

Mesoscopic fluctuations of the density of states and conductivity in the middle of the band for disordered lattices

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The mesoscopic fluctuations of the density of electronic states and of the conductivity of two- and three-dimensional lattices with randomly distributed substitutional impurities are studied. Correlations of the levels lying above (or below) the Fermi surface, in addition to the correlations of the levels lying on opposite sides of the Fermi surface, take place at half filling due to nesting. The Bragg reflections mediate to increase static fluctuations of the conductivity in the middle of the band which change the distribution function of the conductivity at half-filling.

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Recent studies of electronic level statistics in disordered systems have shown¹⁻⁷ that the existence of repulsions between levels in the metallic phase results in a realization of the Wigner-Dyson statistics.⁸ Energy levels in a sufficiently doped $d > 2$ dimensional electron gas become uncorrelated in an insulator phase and the level distribution obeys Poisson statistics.⁹ However, the overlapping of one-particle states with different energies leads to level correlations which change their distribution.

The model of free electrons moving in a field of randomly distributed pointlike impurities has been mainly studied in previous works. A small doping of a lattice usually leads to substitution of the host atoms and does not destroy the periodicity of the Bravais lattice. Our recent studies of weak localization effects in two-dimensional (2D) square lattices and in three-dimensional (3D) cubic lattices with substitutional impurities have revealed that Bragg reflections (BR's) due to commensurability of the electronic wavelength λ and a lattice spacing a strongly change the localization picture.¹⁰⁻¹² The density of states (DOS) vanishes on the Fermi surface for noninteracting electrons in a 2D lattice and it acquires a small dip on the Fermi surface of 3D simple cubic lattices with approaching half-filling.^{10,11} Nevertheless, electron-electron ($e-e$) interactions give a positive quantum correction to the DOS,¹² which compensates the Altshuler-Aronov negative logarithmic corrections to the DOS of 2D systems.^{13,14} Therefore it is interesting to clarify how static fluctuations of physical parameters of 2D disordered lattices with nested Fermi surfaces are changed by BR's as the half-filling is approached.

In this paper we consider the effect of BR's on the level statistics and on conductivity fluctuations of 2D and 3D disordered lattices with a half-filled band. A particular characteristic of level spectra is the two-level correlation function

$$R(\epsilon, \epsilon') = \frac{1}{\rho_{od}} \{ \langle \rho(\epsilon) \rho(\epsilon') \rangle - \langle \rho(\epsilon) \rangle \langle \rho(\epsilon') \rangle \}, \quad (1)$$

where $\langle \dots \rangle$ means averaging over impurity realizations. ρ_{od} is the DOS of the d -dimensional ($d=2,3$) lattice

calculated in the Born approximation: $\rho_{02} = 2/(\pi a)^2 t \ln(\epsilon_F \min\{\tau_0, 1/|\epsilon|\})$, and $\rho_{03} = \text{const} - (2/\pi^2 t^{3/2} a^3) \sqrt{|\epsilon| - t}$ (at $|\epsilon| \approx t$) with t and τ_0 being the tunneling integral for nearest-neighbor sites and the relaxation time for elastic impurity scattering, respectively.

By using the formula for the DOS,

$$\rho(\epsilon) = \frac{1}{2\pi i} \int \frac{d\mathbf{r}}{v} \{ G_A(\mathbf{r}, \mathbf{r}'; \epsilon) - G_R(\mathbf{r}, \mathbf{r}'; \epsilon) \},$$

which relates $\rho(\epsilon)$ to the retarded (G_R) and advanced (G_A) Green's functions, $R(\epsilon, \epsilon')$ can be expressed as

