Characteristics of electromagnetic wave propagation in uniaxially anisotropic left-handed materials

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We investigate the characteristics of electromagnetic wave propagation in uniaxially anisotropic left-handed media. We discuss mainly under what conditions anomalous reflection or refraction shall occur at the interface when propagating waves pass from one isotropic regular medium into another uniaxially anistotropic left-handed medium and under what conditions anomalous transmission shall occur when an evanescent wave is transmitted through a slab of uniaxially anisotropic left-handed medium. We show that the characteristics of electromagnetic wave propagation in uniaxially anisotropic left-handed media are significantly different from that in isotropic left-handed media.

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I. INTRODUCTION

Recently, there has been great interest in a new type of electromagnetic materials called left-handed media.[1] Over thirty years ago, V. G. Veselago predicted that electromagnetic wave propagation in a medium having simultaneously negative permittivity ϵ and permeability μ should give rise to several peculiar characteristics.[2] According to Maxwell's equations, the direction of energy flow of a plane wave is given by the direction of the Poynting vector, which is the cross product of electric field **E** and magnetic field **H**. For plane waves propagated in isotropic regular media having simultaneously positive ϵ and μ, the cross product of electric field **E** and magnetic field **H** gives both the direction of energy flow (the Poynting vector) and the wave itself (that is, the phase velocity, or wave vector), and **E**, **H** and wave vector **k** form a right-handed triplet of vectors. But Veselago predicted that in a medium having simultaneously negative ϵ and μ, while $\mathbf{E}\times\mathbf{H}$ for a plane wave still gives the direction of energy flow, the phase velocity (wave vector) shall be in the opposite direction of energy flow, and **E**, **H** and wave vector **k** shall form a left-handed triplet of vectors. Due to this left-handed characteristic, Veselago termed such type of materials as left-handed medium (LHM), and all regular materials were correspondingly termed right-handed medium (RHM). In addition to this left-handed characteristic, left-handed materials have several other dramatically different electrodynamic properties compared with regular materials, stemming from a simultaneous change of the signs of ϵ and μ, including anomalous refraction, reversal of both the Doppler shift and the Cerenkov radiation, and reversal of radiation pressure to radiation tension.[2] Although these counterintuitive properties follow directly from Maxwell's equations, due to the absence of naturally occurring materials having simultaneously negative ϵ and μ, Veselago's prediction did not attract much attention until very recently, when a system consisting an array of resonators and metallic wires was prepared successfully following the suggestion of Pendry *et al.*[3] and was demonstrated to be left-handed for electromagnetic waves propagating in some special direction and polarization in a narrow microwave frequency region.[4,5] This discovery aroused great interest in the unusual electrodynamic properties of left-handed materials.[6–14] In this paper, we discuss the characteristics of electromagnetic wave propagation in uniaxially anisotropic left-handed media. Veselago's original paper and most of the recent theoretical works discussed mainly the characteristics of electromagnetic wave propagation in isotropic left-handed media, but up to now, the LHM that have been prepared successfully in experiments are actually anisotropic in nature, and it may be very difficult to prepare an isotropic left-handed medium.[2,4,5] In classic electrodynamics, it is well known that the electrodynamic properties of anisotropic materials are significantly different from that of isotropic materials. The simplest and most common form of anisotropy is uniaxial anisotropy, and from analysis of the symmetry of the left-handed medium reported in Refs. 4 and 5, it should also be uniaxially anisotropic. In this paper, we present a detailed investigation on the characteristics of electromagnetic wave propagation in uniaxially anisotropic left-handed media. We shall show that the characteristics of electromagnetic wave propagation in uniaxially anisotropic LHM are significantly different from that in isotropic LHM. The paper is organized as follows: In Sec. II, we present a brief review on the left-handed characteristic of electromagnetic wave propagations in uniaxially anisotropic LHM. In Sec. III, we discuss under what conditions anomalous reflection or refraction shall occur at the interface when a propagating wave passes from one isotropic regular medium into another uniaxially anisotropic left-handed medium. In Sec. IV, we discuss under what conditions anomalous transmission shall occur when an evanescent wave is transmitted through a slab of uniaxially anisotropic left-handed medium.

