit step functions the product is unity until

$$\varepsilon_{lk+\hbar q_z}<\mu+eV/2, \quad \varepsilon_{lk}>\mu-eV/2.$$

These inequalities and the energy conservation law yield that the phonon generation processes will take place in the phonon frequency range[7]

$$2ms^2+2sp_l^-<\hbar\omega_{\mathbf{q}}<eV.$$

Here p_l^- stands for the Fermi momentum shifted by $eV/2$:

$$p_l^-=\sqrt{2m(\mu-\varepsilon_l-eV/2)}.$$

Our approach considers the phonon generating processes as taking place homogeneously inside the whole channel (wire). The phonons emitted from the edges of the channel can be generated only near specific points where the local energy and momentum conservation laws are met. An effective interaction length $\mathcal{L}$ for these processes can be introduced.[7] We assume that relative intensity of these processes compared to uniform generation given by $\min[\mathcal{L}/L,1]$ is small.

We assume the channels have uniform cross sections, the origin of the system of reference being in the center of the current-carrying (drive) wire, the drag wire being displaced by D in x direction. Therefore we need the solution of Eq. (6) only for $s_x>0$.

The solution of Eq. (6) depends on the cross section geometry of the wire. Assuming, for simplicity, that the cross section of the wire is a circle (it is worth noting that the result does not change significantly for other geometries of the cross section) of the radius R we obtain, for the coordinates outside the wire cross section,

$$N_{\mathbf{q}}=N_{\mathbf{q}}^{eq}+\frac{2\mathcal{R}_s}{s_x(1+\alpha^2)}\sqrt{R^2(1+\alpha^2)-(y-\alpha x)^2} \times\Theta[R^2(1+\alpha^2)-(y-\alpha x)^2]. \quad (7)$$

The geometrical interpretation of this solution is physically transparent (see Fig. 1): let us draw a line having the angle φ_s ($\tan\varphi_s=\alpha$) with the x axis through the center of the wire. Now consider a line parallel to the already drawn line and crossing the wire. The distance $|AC|$ from the point x,y outside the wire and lying on the second line to the first line is $|y-\alpha x|\cos\varphi_s=|y-\alpha x|/\sqrt{1+\alpha^2}$. The Θ function in Eq. (7) states that if this distance is smaller than the radius R (i.e., the second line does cross the wire) the result is proportional to the length of the chord cut from the second line by the cross section; otherwise the result is zero. A similar interpretation is valid for the other geometries of the wire cross section.

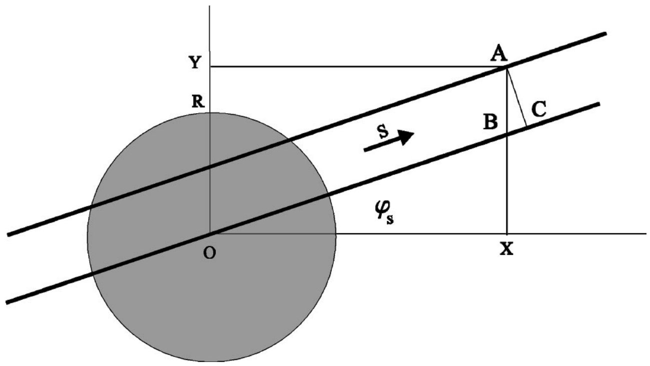

FIG. 1. Schematic representation of the cross section of the drive wire (shaded region) and the phonon propagation direction.

III. PHONON DRAG CONTRIBUTION TO THE DRAG CURRENT

Now in the *drag* wire due to an electron-phonon interaction we have $f_n = f_n^F + \Delta f_n$, with Δf_n satisfying the equation[1]

$$v\frac{\partial \Delta f_n}{\partial z} = I[f_n], \tag{8}$$

where $v = \partial \varepsilon_n / \partial p$ is the electron velocity and $I[f]$ is the electron-phonon collision term. For $p>0$ $(p<0)$, respectively, the solution of this equation is

$$\Delta f_n(z) = (z \pm L/2)\frac{1}{v} I[f_n]. \tag{9}$$

The boundary condition is $\Delta f[p>0\ (p<0)] = 0$ at $z = \mp L/2$. The current then is given by

$$J = e\frac{1}{\mathcal{V}}\sum_{n,p}\int d\mathcal{A} v \Delta f_n = e\sum_n \int \frac{dxdy}{\mathcal{A}}\int_0^\infty \frac{2Ldp}{2\pi\hbar} I[f_n]. \tag{10}$$

