Propagation of acoustic waves through finite superlattices: Transmission enhancement by surface resonance assistance

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(Received 7 June 2002; published 10 September 2002)

We report transmission enhancement of acoustic waves through finite solid superlattices embedded in a fluid medium. The transmission rate increases significantly when the wave vector $\vec{k}$ and frequency ω satisfy the dispersion relation of a surface excitation. We found enhancement of the transmission for both normal and oblique waves. The effect is demonstrated for Al/W, Mo/W, and Pt/Mo multilayers embedded in water. Multilayers supported by solid substrates are also investigated.

DOI: 10.1103/PhysRevB.66.092302 PACS number(s): 43.20.+g, 62.30.+d, 68.35.Ja, 68.65.Cd

The so-called *acoustic*—or *phononic—crystals* are artificial heterogeneous arrays with periodic elastic properties. In recent years the interest in these structures has increased due to the potential applications, and for the expectation of new fundamental phenomena, related to their main property: *they can support frequency gaps for the acoustic wave propagation*.[1–6] In particular for the superlattice, the array of one-dimensional periodicity, theoretical evidence of phenomena associated with the existence of a band gap, such as defect states and surface waves, have been widely studied.[7–10]

With respect to surface waves, a peculiarity of the wave transmission in finite superlattices placed between liquids or solids was recently reported: the displacement intensity of the transmitted waves at frequencies in the gaps can be enhanced due to the resonant excitation of a surface vibration. The enhancement was demonstrated theoretically for the GaAs-$(\mathrm{GaAs/AlAs})_N$-H_2O and GaAs-$(\mathrm{GaAs/AlAs})_N$-^{4}He systems.[11,12] It was concluded that the intensity of the penetrating wave into the transmission region (H_2O or ^{4}He) is amplified when the longitudinal incoming wave (from the GaAs medium) couples to a vibrational mode localized at the superlattice-fluid interface. The field displacement traverses the superlattice slab as a decaying wave reaching the opposite interface with sufficient amplitude to excite the surface vibration. Then, vibrational motion that propagates as a wave is transferred to the fluid. (We found that this *auxiliary* mechanism for wave transmission is not exclusive of the acoustic problem. Extraordinary tunneling magnetoresistence in ferromagnet-insulator-ferromagnet junctions can be explained in terms of metal-induced gap states of surface type.)[13]

In this paper we demonstrate explicitly the connection between the acoustic surface states and the peaks of extraordinary transmission. We found that the dispersion relations of the surface vibrations clearly define the position of the peaks on the frequency axis. Our main purpose is to demonstrate that the amplification of the transmittance occurs not only for normally incident waves (as was in fact already published) but also for oblique waves.

There are two types of vibrational modes with surface character that can amplify the wave transmission.

(1) *Surface waves*. The vibration is characterized by decaying displacement amplitude with an envelope exponential function in the perpendicular direction away from the surface. The surface waves satisfy a dispersion relation $k_x = k_x(\omega)$, where k_x is a real wave vector parallel to the surface (we are not considering absorption effects) and ω is an angular frequency of a bulk gap. The surface modes can exist even at $k_x = 0$. In this limit, the displacement profile remains as a surface wave in the perpendicular direction inside the layered medium but does not oscillate along the surface direction.

(2) *Surface resonances*. These excitations are essentially surface waves of frequencies in the region of an allowed bulk band. The coexistence of surface and bulk oscillations results from the symmetry and polarization differences between the modes. A transverse bulk band can coexist with a surface wave (the resonance) of longitudinal polarization, and vice versa, because they do not couple. The surface resonances have dispersion curve of finite k_x range and can also exist at $k_x = 0$.