$$\begin{aligned} R(\epsilon, \epsilon') = & - \left(\frac{s}{2\pi v \rho_{od}} \right)^2 \int d\mathbf{r} \int d\mathbf{r}' \{ \langle G_A(\mathbf{r}, \mathbf{r}; \epsilon) G_A \\ & \times (\mathbf{r}', \mathbf{r}'; \epsilon') \rangle + \langle G_R(\mathbf{r}, \mathbf{r}; \epsilon) G_R(\mathbf{r}', \mathbf{r}'; \epsilon') \rangle \\ & - \langle G_R(\mathbf{r}, \mathbf{r}; \epsilon) G_A(\mathbf{r}', \mathbf{r}'; \epsilon') \rangle \\ & - \langle G_A(\mathbf{r}, \mathbf{r}; \epsilon) G_R(\mathbf{r}', \mathbf{r}'; \epsilon') \rangle \\ & - 4 \text{Re} \langle G_R(\mathbf{r}, \mathbf{r}; \epsilon) \rangle \text{Re} \langle G_R(\mathbf{r}', \mathbf{r}'; \epsilon') \rangle \}, \quad (2) \end{aligned}$$

where v is the "volume" and s is the factor of spin degeneracy. Far from half-filling the correlators RA and AR in Eq. (2) give only contributions to the two-level correlation function.² However, the existence of electron-hole symmetry for nested Fermi surfaces gives rise to considerable contributions of the RR and AA correlators to $R(\epsilon, \epsilon')$ in Eq. (2). The Fermi surface of a d -dimensional lattice with the energy spectrum of $\epsilon(\mathbf{p}) = t \sum_{i=1}^d [1 - \cos(p_i a)]$ becomes nested at half-filling, when $\epsilon_F = dt$, which permits an electron-hole symmetry, $\epsilon(\mathbf{p} + \mathbf{Q}) - \epsilon_F = -[\epsilon(\mathbf{p}) - \epsilon_F]$, with respect to the nesting vectors $\mathbf{Q} = \{\pm \pi/a, \pi/a\}$ for 2D and $\mathbf{Q} = \{\pm \pi/a, \pm \pi/a, \pi/a\}$ for 3D lattices. New singular impurity blocks take place at half-filling with particle-hole symmetry, which are referred to as the π -diffuson (D_π) and the π -Cooperon (C_π). The π -diffuson (π -Cooperon) has a dif-

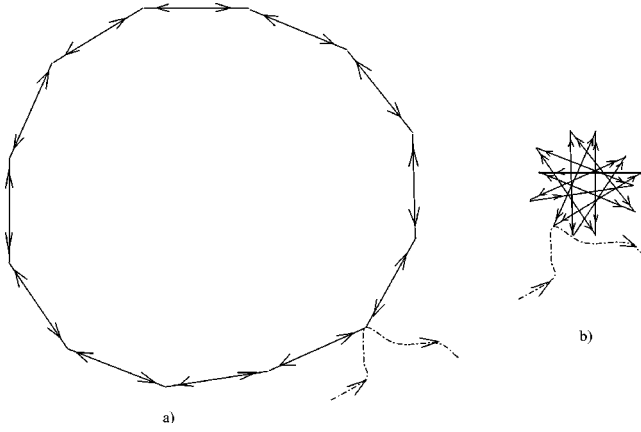


FIG. 1. (a) Self-intersecting trajectory due to multiple scattering on impurities with small tilted angles α_i in the i th act of scattering. (b) The same trajectory that is drawn in (a) with the exception that each scattering is accompanied by BR's, resulting in angles $\alpha_i + \pi$. In both figures the magnitudes of the velocity vectors are chosen to be the same.

fusion pole at large $\propto \mathbf{Q}$ momenta differences (total momenta) and small total energies of the electron and the hole (of two electrons).^{10,11}

As is known, an interference between the self-intersecting trajectories leads to weak localization of an electronic wave

$$C_\pi(\mathbf{q}, \epsilon + \epsilon') = \frac{1}{2\pi\rho_0 d \tau_0} \left\{ \theta(-\epsilon\epsilon') + \frac{\theta(\epsilon\epsilon')}{(1 - i\tau_0|\epsilon + \epsilon'| + \gamma\tau_0)^2 + \frac{2}{d}(ql)^2 - 1} \right\}, \quad (3)$$

where the phenomenological parameter γ is introduced to signify an inelastic processes rate. The π -diffuson $D_\pi(\mathbf{q}, \epsilon + \epsilon')$ is also expressed by Eq. (3) with the exception that \mathbf{q} will now be the momentum difference of a particle and a hole with the accuracy of the nesting vector \mathbf{Q} . Notice that the “normal” Cooperon and diffuson blocks depend on the difference of the energies ϵ and ϵ' instead of sum in Eq. (3).