Here the integration is over the cross section $\mathcal{A}$ of the drag wire. The electron-phonon collision term is

$$I[f_n] = \sum_{n'}\int \frac{dp'}{2\pi\hbar}\int \frac{d\mathbf{q}_\perp}{(2\pi)^2} W_\mathbf{q} |C_{nn'}(\mathbf{q}_\perp)|^2 \{[f_{n'p'}(1-f_{np}) \times (N_{p'-p}+1) - f_{np}(1-f_{n'p'})N_{p'-p}]\delta(\varepsilon_{np}-\varepsilon_{n'p'} + \hbar\omega_{p'-p}) + [f_{n'p'}(1-f_{np})N_{p-p'} - f_{np}(1-f_{n'p'}) \times (N_{p-p'}+1)]\delta(\varepsilon_{np}-\varepsilon_{n'p'}-\hbar\omega_{p-p'})\}. \tag{11}$$

In $N_\mathbf{q}$ and $\omega_\mathbf{q}$ we indicate explicitly only the longitudinal component of quasimomenta of phonons. The first and second terms in Eq. (11) describe the $N_{p'\text{-}p}$ phonon generation in the electron transition $\varepsilon_{n'p'} \to \varepsilon_{np}$ and absorption of the phonon $N_{p'\text{-}p}$ via the electron transition $\varepsilon_{np} \to \varepsilon_{n'p'}$ respectively. The third and fourth terms describe absorption of the phonon $N_{p\text{-}p'}$ $(\varepsilon_{n'p'} \to \varepsilon_{np})$ and generation $(\varepsilon_{np} \to \varepsilon_{n'p'})$ of the phonon $N_{p\text{-}p'}$. In what follows the notation $N_\mathbf{q}$ stands only for the nonequilibrium part of the phonon distribution in Eq. (7) since the equilibrium part yields zero. For the current induced in the *drag* wire by the phonons emitted by the *drive* wire we obtain

$$J = e\sum_{n,n'}\int \frac{dxdy}{\mathcal{A}}\int_0^\infty \frac{2Ldp}{2\pi\hbar}\int_{-\infty}^\infty \frac{dp'}{2\pi\hbar}\int \frac{d\mathbf{q}_\perp}{(2\pi)^2} \times W_\mathbf{q}|C_{nn'}(\mathbf{q}_\perp)|^2 [f_{n'p'} - f_{np}]\{N_{p'\text{-}p}\delta(\varepsilon_{np}-\varepsilon_{n'p'} + \hbar\omega_{p'\text{-}p}) + N_{p\text{-}p'}\delta(\varepsilon_{np}-\varepsilon_{n'p'}-\hbar\omega_{p\text{-}p'})\}. \tag{12}$$

Here the distribution functions $f_{(n,n')}$ are equilibrium Fermi functions $f^F(\varepsilon_{np}-\mu)$. Assuming that the angle dependence is involved only through the phonon distribution we can take an average over the cross section of the drag wire and integrate over the angles φ_q:

$$\int \frac{dxdy}{\pi R^2}\int \frac{q_\perp dq_\perp}{(2\pi)^2} d\varphi_q N_\mathbf{q} = \frac{2R}{(2\pi)^2 s\pi}\int q dq_\perp \mathcal{R}_s \int_0^1 \rho d\rho \int_0^{2\pi} d\varphi \int_{-\pi/2}^{\pi/2} d\varphi_q \times \sqrt{1-\left(\rho\sin\varphi - \frac{D}{R}\sin\varphi_q\right)^2}. \tag{13}$$

Since the distance between the centers of the wires is assumed to be much bigger than the radius of each wire,

$$\frac{R}{D} \ll 1,$$

we see that only small angles contribute to the integral:

$$\frac{R}{D}(\rho\sin\varphi - 1) < \sin\varphi_q \simeq \varphi_q < \frac{R}{D}(\rho\sin\varphi + 1).$$