We are interested in the system shown in Fig. 1. A longitudinal wave is incident from the substrate (fluid or solid) and the transmission medium is a fluid. For oblique incidence the waves inside the multilayer acquire a mixed character because an additional transverse component appears. As we shall see, the transmittance T, which is obtained directly from the boundary conditions for the stress $\sigma_{\alpha\beta}$ and the displacement $\vec{u}$ at each interface, reveals that in addition to the nonoscillatory modes of surface character discussed above, the layered slab also supports oscillatory modes. When one of these modes is excited, a maximum of transmission appears. Thus, as a function of the frequency, the transmittance oscillates. Some authors refer to this effect as one of Fabry-Perot type. Under favorable conditions (appropriate structural and material parameters) these maxima are of much minor intensity than the peaks of transmission assisted by surface vibrations.

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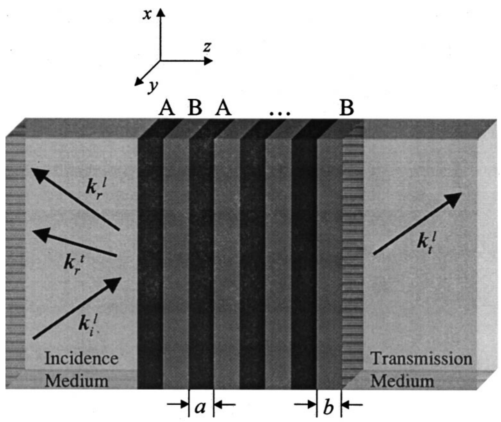

FIG. 1. The superlattice slab surrounded by the incidence and transmission media. A longitudinal wave with wave vector k_i^l comes in from the left side. A part of the acoustic energy is transmitted through the slab. The reflected longitudinal wave shares the total reflected energy with a transverse wave only when the incidence medium is a solid. The sagittal plane is the x-z plane and the thickness of the slab is $L_s=nd$. n is an integer number and $d=a+b$.

We know that the dispersion relations of the surface modes lie in the frequency gaps of bulk vibration. For a finite superlattice of large enough number of cells n, the oscillatory modes group to form such bands and the surface solutions correspond to those of the semi-infinite case. In order to define the frequency regions where surface modes can exist and to know the characteristics of the bulk vibrations for the possible occurrence of surface resonances, we must evaluate the band structure of the superlattice under study. Let us consider the infinite array of two alternated layers of homogeneous solid materials. The layers of thicknesses a and b have the material parameters ρ_a, c_{ta}, c_{la} and ρ_b, c_{tb}, c_{lb}, respectively. Thus the medium is a periodical structure of mass density $\rho(z)=\rho(z+d)$, transverse sound velocity $c_t(z)=c_t(z+d)$, and longitudinal sound velocity $c_l(z)=c_l(z+d)$. $d=a+b$ is the period. The normal modes of the mass element in this infinite structure are obtained from the Newton's second law,

$$\rho\frac{\partial^2 u_i}{\partial t^2}=\frac{\partial\sigma_{iK}}{\partial x_K}, \tag{1}$$

where u_i and σ_{iK} are components of the deformation and stress tensor, respectively. The stress is related only to the elastic properties of the medium (no external forces are considered). By taking into account the periodicity along the z direction and allowing oscillation in the direction of homogeneity, we look for harmonic solutions for u_x and u_z (the amplitudes in the sagittal plane) satisfying the Bloch theorem along the z axis. By a straightforward calculation, it is easy to show that Eq. (1) takes the form of an ordinary eigenvalue problem, $Au=\omega^2u$. Matrix A includes the reciprocal vectors of the one-dimensional lattice $\vec{G}=(2\pi l/d)\hat{z}$ with l integer, the wave vector $\vec{k}=k_x\hat{x}+k_z\hat{z}$ with the Bloch component k_z, and the Fourier coefficients $\rho(G)$, $\zeta(G)$, and $\xi(G)$ of the expansions for ρ, ρc_l^2, and ρc_t^2, respectively (for details see Refs. 1 and 9).