II. LEFT-HANDED CHARACTERISTIC OF ELECTROMAGNETIC WAVE PROPAGATIONS IN UNIAXIALLY ANISOTROPIC LEFT-HANDED MEDIA

In this section, we present a brief review on the left-handed characteristic of electromagnetic wave propagations in uniaxially anisotropic left-handed media. For anisotropic materials, one or both of the permittivity and permeability are second-rank tensors.[15] In the following we assume that both the permittivity and permeability are uniaxially anisotropic. The results for the case that one of the permittivity and permeability is uniaxially anisotropic but the other one is isotropic can be obtained similarly. For uniaxially anisotropic media, if we take the optical axis as the z axis, the permittivity and permeability tensors will have following forms:[15]

$$\hat{\epsilon}=\begin{bmatrix} \epsilon_{\perp} & 0 & 0 \\ 0 & \epsilon_{\perp} & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}, \tag{1}$$

$$\hat{\mu}=\begin{bmatrix} \mu_{\perp} & 0 & 0 \\ 0 & \mu_{\perp} & 0 \\ 0 & 0 & \mu_z \end{bmatrix}, \tag{2}$$

where ϵ_z, μ_z and $\epsilon_{\perp}$, $\mu_{\perp}$ are the permittivity and permeability constants in the directions parallel and perpendicular to the optical axis, respectively. (For uniaxial anisotropy, the media are isotropic in the plane perpendicular to the optical axis). Consider the propagation of a plane wave of frequency ω with $\mathbf{E}=\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}$, $\mathbf{H}=\mathbf{H}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}$, from Maxwell's equations, it can be easily shown that there are two types of linearly polarized plane waves, namely E-polarized and H-polarized plane waves.[15] E-polarized plane waves satisfy the conditions $\mathbf{k}\cdot\mathbf{E}=0$ and $E_z=0$, H-polarized plane waves satisfy the conditions $\mathbf{k}\cdot\mathbf{H}=0$ and $H_z=0$. Since the media are isotropic in the x–y plane, we can assume that the wave vectors are in the x–z plane without losing any generality, then the electric and magnetic fields of E-polarized plane waves can be expressed as

$$\mathbf{E}=E_0\mathbf{e}_y \exp(ik_x x+ik_z z-i\omega t), \tag{3}$$

$$\mathbf{H}=\left[-\frac{E_0 k_z}{\omega\mu_{\perp}}\mathbf{e}_x+\frac{E_0 k_x}{\omega\mu_z}\mathbf{e}_z\right]\exp(ik_x x+ik_z z-i\omega t), \tag{4}$$

where $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ are unit vectors along the x, y, and z axis. Similarly, the electric and magnetic fields of H-polarized plane waves can be expressed as

$$\mathbf{H}=H_0\mathbf{e}_y \exp(ik_x x+ik_z z-i\omega t), \tag{5}$$

$$\mathbf{E}=\left[\frac{H_0 k_z}{\omega\epsilon_{\perp}}\mathbf{e}_x-\frac{H_0 k_x}{\omega\epsilon_z}\mathbf{e}_z\right]\exp(ik_x x+ik_z z-i\omega t). \tag{6}$$

The dispersion relation for E-polarized plane waves is determined by

$$\frac{k_x^2}{\mu_z}+\frac{k_z^2}{\mu_{\perp}}=\omega^2\epsilon_{\perp}, \tag{7}$$

and the dispersion relation for H-polarized plane waves is determined by

$$\frac{k_x^2}{\epsilon_z}+\frac{k_z^2}{\epsilon_{\perp}}=\omega^2\mu_{\perp}. \tag{8}$$

For E-polarized plane waves, the energy current density (i.e., the time-averaged Poynting vector) and its inner product with wave vector $\mathbf{k}$ are given by

$$\mathbf{S}=\mathrm{Re}\left[\frac{E_0^2 k_x}{2\omega\mu_z}\mathbf{e}_x+\frac{E_0^2 k_z}{2\omega\mu_{\perp}}\mathbf{e}_z\right], \tag{9}$$

$$\mathbf{k}\cdot\mathbf{S}=\mathrm{Re}\left[\tfrac{1}{2}\omega\epsilon_{\perp}E_0^2\right], \tag{10}$$

and for H-polarized plane waves, we can get that

$$\mathbf{S}=\mathrm{Re}\left[\frac{H_0^2 k_x}{2\omega\epsilon_z}\mathbf{e}_x+\frac{H_0^2 k_z}{2\omega\epsilon_{\perp}}\mathbf{e}_z\right], \tag{11}$$

$$\mathbf{k}\cdot\mathbf{S}=\mathrm{Re}\left[\tfrac{1}{2}\omega\mu_{\perp}H_0^2\right]. \tag{12}$$