Therefore, the result

$$\int_{-(R/D)(1-\rho\sin\varphi)}^{(R/D)(\rho\sin\varphi+1)} d\varphi_q \sqrt{1-\left(\rho\sin\varphi - \frac{D}{R}\varphi_q\right)^2} = \frac{\pi}{2}\frac{R}{D}$$

is proportional to the (solid) angle R/D of the cross section of the drag wire relative to the drive wire. This factor simply reflects the fact that phonons emitted by the drive wire should pass through the drag wire:

$$\int \frac{dxdy}{\pi R^2}\int \frac{q_\perp dq_\perp}{(2\pi)^2} d\varphi_q N_\mathbf{q} = \frac{1}{4\pi}\frac{R}{s}\frac{R}{D}\int q dq_\perp \mathcal{R}_s. \tag{14}$$

In what follows we consider only intraband phonon emission and absorption processes, i.e., we put $n=n'$ and $l=l'$. Inserting this expression into Eq. (12) and taking into account energy conservation laws

$$\delta(\varepsilon_{np}-\varepsilon_{np'}+\hbar\omega_{p'-p})\delta(\varepsilon_{lk+p'-p}-\varepsilon_{lk}-\hbar\omega_{p'-p}) = \frac{m}{\hbar s|p-p'|}\delta(k-p)\delta\left(q-\frac{\varepsilon_{np'}-\varepsilon_{np}}{\hbar s}\right), \tag{15}$$

that allow the integration over k (and q), we obtain

$$J=e\frac{Lm\Lambda^4}{8D\rho^2\pi^3(s\hbar)^7}\sum_{nl}\int_0^\infty dp\int_{-\infty}^{\infty}dp'[f_{np'}-f_{np}]$$
$$\times\frac{P_{nl}(p,p')}{|p-p'|}\{f_{lp'}(1-f_{lp})\Theta(\varepsilon_{np'}-\varepsilon_{np}-s|p-p'|)$$
$$+f_{lp}(1-f_{lp'})\Theta(\varepsilon_{np}-\varepsilon_{np'}-s|p-p'|)\}. \tag{16}$$

Here we introduced notations

$$P_{nl}(p,p')=\frac{(\varepsilon_{np'}-\varepsilon_{np})^4}{\phi_n(p,p')}\left|C_n\left(\frac{1}{\hbar s}\phi_n(p,p')\right)\right|^2$$
$$\times\left|C_l\left(\frac{1}{\hbar s}\phi_n(p,p')\right)\right|^2, \tag{17}$$

$$\phi_n(p,p')=\sqrt{(\varepsilon_{np'}-\varepsilon_{np})^2-s^2(p-p')^2}. \tag{18}$$

Now taking into account that the phonons are emitted by electrons having a negative initial momentum [the nonequilibrium functions $f_{l,p}$ are Fermi functions shifted by $eV/2>0$ chemical potentials, i.e., $f_{lp}=f_{lp}^L$ for $p>0$ and $f_{lp}=f_{lp}^R$ for $p<0$ ($f^L=f^F(\varepsilon-[\mu-eV/2])$, $f^R=f^F(\varepsilon-[\mu+eV/2])$)] and that the integrals vanish unless the Fermi functions f and $1-f$ under the integral overlap, one can conclude that p', satisfying $-\infty<p'<-p-2p_s$, $p_s=ms$, contributes to the current. To avoid further confusion about the drag current direction note that we consider $eV>0$, and therefore the first term on the right-hand side of Eq. (16) survives. Otherwise, if $eV<0$ only the second term in this equation will contribute to the current and due to $f_{np'}-f_{np}$ the current will change the sign.

We consider the case $T\ll\hbar\omega_{\mathbf{q}}<eV$, under these conditions electron distribution functions can be replaced by the step functions and we obtain

$$J=e\frac{Lm\Lambda^4}{8D\rho^2\pi^3(s\hbar)^7}\sum_{nl}\int_0^\infty dp\int_{p+2p_s}^{\infty}dp'[\Theta(p_n-p')$$
$$-\Theta(p_n-p)]\frac{P_{nl}(p,-p')}{p+p'}\Theta(p-p_l^-)\Theta(p_l^+-p'), \tag{19}$$

where

$$p_n=\sqrt{2m(\mu-\varepsilon_n)},\quad p_n^{\pm}=\sqrt{2m(\mu-\varepsilon_n\pm eV/2)}.$$