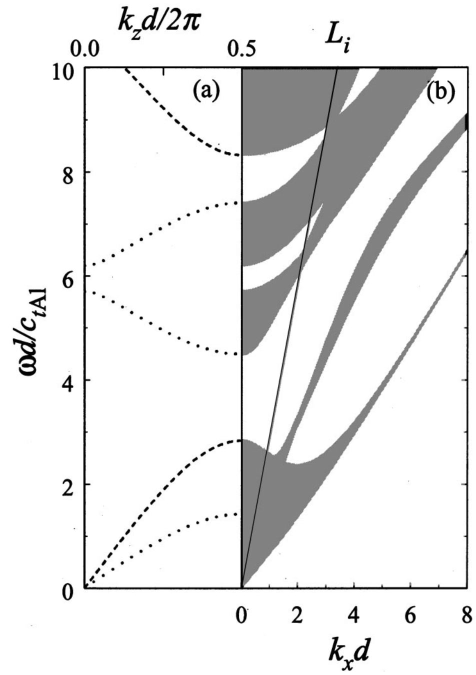

FIG. 2. Bulk band structure of an Al/W superlattice with $b=a$. The dimensionless quantities on the frequency axis are written in terms of the sound velocity in Al, $c_{t\mathrm{Al}}$. Panel (a) shows the dispersion curves of waves traveling along the superlattice axis—the z direction. Dotted (dashed) curves correspond to waves of transverse (longitudinal) polarization. The shaded regions in panel (b) represent the bulk bands with finite wave vector k_x.

As an example we present in Fig. 2 the bands of an Al/W superlattice. Only with $k_x=0$, which means energy propagation along the z axis, the equations that describe the mixed modes separate and two independent longitudinal and transverse modes exist [see Fig. 2(a)]. We found that the modes of the second and fifth bands are longitudinal while the modes in the first, third, and fourth bands have transverse character (they are shear waves). However, in the regime of $k_x\neq 0$ [Fig. 2(b)], where the sagittal modes are a complicated mix of displacements on the x and z directions, several frequency gaps appear (white regions). Within these gaps the surface waves can exist once the superlattice has been truncated.

The dispersion relation of the surface excitations were evaluated directly for a layered slab of n cells satisfying the boundary condition of zero stress at the surfaces. As was expected, we found a series of oscillatory solutions of frequencies in the regions of bulk propagation. As n was increased, more and more oscillatory modes were grouped to form the bands and some solutions corresponding to surface modes separate to lie in the gaps. More complicate was the identification of the surface resonances. They were found when we plotted the intensities of the modes in the bands. In general, the surface resonances exist for finite wave vector ranges and their curve of dispersion $\omega=\omega(k_x)$ does not necessarily begins at $k_x=0$.

In Fig. 3 we present the transmission intensity through a sample of six units cells of a Pt/Mo superlattice for normally incident waves. The substrate is Mo and the transmission medium is water. We show the sagittal bulk bands (left side