From Eqs. (3) to (12), we can see that if $\epsilon_{\perp}\neq\epsilon_z$ and $\mu_{\perp}\neq\mu_z$, the electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$ and wave vector $\mathbf{k}$ cannot form a strictly left-handed triplet of vectors and the directions of energy flow cannot be in the exactly opposite directions of wave vectors for both E- and H-polarized plane waves, except that the wave propagation is in the direction of the optical axis. But if some conditions are satisfied, both E- and H-polarized waves can be approximately left-handed waves (i.e., $\mathbf{E}$, $\mathbf{H}$, and $\mathbf{k}$ form an approximately left-handed triplet of vectors and the direction of energy flow is in the backward but not exactly opposite direction of wave vector). The condition for E-polarized waves being approximately left-handed waves is $\epsilon_{\perp}<0$ but other elements of both $\hat{\epsilon}$ and $\hat{\mu}$ do not need to be negative. The condition for H-polarized waves being approximately left-handed waves is $\mu_{\perp}<0$ and other elements of both $\hat{\epsilon}$ and $\hat{\mu}$ do not need to be negative. If the wave propagation is in the direction of the optical axis, both E- and H-polarized waves can be exactly left-handed waves if $\epsilon_{\perp}$ and $\mu_{\perp}$ are simultaneously negative, and ϵ_z and μ_z (including their signs) have no effect on the propagation and the left-handedness of both E- and H-polarized waves. It can also be noted that although the conditions for E- and H-polarized waves being approximately left-handed waves do not require that all elements of the permittivity and permeability tensors are negative, if some elements of $\hat{\epsilon}$ and $\hat{\mu}$ are positive, in some directions the waves may not be able to propagate. For example, if $\epsilon_{\perp}<0$ but $\mu_{\perp}$ and μ_z are both positive, E-polarized waves cannot propagate in any direction; if $\epsilon_{\perp}<0$, $\mu_{\perp}<0$ and $\mu_z>0$, E-polarized waves can propagate only if the angle between wave vector $\mathbf{k}$ and the optical axis is smaller than a critical angle $\theta_c=\arctan(|\mu_z/\mu_{\perp}|)$, or else wave vector shall be imaginary and E-polarized waves cannot propagate; if $\epsilon_{\perp}<0$, $\mu_{\perp}>0$, and $\mu_z<0$, E-polarized waves can propagate only if the angle between wave vector $\mathbf{k}$ and the optical axis is larger than a critical angle $\theta_c=\arctan(|\mu_z/\mu_{\perp}|)$. From these results, we can see that the characteristics of anisotropic left-handed media are significantly different from that of isotropic left-handed media. Similar conclusions as mentioned previously were also obtained in Ref. 13. In that paper, it was argued that the name "left-handed medium" is misleading and the recently realized so-called left-handed medium should be called backward medium. But considering that the name "left-handed medium" has been widely used and accepted in most of the recent related literature,[3–12] in this paper we will still call such materials left-handed media, but we think that the original concepts on left-handed media should be extended in the presence of anisotropy since the electric and magnetic fields and wave vector cannot not form a truly left-handed triplet of vectors except in some special propagation direction.

III. ANOMALOUS REFLECTION AND REFRACTION OF ELECTROMAGNETIC WAVES AT THE INTERFACES OF ISOTROPIC REGULAR MEDIA AND UNIAXIALLY ANISOTROPIC LEFT-HANDED MEDIA

When a beam of light passes from one regular medium into another regular medium, the ray shall undergo reflection and refraction at the interface between two media, and the refracted ray should bend toward the normal of the interface but never emerge on the same side of the normal as the incident ray (Snell's law). But Veselago predicted that, if the second medium is an isotropic left-handed medium (LHM), the refracted ray should lie on the same side of the normal of the interface as the incident ray.[2] This anomalous refraction phenomenon is one of the most interesting peculiar properties of isotropic left-handed media and has been verified experimentally very recently.[7] In this section, we discuss the characteristics of reflection and refraction of electromagnetic waves at the interface between one isotropic regular medium and another uniaxially anisotropic left-handed medium. We shall show that, in addition to the anomalous refraction phenomenon, in some conditions anomalous total reflection phenomenon can also occur at the interface between one isotropic regular medium and the second uniaxially anisotropic left-handed medium. This anomalous total reflection phenomenon is very different from the usual total internal reflection phenomenon which is well known in classic electrodynamics. This anomalous total reflection phenomenon occurs when the incident angles are *smaller but not larger* than a critical angle. In contrast, the usual total internal reflection phenomenon can occur only if the incident angles are *larger but not smaller* than a critical angle. In what follows two cases will be discussed. The first case is that the optical axis of the left-handed medium (LHM) is normal to the interface of two media. The second case is that the optical axis of the LHM is parallel to the interface of two media.

