Big fast vortices in the *d*-wave resonating valence bond theory of high-temperature superconductivity

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The effect of proximity to a Mott insulating phase on the superflow properties of a *d*-wave superconductor is studied using the slave-boson U(1)-gauge-theory model. The model has two limits corresponding to superconductivity emerging either out of a "renormalized Fermi-liquid" or out of a non-Fermi-liquid regime. Three crucial physical parameters are identified: the size of the vortex *as determined from the supercurrent it induces*, the coupling of the superflow to the quasiparticles, and the "nondissipative time derivative" term. As the Mott phase is approached, the core size as defined from the supercurrent diverges, the coupling between superflow and quasiparticles vanishes, and the magnitude of the nondissipative time derivative dramatically increases. The dissipation due to a moving vortex is found to vary as the third power of the doping. The upper critical field and the size of the critical regime in which paraconductivity may be observed are estimated and found to be controlled by the supercurrent length scale.

I. INTRODUCTION

High-T_c superconductors are created by doping an antiferromagnetic "Mott insulating" parent material, and the effect of proximity to the Mott phase on their superconducting properties remains a crucial and still incompletely understood issue.[1] One expects on general grounds that the suppression of the current response near a Mott insulator leads to "type-II" behavior, so a fundamental issue is the physics associated with vortices in the superconducting order parameter. An isolated vortex involves a quantized flux ($hc/2e$ in conventional superconductors), a circulating supercurrent pattern, and a "core region" in which the quasiparticle excitation spectrum differs from that observed far from the vortex. The possibility (apparently not realized in known superconductors) that proximity to the Mott phase could induce an unconventional value of the flux quantum has been discussed.[2,3] An extensive literature exists on quasiparticle properties (including the possibility of interesting discrete core states[4] and whether an antiferromagnetic[5,6] or other[7] state is induced in or near the vortex core). However, apart from the pioneering phenomenological work of Lee and Wen[8] and an analysis of the resistive transition in overdoped Tl-based materials,[9] little theooretical attention has been paid to the superflow properties even though these in fact control many physically important quantities including H_{c2} and the size of the "critical regime" in which superconducting fluctuation properties may be observed in the conductivity.

Theoretical analysis of vortex properties requires a model. Conventional models of superconductivity in interacting electron systems are based on Landau's Fermi-liquid theory, but as we show in the Appendix, analysis of the changes occurring as the Mott phase is approached requires a model which goes beyond Fermi-liquid theory, at minimum by including effects corresponding to a scale dependence of a Landau parameter and perhaps more fundamentally by allowing for superconductivity to emerge from a non-Fermi-liquid state or regime. One widely studied theoretical model of a doped Mott insulator is the U(1)-gauge-theory implementation[10] of the resonating valence bond (RVB) ideas of Anderson.[11] This theory and its variants have been extensively studied as an approximation to the low-energy physics of the *t-J* model believed[1] to capture the essential aspects of the low-energy physics of high-T_c materials. It exhibits (at least in a large-*N* limit) a non-Fermi-liquid regime[10,12] involving exotic excitations ("spinons" and "holons" coupled by a gauge field) and a Fermi-liquid regime in which the spinon and holon are bound together into a conventional electron and the gauge field effects produce a nontrivial doping dependence of the Landau parameter F_{1S}.[13,14] The model also possesses a *d*-wave superconducting state[15] which may emerge either from the Fermi-liquid or non-Fermi-liquid regimes. Quasiparticle properties (including a possible antiferromagnetic[6] or staggered flux[7] state in the core of the vortex) have been studied and the model has been shown to admit $h/2e$ vortices,[16] but superfluid properties such as the supercurrent distribution in the vortex state and the dissipation occurring when a vortex moves have been less well studied.

As discussed at length elsewhere[8,14,18] this theory disagrees with experiment in a number of ways. Most problematically, the model predicts a strong doping dependence to the leading low-T correction to the London penetration depth $[d\lambda^{-2}/dT \sim (\text{doping})^2]$ which is not observed. We therefore do not believe the theory is a realistic representation of high-temperature superconductors; however, it is a very useful model system. We stress that although as usually formulated the model involves exotic excitations such as holons and spinons, for the properties we discuss these can be completely eliminated: as shown in the text and Appendix, the model can be viewed simply as a method of calculating the behavior of a Fermi-liquid-based system at length scales

short enough that the scale dependence of the Landau parameters becomes important.

The importance of the model is that it provides an explicit realization of a situation (which, we believe, is generically realized in lightly doped Mott insulators) where the supercurrent-defined and quasiparticle-defined length scales are parametrically different. We show in this paper that the longer length scale is in fact the one relevant to the conventional superfluid properties such as $T_c(H)$ and the size of the "fluctuation regime" in which the model exhibits (for example) a nontrivial paraconductivity.

The rest of this paper is organized as follows. Section II reviews necessary aspects of the U(1) gauge theory formalism. Section III discusses in detail the current distribution around a vortex. Section IV calculates the dissipation induced when a vortex moves and uses this information to estimate the size of the "critical regime" in which superconducting fluctuation effects are visible in the conductivity. Section V considers the low-temperature limit of the upper critical field. Section VI is a conclusion, summarizing the results and their implications. The Appendix explicates the relation between the results presented here and the conventional Fermi-liquid analysis.

II. FORMALISM

This subsection reviews results obtained in the early days of the gauge theory,[10,12,15,16] in order to establish notation and introduce important concepts.