We obtain the nonzero result for the current in Eq. (19) only if the following inequalities are met

$$p_l^+-p_l^->2p_s,\quad p_l^-<p_n<p_l^+. \tag{20}$$

The last inequality is equivalent to

$$eV/2>|\varepsilon_n-\varepsilon_l|=|\varepsilon_{nl}|.$$

Assuming $eV/2\ll p_n^2/2m,|\varepsilon_{nl}|\ll p_n^2/2m$ the first inequality in Eq. (20) can be simplified too, and we may summarize the inequalities by stating that the drag current is generated provided the bias voltage exceeds the threshold given by

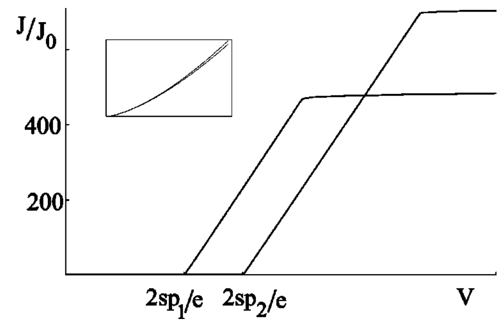

FIG. 2. Drive voltage dependence of the phonon contribution to the drag current for different values of p_n.

$$eV/2>\max\{sp_n,|\varepsilon_{nl}|\}.$$

For simplicity we consider the case of aligned subbands, i.e., we put $\varepsilon_{nl}=0$. Then the result depends on the parameter

$$\alpha=eV/4sp_n,$$

and we discriminate between two regions of α determined by $\alpha<1$ and $\alpha>1$. In the first region $1/2<\alpha<1$ we obtain

$$J=J_0\sum_n\left(\frac{v_n}{s}\right)^2\int_1^{2\alpha}dt(2\alpha-t)T(t), \tag{21}$$

where we introduced

$$J_0=-e\frac{2Lm^5\Lambda^4}{\pi^3D\rho^2\hbar^7}, \tag{22}$$

$$T(t)=\frac{t^4}{\sqrt{t^2-1}}\left|C_n\left(\frac{2p_n}{\hbar}\sqrt{t^2-1}\right)\right|^2\left|C_l\left(\frac{2p_n}{\hbar}\sqrt{t^2-1}\right)\right|^2. \tag{23}$$

In the second region $\alpha>1$ ($eV/2>2sp_n$) we obtain

$$J=J_0\sum_n\left(\frac{v_n}{s}\right)^2\left\{\int_1^{\alpha}dtt+\int_\alpha^{2\alpha}dt(2\alpha-t)\right\}T(t). \tag{24}$$

Assuming the following model dependence for C_n:

$$|C_n(q)|^2=\frac{1}{[1+q^2R^2]^2}, \tag{25}$$

in Fig. 2 we plot the drag current versus the voltage applied along the drive nanowire for different p_n values $p_1R/\hbar=3.33$ and $p_2R/\hbar=5$. Near the threshold $\alpha-1/2\ll1$ we obtain, assuming $2\alpha-1\ll(\hbar/2p_nR)^2$ so that the argument of the function C_n is small and $C_n\sim1$,

$$J=-e\frac{4\sqrt{2}Lm^5\Lambda^4}{3D\rho^2\pi^3\hbar^7}\left(\frac{v_n}{s}\right)^2\left(\frac{eV}{2sp_n}-1\right)^{3/2},\quad 1<\frac{eV}{2sp_n}(<2), \tag{26}$$

i.e. at the threshold the current increases nonlinearly with the applied voltage. This dependence is illustrated by the small inlet in Fig. 2 since it can be noted only in very small vicinity near the threshold. However, the second derivative of the drag current with respect to the bias voltage diverges as

$$\frac{d^2J}{dV^2}\sim\frac{1}{\sqrt{eV/2sp_n-1}}$$

at the threshold for any new subband n, this fact can be instrumental for the experimental investigation of the phonon drag.

At large bias voltages $\alpha\gg 1$ (taking into account $2p_nR/\hbar > 1$) we obtain from Eq. (24) that the current saturates at the value

$$J=J_0\left(\frac{v_n}{s}\right)^2\frac{5\pi}{32}\frac{\hbar}{2p_nR}. \tag{27}$$

Therefore the phonon drag current is a steplike function of the bias voltage. New subbands will contribute to the drag if the voltage is increased.

