Soluble model to treat the quantum spin glass

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The van Hemmen model with a transverse field is studied to describe the quantum Ising spin glass. The free energy and phase diagrams (T versus Ω, and T versus J_o/J, where Ω is the transverse field, J_o and J are the ferromagnetic and random exchange interactions, respectively) are calculated for the model with two-peaked and Gaussian exchange distribution. The system presents three ordered phases, namely, spin glass, mixed, and ferromagnetic phases, besides the paramagnetic disordered phase. The influence of the transverse field Ω is to destroy the ordered phases. In the (T,Ω) plane our results are compared with those obtained by the replica-symmetry-breaking solution and the same qualitative behavior is observed for the spin-glass transition.

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The role of quantum fluctuations in spin glass (SG) remains a long standing theoretical problem.[1] Recently, there has been growing interest in theoretical and experimental investigations of the Ising spin glass in a transverse field to treat the phase transition in quantum spin glasses.[2–6] Experimentally, results for the nonlinear susceptibility provide strong evidence for a finite transition temperature, T_c, and as an example we have the so-called proton glasses,[7,8] being a random mixture of ferroelectric and antiferroeletric materials such as $Rb_{1-x}(NH_4)_xH_2PO_4$, where proton tunneling in the glass state can be represented by transverse field in the pseudospin Ising model.[9] The transverse Ising spin-glass model has also been used to treat the quantum spin-glass phase transition of the diluted dipole coupled magnet $LiHo_xY_{1-x}F_4$.[10–12]

The spin-glass problem represents a quite difficult task in statistical mechanics. For many years there has been great controversy on whether the spin-glass transition is of thermodynamic or of dynamic nature. However, simulations[13] and phenomenological scaling arguments at zero temperature[14] suggested the existence of a true thermodynamic phase. Until now, only mean-field models are exactly tractable, but they require sophisticated mathematical tools.[15] The treatment of the quantum spin-glass problem is more complicated than its classical counterpart mainly by two factors: (i) the system has a dynamic nature from the outset and cannot be simplified to calculation of static quantities even while evaluating statistical mechanical averages; (ii) quenched disorder and associated very complicated energy landscape resulting in a huge number of local minima of free energy as in the case of the classical spin glass.

In the present paper, we study the quantum influence of the transverse field on the spin glass mean-field model introduced by van Hemmen (VH).[16] This model is exactly soluble, and, unlike the Sherrington-Kirkpatrick[15] model (SK), its solution does not require the use of the replica trick. It spite of being nonrealistic, mean-field models give a first qualitative understanding of the thermodynamic behavior. Among these are the susceptibility cusp at the freezing temperature T_f and the field-induced transition away from the spin-glass phase, at finite magnetic field. To treat the spin-glass Ising in a transverse field, usually, an approximate analytically tractable solution is obtained by replacing the dynamic self-interaction by an appropriate time average. In the context of the Matsubara imaginary time and replica approach this model is referred to as the static approximation.[14]

In the literature, some controversy regarding the nature of the spin-glass phase of the SK with a transverse field has been observed. Using the static approximation,[14,17] there is a small region in the spin-glass phase where a replica-symmetric solution is stable, unlike the classical SK model with no transverse field. The SK model in the presence of a transverse field was also treated by other methods[18] which does not use the static approximation, and predicts that the replica-symmetric solution is always unstable in the whole spin-glass phase. On the other hand, Yokota[6] treated the SK model by a pair approximation and showed that the spin-glass transition temperature increases linearly with Ω for small transverse fields [i.e., $T_c(0)-T_c(\Omega)\simeq\Omega$]. The results of the stability of the replica-symmetric solution[18] and the linear dependence of T_c for small Ω (Ref. 19) has been verified by some Monte Carlo simulation.[19] The first step of replica-symmetry-breaking (RSB) solution[20] in the infinite-range Ising spin glass with a transverse field (quantum spin glass) has showed that the phase diagram is not in accordance with the results of pair approximation[6] and Monte Carlo simulation.[19]

