Disorder-induced modification of the transmission of light in a two-dimensional photonic crystal

M. A. Kaliteevski, J. Manzanares Martinez, D. Cassagne, and J. P. Albert
Groupe d'Etude des Semiconducteurs, CC074 Universite de Montpellier II, Place Bataillon, 34095 Montpellier Cedex 05, France

(Received 15 February 2002; revised manuscript received 8 April 2002; published 3 September 2002)

We report on the modeling of light transmission in disordered two-dimensional photonic crystals with an incomplete photonic band gap. The disorder leads to a decrease in light attenuation in the transmission dip corresponding to the photonic band gap. We show that the scattered transmission can differ substantially from the straight one and that the minimum of this scattered transmission does not correspond to the center of the photonic band gap. The disorder smoothes and broadens the attenuation dip that appears in transmission spectra and can create an asymmetric shape. The straight transmission has a thresholdlike behavior as a function of disorder and sustains a certain amount of disorder before changing.

DOI: 10.1103/PhysRevB.66.113101 PACS number(s): 42.70.Qs, 42.25.Dd

I. INTRODUCTION

Photonic crystals (PC's) have attracted considerable interest for their ability to confine, manipulate, or guide light and also for their potential to inhibit spontaneous emission.[1–5] One of the main technological problems which prevent the use of three-dimensional (3D) self-organized PC's like opals for the fabrication of optoelectronic devices is the occurrence of *disorder* inherent to the growth process:[6–9] the spheres forming opals exhibit a certain dispersion in radius and displacement of their position in the crystalline lattice.[10] Also opals include stacking faults.[11] It is also worth noting that structures obtained by lithography in the 2D case always contain roughness of the surfaces and imperfections due to the etching process.[12] The presence of defects in the crystalline photonic lattice can then damage the photonic band gap (PBG) by filling it with localized photonic states.[10]

In a PC with a finite thickness, a PBG creates an attenuation of the transmitted light which results in a dip in the transmission spectrum. One of the characteristics of the appearance of localized photonic states inside the PBG (which could be experimentally measured) is a decrease in the attenuation of the dip.[10] The aim of the present work is to analyze the influence of disorder on this dip. For the one-dimensional case, it is known that a certain critical threshold ''amount of disorder'' is required for the damaging of the dip.[13] A natural question is, will the same behavior be observed in the case of 2D systems? In the 2D case, the properties of the PC according to the different directions of light propagation can differ substantially. Recent numerical modeling studies have obtained important results on the disorder-induced modification of the light propagation and density of modes in PC's,[14–17] but the explanation of the physical nature of the obtained phenomena remains an actual task. In this paper, we perform a systematic analysis where we separate straight and scattered transmissions. The subject of our study is the case of a PBG that exists only for certain particular directions, the so-called incomplete PBG. This situation is particularly important in opal-based PC's because it corresponds to the lowest PBG which is generally studied along the [111] natural growth direction of opals.[18] Such a situation can be implemented in the case of hexagonal 2D PC's formed by air cylinders,[19] as shown in Fig. 1.

II. RESULTS AND DISCUSSION

Figure 2 shows the photonic band structure of a 2D hexagonal PC for the TM electromagnetic modes (for which the electric field is parallel to the cylinders and the magnetic field lies in the plane perpendicular to the cylinders), calculated by the plane-wave method.[19] The refractive index of the material is 3.6 and the radius of the cylinder $r_0 = 0.4d$ (where d is the distance between the centers of the two neighboring cylinders). An incomplete PBG exists in the Γ-M direction of the Brillouin zone, centered at the frequency $fd/c = 0.2212\ldots$, with a relative width of about 0.15. When the propagation direction shifts toward the Γ-K direction, the PBG becomes smaller and closes completely for the Γ-K direction.

In experimental studies, there are two main schemes which analyze either *straight* transmission (when the transmitted wave has the same direction of propagation as incident light) or total transmission (all the waves, emerging from the rear side of the sample, which include *straight* and *scattered* waves, for which direction of the propagation differs from the direction of the incident light), as shown in Fig. 1. Conducting the modeling of the light transmission, we will

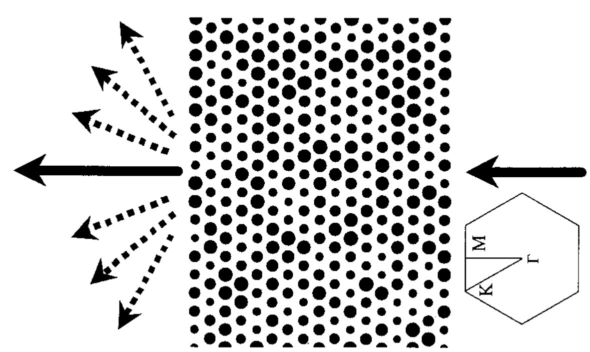

FIG. 1. Schematic view of the supercell of the disordered hexagonal structure. The black circles correspond to air cylinders. Solid arrows illustrate incident light and light transmitted in the same direction as incident light. Dotted arrows indicate the scattered light. For the particular configuration of the structure the deviation of the cylinder radius $\delta = 0.25$. The Brillouin zone of ideal hexagonal structure is also shown.