A. The optical axis of the LHM is normal to the interface

In this section we discuss the characteristics of reflection and refraction of electromagnetic waves passing from one isotropic regular medium into the second uniaxially anisotropic left-handed medium and the optical axis (the z axis) of the LHM is normal to the interface of the two media. (A schematic illustration of the system is shown in Fig. 1.) In what follows we will denote the permittivity and permeability of the regular medium as $\epsilon_r(>0)$ and $\mu_r(>0)$. The permittivity and permeability tensors of the LHM have been denoted in Eqs. (1) and (2). Since the system is isotropic in the x–y plane, we can assume that wave vectors of the incident, reflected, and refracted waves are in the x–z plane without losing generality. The electric fields of the E-polarized incident and reflected waves are given by

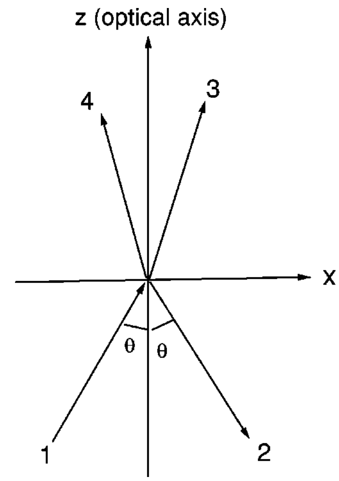

FIG. 1. Illustration of ordinary and anomalous refraction when a beam of wave passes from one isotropic regular medium into the second uniaxially anisotropic left-handed medium. (1) incident ray; (2) reflected ray; (3) refracted ray if refraction is ordinary; (4) refracted ray if refraction is anomalous.

$$E_i = E_0 \mathbf{e}_y \exp(ik_x x + ik_z z - i\omega t), \tag{13}$$

$$E_r = rE_0 \mathbf{e}_y \exp(ik_x x - ik_z z - i\omega t), \tag{14}$$

where r is the reflection coefficient and $k^2 = k_x^2 + k_z^2 = \omega^2 \epsilon_r \mu_r$. Similarly, the magnetic fields of the H-polarized incident and reflected waves can be given by

$$H_i = H_0 \mathbf{e}_y \exp(ik_x x + ik_z z - i\omega t), \tag{15}$$

$$H_r = r' H_0 \mathbf{e}_y \exp(ik_x x - ik_z z - i\omega t). \tag{16}$$

In the case that the second medium is uniaxially anisotropic, from the boundary conditions it can be easily shown that the refracted waves should maintain the same polarization as the incident waves, and the tangential components of the wave vectors of the refracted waves are equal to that of the incident waves. So for E-polarized incident waves, the refracted waves are also E-polarized, and the electric fields of the refracted waves can be expressed as

$$E_t = tE_0 \mathbf{e}_y \exp(ik_x x + ik_z' z - i\omega t), \tag{17}$$

where t is the transmission coefficient and k_z' can be obtained from Eq. (7):

$$k_z'^2 = \omega^2 \epsilon_\perp \mu_\perp - \frac{\mu_\perp}{\mu_z} k_x^2. \tag{18}$$

For H-polarized incident waves, the refracted waves are also H-polarized, and the magnetic fields of the refracted waves can be expressed as

$$H_t = t' H_0 \mathbf{e}_y \exp(ik_x x + ik_z'' z - i\omega t), \tag{19}$$

where k_z'' can be obtained from Eq. (8):

$$k_z''^2 = \omega^2 \epsilon_\perp \mu_\perp - \frac{\epsilon_\perp}{\epsilon_z} k_x^2. \tag{20}$$

The reflection and transmission coefficients can be obtained from the boundary conditions, from which we can get that

$$r=\frac{\mu_\perp k_z-\mu_r k_z'}{\mu_\perp k_z+\mu_r k_z'},\quad t=\frac{2\,\mu_\perp k_z}{\mu_\perp k_z+\mu_r k_z'},\tag{21}$$

$$r'=\frac{\epsilon_\perp k_z-\epsilon_r k_z''}{\epsilon_\perp k_z+\epsilon_r k_z''},\quad t'=\frac{2\,\epsilon_\perp k_z}{\epsilon_\perp k_z+\epsilon_r k_z''}.\tag{22}$$