In the gauge theory one writes the electron $c_{i\alpha}$ in terms of a charge-e boson b (representing a hole) and a fermionic "spinon" representing a spin degree of freedom, thus $c_{i\sigma}^\dagger = b_i f_{i\sigma}^\dagger$. The superconducting state is described by a d-wave BCS pairing of spinons[15] (involving a d-symmetry pairing gap with maximum value Δ) and a condensation of the bosons. The low-energy, long-wavelength physics is controlled by the Hamiltonian

$$H_{gauge} = \frac{1}{2}\rho_B(\nabla\phi_B - a - A)^2 + \frac{1}{2}\rho_F\left(\frac{1}{2}\nabla\phi_F - a\right)^2$$
$$+ H_D + H_{mix} + \cdots. \qquad (1)$$

Here a is an internal gauge field which enforces the constraint, arising because the physical fermion $c_i = f_i b_i^\dagger$, which a longitudinal spinon current must cause an equal and opposite boson current. (It is possible to have transverse currents of spinons with no holon motion but these are not relevant here). ϕ_B is the phase of the boson field, ρ_B is the $T=0$ boson superfluid stiffness, ϕ_F is the phase of the spinon (fermion) pairing amplitude, and ρ_F is the corresponding $T=0$ spinon "superfluid" stiffness. H_D is the usual (normal-ordered) Dirac Hamiltonian describing the quasiparticle part of the spinon degrees of freedom and the ellipsis expresses terms irrelevant to the present discussion.

H_D has eigenvalues $E_p = \sqrt{v_1^2 p_1^2 + v_2^2 p_2^2}$ with v_1 the spinon Fermi velocity and v_2 related to the d-wave gap in the usual way. The spinons are coupled to the gauge field and thus to the "holons" via the term

$$H_{mix} = \sum_{\alpha,\sigma}\left(i\frac{1}{2}\partial_\mu\phi_F - ea_\mu\right)\cdot\vec{v}_1 c_{\alpha\sigma}^\dagger c_{\alpha\sigma}. \qquad (2)$$

We begin our analysis of H_{gauge} by considering length and energy scales. The fermionic part of the Hamiltonian involves the length scale

$$\xi_F = \frac{v_1}{\Delta} \qquad (3)$$

and two energy scales Δ and

$$\rho_F \sim v_1 p_F. \qquad (4)$$

Here ξ_F is relatively short and does not diverge as the Mott phase is approached, and ρ_F is relatively large (of order J in the t-J model) and does not vanish as the Mott phase is approached.

The boson stiffness ρ_B has dimension of energy (in two spatial dimensions) and is proportional to the doping x and to the basic electronic hopping parameter t.[10,12,15] ρ_B is expected to vary[19] on the length scale $x^{-1/2}$ which is the distance between charge carriers. We shall be interested primarily in the limit $x^{-1/2} > \xi_F$. We note that as the doping is increased, Δ decreases[15] and eventually becomes smaller than an energy of order $x^{1/2}J$, so the inequality is reversed. For larger dopings the theory becomes essentially the familiar BCS one, with only one important length scale ξ_F.

The currents carried by boson and fermion degrees of freedom are, respectively,

$$j_B = \rho_B^*(\nabla\phi_B - a - A), \qquad (5)$$

$$j_F = \rho_F\left(\frac{1}{2}\nabla\phi_F - a\right). \qquad (6)$$

Here the * denotes convolution and is to remind the reader that ρ_B is scale dependent on scales relevant to the subsequent discussion.

The physical current $j_{phys} = j_B$ and the constraint enforced by the gauge field a is $j_B + j_F = 0$, i.e.,

$$\rho_B^*(\nabla\phi_B - a - A) + \rho_F\left(\frac{1}{2}\nabla\phi_F - a\right) = 0. \qquad (7)$$

This implies

$$a = (\rho_B + \rho_F)^{-1} * [\rho_B^*(\nabla\phi_B - A) + \rho_F\nabla\phi_F]. \qquad (8)$$

In the long-wavelength limit the nonlocality of ρ_B may be neglected. As $T\to 0$ and assuming no fermions are excited, elimination of a leads to[10,16]

$$H_{phase} = \frac{\rho_B\rho_F}{\rho_B + \rho_F}\left(\nabla\phi_B - \frac{1}{2}\nabla\phi_F - A\right)^2. \qquad (9)$$

The meaning of this equation is that in a state with paired spinons and condensed bosons at long wavelengths only the combination $\nabla\phi_B - \frac{1}{2}\nabla\phi_F$ couples to an external vector potential or is relevant to the energy, and the physical superfluid stiffness $\rho_S = \rho_B\rho_F/(\rho_B + \rho_F)$. Similarly one finds

$$H_{mix} = \sum_{\alpha,\sigma} \frac{-\rho_B}{\rho_B + \rho_F}\left(\nabla\phi_B - \frac{1}{2}\nabla\phi_F - A\right)\cdot\vec{v}_1 c_{\alpha\sigma}^\dagger c_{\alpha\sigma}.$$

(10)

Because the gauge field has been eliminated, the fermionic degrees of freedom should be regarded not as spinons but as Bogoliubov quasiparticles of the superfluid (or near-superfluid) state. They couple only to the combination $\nabla\phi_B - \frac{1}{2}\nabla\phi_F$ and the coupling is via an effective charge $Z^e = -\rho_B/(\rho_B + \rho_F)$, which is negative (hole like) and vanishes as the Mott insulator is approached.

Equations (9) and (10) constitute a derivation, from the U(1) gauge theory, of phenomenological equations discussed in Ref. 18. The derivation makes it clear that deviations from the phenomenological equations of Ref. 18 occur at length scales shorter than that specified by the scale dependence of the physical superfluid stiffness, i.e., than the longer of ξ_F and $x^{-1/2}$. The derivation also makes it manifest that the phenomenological action, discussed in Ref. 14 on the basis of Fermi-liquid theory, is more general and may apply also to situations in which the normal state is not described by Fermi-liquid theory.