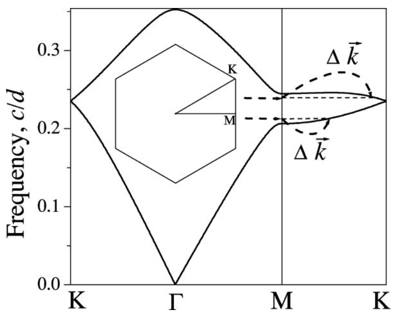

FIG. 2. Photonic band structure of the ideal (without disorder) hexagonal structure under study for the TM polarization. The inset shows the hexagonal Brillouin zone. Light scattering from evanescent photonic states into passed photonic states is illustrated by dashed arrows.

then separate the contribution of straight and scattered transmissions.

Figure 3 shows the result of the modeling of the *straight* and *scattered* light transmission through the hexagonal structure shown in Fig. 1, for various ''amounts of the disorder,'' described by a deviation δ in the cylinder radius, which is uniformly distributed in the interval from $r_0(1-\delta)$ to $r_0(1+\delta)$. Calculations have been conducted by a combination of the transfer matrix method and the multiple-scattering technique, using the publicly available computer code by Bell *et al.*[20] Each cylinder was described by a 7×6 mesh. Periodic boundary conditions were used at the left and right sides of the structure which consist of a supercell of 323 cylinders (17 rows by 19 cylinders) and ensure sufficient convergence in the frequency range under study. The value of deviation used in our modeling was $\delta=0.05, 0.1, 0.15, 0.2$, and 0.25. For comparison, the straight transmission spectra of an ideal structure are shown by a dashed line, and it can be seen that at the minimum of the transmission dip light attenuation exceeds 10^4. Note that the light does not experience scattering in the case of the ideal structure, since this gap is below the diffraction cutoff.

Insertion of the disorder into the PC leads to substantial modifications of the transmission spectra. In the case of a single configuration of disorder, as can be seen from Fig. 3, small peaks disrupt the transmission dip of the *straight* light. These spikes are associated with photonic states localized in the direction of the light propagation. Furthermore, in contrast to the case of the ideal structure, scattered light appears, and the transmission dip for the scattered light has an asymmetric shape, with the minimum shifted towards the high-frequency edge of the PBG. It is worth noting that the scattered transmission is more than 10 times as large as the straight one, so its effect will be largely dominant in the total transmission. In order to simulate experimentally observed spectra of real PC's, which are always (more or less) disordered and usually exhibit a smooth dip in the area of the PBG,[13] we have averaged such calculated spectra over different disorder configurations. Thick lines show the transmission spectra averaged over 20 configurations and additionally smoothed down, to remove remaining traces of the spikes. For $\delta=0.05$, we find that the averaged transmission spectra of the straight light almost coincides with that of the ideal structure.

With the increase of the disorder, more and more spikes appear in the transmission dip, their amplitude increases, and, on average, the attenuation decreases. Averaged spectra for the *straight* light exhibit a small distortion, and the minimum moves slightly toward higher frequencies. Averaged spectra for the *scattered* light retain their asymmetric, triangular shape. Note also the broadening of the transmission dip in the averaged spectra with increase of disorder. When the deviation δ becomes as large as 0.25, the triangular shape of the averaged spectra of scattered light begins to smoothen out.

The shape modification of the scattered spectra can be explained by the following simple physical arguments. Suppose that the frequency of an incident light, propagating along the Γ-M direction, corresponds to the PBG. Consequently the states excited by this light inside the PC are evanescent states, experiencing an exponential attenuation when propagating through the PC. Due to disorder, the propagation direction of these states can change and excite propagating states with the wave vector lying on the M-K