After the transmission coefficients are determined, the energy current density **S** of the refracted waves can be obtained, which determines the directions of the refracted waves. For E- and H-polarized refracted waves, **S** is given by

$$\mathbf{S}^{(E)}=\mathrm{Re}\left[\frac{t^2E_0^2k_x}{2\,\omega\mu_z}\mathbf{e}_x+\frac{t^2E_0^2k_z'}{2\,\omega\mu_\perp}\mathbf{e}_z\right],\tag{23}$$

$$\mathbf{S}^{(H)}=\mathrm{Re}\left[\frac{t'^2H_0^2k_x}{2\,\omega\epsilon_z}\mathbf{e}_x+\frac{t'^2H_0^2k_z''}{2\omega\epsilon_\perp}\mathbf{e}_z\right].\tag{24}$$

The basic features of reflection and refraction can be seen from Eqs. (23) and (24). First let us see under what conditions refraction will occur at the interface. In principle, the occurrence of refraction requires that the z component of the wave vectors of the refracted waves must be real, or else the incident waves shall be totally reflected. This is due to the fact that if the z component of the wave vectors of the refracted waves is imaginary, the normal component of the energy current density of the refracted waves will be zero, i.e., $\mathbf{S}\cdot\mathbf{e}_z=0$, as can be seen from Eqs. (23) to (24), hence no power will be transmitted into the second medium and the incident waves shall be totally reflected. So from Eqs. (18) and (20), we can see that the refraction shall occur only if the incident angles satisfy the following inequality:

$$\frac{\mu_\perp}{\mu_z}k^2\sin^2\theta<\omega^2\epsilon_\perp\mu_\perp\tag{25}$$

for E-polarized incident waves, and

$$\frac{\epsilon_\perp}{\epsilon_z}k^2\sin^2\theta<\omega^2\epsilon_\perp\mu_\perp\tag{26}$$

for H-polarized incident waves. As to the directions of the refracted waves, they can be determined by following principles. First, the boundary conditions require that the x component of the wave vectors of the refracted waves should be equal to the x component of the wave vectors of the incident waves, then from Eqs. (23) to (24), we can see that the x component of the energy current density of the refracted waves will be determined by μ_z(for E-polarized incident waves) and ϵ_z(for H-polarized incident waves). Second, causality requires that in the second medium, energy current of the refracted waves should be transmitted away from the interface of the two media but never toward the interface. This requires that the z component of the energy current density of the refracted waves must always have the same signs as the z component of the energy current density of the incident waves. Following these arguments, for the case illustrated in Fig. 1, we can get the following conclusions.

(1). For E-polarized incident waves, anomalous total reflection will occur if $\mu_\perp/\mu_z$ and $\epsilon_\perp\mu_\perp$ are simultaneously negative. From Eq. (18), we can see that if $\mu_\perp/\mu_z$ and $\epsilon_\perp\mu_\perp$ are simultaneously negative, the occurrence of refraction shall require that the incident angle must be *larger* than a critical angle $\theta_c=\arcsin\sqrt{(\epsilon_\perp\mu_z)/(\epsilon_r\mu_r)}$. If the incident angle is *smaller* than the critical angle θ_c, k_z' will be imaginary and the incident waves shall be totally reflected. If $\epsilon_\perp\mu_z>\epsilon_r\mu_r$, the critical angle θ_c will be equal to $\pi/2$. In this case, for any incident angles, the E-polarized incident waves shall be totally reflected. This anomalous total reflection cannot occur if the second medium is an isotropic left-handed medium.

(2). For H-polarized incident waves, anomalous total reflection will occur if $\epsilon_\perp/\epsilon_z$ and $\epsilon_\perp\mu_\perp$ are simultaneously negative. From Eq. (20), we can see that if $\epsilon_\perp/\epsilon_z$ and $\epsilon_\perp\mu_\perp$ are simultaneously negative, the occurrence of refraction shall require that the incident angle must be *larger* than a critical angle $\theta_c=\arcsin\sqrt{(\mu_\perp\epsilon_z)/(\epsilon_r\mu_r)}$. If the incident angle is *smaller* than the critical angle θ_c, k_z'' will be imaginary and the incident waves shall be totally reflected. If $\mu_\perp\epsilon_z>\epsilon_r\mu_r$, the critical angle θ_c will be equal to $\pi/2$. In this case, for any incident angles, the H-polarized incident waves shall be totally reflected. This anomalous total reflection phenomenon cannot occur if the second medium is an isotropic left-handed medium.