Assuming the parameters $eV/4sp_n\gg 1$ (for $s\simeq 3\times 10^5$ cm/s,$\hbar/p_n\simeq 0.5\times 10^{-6}$ cm this means voltages $V\sim$ mV), $p_nR/\hbar\simeq 1$, $L\simeq 2$ μm, $D\sim 0.1$ μm, $\Lambda\sim 8$ eV, and $m=0.07m_0$ we obtain the following estimation for the contribution of any subband to the phonon-mediated drag current:

$$J\sim 3\cdot 10^{-12}A.$$

IV. CONCLUSION

We note three essential differences between the phonon drag and the Coulomb drag in the nonlinear regime $eV\gg T$. First, there is the existence of the threshold $eV/2>\max\{sp_n,|\varepsilon_{nl}|\}$ (that can be achieved changing either the bias voltage or the gate voltage).

Second, the weak dependence $J\sim 1/D$ on the distance D between the centers of the drive and drag wires rather than the exponential one in the CD case. On the other hand, note that the distance dependence of the phonon drag current in our case is stronger than the distance dependence of the phonon drag between two 2DEG layers.[8]

Third, in contrast to the CD case the phonon drag current saturates at large $eV\gg sp_n$. The current is a steplike function of the bias voltage since the bias voltage increasing will involve new subbands in the phonon drag. The width of each step can be estimated as $\Delta V\sim sp_n/e$, the height as $J_0(v_n/s)^2[\hbar/(p_nR)]$.

The piezoelectric coupling coefficient for GaAs having the cubic $\mathbf{T_d}$ symmetry can be written as

$$W_{\mathbf{q}}=\frac{\pi}{\rho\omega_{\mathbf{q}}}\left[\frac{4\pi e\beta}{\epsilon}\right]^2[F(\theta_q,\varphi_q)]^2,$$

where β is the piezoelectric constant, ϵ is the dielectric susceptibility, and F is a function of angles. The estimations for GaAs ($\beta=10^5$ Gaussian units, $\epsilon=12$, $\Lambda=8$ eV, $\rho=5$ g/cm^3, and $s=3\times 10^5$ cm/s) show that the piezoelectric interaction is the dominating one for frequencies $\omega_{\mathbf{q}}\leqslant 10^{11}$ s^{-1}.

Frequencies of transmitted phonons in the phonon drag are $\omega_q>2s/(\hbar/p_n)$, i.e., are greater than 10^{12} s^{-1}. Therefore, the contribution of the piezoelectric coupling is smaller than the considered deformation potential coupling contribution.

Let us finally briefly discuss the temperature dependence of the phonon drag current. In the nonlinear regime $T\ll eV\sim sp_n$ the drag current does not depend on the temperature. The temperature dependence in the linear response regime $eV\ll T$ was investigated in Ref. 10. Under the condition $2p_nD/\hbar\gg 1$ it was found that the phonon drag current with increasing temperature evolves from a power-law dependence to an exponential dependence and then in a region of relatively high temperatures $T\gg sp_n$ becomes temperature independent (if $2p_nR/\hbar\sim 1$).

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APPENDIX: DIAGRAMMATIC DERIVATION OF THE NONLINEAR PHONON DRAG

One can express the drag current also as an integral over the transferred phonon momentum q_z in an electron-electron interaction. In the screened Hartree-Fock approximation for the phonon self energy we obtain, for the current in the drag wire,

$$J=-\frac{e}{2}\sum_{nl}\int\frac{d\omega}{2\pi}\frac{Ldq_z}{2\pi}\Pi^K_{0\omega,q_z}\times\left\{\Pi^K_{\omega,q_z}\tanh\frac{\omega}{2T}-[\Pi^R_{\omega,q_z}-\Pi^A_{\omega,q_z}]\right\}\frac{|D^R_{0\omega,q_z}|^2}{|\epsilon(\omega,q_z)|^2}, \tag{A1}$$

where according to usual notations Π^K, Π^R, $\Pi^A(\Pi^K_0,\Pi^R_0)$ are Keldysh, retarded, and advanced components of the polarization operator for the nonequilibrium drive (equilibrium drag) wire, $D^R_{0\omega,q_z}$ is the summed over $\mathbf{q}_\perp$ retarded component of the (free) phonon Green's function, and ϵ is the dielectric function describing the screening by the Coulomb and phonon interactions. This form demonstrates that the drag current is a convolution of the spontaneous polarizations within each quantum wire. We include the electron–phonon coupling constants into D^R. Polarization operators