III. VORTEX SOLUTION: STATIC CASE

Consider a vortex. Far from the vortex core the fields are found by minimizing H_{phase} [Eq. (9)] which implies that $\nabla^2(\phi_B - \frac{1}{2}\phi_F - A) = 0$. Single valuedness of the wave function implies that both ϕ_B and ϕ_F must have circulation which is an integer multiple of 2π so that in a mean-field approximation one would write

$$\nabla\phi_B = \frac{m\hat{\theta}}{r},$$

(11)

$$\nabla\phi_F = \frac{n\hat{\theta}}{r}.$$

(12)

The energy associated with a vortex is thus, approximately,

$$E_V = E_{core}(n,m) + \frac{1}{2}\rho_S\left(m - \frac{1}{2}n - A\right)^2 \ln\left(\frac{R}{\xi}\right),$$

(13)

where ρ_S is the physical superfluid stiffness defined below Eq. (9), R is of the order of the intervortex separation, and ξ is the length scale below which the supercurrent magnitude deviates from $1/r$ and will be discussed more fully below. E_{core} is the core energy of the vortex, i.e., the contribution to the energy arising from scales less than ξ.

The superflow contribution is clearly minimized by the choice $m = 0$, $n = 1$, corresponding to a conventional $h/2e$ vortex. The core energy term requires more discussion. If $n = 1$, then a singularity in the fermion pairing amplitude is required. In a clean conventional superconductor one would estimate the energy cost of this singularity as the product of the condensation energy per unit area ($N_0\Delta^2$ with N_0 the density of states) and the area of the core ($\xi_F^2 = v_F^2/\Delta^2$), leading to $E_{core} \sim v_F^2 N_0$. In the present problem this implies an E_{core} of the order of the effective fermi energy J. On the

other hand, if $n = 0$ and $m = 1$, then no singularity is required in the fermion field and a calculation very similar to that given in Eq. (19) then shows that the core energy is of the order of the boson or superflow energy xt and can be absorbed into the definition of ξ. These considerations suggest that in the gauge theory the vortex energy may be estimated by

$$E_V \approx C_{core}J(1 - \delta_{n,0}) + C_{sf}xt \ln\left(\frac{R}{\xi}\right)\left(m - \frac{1}{2}n - A\right)^2,$$

(14)

with $C_{core,sf}$ constants. This estimate (proposed and presented in more sophisticated form by Sachdev[2]) suggests that when the superflow energy is dominant (low vortex density or high doping) one has conventional $h/2e$ vortices but that as x is reduced or R is decreased a transition to doubly quantized vortices may occur.

This argument, however, is vitiated by recent work on the structure of the vortex core. From different points of view the authors of Refs. 5–7 show that (within certain reasonable assumptions) some other ordered state, also characterized by an electronic gap of the order of Δ, is very nearby in energy and indeed becomes favored as $x \to 0$. The consequence is that C_{core} in Eq. (14) decreases rapidly as $x \to 0$ and may even become negative, implying that conventional ($h/2e$) vortices are always favored. In more physical terms, within the classes of models (including the gauge model) considered by Refs. 5–7 the reason that the ground state has superconducting rather than some other sort of order is the gain in energy associated with establishing superfluid phase coherence, so it is natural that even the vortex core energy is set by the phase stiffness. The conventional nature of vortices in this theory was stressed in Ref. 16, which, however, did not consider the core energy explicitly. We note, however, that even if not favored in the superfluid state, doubly quantized (h/e) vortices may be easily excited thermally once the long-ranged superfluid order is disrupted.

We now study the structure of the vortex at shorter length scales. We consider the case of very weak applied field, so we may choose a gauge in which $A = 0$. We assume $\rho_B \ll \rho_F$ and expand in powers of ρ_B. We find from Eqs. (8) and (11)

$$a = \frac{1}{2}\nabla\phi_F - \frac{1}{4\rho_F}\rho_B^*\nabla\phi_F + \cdots,$$

(15)

where the ellipsis denotes both terms higher order in ρ_B/ρ_F and fluctuations about the mean-field solution for $\phi_{F,B}$.

We must now determine the behavior of the boson field and the fermion pairing amplitude. The fermion pairing amplitude varies on the scale ξ_F which by assumption remains finite as the Mott phase is approached, whereas one expects the Bose amplitude to vary on the scale set by the spacing between carriers, which diverges as the Mott phase is approached. We therefore focus on the Bose field. In a lightly doped Mott insulator the density of bosons is low. In the limit of dilute bosons one expects[19] that the bose amplitude is described by the two-dimensional Hamiltonian density

$$H_{Bose} = \frac{1}{2m_B}[(\nabla - a - A)\psi]^2 + \frac{1}{2}\mu\psi^2 + \frac{1}{4}U\psi^4. \quad (16)$$

In the dilute limit, the parameters μ and U are universal, given in terms of the boson density x and mass m_B by

$$\mu = Ux, \quad (17)$$

$$U = \frac{4\pi}{m_B \ln(1/x)}. \quad (18)$$

We see that $m_B \sim 1/tb^2$ with b the underlying lattice constant is required to reproduce the established result for the boson stiffness cited above (for a more detailed derivation see Ref. 10). Also, the dilute limit means that a mean-field approximation for the boson field is reliable.

We must, however, consider the mean-field approximation for the gauge field in more detail. In the d-wave RVB (d-RVB) state, fluctuations in a are controlled by the stiffness corresponding to fermion pairing. This stiffness is large at length scales longer than ξ_F or energy scales less than the fermion pairing amplitude Δ, so a mean-field approximation is expected to be reliable at long-length and low-energy scales. However, if ξ_F, Δ^{-1} are shorter than the relevant bosonic length scale $x^{-1/2}$ and time scale $(xt)^{-1}$, then fluctuations in a may appreciably renormalize the parameters in L_{Bose}. Because the decay with q of the fermionic stiffness is slow $[\sim(\xi_F q)^{-1}]$, we focus here on the energy scale. As carriers are added to a lightly doped Mott insulator, Δ decreases and xt increases. When Δ/xt becomes less than unity we expect that fluctuation corrections to the various parameters become large. We thus distinguish two regimes: a ''superdilute regime'' in which $\Delta > xt$ and $\gamma \sim 1$ and a ''dilute boson regime'' in which $x \ll 1$ but $\Delta < xt$ and $\gamma \ll 1$.