(3). For E-polarized incident waves, anomalous refraction will occur if $\mu_z<0$ and the incident angles satisfy the inequality (25), and other elements of the permittivity and permeability tensors do not need to be negative; for H-polarized incident waves, anomalous refraction will occur if $\epsilon_z<0$ and the incident angles satisfy the inequality (26), and other elements of the permittivity and permeability tensors do not need to be negative. A similar conclusion was also obtained in Ref. 13. An important fact that should be pointed out is that in the presence of anisotropy, the refracted waves do not need to be left-handed waves or backward waves even if the refraction is anomalous, or though the refracted waves are left-handed waves or backward waves, the refraction may be ordinary but not anomalous. For example, for E-polarized incident waves, if $\mu_z<0$ and $\epsilon_\perp>0$, the refraction shall be anomalous but the refracted waves shall not be left-handed waves or backward waves since in this case energy flow of the refracted waves shall be in the forward but not backward direction of the wave vectors. This shows once more that the original concepts about left-handed medium need to be extended in the presence of anisotropy.

B. The optical axis of the LHM is parallel to the interface

If the optical axis of the uniaxially anisotropic left-handed medium is parallel to the interface of the two media, we can show that anomalous reflection or refraction can still occur under certain conditions. For convenience, in the following we will choose the x axis to be normal to the interface and the z axis to be along the optical axis (parallel to the interface) and assume that the wave vectors of the incident waves are in the x–z plane, then the fields of the incident, reflected, and refracted waves can still be expressed by similar forms as those given in Sec.III A, and the energy current density of the refracted waves can by expressed by same forms as those given in Eqs. (23)–(24), but in this case the boundary conditions will require that the z components of the wave vectors of the refracted waves be equal to the z component of the wave vectors of the incident waves, and from Eqs. (7) and (8), we can see that the x component of the wave vectors of the refracted waves will be given by

$$k_x'^2=\omega^2\epsilon_\perp\mu_z-\frac{\mu_z}{\mu_\perp}k_z^2 \tag{27}$$

for E-polarized incident waves, and

$$k_x''^2=\omega^2\epsilon_z\mu_\perp-\frac{\epsilon_z}{\epsilon_\perp}k_z^2 \tag{28}$$

for H-polarized incident waves. The occurrence of refraction will require that k_x' and k_x'' must be real. This means that for E-polarized incident waves, the incident angle θ should satisfy the following inequality:

$$\frac{\mu_z}{\mu_\perp}k^2\sin^2\theta<\omega^2\epsilon_\perp\mu_z, \tag{29}$$

where $\mathbf{k}$ is the wave vectors of the incident waves and is given by $k^2=\omega^2\epsilon_r\mu_r$. For H-polarized incident waves, the incident angle θ should satisfy the following inequality

$$\frac{\epsilon_z}{\epsilon_\perp}k^2\sin^2\theta<\omega^2\epsilon_z\mu_\perp. \tag{30}$$

The directions of the refracted waves can be determined by following principles. First, causality will require that in the second medium, the x component of the energy current density of the refracted waves should have the same signs as the x component of the energy current density of the incident waves. Second, from the boundary conditions and Eqs. (23) and (24), we can see that the z component of the energy current density of the refracted waves will be determined by $\mu_\perp$(for E-polarized incident waves) and $\epsilon_\perp$ (for H-polarized incident waves). Following these arguments, we can get the following conclusions.

(**1**). For E-polarized incident waves, anomalous total reflection will occur if $\mu_z/\mu_\perp$ and $\epsilon_\perp\mu_z$ are simultaneously negative. From Eq. (27), we can see that if $\mu_z/\mu_\perp$ and $\epsilon_\perp\mu_z$ are both negative, the occurrence of refraction shall require that the incident angle must be *larger* than a critical angle $\theta_c=\arcsin\sqrt{(\epsilon_\perp\mu_\perp)/(\epsilon_r\mu_r)}$. If the incident angle is *smaller* than the critical angle θ_c, k_x' will be imaginary and the incident waves shall be totally reflected. If $\epsilon_\perp\mu_\perp>\epsilon_r\mu_r$, the E-polarized incident waves shall be totally reflected for any incident angles.