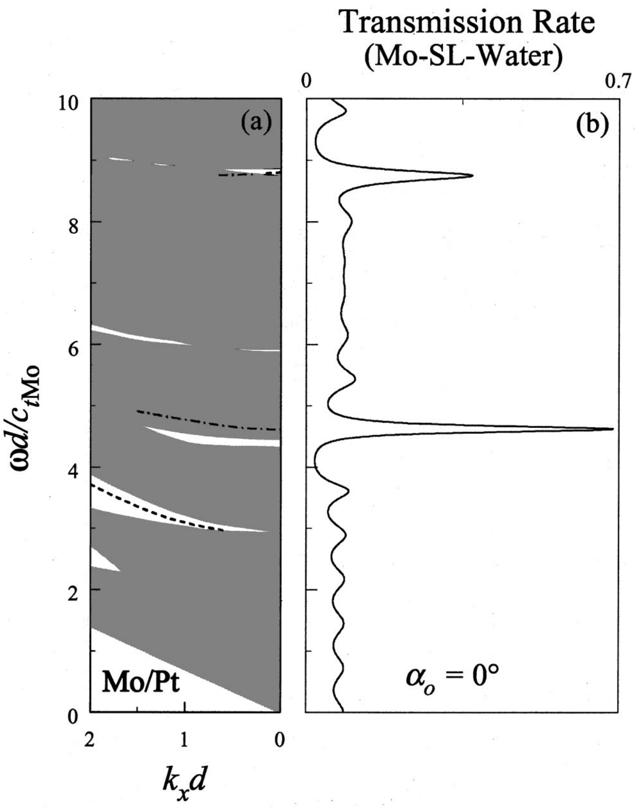

FIG. 3. Transmission enhancement assisted by surface excitations. (a) band structure of the infinite superlattice Pt/Mo with $a = b$ (shaded regions) and the dispersion curves of the surface vibrations occurring at the interface superlattice-vacuum when the superlattice terminates with a Mo layer (discontinuous curves). (b) Transmission rate through the slab of 13 layers ($n=6$ plus one layer) supported by a Mo substrate from which the waves come in. The last layer (in contact with water) is one of Mo.

of the figure) and the dispersion relations of the surface sagittal excitations at the interface superlattice-vacuum (the last layer of the superlattice is a Mo layer). The dashed curve corresponds to surface waves and the dash-dotted curves represent surface resonances (surface waves of longitudinal polarization at $k_x=0$ coexisting with bulk bands of transverse polarization). On the right side of the figure, we observe that the transmission rate of longitudinal waves into the water presents resonant peaks. The frequencies of the peaks clearly coincide with the dispersion curves of the two surface vibrations at $k_x=0$.

The most important characteristic of this assisted transmission is that the peak intensities are higher than the transmittance through the single interface between the incidence and transmission media. Thus we are treating the phenomenon of transmission enhancement. In Fig. 3(b), the limit for the ordinary transmission is $T=0.1$. As a consequence of the surface mode assistance, the peak at $\omega d/c_{t\mathrm{Mo}}=4.62$ reaches the transmission rate $T=0.68$.

This type of resonant transmission was found previously only for normally incident waves with frequencies in the band gaps (as the transmission associated with the mode in the upper tiny gap in Fig. 3).[11,12] The involved vibrations

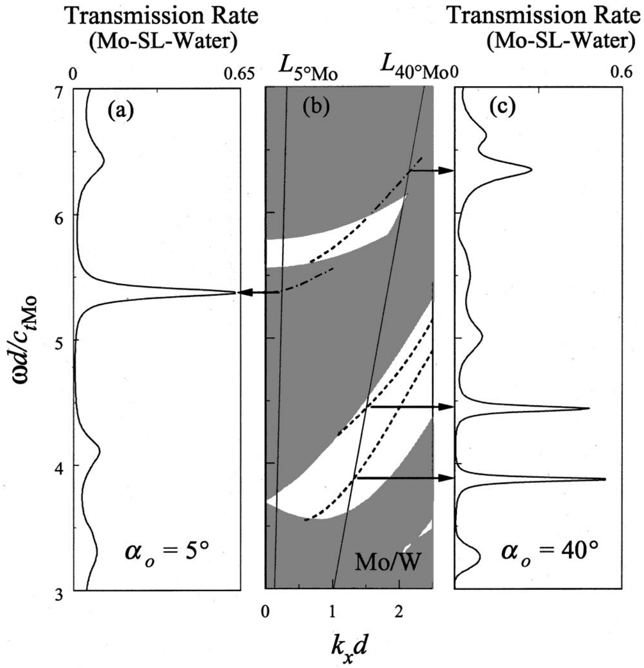

FIG. 4. Transmission enhancement at oblique incidence. The calculations correspond to a W/Mo superlattice with $a=b$. Panels (a) and (c) show the enhanced transmission at frequencies corresponding to the surface vibrations. Panel (b) shows the frequency band structure.

were never described as the limit at $k_x=0$ of a curve of dispersion that defines the surface modes available for direct excitation. Figure 3 shows that the energy transmission can be also enhanced by surface resonance assistance with frequency lying inside an allowed bulk band (see in Fig. 3 the resonance at $\omega d/c_{t\mathrm{Mo}}=4.62$).