In the limit of interest $\xi_F < x^{-1/2}$ the boson ground state in the presence of a vector potential is given by the solution of

$$\frac{-1}{2m}(\nabla + a + A)^2\psi + U\psi^3 = \mu\psi. \quad (19)$$

Equations (19), (12), and (15) imply that in radial coordinates we have (up to terms of relative order x)

$$\frac{-1}{2m}\left(\partial_r^2 + \frac{1}{r}\partial_r + \frac{1}{4r^2}\right)\psi + U\psi^3 = \mu\psi. \quad (20)$$

Defining

$$\psi = x^{1/2}f(r/\xi), \quad (21)$$

with

$$\xi^{-2} = 2m\mu, \quad (22)$$

leads to the solution shown in Fig. 1. In particular, at large distance $f \to 1$ while at small distance

$$f = \sqrt{r}f_0, \quad (23)$$

with $f_0 = 0.886$. In other words, as the core of the vortex is approached, the Bose amplitude decreases as the square root

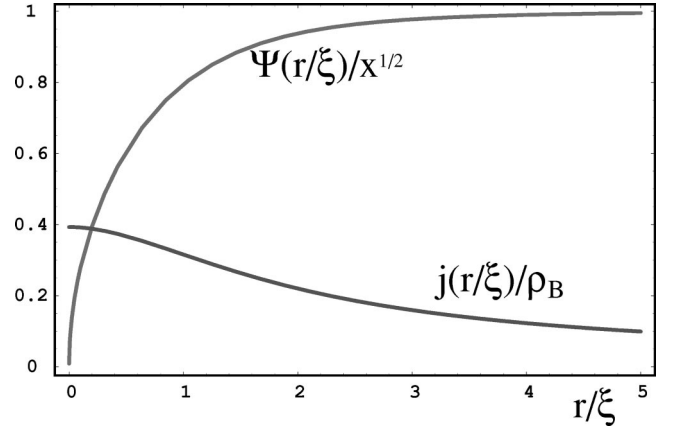FIG. 1. Variation with distance from the vortex core of the boson amplitude ψ and supercurrent j.

of the distance from the vortex core. This result was previously obtained by Franz and Tesanovic.[20]

The supercurrent is given from Eqs. (5) and (16) as

$$j(r) = \frac{|\psi(r)|^2}{m_B}a, \quad (24)$$

so in particular at small distances, in physical units

$$j = \frac{f_0}{2\xi}\rho_B(\infty). \quad (25)$$

The resulting current profile is also shown in Fig. 1; we see that the supercurrent varies as $1/r$ for $r > x^{-1/2}$ and is constant for smaller r, justifiying the qualitative statements made in Ref. 16.

The fermion spectrum retains its long-distance value down to a length $\xi_F = v_1/\Delta$ which is parametrically less than ξ as $x \to 0$; below this length a variety of interesting physical effects[6,7] may occur. The physical electron spectrum, observable (in principle) via tunneling, is calculated in the U(1) gauge theory as a convolution of a holon and a spinon,[11] and so is relatively broad in the nonsuperconducting phase of the model. In the superconducting state the $q = 0$ boson amplitude develops an expectation value and so the electron spectral function acquires a sharp ''quasiparticle pole'' feature. In the limit $x \to 0$ the short length scales of the spinon spectrum control the convolution so the coherent part of the spectrum at distance r from the vortex core is proportional to the boson amplitude at distance r. In other words, in this theory the strength of the quasiparticle peak measured at a distance r from the vortex core should begin to decrease for as r is reduced below ξ. This effect is not visible in published tunneling data.[21]

To summarize, in the U(1) gauge theory of a lightly doped Mott insulator, a vortex is characterized by two length scales: $\xi \sim x^{-1/2}$, below which the supercurrent ceases to vary as $1/r$ (and in fact becomes essentially r independent), and the scale ξ_F, which does not diverge as the Mott phase is approached and which controls the quasiparticle properties. The core energy is small (of order of the superfluid stiffness); however, the state in the core possesses a gap very similar to the su-

perconducting gap. This behavior should be contrasted with that of a conventional (BCS) superconductor, in which the length defined by the supercurrent is essentially the same as the length defined by the quasiparticle properties, the core energy is large (of the order of the Fermi energy), and the core is gapless apart from the "finite-size effects" which lead to the Caroli-Matricon states.

IV. MOVING VORTEX

We consider a slowly moving vortex with center position $\vec{X}_v(t)$, so that time derivatives of fields may be replaced by the dot product of a field gradient and the vortex velocity: for example, $\partial_t \psi(r,t) = \partial_t \vec{X}_v(t) \cdot \vec{\nabla}\psi$. The contribution to the action from vortex motion has two terms: a nondissipative term corresponding to motion in an effective magnetic field B_{eff} and a dissipative term arising because vortex motion excites fermionic excitations. These terms imply a classical equation of motion

$$B_{eff}\hat{z} \times \partial_t X_V + \eta \partial_t X_V = F_V, \qquad (26)$$

where F_V represents the forces acting on the vortex (arising for example from an imposed current and from vortex-vortex interactions). Equation (26) applies only for frequencies less than a cutoff frequency which is the minimum of Δ and the boson frequency scale xt.