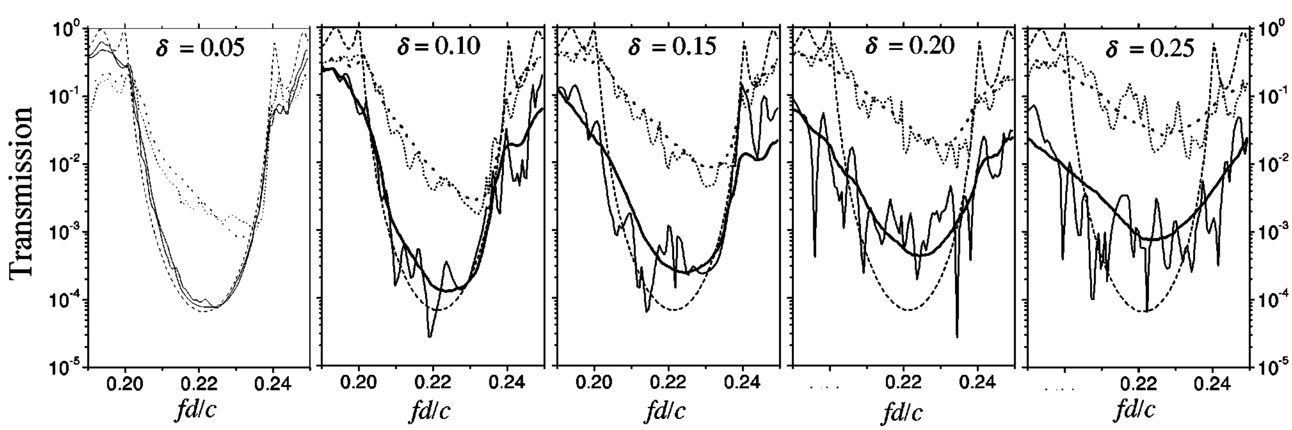

FIG. 3. Transmission spectra for the ideal (dashed lines) and disordered structures, characterized by different deviation in radii of cylinders δ:0.05;0.1;0.15;0.2;0.25. An example of the spectrum for a single configuration of disorder is shown by thin lines. Averaged transmission corresponds to thick lines. Solid and dotted lines refer to the straight and scattered transmissions, respectively.

branch of dispersion curves, as illustrated in Fig. 2. In other words, due to disorder, light can be scattered from evanescent states into propagating states and thus propagate toward the rear side of the sample without attenuation. Another (more probable) possibility is that the light experiences scattering from one evanescent state into another evanescent state, which is characterized by a lower attenuation length. It can be seen from Fig. 2, that for the structure under study, scattering of light from evanescent states into propagating states (or into evanescent states closer to the gap edge, where attenuation is weaker) can be more easily achieved at the low-frequency side of the PBG, since a smaller modification $\Delta\vec{k}$ of the wave vector in the M-K branch is required for such scattering as shown in Fig. 2. This is the reason why the averaged *scattered* transmission coefficient increases faster on the low-frequency side than on the high-frequency side of the dip. This leads to a shift of the minimum towards the high-frequency side. In general the direction of the shift of the spectral minimum will depend on the relative curvature of the lower and upper bands.

The same explanation can be given for the slight distortion of the dip in the straight case: light propagating in the Γ-M direction scatters from the evanescent state into a propagating state, allowing propagation without attenuation toward the rear side of the sample, and then scatters a second time and goes back into the mode propagating in the Γ-M direction. Since this is a second-order process, only minor modifications of the dip shape occur and a slight shift of the minimum takes place.

Figure 4(a) shows the dependences as a function of disorder of the frequency of the minimum for *straight* and *scattered* lights. When the disorder is small $\delta = 0.05$, the frequency of the minimum of straight light corresponds to the center of the PBG, while for the *scattered* light this minimum is shifted by more than 4% towards the high-frequency edge of the PBG. With an increase of disorder, the frequency of the minimum for the *scattered* light decreases, while for the straight light this minimum moves towards slightly higher frequencies until the larger value of disorder $\delta = 0.20$ and then pulls back.

In Fig. 4(b) we plot the corresponding transmission coefficient at the minimum (TCAM). Two fundamentally different behaviors of the TCAM are found according to the straight or scattered character of the transmission. For the scattered light, the TCAM has a huge variation of more than two orders when δ varies between 0.02 and 0.25. It grows rapidly with an increase of disorder when δ is small, but further growth slows down. For the straight light, the TCAM exhibits a thresholdlike behavior, similar to the one-dimensional case.[13] When δ is below 0.05, there is almost no variation of the TCAM. Above $\delta = 0.05$, the TCAM increases but with a slower increase than in the scattered case. Here is evidence that a certain amount of disorder, which should be related to the relative gap width, does not damage the straight attenuation due to the PBG.