It is important to remark that the involved surface vibrations that we are considering correspond to a free surface superlattice slab. We found the boundary conditions for the superlattice-vacuum problem sufficient when a fluid substitutes the vacuum, because the fluid medium does not produce appreciable stress at the surface. [It is easy to show that between two elastic media the continuity of the normal stress leads to the identity $Z_A(A_i-A_r)=Z_B(A_t)$, where Z_A and Z_B are the impedances of the media and A_i, A_r, and A_t the incident, reflected, and transmitted amplitudes, respectively. When $Z_A \gg Z_B$, our case if we consider Z_A as the average impedance of the superlattice, the solutions with finite amplitude inside the superlattice are obtained approximately from the solid-vacuum condition $Z_A(A_i-A_r)=0$.]

The extraordinary transmission is not restricted to normal incident waves. Figure 4 shows that such effect remains for oblique waves. The lines of incidence defined as $\omega/k_x = c_{t\mathrm{Mo}}/\sin\alpha_0$ for $\alpha_0=5°$ and $\alpha_0=40°$ [solid lines in panel (b)] cross the dispersion curves of four excitations producing transmission amplification at the respective frequencies. This example corresponds to a Mo/W superlattice and the sample for transmission has six unit cells with Mo as substrate. Again the last layer is that of Mo and the transmission medium is water. While the lower three peaks (one on the left panel and two on the right panel) have well-defined profiles,

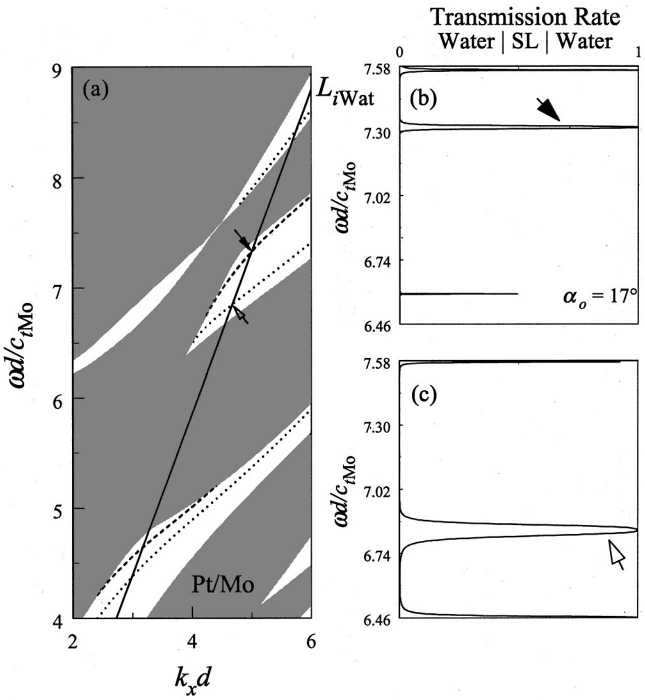

FIG. 5. Transmission peak dependence on the superlattice termination. (a) Band structure of the same superlattice of Fig. 3 but plotted in a different range. The line of incidence from water at 17° crosses the dashed and dotted curves of the surface vibrations when the multilayer terminates with Pt and Mo, respectively. Panels (b) and (c) show the corresponding transmission peaks at frequencies indicated by the black and white arrows.

the peak at the upper frequency on the right panel seems almost vanished. The reason is that this peak is produced by a weak surface resonance that lies near the right limit of the corresponding dispersion curve, where such resonances disappear. All the lower three peaks correspond to well-defined excitations.