B_{eff} may be obtained by considering the action arising from moving a vortex around a closed loop. For orientation we first consider the related purely bosonic problem of a vortex in a two-boson condensate. The standard bosonic Lagrangian density[19] includes a "nondissipative time derivative" term $i\gamma_B \psi^\dagger \partial_t \psi$ with coefficient $\gamma_B = 1$. A superfluid state is described by a condensate amplitude $n_s = |\langle\psi\rangle|^2$ and condensate phase ϕ, leading to a term $in_S \partial_\tau \phi$ in the action. This term ensures that dragging a long straight vortex in the boson condensate around a loop enclosing an area s leads to a contribution to the action of $\Delta S = 2\pi n_S s$. This contribution is just the action appropriate to a particle in a magnetic field of strength $h_{eff} = 2\pi n_S$. For particles on a lattice, a magnetic flux of 2π per unit cell has no dynamical consequences, so that one measures n_S modulo 1 per lattice site. Finally, we consider the relation between the condensate density n_S and the total boson density n_B. Gauge invariance means that $-i\partial_t \phi$ is a chemical potential and therefore couples to the total particle density n_B. In simple boson problems, at $T=0$ the only gapless excitation is the phase mode of the superfluid state and therefore $n_S = n_B$. As T is increased from $T=0$ gapless "normal fluid" excitations occur. The presence or absence of Galilean invariance then becomes crucial. One may interpret the $\partial_t \phi$ term in the vortex action in terms of the acceleration of a vortex in a given force. In a Galilean-invariant situation one expects that if a vortex is accelerated, it will drag all particles in the system with it, so that the coefficient of $\partial_t \phi$ is simply the total particle density in the system. However, in a non-Galilean-invariant system the superfluid component may accelerate independently of the normal component and $n_S \lesssim n_B$.

We next consider a superconducting condensate made of paired electrons. The important different here is that "two-fluid" effects may be important even at very low T, for example, because of impurity-induced pair breaking or of vortices. The conventional result is that if a Landau expansion may be constructed about a nonsuperconducting state (for example, very near to T_c or in the presence of strong pair breaking), then one has

$$S_{sc} = \int dt d^d x N_0 T_0^2$$

$$\times \left[\gamma_F \frac{\Delta^+ \partial_t \Delta}{T_0^3} + \frac{\xi_0^2 (\nabla\Delta)^2}{T_0^2} + \frac{T-T_c}{T_0} \frac{\Delta^2}{T_0^2} + u \frac{\Delta^4}{T_0^4} \right]. \qquad (27)$$

Here N_0 is the electronic density of states, T_0 is an energy scale of the order of the transition temperature or the pair-breaking scattering rate u is a coefficient of the order of unity, and the coefficient γ_F of the time derivative term has both real and imaginary parts, $\gamma_F = \gamma_F' + i\gamma_F''$. In a usual superconductor the dissipative (real) part of γ_F is of the order of unity (the conventional result is $\gamma_F'' = \pi/8$). Because a time-dependent phase is a contribution to the chemical potential, we may identify the imaginary part γ_F'' with $-\partial T_c/\partial\mu$. In conventional superconductors this is very small (of order T_c/E_F), so the total coefficient of the dissipationless time derivative, $N_0 T_0 \gamma_F''' \sim (T_c/E_F)^2$, is extremely small and for most purposes may be neglected (for exceptions see, e.g., Refs. 17 and 22). Of course in a conventional superconductor with weak pair breaking, a Landau expansion only applies for temperatures very near to T_c and as $T \to 0$ one expects $\gamma_F'' \to 0$ while the dissipationless term $N_0 T_0 \gamma_F''$ must approach the total fermion density n. For a type-II superconductor in a magnetic field one similarly expects that $N_0 T_0 \gamma_F'' \to n$ only for temperatures of the order of the core-state level spacing T_c^2/E_F and only in the "super-clean" limit. To summarize, known results from simple fermion and boson problems imply that the dissipationless time derivative term in the superfluid action involves a nonuniversal coefficient which depends on the interplay between the superfluid and non-superfluid components of the system, is in general quite small for fermion-based superfluids, and is of the order of the particle density for boson-based systems.

We now turn to the boson-fermion-gauge-field problem of interest here. The discussion above shows that there is no simple, generally valid expression for the nondissipative coefficient, except in the $T \to 0$ no-pair-breaking limit, in which the coefficient is the total particle density. Nevertheless, a few remarks can be made and limits can be estimated. At high dopings ($xt > J$) the bosons condense (or quasicondense) at temperatures well above T_c, so in this limit superconductivity arises out of a more or less Fermi-liquid-like state, so near T_c one expects the resulting superfluid state to be described by a nondissipative coefficient γ which is of a fermionic order of magnitude and thus much less than unity. We note, though, that in the present model the spinon Fermi energy is of order J and the pairing amplitude varies from a

(not too small) fraction of J at low doping to a very small value at high doping,[15] so $\partial\Delta/\partial\mu$ is not particularly small; the main smallness is provided by the factor $N_0 T_0 \sim T_c/J$. On the other hand, as doping is reduced, the physics changes. The fermions pair at a higher scale and the superconducting transition is set by the condensation scale of the bosons. An upper bound on the nondissipative coefficient is then set by the total particle density (modulo 1), i.e., $\gamma < x$. In this limit one expects that by temperatures of the order of T_c the fermions are mostly paired, so that a Landau expansion is not appropriate and the coefficient is set by bosonic physics; i.e., γ is not that different from its $T=0$ no-pair-breaking value. We therefore propose the following approximate interpolation formula for the eff[f]ecitve magnetic field B_{eff} implied by the nondissipative terms in the action of our problem:

$$B_{eff} \approx \frac{1}{(\pi x)^{-1} + (\Delta/J)^{-1}}. \quad (28)$$

We turn next to the dissipative term. In the gauge model, the dissipation arises from the continuum of spinon excitations, and we assume that the temperature, magnetic field, or impurity density is large enough that an appreciable number of these exist and may be characterized by a spinon conductivity σ_{sp} which we take to be local on the scales of interest. The "electric field" felt by the spinons is $\partial_t(\frac{1}{2}\nabla\phi - a)$ so that the dissipative contribution to the action is

$$S_{diss} = \frac{\sigma_{sp}}{2} \int d^2r\,dt\,dt'$$
$$\times \partial_t\left(\frac{1}{2}\nabla\phi - a\right)_t K(t-t')\partial_t\left(\frac{1}{2}\nabla\phi - a\right)_{t'}, \quad (29)$$

with K the Fourier transform of $1/|\omega|$. In a slowly moving vortex with center position $\vec{X}_v(t)$, $\frac{1}{2}\nabla\phi - a$ is a function $\tilde{a}[\vec{r} - \vec{X}_v(t)]$ Substitution into Eq. (29) and some rearrangement leads to the standard form for a particle moving in a dissipative medium, namely,

$$S_{diss} = \frac{\eta}{2} \int dt\,dt'[\partial_t\vec{X}_v(t)]K(t-t')[\partial_{t'}\vec{X}_v(t')], \quad (30)$$

with viscosity η given by

$$\eta = \frac{\sigma_{sp}}{2}\int d^2r \sum_{ij} \left(\frac{\partial\left(\frac{1}{2}\nabla\phi - a\right)_i}{\partial r_j}\right)^2. \quad (31)$$