New diagrams (see Fig. 2) appear at half-filling due to BR's which give contributions to $R(\epsilon, \epsilon')$ in addition to those coming from normal scattering in a diffusive system. The study of the diffusive regime assumes that the linear

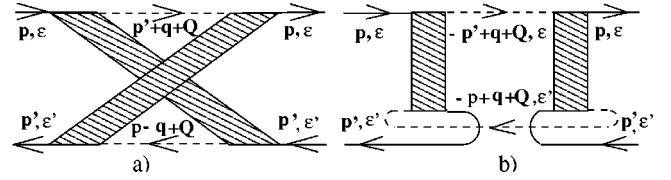


FIG. 2. First-order additional corrections to the DOS correlator $R(\epsilon, \epsilon')$ in the metallic regime due to BR's from (a) π -diffuson and (b) π -Cooperon blocks.

function (see Ref. 13). The electron passing the loop [e.g., in Fig. 1(a)], which is formed on the trajectory due to multiple scattering on impurities with small tilted angles, clockwise and counterclockwise, reduces its transmission probability. However, for a nested Fermi surface each act of impurity scattering is accompanied by BR's with large ($\sim \pi$) scattering angle [see Fig. 1(b)], which strongly changes the weak-localization picture of a free electron gas. It is worthwhile to notice that the electron-hole symmetry effects in the 2D case strongly differ from those in the 1D case, where BR's act as a destructive factor of localization and result in big effects [like the Dyson singularity in the 1D DOS (Ref. 15)] only in the absence of forward scattering (see, e.g., Ref. 16), reversing the backward scattering to a forward one.

The expression for the two-particle impurity block $C_\pi(\mathbf{q}, \epsilon + \epsilon')$ in the particle-particle channel due to umklapp scatterings with the particle energies ϵ and ϵ' is given as¹⁰⁻¹²

length L of the system is larger than the elastic mean free path l and smaller than the localization length, the latter of which is exponentially large. By considering the energy scales as the Thouless energy $E_c = \hbar D/L^2$ with diffusion coefficient $D = v_F^2 \tau_0 / d$ of a d -dimensional system and average level spacing $\Delta = 1/\rho_0 d L^d$, it is possible to see that $E_c/\Delta = \hbar \rho_0 d D L^{d-2} = [\sigma/(e^2/\hbar)] L^{d-2} = g$ is a dimensionless conductance. Since $g > 1$ for the metallic case, the diffusive system can be characterized by the condition $\Delta \ll E_c \ll \hbar/\tau_0$. By summing up contributions of the diagrams in Fig. 2 the correlator $R(\epsilon, \epsilon')$ is expressed as

$$R(\epsilon, \epsilon') = \frac{(s\Delta\tau_0)^2}{\beta\pi^2} \text{Re} \sum_{\mathbf{q}} \left\{ \frac{\theta(-\epsilon\epsilon')}{\tau_0^2(-i|\epsilon - \epsilon'| + Dq^2 + \gamma)^2} - \frac{\theta(\epsilon\epsilon')}{\left[(1 - i\tau_0|\epsilon + \epsilon'| + \gamma\tau_0)^2 + \frac{2}{d}(ql)^2 - 1 \right]^2} \right\}, \quad (4)$$

where $q^2 = \sum_{\alpha=1}^d q_{\alpha}^2$ and $q_{\alpha} = (2\pi/aN_{\alpha})n_{\alpha}$ with $(-N_{\alpha}/2) < n_{\alpha} \leq (N_{\alpha}/2)$, and β is the Dyson index classifying the orthogonal, unitary, and symplectic ensembles with $\beta=1, 2$, and 4, respectively.⁸ Far from half-filling where BR's are suppressed, only the first term in the brackets of Eq. (4) contributes to R .²