On the other hand, to treat the influence of the transverse field in the spin-glass phase, we generalize in this paper the van Hemmen model of spin glass, which consists of a fully connected net of N Ising spins with a transverse field described by the following Hamiltonian:

$$\mathcal{H}=-\frac{J_o}{N}\sum_{(i,j)}\sigma_i^z\cdot\sigma_j^z-\sum_{(i,j)}J_{ij}\sigma_i^z\cdot\sigma_j^z-\Omega\sum_i\sigma_i^x, \qquad (1)$$

where (i,j) denotes a sum over all possible pairs of spins, J_o represents a ferromagnetic interaction, Ω is the transverse field, σ_i^x, σ_i^z are the Pauli matrices for spin at site i, J_{ij} is the spin-glass random coupling given by

$$J_{ij}=\frac{J}{N}(\xi_i\eta_j+\xi_j\eta_i), \qquad (2)$$

where the ξ_i's and η_i's are independent, identically distributed random variables with even distribution around zero and a finite variance, say 1. In particular, we restrict ourselves to the case in which they can take the values $+1$ and -1, i.e.,

$$\mathcal{P}(x_i)=\tfrac{1}{2}[\delta(x_i-1)+\delta(x_i+1)], \tag{3}$$

where $x_i\equiv\xi_i$ or η_i.

In the thermodynamic limit $N\rightarrow\infty$, the van Hemmen model has three order parameters,[16]

$$m=\lim_{N\rightarrow\infty}\left\{\frac{1}{N}\sum_{i=1}^{N}\langle\sigma_i^z\rangle\right\}, \tag{4}$$

$$q_1=\lim_{N\rightarrow\infty}\left\{\frac{1}{N}\sum_{i=1}^{N}\langle\xi_i\sigma_i^z\rangle\right\}, \tag{5}$$

and

$$q_2=\lim_{N\rightarrow\infty}\left\{\frac{1}{N}\sum_{i=1}^{N}\langle\eta_i\sigma_i^z\rangle\right\}, \tag{6}$$

which have to be chosen in such a way that $\vec{M}=(m,q_1,q_2)$ minimizes a certain free-energy functional. The number of independent random variables is $2N$ in contrast with $N^2/2$ of SK model. Thus we have a random-site problem and not a random-bond problem as in most other SG models, in agreement with the experimental situation.

The part second of the Hamiltonian (1) can be separable, i.e.,

$$\sum_{i\neq j}(\xi_i\eta_j+\xi_j\eta_i)\sigma_i^z\sigma_j^z=\left[\sum_{i=1}^{N}((\eta_{i+}\xi_i)\sigma_i^z)\right]^2-\left(\sum_{i=1}^{N}\eta_i\sigma_i^z\right)^2-\left(\sum_{i=1}^{N}\xi_i\sigma_i^z\right)^2-2\sum_i\xi_i\eta_i, \tag{7}$$

using Eq. (8) the Hamiltonian (1) can be rewritten as

$$\mathcal{H}=-\frac{J_o}{N}\sum_{(i,j)}\sigma_i^z\cdot\sigma_j^z-\frac{J}{N}\left\{\left[\sum_{i=1}^{N}((\eta_{i+}\xi_i)\sigma_i^z)\right]^2-\left(\sum_{i=1}^{N}\eta_i\sigma_i^z\right)^2-\left(\sum_{i=1}^{N}\xi_i\sigma_i^z\right)^2\right\}+2\frac{J}{N}\sum_i\xi_i\eta_i-H\sum_i\sigma_i^z-\Omega\sum_i\sigma_i^x. \tag{8}$$

In order to study theoretically the thermal properties of the relevant system described by the Hamiltonian (1), we have to calculate the partition function

$$Z=\mathrm{Tr}\{\exp(-\beta\mathcal{H})\}=\sum_\mu\langle\mu|\exp(-\beta\mathcal{H})|\mu\rangle, \tag{9}$$

for the orthogonal complete set of states $|\mu\rangle$, where we use the Gaussian identity

$$\exp(\alpha x^2)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}dy\,\exp\left(-\frac{y^2}{2}+\sqrt{2\alpha}xy\right). \tag{10}$$

Performing the trace and using steepest descent integrations, we obtain, after some algebra, the following expression for the free-energy per spin:

$$\beta f(m,q)=\frac{\beta}{2}(J_om^2+2Jq^2)-\langle\ln[2\cosh(\beta W)]\rangle_c, \tag{11}$$

whose minimum corresponds always to $q_1=q_2=q$ and the mean-field equations take the form

$$m=\left\langle\frac{[J_om+Jq(\xi+\eta)]}{W}\tanh(\beta W)\right\rangle_c \tag{12}$$

and

$$q=\left\langle\frac{(\eta+\xi)[J_om+Jq(\xi+\eta)]}{2W}\tanh(\beta W)\right\rangle_c, \tag{13}$$

where $W=\sqrt{\Omega^2+[J_om+Jq(\xi+\eta)]^2}$ and the notation $\langle\cdots\rangle_c$ denotes the average over the random variable η and ξ.