(**2**). For H-polarized incident waves, anomalous total reflection will occur if $\epsilon_z/\epsilon_\perp$ and $\epsilon_z\mu_\perp$ are simultaneously negative. From Eq. (28), we can see that if $\epsilon_z/\epsilon_\perp$ and $\epsilon_z\mu_\perp$ are both negative, the occurrence of refraction shall require that the incident angle must be *larger* than a critical angle $\theta_c=\arcsin\sqrt{(\epsilon_\perp\mu_\perp)/(\epsilon_r\mu_r)}$. If the incident angle is *smaller* than the critical angle θ_c, k_x'' will be imaginary and the incident waves shall be totally reflected. If $\epsilon_\perp\mu_\perp>\epsilon_r\mu_r$, the H-polarized incident waves shall be totally reflected for any incident angles.

(**3**). For E-polarized incident waves, anomalous refraction will occur if $\mu_\perp<0$ and the incident angles satisfy the inequality (29), but other elements of the permittivity and permeability tensors do not need to be negative, and the refracted waves shall be approximately left-handed waves. For H-polarized incident waves, anomalous refraction will occur if $\epsilon_\perp<0$ and the incident angles satisfy the inequality (30), but other elements of the permittivity and permeability tensors do not need to be negative, and the refracted waves will also be approximately left-handed waves.

IV. ANOMALOUS TRANSMISSION OF EVANESCENT WAVES THROUGH A SLAB OF UNIAXIALLY ANISOTROPIC LEFT-HANDED MEDIA

When an evanescent wave is transmitted through a slab of regular media with simultaneously positive permittivity and permeability, the amplitude of the transmitted wave will decay exponentially as the thickness of the slab increases. This is well known in classic electrodynamics. But recently it was shown that, when an evanescent wave incident from a surrounding isotropic regular medium is transmitted through a slab of isotropic left-handed media, the amplitude of the transmitted wave would be amplified exponentially by the LHM slab if the permittivity and permeability constants of the LHM slab are equal to the negative values of the permittivity and permeability constants of the surrounding regular medium.[8] Though evanescent waves transport no energy, this anomalous transmission of evanescent waves is a very interesting peculiar property of isotropic left-handed media and it may lead to several strange optics.[8,9] For example, lenses prepared by a slab of isotropic left-handed media will have the power to focus all Fourier components of a two-dimensional image, not only the propagating waves but also the evanescent waves which cannot be accessed by conventional imaging optics, thus all the information about the source could thereby be brought to the focus.[8,9] In this section, we discuss under what conditions anomalous transmission will occur when an evanescent wave incident from a surrounding isotropic regular medium with simultaneously positive permittivity ϵ_r and permeability μ_r is transmitted through a slab of uniaxially anisotropic left-handed media. In the following the thickness of the LHM slab will be denoted as d. The permittivity and permeability tensors of the LHM slab have been denoted in Eqs. (1) and (2). The optical axis (the z axis) of the LHM slab is normal to its interfaces with the surrounding medium, and the two interfaces are located at the planes of $z=0$ and $z=d$. Let us first assume that the incident wave in the surrounding medium is E-polarized. The electric field of the E-polarized incident evanescent wave is given by

$$\mathbf{E}_i=E_0\mathbf{e}_y\exp(ik_xx+ik_zz-i\omega t), \tag{31}$$

where k_z is imaginary implying exponential decay:

$$k_z = i\sqrt{k_x^2 - \omega^2\epsilon_r\mu_r},\omega^2\epsilon_r\mu_r < k_x^2. \tag{32}$$

At the interface between the surrounding medium and the LHM slab, some of the incident wave is reflected, and the reflected wave is given by

$$\mathbf{E}_r = rE_0\mathbf{e}_y \exp(ik_x x - ik_z z - i\omega t), \tag{33}$$

where r is the overall reflection coefficient of the slab. Some of the incident wave is transmitted into the LHM slab, and conversely a wave inside the LHM slab incident on its interfaces with the surrounding medium also experiences transmission and reflection, so the electric field of the wave inside the slab (i.e., inside the region of $0 < z < d$) should be given by[16]

$$\mathbf{E}' = aE_0\mathbf{e}_y \exp(ik_x x + ik_z' z - i\omega t) + bE_0\mathbf{e}_y \exp(ik_x x - ik_z' z - i\omega t), \tag{34}$$

where a and b are coefficients which need to be determined by boundary conditions, and k_z' is given by [see Eq. (7)]

$$k_z' = i\sqrt{\frac{\mu_\perp}{\mu_z}k_x^2 - \omega^2\epsilon_\perp\mu_\perp}. \tag{35}$$