Use of Eq. (5) shows that $\rho_F(\frac{1}{2}\nabla\phi - a)$ is just the physical current $\vec{j}_{phys} = \rho_B f^2(r/\xi)\hat{\theta}/r$. Substitution of our result for f leads to (note that the logarithm comes from the angular derivative $[\nabla\to(1/r)\partial_\theta]$)

$$\eta = \frac{\pi\sigma_{sp}}{\xi^2}f_0^2\left(\frac{\rho_B}{\rho_F}\right)^2 \ln(\xi/\xi_F). \quad (32)$$

We see that as the insulator is approached, the vortex viscosity vanishes very rapidly, indeed as x^3. One factor of x comes from the large size of the vortex (proportional to ξ^{-2}); the other two factors $(\rho_B/\rho_S)^2$ come from the decreased coupling of the vortex motion to the spinons; we interpret this as a signature of the vanishing of the quasiparticle charge in this theory (for a discussion of other signatures see Refs. 14 and 18).

It is instructive to view this result in a slightly different way. The "internal electric field" e felt by the spinons is $\partial_t(a - \frac{1}{2}\nabla\phi_F) = (1/e\rho_F)\partial_t(j_{phys}(r))$. Application of the usual composition rules of the gauge theory[10] shows that the physical electric field $E = e\rho_F/\rho_B$ (in the limit $\rho_F \gg \rho_B$) so that the physical electric field E generated by a moving vortex is

$$E(r) = \frac{1}{e\rho_B}\partial_t(j_{phys}(r)) = \partial_t(\vec{X}_v(t))\cdot\vec{\nabla}\left(\frac{f^2\left(\frac{r-X_V}{\xi}\right)\hat{\theta}}{r-X_V}\right). \quad (33)$$

This shows that a moving vortex generates an electric field which varies on the length scale set by the physical current. $E(r)$ varies as $1/r^2$ far from the vortex and as $1/r$ for $\xi_F < r < x^{-1/2}$. A standard argument[23] says that the dissipative contribution may be estimated by multiplying the square of the electric field by the physical conductivity, which from Eq. (10) is of order $(\rho_B/\rho_F)^2$, leading again to Eq. (32) for η.

The result for η, along with the estimates of the core energy, has implications for the width of the resistive superconducting transition in this model. In a two-dimensional superconductor, the resistive transition is of the Kosterlitz-Thouless (KT) vortex unbinding type. Near to the transition point, but in the normal state, one has a dilute gas of vortices and antivortices. The physical conductivity is the sum of the conductivity due to the moving vortices and the conductivity due to the quasiparticles. Use of Eqs. (10) and (32) leads (at leading order in the small parameter ρ_B/ρ_F) to

$$\sigma_{phys} = \left(\frac{\rho_B}{\rho_F}\right)^2\sigma_{sp}\left(1 + \frac{\pi f_0^2\ln(\xi/\xi_F)}{n_V\xi^2}\right). \quad (34)$$

Thus, recalling that in the pseudogap regime of this model the quasiparticle contribution to the conductivity is $(\rho_B/\rho_F)^2\sigma_{sp}$ we see that the total conductivity is dominated by proximity to the superconducting phase only if (up to a logarithm) vortices are dilute on the length scale set by the supercurrent pattern; as n_V increases further the superconducting fluctuation contribution to the conductivity quickly becomes negligible. However, even when vortices overlap from the "current" point of view (so the superfluid condensate does not make an appreciable contribution to the conductivity), they may still be dilute on the scale set by the "quasiparticle core size" ξ_F, so we would therefore expect the gap in the fermionic excitation spectrum to persist to much higher temperatures than does the "superfluid" contribution to the conductivity.

The density (and nature) of thermally excited vortices bears further discussion. The factors influencing the density of vortices may be understood from simple mean-field free energy arguments. For conventional ($h/2e$) vortices the mean-field free energy as a function of conventional vortex density n_1 is (note that the position of the conventional vortex may be defined to within ξ_F but the logarithm from the superfluid stiffness is cut off by ξ)

$$E_{n=1,m=0} = E_{core}n_1 - \rho_S n_1 \ln(n_1 \xi^2) - Tn_1 \ln(n_1 \xi_F^2). \quad (35)$$

Away from the KT fluctuation regime, ρ_S becomes less than T and we have, approximately,

$$n_1 \xi^2 = \frac{\xi^2}{\xi_F^2} e^{-E_{core}/T}. \quad (36)$$

The factor $= \xi^2/\xi_F^2$ makes it easier than one might guess to obtain a reasonable density of conventional vortices; of course the small value of the core energy, arising as discussed above from the proximity in energy of other gapped states within the *d*-RVB theory, is also important.

It is also of interest to consider the density n_2 of doubly quantized vortices (i.e., vortices in the boson field only), because their motion leads to dissipation too. As noted above, in this case the core energy is entirely determined by the superfluid stiffness, so we have (Φ is a scaling function which is exponentially small at large argument and becomes of order unity when its argument becomes of order unity and the factor of 4 comes from the doubly quantized nature of the vortex)

$$n_2 \xi^2 = \Phi(4\rho_S/T). \quad (37)$$

Therefore, even if the core energy of a conventional vortex is very high (which in the *d*-RVB theory it is not), above a temperature scale set by ρ_S double-quantized vortices will proliferate and will suppress the superfluid contribution to the conductivity.