$$\Pi^K_{\omega,q}=-2i\pi\sum_p\delta(\omega+\varepsilon_{lp-q}-\varepsilon_{lp})[f_{lp}(1-f_{lp-q})+f_{lp-q}(1-f_{lp})], \tag{A2}$$

$$\Pi^R_{\omega,q}=\sum_p\frac{f_{lp-q}-f_{lp}}{\omega+\varepsilon_{lp-q}-\varepsilon_{lp}+i0}, \tag{A3}$$

$$\Pi^A_{\omega,q}=(\Pi^R_{\omega,q})^* \tag{A4}$$

involve nonequilibrium electron distribution functions. Here the spin summation is implied. Π^K_0 involves equilibrium distribution functions and is restricted to $p>0$ [cf. Eq. (10)].

According to our convention we insert into Eqs. (A2) and (A3) shifted Fermi distribution functions $f^{L,R}$. Since in the equilibrium there is an identity

$$\Pi^K_{\omega,q}=(\Pi^R_{\omega,q}-\Pi^A_{\omega,q})\coth\frac{\omega}{2T},$$

we can rewrite Eq. (A1) as

$$J=-\frac{e}{2}\int\frac{d\omega}{2\pi}\frac{L\,dq_z}{2\pi}[\Pi^K_{0,\omega,q_z}\Pi^K_{2,\omega,q_z}-\Pi^K_{0,-\omega,q_z}\Pi^K_{1,-\omega,q_z}]|D^R_{0\omega,q_z}|^2\left(\tanh\frac{\omega}{2T}-\tanh\frac{\omega-eV}{2T}\right),\tag{A5}$$

where we neglected the screening of the interaction and put $\epsilon=1$. In the end of this section we briefly discuss when the screening can be neglected.

In Eq. (A5) we introduced the restricted to $0<p<q$ operator

$$\Pi^K_{1,\omega,q}=-2i\pi\sum_{0<p<q}\delta(\omega+\varepsilon_{lp-q}-\varepsilon_{lp})\times[f^L_{lp}(1-f^R_{lp-q})+f^R_{lp-q}(1-f^L_{lp})].\tag{A6}$$

In $\Pi^K_{2,\omega,q}$ the sum is restricted by $q<p<0$ and the replacements $R\to L$, $L\to R$ are made.

In the linear response case $eV\ll T$ Eq. (A5) can be written in the form formally coinciding with the formula derived in Ref. 9 for the phonon-mediated drag in the 2DEG situation:

$$J=-\frac{e^2V}{T}\int\frac{d\omega}{2\pi}\frac{L\,dq_z}{2\pi}\frac{\mathrm{Im}\,\Pi^R_{0,\omega,q_z}\mathrm{Im}\,\Pi^R_{1,\omega,q_z}}{\sinh^2\omega/2T}|D^R_{0\omega,q_z}|^2.\tag{A7}$$

Here in Π^R_1 the summation is restricted by $0<p<q$. This case has been considered in a slightly different manner in Ref. 10, and corresponding results can be restored.

Let us concentrate on the nonlinear regime $eV\gg T$, i.e., the case of low temperatures. First, we note that the difference of $\tanh\omega/2T$ and $\tanh(\omega-eV)/2T$ in Eq. (A5) imposes the constraint

$$0<\omega<eV.$$

Under this condition the term $\Pi^K_2\Pi^K_0$ does not contribute, the polarization operator $\Pi^K_{0,-\omega}$ is equal to

$$\Pi^K_{0,-\omega,q_z}=-2i\frac{mL}{q_z}\Theta(\omega-v_n|2p_n-q_z|)\tag{A8}$$

provided $\omega\ll p_n^2/2m$ and the product $\Pi^K_1\Pi^K_0$ in the case of aligned subbands in the two wires $p_n=p_l$ is reduced to

$$\Pi^K_{1,-\omega}\Pi^K_{0,-\omega}=-\frac{4m^2L^2}{q_z^2}\Theta(eV/2-\omega)\Theta(\omega-v_n|2p_n-q_z|)+[\omega\to eV-\omega].\tag{A9}$$

Then after inserting the product into Eq. (A5) we obtain

$$J=J_0\sum_n\left(\frac{v_n}{s}\right)^2 8\pi(2p_nD/\hbar)\times\left\{\int_0^\alpha d\omega\,\omega+\int_\alpha^{2\alpha}d\omega(2\alpha-\omega)\right\}|D^R(\omega)|^2.\tag{A10}$$