In the limit of null transverse field, the above equations reduce to the same expressions obtained by van Hemmen.[16] The phase diagram in the $(T/J,\alpha)$ plane, where $\alpha=J_o/J$ and we have set the Boltzmann constant $k_B=1$, presents three ordered phases: (i) *ferromagnetic* $F(m>0,\ q=0)$, (ii) *spin glass* (SG) $(m=0,\ q>0)$, and *mixed* M $(m,q>0)$ phases. We have also the paramagnetic phase (P) corresponding to the trivial solution $m=q=0$.

In Fig. 1 we present the phase diagram in the $(T/J,\alpha)$ plane for some values of $\delta=\Omega/J$. The critical lines (SG-P, SG-M, SG-F, and F-P) are obtained by analyzing the stability limit of Eqs. (11)–(13) in the respective phases. It can be seen that the effect of the transverse field is to destroy the ordered phases. For $\delta>\delta_{1c}=0.42$ the mixed (M) phase disapears and for $\delta>\delta_{2c}=1$ all ordered phases are destroyed in the phase diagram. We have also considered a Gaussian distribution for the random variables ξ and η, with $\langle\eta\rangle=\langle\xi\rangle=0$ and $\langle\eta^2\rangle=\langle\xi^2\rangle=1$. In this case we do not observe the mixed phase and the qualitative behavior of the phase diagram is similar to that in Fig. 1 for $\delta>\delta_{1c}$.

Figure 2 shows the reduced transition temperature $T_c(\Omega)/T_c(0)$ from the paramagnetic to the spin-glass phase as a function of Ω/Ω_c. For a simple comparison we can indicate qualitatively other models and approximations that have been discussed in the literature, for example, the present results are in qualitative agreement with those from replica symmetric (RS) without using the static approximation[1] and RSB (Ref. 20) solutions in the high-temperature regime (small Ω). It is, however, qualitatively similar to the ones recently obtained using the quantum spherical description to treat the Ising spin glass in a transverse field.[3] Our results do not agree with those from pair approximation,[6] Monte Carlo simulation,[19] and renormalization group[5] which predict an increase of T_c for small fields Ω. The corresponding quantum-critical point above which no spin-glass transition occurs in the present model reads

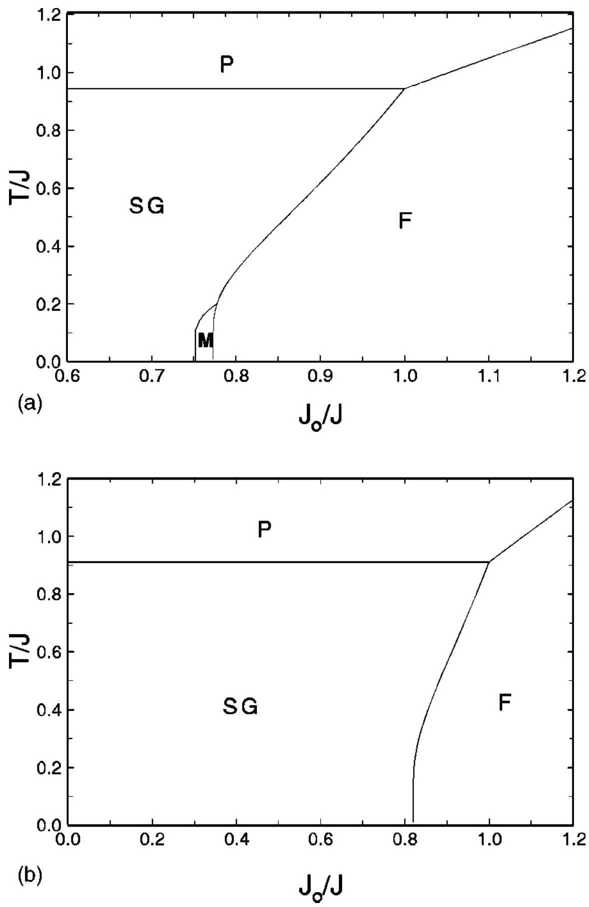

FIG. 1. Phase diagram in the $(T/J, J_o/J)$ plane for the quantum van Hemmen model with transverse field $\Omega/J=0.40$ (a) and $\Omega/J=0.50$ (b).