V. QUANTAL FLUCTUATIONS AND THE MELTING OF THE VORTEX LATTICE

We now consider the physics at low temperature in an applied magnetic field. An applied field induces a vortex lattice and, ultimately, a nonsuperconducting state. In a conventional (BCS) superconductor the transition is driven by the collapse of the superconducting gap. In the present model the important physics involves quantal fluctuations in the positions of the vortices, leading via a first-order transition to a "quantal vortex liquid" state. In particular, although an applied field leads to pair breaking and thus to a nonvanishing density of quasiparticles and to a reduction of the superfluid stiffness and gap amplitude, in the gauge model of interest here these effects are small (order x^{-2}),[18] so we neglect them.

We estimate the magnitude of the quantal fluctuations of the vortices by considering the fluctuation in position of one vortex about its ideal Abrikosov lattice position. To do this

we note that in a vortex lattice the force term in Eq. (26) may be written for small amplitude displacements as $F_V = K_{latt}X_V$, where K_{latt} is the coefficient of the quadratic term in the restoring potential acting on the vortex and arising from the other vortices in the vortex lattice (plus any pinning forces which may exist and which we do not treat). Equation (26) applies only for frequencies less than a cutoff frequency which is the minimum of Δ and the boson frequency scale xt. Here K_{latt} is of the order of the physical superfluid stiffness ρ_B divided by the square of the intervortex spacing, i.e., $K_{latt} = K_0 n_V \rho_B$. The long-range (logarithmic) form of the intervortex potential means that $K_0 \ll 1$. We may now quantize Eq. (26) and thereby estimate the zero-point fluctuations of a vortex as (ω_n is a Matusbara frequency and σ^y is a Pauli matrix)

$$\langle X_V^2 \rangle = Tr \sum_n [\eta|\omega_n| + \sigma^y B_{eff}\omega_n + K]^{-1}. \quad (38)$$

We estimate that the vortex lattice melts when the mean-square vortex displacement divided by the square of the intervortex distance b, i.e., $\langle X_V^2 \rangle/b^2$, becomes of the order of the square of the Lindemann number c_L^2. The value of the Lindemann number depends on the physical situation and varies from $c_L^2 = 0.01$ to $c_L^2 = 0.1$.[24] The estimate of $\langle X_v^2 \rangle$ depends on the values of η and B_{eff} and in particular on whether the system is in the superdilute regime ($B_{eff} \sim \pi n_B$ and $\eta \to 0$) or the dilute regime (B_{eff} unimportant, η dominant). In the superdilute regime we neglect the η and find (the numerical factors give the relation between b^2 and n_V for a triangular vortex lattice)

$$\frac{\langle X_V^2 \rangle}{b^2} = \frac{3\sqrt{3}n_V}{8B_{eff}}. \quad (39)$$

In fact, if dissipation is negligible, then we may look at the problem in a different way. The vortices move in an effective field which is large, leading to Landau-level quantization. If the density of vortices is much less than the density of bosons, $n_V \ll n_B$, then only the lowest Landau level is populated and we may use known results for the melting of the Wigner crystal in a high magnetic field to argue that lattice melting occurs when $n_V/n_B \approx 1/10$ corresponding to a Lindemann number $c_L^2 \approx 0.01$, reasonably consistent with known results for two-dimensional triangular lattices.[24]

In the dilute regime, the frequency cutoff is needed. If $\rho_B > \Delta$, then this scale is Δ and we obtain

$$\frac{\langle X_V^2 \rangle}{b^2} = \frac{3\sqrt{3}n_V}{4\pi\eta} \ln\left[1 + \frac{\Delta\eta}{K}\right]. \quad (40)$$

Use of Eq. (32) and our estimate for K shows that the important dimensionless parameter is given by

$$\frac{\Delta\eta}{K} \sim \frac{\Delta}{\rho_B}\left(\frac{\xi_V^2}{\xi_B^2}\right)\frac{\pi\sigma_{sp}}{K_0}\left(\frac{\rho_B}{\rho_F}\right)^2 \ln\left(\frac{\xi_B}{\xi_F}\right). \quad (41)$$

This parameter may be larger or smaller than unity, because $\rho_B/\rho_S \sim x$ while $K_0 \ll 1$ and σ_{sp} must by consistency be of order $1/x$ (because we have written clean-limit formulas which require that $\sigma_{sp} = p_F l_F/2\pi > p_F \xi_F/2\pi > \rho_S/\rho_B \sim 1/x$). However, as one goes more deeply into the small-Δ regime, the parameter shrinks and we obtain

$$\frac{\langle X_V^2 \rangle}{b^2} = \frac{3\sqrt{3}\Delta}{4\pi K_0 \rho_B}\left(\frac{\xi_V^2}{\xi_B^2}\right). \tag{42}$$

Thus in the dilute limit the lattice still melts when the flux per boson is of the order of the square of the Lindemann number up to a factor of order ρ_B/Δ.

The melted phase is an interesting example of a "non-Fermi liquid" the vortex motion is damped (albeit weakly), so the model is characterized by a nonvanishing σ_{xx} and σ_{xy}, as in a normal metal (although the σ_{xy} value is rather large), but also by a "non-Luttinger" Fermi surface with a gap over large regions of the nominal Fermi surface.