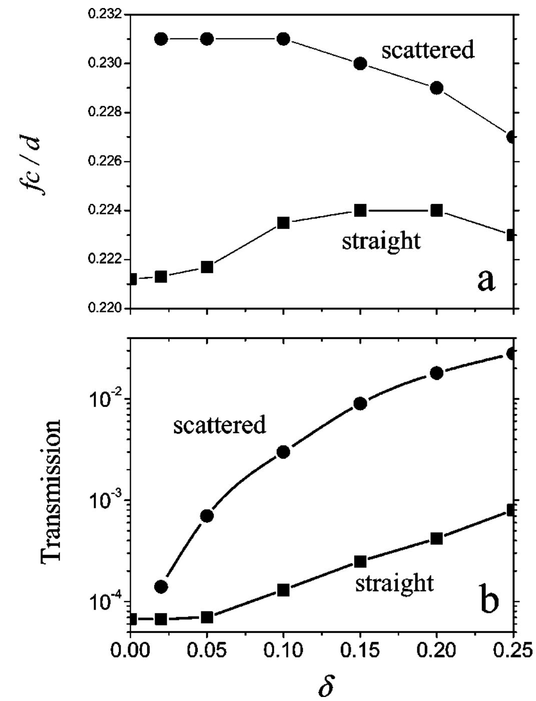

FIG. 4. (a) Spectral position of minimum in the averaged transmission spectra for the straight (squares) and scattered (circles) light. (b) Averaged transmission coefficient at the minimum for the straight (squares) and scattered (circles) light as a function of deviation of the cylinder radii δ.

III. CONCLUSION

To conclude, we have shown that the *scattered* transmission (which is dominant in the total transmission) of disordered PC's with an incomplete PBG can differ substantially from the *straight* one. The *scattered* transmission presents a strong modification as a function of disorder. On the contrary, the *straight* transmission is more tolerant since it has a thresholdlike behavior as a function of disorder and then sustains a certain amount of disorder before changing. Due to *disorder*, transmission spectra present an asymmetrical shape which is related to the dispersion of the incomplete PBG in reciprocal space. Hence, the minimum of the *scattered* transmission does not correspond to the center of the PBG. This effect can lead to a substantial error in the positioning of incomplete PBG's (like the lower PBG's of opal-based PC's) when assigning the gap center to the minimum of transmission as is usually done in transmission studies.

ACKNOWLEDGMENTS

This work was funded by EC, Project No. IST-1999-19009 PHOBOS. The authors thank the "Center Informatique National de l'Enseigement Superior" for an allowance of computer time and Professor Lees for useful discussion.

[1] *Photonic Bandgaps and Localization*, edited by C.M. Soukoulis (Plenum, New York, 1993).
[2] *Confined Electrons and Photons: New Physics and Applications*, edited by E. Burstein and C. Weisbuch (Plenum, New York, 1995).
[3] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).
[4] R.D. Meade, A.M. Rappe, K.D. Brommer, J.D. Joannopoulos, and O.L. Alherhand, Phys. Rev. B **48**, 8434 (1993).
[5] K.M. Ho, C.T. Chan, and C.M. Soukoulis, Phys. Rev. Lett. **65**, 3152 (1990).
[6] A.Z. Genack and N. Garcia, J. Opt. Soc. Am. B **10**, 408 (1993).
[7] M.M. Sigalas, C.M. Soukoulis, C.T. Chan, R. Biswas, and K.M. Ho, Phys. Rev. B **59**, 12 767 (1999).
[8] Z.Y. Li and Z.Q. Zhang, Phys. Rev. B **62**, 1516 (2000).
[9] A.A. Asatryan, P.A. Robinson, L.C. Botten, R.C. McPhedran, N.A. Nicorovici, and C.M. de Sterke, Phys. Rev. E **60**, 6118 (1999).
[10] Yu.A. Vlasov, V.N. Astratov, A.V. Baryshev, A.A. Kaplyanskii, O.Z. Karimov, and M.F. Limonov, Phys. Rev. E **61**, 5784 (2000).
[11] V. Yannopapas, N. Stefanou, and A. Modinos, Phys. Rev. Lett. **86**, 4811 (2001).
[12] T.F. Krauss, R.M. De La Rue, and S. Brand, Nature (London) **383**, 699 (1996).
[13] Yu.A. Vlasov, M.A. Kaliteevski, and V.V. Nikolaev, Phys. Rev. B **60**, 1555 (1999).
[14] A.A. Asatryan, P.A. Robinson, L.C. Botten, R.C. McPhedran, N.A. Nicorovici, and C.M. de Sterke, Phys. Rev. E **62**, 5711 (2000).
[15] E. Lidorikis, M.M. Sigalas, E.N. Economou, and C.M. Soukoulis, Phys. Rev. B **61**, 13 458 (2000).
[16] Z.Y. Li, X.D. Zhang, and Z.Q. Zhang, Phys. Rev. B **61**, 15 738 (2000).
[17] S.H. Fan, P.R. Villneuve, and J.D. Joannopoulos, J. Appl. Phys. **78**, 1415 (1995).
[18] A.L. Reynolds, D. Cassagne, C. Jouanin, and J.M. Arnold, Synth. Met. **116**, 453 (2001).
[19] M.A. Pihal, Shambrook, A.A. Maradudin, and P. Sheng, Opt. Commun. **80**, 199 (1994).
[20] P.M. Bell, J.B. Pendry, L.M. Moreno, and A.J. Ward, Comput. Phys. Commun. **85**, 306 (1995).