$\delta_c=1$. In Table I, we present some values for δ_c obtained by other methods. The value $\delta_c=1$ is the same value found for the quantum spin 1/2 XY spin glass in a transverse field,[21] and also with the quantum Ising spin glass using the static approximation.[1]

The behavior of the critical temperature of the infinite-ranged quantum Heisenberg spin glass[21] for strong field and

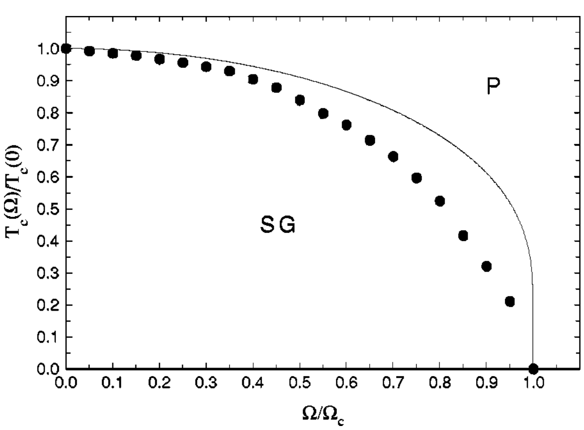

FIG. 2. Critical transition temperature between the paramagnetic and spin-glass phases for the transverse van Hemmen model in the reduced variables $T_c(\Omega)/T_c(0)$ and Ω/Ω_c.

TABLE I. Values of the quantum-critical point δ_c for the Ising spin glass with a transverse field.

Method	δ_c
Static approximation (Ref. 1)	1.00
Replica-symmetry-breaking (RSB) solution (Ref. 20)	1.60
Quantum spherical description (Ref. 3)	1.39
Renormalization group (Ref. 5)	1.58
Present paper	1.00

low temperature is given by $T_c \simeq \exp(-2\delta^2)$, and therefore presents no phase transition at $T=0$. The phase transition at $T=0$ is due to the absence of random couplings in the z direction and hence the local effective field is equal to the applied field. These fields order the spins to point in the z direction, and hence their transverse component becomes small for larger Ω which suppresses the transverse spin glass ordering and is enough to destroy it for some finite field (Ω_c). For the Heisenberg model, there is an effective field at each site, the z component of which is equal to the sum of the applied field Ω plus a random Gaussian field. Recently, Nogueira *et al.*[22] studied the infinite-range classical $O(n)$ spin glass in the presence of a Gaussian random field (with $\langle h_i \rangle = \Omega$ and $\langle h_i^2 \rangle - \langle h_i \rangle^2 = \Delta^2$) through the replica-symmetric solution. For high fields and low temperatures, with $\Delta=0$ (uniform magnetic field) the critical temperature of the classical Heisenberg spin glass $(n=3)$ present the same qualitative behavior obtained by Goldschmidt and Lai.[21]

The comparison of the present results with other models and approximations is important to show that a simple treatment using the version of the VH model with a transverse field obtains correct qualitative behavior for the phase diagram (SG-P) in the $(T\text{-}\Omega)$ plane of a quantum spin glass model. On the other hand, we also observed that the quantum influence destroys the mixed phase for a certain critical value of the transverse field. This result has not been observed through other models (e.g., the SK model) and approches. The VH model presents some limitations and has been discussed in detail by Choy and Sherrington;[23] for example, it does not predict the great multiplicity of states in the free energy.

In conclusion, we have studied the effect of a transverse field in the van Hemmen spin-glass model. The phase diagram in the temperature-field plane, by using a two-peaked distribution for the random variables η and ξ, exhibits qualitatively the same behavior as obtained by other methods[1,3,6,20] and we obtain a quantum-critical point at $T=0$. Using a Gaussian distribution for η and ξ, we obtain the same phase diagram presented in Fig. 2 for the van Hemmen model. It would be very interesting to investigate the problem of the generalized $O(n)$ spin glass in the version of the van Hemmen model and analyze the behavior of T_c with the magnetic field Ω for various values of $n=1,2,3,\ldots\infty$. These are areas for future research.

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