VI. CONCLUSION

One of the most interesting aspects of high-temperature superconductivity is the "pseudogap" regime of underdoped materials. This regime is characterized by a gap (of approximately d-wave form) in the quasiparticle spectrum but neither long-ranged superconducting order nor particularly noticeable superconducting fluctuations. One possibility is that this regime involves electron pairing (as in a conventional superconductor) but with long-ranged superconducting order disrupted by strong phase fluctuations, arising physically from the strong suppression of charge response expected mear a Mott insulator. A difficulty with this idea is the absence of noticeable "paraconductivity": transport measurements indicate a critical regime of order 10 K at most.[25,26]

In this paper we studied one theoretical implementation of the "phase fluctuation" scenario for the pseudogap: namely, the d-RVB regime of the $U(1)$ gauge theory of lightly doped Mott insulators. In this approach the spin degrees of freedom are mostly paired into a d-wave pairing state and the low-T charge response is essentially that of a superfluid, but with properties strongly affected by proximity to the Mott transition. One important feature of the model is that the vortex excitations are characterized by two length scales: the "quasiparticle coherence length" $\xi_F = v/\Delta$, which controls the distance over which the excitation spectrum differs from that far from a vortex, and the "current coherence length" ξ, which varies as the inverse of the square root of the doping and cuts off the familiar $1/r$ divergence of the supercurrent near a vortex. We studied the charge transport properties (many of which are dominated by vortices) and showed in particular that the electric field created by a moving vortex, the dissipation due to moving vortex, the value of H_{c2} and the size of the fluctuation regime near the resistive transition are all controlled by ξ which diverges near the Mott transition, rather than by the "quasiparticle length" ξ_F which does not.

As noted elsewhere[8,14,18] the theory disagrees in a number of ways with experiments; the most significant difficulty is the small value and strong doping dependence of the "quasiparticle charge" defined in Eq. (10). Further, our calculation has a number of phenomenological aspects. For example, we assumed a finite "spinon conductivity" which could reasonably be expected to arise from the "gapless Fermi arcs" induced by a nonvanishing temperature or applied magnetic field, but we did not attempt to calculated this from first principles; nor did we investigate the subtle quantum mechanics of fermions in the presence of conventional vortices. However, we believe the results presented here are useful because they provide an explicit demonstration in a well-defined model that if the supercurrent-defined correlation length is parametrically larger than the quasiparticle-defined length, then the resistive properties are controlled by the length scale over which the supercurrent varies.

Other workers have observed that the theory admits doubly quantized vortices. We have noted that they proliferate above a scale defined by the physical superfluid stiffness and (if the core energy of conventional vortices were large, which it is not in this model) would suppress the superconducting fluctuation contribution to the conductivity. We also showed how, in this model, the "nondissipative time derivative," whose importance was stressed in Ref. 17, is important for the estimation of the upper critical field and argued that it crosses over from the "fermionic" value T_c/E_F to the "bosonic" value proportional to the density of charges x. A subsequent paper will apply these ideas to a different model of high-T_c superconductivity.

APPENDIX: FERMI-LIQUID-BASED APPROACHES TO DOPED MOTT INSULATORS

This appendix treats the case of superconductivity developing out of a state which is well described by the usual Fermi-liquid theory. The necessary formalism was developed by Larkin and Leggett, and some of the results were sketched elsewhere.[27] A Fermi-liquid state is characterized by a quasiparticle velocity $v^*(\theta)$ which may depend on position (θ) on the Fermi surface, a characteristic energy scale E^*, and a Landau interaction function $T(\theta, \theta')$. In order for Fermi-liquid theory to be applicable, the maximum superconducting gap Δ_0 must be less than E^*. Transcription of the standard results to the langauge of the section above leads to

$$\rho_{s0} = \langle v^*(\theta)(1-T)^{-1}_{\theta,\theta'}v^*(\theta')\rangle, \tag{A1}$$

$$v_F = v^*, \tag{A2}$$

$$\frac{d\rho_S}{dT} = \frac{\langle v^*(\theta)(1-T)^{-1}_{\theta,\theta_1}L(\theta_1)(1-T)^{-1}_{\theta_1,\theta'}v^*(\theta')\rangle}{\langle v^*(\theta)^2\rangle}, \tag{A3}$$

where the angular backets mean multiplication by density of states and average over the Fermi line and

$$L(\theta) = \int \frac{d\omega d\theta}{2\pi^2} \frac{\omega}{2T^2 \coth\left(\frac{\omega}{2T}\right)} \frac{\Delta(\theta)^2}{[\omega^2 + \Delta(\theta)^2]^{3/2}}.$$

Comparison of the these results with those presented in Sec. II shows that the U(1) theory corresponds to a fermi liquid with a weakly angle-dependent Landau interaction function whose "current-channel" value is of order $1/x$.[27]

We now consider the situation in more detail by calculating the low-T current-current correlation function for a superconducting Fermi liquid, making the usual assumption that the Landau interaction function may be decomposed into angular channels in the conventional way and that the maximum value of the superconducting gap is small compared to the characteristic quasiparticle energy scale E^* so that quasiparticle damping effects may be neglected. The gauge-invariant current-current correlation function is then[27]

$$\chi_{jj}(q,\Delta) = \frac{\chi_{qp}(q,\Delta)}{1 + I_1\chi_{qp}(q,\Delta)}, \qquad (A4)$$

with I_1 the current-channel Landau interaction parameter [so that the Landau parameter $F_{1s} = 2I_1\chi_{qp}(0,\Delta)$] and

$$\chi_{qp}(q,\Delta) = T\sum_\omega \int \frac{d\theta}{(2\pi)^2 v^*(\theta)} v_x^{*2} \frac{\Delta(\theta)^2}{\sqrt{\Delta(\theta)^2 + \omega^2}}$$

$$\times \frac{1}{\omega^2 + \Delta(\theta)^2 + [v^*q\cos(\theta - \theta_q)]^2/4}.$$

$$(A5)$$

(Note that the result depends on the angle θ_q between the direction of q and the nodes in the gap.)

Standard calculations[28] show that at $vq \gg \Delta_0$, $\chi_{qp} \sim \Delta_0/vq$, so that we expect an appreciable change in χ_{jj} when $vq \sim \Delta_0/x$. In other words, a naive application of Fermi-liquid theory would predict a very short characteristic length scale of order xv/Δ. However, from the usual physical picture of the doped Mott insulator as a dilute collection of holes one might expect the interparticle spacing $x^{-1/2}$ to be an important scale. The discrepancy is resolved by noting that the Landau parameter presumably varies on the scale $x^{-1/2}$. This effect is beyond the scope of Landau theory but is captured in the U(1) approach.

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