Here $\alpha=eV/(4sp_n)$ and $D^R(\omega)$ is the dimensionless phonon Green function

$$D^R(\omega)=\int\frac{d\mathbf{q}_\perp}{(2\pi)^2}\frac{1+q_\perp^2}{(\omega+i0)^2-1-q_\perp^2}\times C^{(1)}(2p_n\mathbf{q}_\perp)C^{(2)}(-2p_n\mathbf{q}_\perp),\tag{A11}$$

where $C^{(1,2)}$ stand for matrix elements of phonon-induced transitions in the drive and drag wire. Note that in this approach the product of the matrix elements depends on the spatial displacement D of the centers of the wires. This dependence is determined independently of the shapes of the channels and is described by $\cos(q_xD)$ provided $R\ll D$, where R is the maximal transverse length of the wire(s). Further calculations can be made if one specifies the matrix elements depending on the shape of the channel. If we model them according to Eq. (25) our results are restored except for that the threshold is determined with $\hbar^2/(2p_nD)^2\ll 1$ accuracy. The threshold is the consequence of the fact that $D^R(\omega)$ for $\omega<1$ is exponentially decreasing function of the parameter D/R. For $\omega>1$ $D^R(\omega)$ can be reduced to[10]

$$D^R(\omega)=-\frac{\omega^2}{4[1+R_d^2(\omega^2-1)]^2}\{iJ_0(D_d\sqrt{\omega^2-1})+N_0(D_d\sqrt{\omega^2-1})\},\tag{A12}$$

where $J_0(x)$ is the Bessel function and $N_0(x)$ is the Neumann function.[13] In the last equation we introduced the dimensionless parameters $R_d=2p_nR/\hbar$ and $D_d=2p_nD/\hbar$. Therefore, for $1/2<\alpha<1$ the current is given by

$$J=J_0\sum_n\left(\frac{v_n}{s}\right)^2 8\pi(2p_nD/\hbar)\int_1^{2\alpha}d\omega(2\alpha-\omega)|D^R(\omega)|^2,\tag{A13}$$

with $D^R(\omega)$ given by Eq. (A12) [cf. Eq. (21)]. For $\alpha>1$ we have

$$J=J_0\sum_n\left(\frac{v_n}{s}\right)^2 8\pi(2p_nD/\hbar)\times\left\{\int_1^\alpha d\omega\,\omega+\int_\alpha^{2\alpha}d\omega(2\alpha-\omega)\right\}|D^R(\omega)|^2,\tag{A14}$$

again with $D^R(\omega)$ from Eq. (A12) [cf. Eq. (24)].

Let us discuss when the screening can be neglected. In the two-subband approximation the dielectric function is

$$\epsilon(\omega,q_z)=[1-(D_f^R+U_f(q_z))\Pi_f^R][1-(D_s^R+U_s(q_z)\Pi_s^R]-(D^R+U(q_z))^2\Pi_f^R\Pi_s^R.$$

Here the first term on the right-hand side includes the intrawire D_{fs}^R and $U(q_z)$ [for the first (f) and second wire (s)], while D^R and $U(q_z)$ in the last term stand for interwire (phonon and Coulomb) interactions. We note that the dielectric function with $q_z=2p_n$ enters the final expression for the phonon drag current and assume that the intrawire phonon interaction is smaller than the Coulomb intrawire interaction. Then (omitting also the small interwire terms) we are left with

$$\epsilon(\omega,2p_n)=[1-U(2p_n)\Pi^R]^2. \tag{A15}$$

The intrawire Coulomb interaction can be estimated as

$$U(2p_n)\simeq\frac{e^2}{LR_d^2}, \tag{A16}$$

where we assumed that $R_d\gg 1$. The polarization operator at $\omega\sim eV$ can be estimated as

$$\Pi^R\simeq-\frac{mL}{2p_n\hbar\pi}\left[i\pi+2\ln\left\{\frac{8p_n^2}{2meV}\right\}\right]. \tag{A17}$$

Therefore we obtain the following restriction to neglect the screening:

$$\frac{e^2m}{p_n\hbar}\frac{1}{R_d^2}\ln\left\{\frac{p_n^2}{2meV}\right\}\ll 1. \tag{A18}$$

Otherwise the phonon drag current will be reduced by the factor $|\epsilon(eV,2p_n)|^2$.

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