Sharp peaks of total transmission were found only for superlattice slabs embedded in a fluid. Figure 5 shows the peaks for the system water-superlattice-water. The bulk bands in panel (a) correspond to a Pt/Mo superlattice. The dashed and dotted curves are the dispersion relations of the surface states in a truncated superlattice terminated with a Mo and Pt layer, respectively. Reference 8 shows how the position of the dispersion curve changes when the layered slab is terminated with one or the other one layer at the surface. The continuum line is the line of incidence at 17°. The two arrows indicate the points (k_x, ω) inside the band gap for which transmission enhancement occurs. Panels (b) and (c) present the transmission profiles at these two frequencies in a sample of six unit cells. As can be seen, the surface modes open a finite range (very short) of frequencies within the band gap for wave transmission. The other peaks in these panels correspond to the oscillatory (Fabry-Perot) modes that the finite sample supports, as was discussed previously. They lay inside the bulk bands that delimit the gap.

In conclusion, we have demonstrated the transmission enhancement of longitudinal waves traversing solid superlattices. The phenomenon is caused by the resonant assistance of surface excitations. In order to establish the occurrence of this phenomenon even for oblique angles of incidence, we calculated the dispersion curves of such excitations. Peaks of transmission appear when the frequency and the parallel wave vector of the incident wave satisfy such dispersion relations.

We found that the transmission enhancement is strongly dependent on n, the number of cells in the slab. For example, the peak in Fig. 5(c) that corresponds to a symmetric Pt/Mo multilayer terminated with Mo layers on both sides and embedded in water, changes from $T=0.99$ to $T=0.12$ when n varies from $n=5$ to $n=15$. The reason of this behavior is the exponential decay that the surface vibration suffers inside the multilayer. The larger the thickness of the slab, the smaller the tunneled vibration available to excite the surface vibration. The surface modes studied here have the decaying distance $\beta=3d$ and our calculations show that maximum enhancement results for samples with $n=6$. Thus, optimum enhancement is obtained for samples of thickness $L_s=2\beta$.

This work was supported by Consejo Nacional de Ciencia y Tecnología, CONACyT México, Grant No. 489100-5-35541-E. B.M.M. is grateful to CONACyT for support.

[1] M. S. Kushwaha, P. Halevi, L. Dobrzynski, and B. Djafari-Rouhani, Phys. Rev. Lett. **71**, 2022 (1993).
[2] M. Sigalas and E. N. Economou, Solid State Commun. **86**, 141 (1993).
[3] M. S. Kushwaha, Int. J. Mod. Phys. B **10**, 977 (1996).
[4] J. V. Sánchez-Pérez, D. Caballero, R. Martínez-Sala, C. Rubio, J. Sánchez-Dehesa, F. Meseguer, J. Linares, and F. Gálvez, Phys. Rev. Lett. **80**, 5325 (1998).
[5] Zhengyou Liu, Xixiang Zhang, Yiwei Mao, Y. Y. Zhu, Zhiyu Yang, C. T. Chang, and Ping Sheng, Science **289**, 1734 (2000).
[6] M. M. Sigalas and C. M. Soukoulis, Phys. Rev. B **51**, 2780 (1995).
[7] J. Sapriel and B. Djafari-Rouhani, Surf. Sci. Rep. **10**, 189 (1989).
[8] D. Bria, E. H. El Boudouti, A. Nougaoui, B. Djafari-Rouhani, and V. R. Velasco, Phys. Rev. B **61**, 15 858 (2000).
[9] B. Manzanares-Martínez and F. Ramos-Mendieta, Phys. Rev. B **61**, 12 877 (2000).
[10] Shin-ichiro Tamura, Phys. Rev. B **39**, 1261 (1989).
[11] Hatsuyoshi Kato, Phys. Rev. B **59**, 11 136 (1999).
[12] Seiji Mizuno, Phys. Rev. B **63**, 035301 (2001).
[13] Ph. Mavropoulos, N. Papanikolaou, and P. H. Dederichs, Phys. Rev. Lett. **85**, 1088 (2000).