On the far side of the LHM slab (i.e., in the region of $z > d$), the electric field of the transmitted wave is given by

$$\mathbf{E}_t = tE_0\mathbf{e}_y \exp(ik_x x + ik_z(z-d) - i\omega t), \tag{36}$$

where t is the overall transmission coefficient of the slab. By matching the electric and magnetic fields at the two interfaces between the LHM slab and the surrounding medium, the unknown coefficients in Eqs. (33), (34), and (36) can be determined, and we can get that the overall transmission through both surfaces of the LHM slab is given by

$$t = \frac{4\mu_r\mu_\perp k_z k_z' \exp(ik_z' d)}{(\mu_\perp k_z + \mu_r k_z')^2 - (\mu_r k_z' - \mu_\perp k_z)^2 \exp(2ik_z' d)}. \tag{37}$$

From Eqs. (37), (32), and (35), we can see that in general cases, when an evanescent wave is transmitted through a LHM slab, its amplitude will decay exponentially as the thickness of the slab increases, i.e., $t \propto \exp(-|k_z' d|)$ when $d \to \infty$. But if the following conditions are satisfied:

$$\mu_\perp < 0, \mu_z < 0, \epsilon_\perp < 0, \tag{38}$$

$$\epsilon_\perp\mu_z = \epsilon_r\mu_r, \mu_\perp\mu_z = \mu_r^2, \tag{39}$$

the amplitude of the transmitted evanescent wave will be amplified exponentially by the transmission process through the LHM slab. From Eqs. (37), (32), and (35), we can see that if the conditions (38) and (39) are satisfied, the overall transmission coefficient t will be equal to $\exp(|k_z' d|)$, hence the amplitude of the transmitted evanescent wave will be amplified exponentially as the thickness of the LHM slab increases. So for E-polarized evanescent waves, if the conditions (38) and (39) are satisfied, a slab of uniaxially anisotropic left-handed medium does amplify the transmitted waves. As for H-polarized evanescent waves, the overall transmission through both surfaces of a slab of uniaxially anisotropic medium can be obtained by similar procedures as noted earlier, and we can get that the overall transmission coefficient is given by

$$t' = \frac{4\epsilon_r\epsilon_\perp k_z k_z'' \exp(ik_z'' d)}{(\epsilon_\perp k_z + \epsilon_r k_z'')^2 - (\epsilon_r k_z'' - \epsilon_\perp k_z)^2 \exp(2ik_z'' d)}, \tag{40}$$

in which

$$k_z'' = i\sqrt{\frac{\epsilon_\perp}{\epsilon_z}k_x^2 - \omega^2\epsilon_\perp\mu_\perp}. \tag{41}$$

From Eq. (40), we can see that in general cases, the amplitude of the transmitted H-polarized evanescent waves will also decay exponentially as the thickness of the LHM slab increases, i.e., $t' \propto \exp(-|k_z'' d|)$ when $d \to \infty$. But if the following conditions are satisfied,

$$\epsilon_\perp < 0, \epsilon_z < 0, \mu_\perp < 0, \tag{42}$$

$$\epsilon_z\mu_\perp = \epsilon_r\mu_r, \epsilon_z\epsilon_\perp = \epsilon_r^2, \tag{43}$$

the overall transmission coefficient t' will be equal to $\exp(|k_z'' d|)$, and the amplitude of the transmitted H-polarized evanescent wave will be amplified exponentially by the transmission process through the LHM slab. So for H-polarized evanescent waves, if conditions (42) and (43) are satisfied, a slab of uniaxially anisotropic left-handed medium will enhance exponentially the transmitted waves. From (38) and (39) and (42) and (43), we note that in the presence of uniaxial anisotropy, for E- and H-polarized evanescent waves, the conditions for the occurrence of this type of anomalous transmission are different. If the LHM slab is isotropic(i.e., $\epsilon_\perp = \epsilon_z$ and $\mu_\perp = \mu_z$), the conditions (38) and (39) and (42) and (43) will both reduce to $\epsilon_\perp = \epsilon_z = -\epsilon_r$ and $\mu_\perp = \mu_z = -\mu_r$, in agreement with the results obtained in Refs. 8 and 9.

In conclusion, we have presented a detailed investigation on the characteristics of electromagnetic wave propagation in uniaxially anisotropic left-handed media. We have discussed in detail under what conditions anomalous reflection and anomalous refraction shall occur at the interface between one isotropic regular medium and another uniaxially anisotropic media and under what conditions anomalous transmission will occur when an evanescent wave is transmitted through a slab of uniaxially anisotropic left-handed media. We have shown that the characteristics of electromagnetic wave propagation in uniaxially anisotropic left-handed media are significantly different from that in isotropic left-handed media.

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