In the ergodic regime $|\epsilon \pm \epsilon'| \ll E_c$, only the $\mathbf{q}=0$ term needs to be retained in the summation over \mathbf{q} in Eq. (4). So

$$R(\epsilon, \epsilon') = -\frac{(s\Delta)^2}{\beta\pi^2} \text{Re} \left\{ \frac{\theta(-\epsilon\epsilon')}{(\epsilon - \epsilon' + i\gamma)^2} - \frac{\theta(\epsilon\epsilon')}{4(\epsilon + \epsilon' + i\gamma)^2} \right\}, \quad (5)$$

which shows that two levels on the opposite sides of the Fermi surface with energy difference $|\epsilon - \epsilon'| > \gamma$ repel each

other and they attract at an energy difference of $|\epsilon - \epsilon'| < \gamma$. On the other hand, two levels with energies $|\epsilon + \epsilon'| > \gamma$ on the same side of the Fermi surface attract each other and they repel for energies $|\epsilon + \epsilon'| < \gamma$. Furthermore, the attraction of two levels on the Fermi surface ($\epsilon = \epsilon' = 0$) weakens with approaching half-filling and the correlator $R(0,0)$ reaches its 3/4 value at half-filling. Notice that far from half-filling the levels lying only on the opposite sides of the Fermi surface interact with each other. An additional interaction of the levels on the same side of the Fermi surface appears due to BR's at half-filling.

The correlator of two levels centered at ϵ_0 and ϵ'_0 and averaged in an energy interval of $E \leq W$, where $W = 2dt$ is the bandwidth, can be obtained by integrating $R(\epsilon, \epsilon')$ given by Eq. (4) over the energy interval E :

$$\begin{aligned} \langle \delta\rho_{\epsilon_0}(E) \delta\rho_{\epsilon'_0}(E) \rangle &= \int_{\epsilon_0 - E/2}^{\epsilon_0 + E/2} d\epsilon \int_{\epsilon'_0 - E/2}^{\epsilon'_0 + E/2} d\epsilon' R(\epsilon, \epsilon') \\ &= \frac{s^2}{\beta\pi^2 \rho_{0d}^2} \text{Re} \left\{ \ln \frac{\gamma^2 [(\epsilon_0 - \epsilon'_0)^2 - (E + i\gamma)^2]}{[\epsilon_0^2 - (E/2 + i\gamma)^2][\epsilon_0'^2 - (E/2 + i\gamma)^2]} \right. \\ &\quad \left. - \frac{1}{4} \ln \frac{\gamma^2 [(\epsilon_0 + \epsilon'_0)^2 - (E + i\gamma)^2]}{[\epsilon_0^2 - (E/2 + i\gamma)^2][\epsilon_0'^2 - (E/2 + i\gamma)^2]} \right\} \quad (|\epsilon_0|, |\epsilon'_0| \leq E/2) \quad (6) \end{aligned}$$

$$\begin{aligned} &= \frac{s^2}{\beta\pi^2 \rho_{0d}^2} \text{Re} \ln \left[1 - \frac{E^2}{(|\epsilon_0| + |\epsilon'_0| + i\gamma)^2} \right] \\ &\quad \times \left\{ \theta(-\epsilon_0\epsilon'_0) - \frac{1}{4} \theta(\epsilon_0\epsilon'_0) \right\} \quad (|\epsilon_0|, |\epsilon'_0| \geq E/2). \quad (7) \end{aligned}$$

Far from half-filling the second contribution in the brackets of Eqs. (6) and (7) vanishes. This case corresponds to the continuum model.² However, the fact that interactions of levels lying on opposite sides of the Fermi surface do give contributions to $\langle \delta\rho_{\epsilon_0}(E) \delta\rho_{\epsilon'_0}(E) \rangle$ has not been taken into account in Ref. 2. As is seen from Eqs. (6) and (7) the two-level correlation function strongly depends on the center-of-energy strip E even if $\epsilon_0 = \epsilon'_0$. A logarithmical energy dependence of the variance $\langle [\delta\rho_{\epsilon_0}(E)]^2 \rangle$ takes place for a strip centered around the Fermi level:

$$\begin{aligned} \langle [\delta\rho_{\epsilon_0=0}(E)]^2 \rangle &= \frac{2s^2(1-f)}{\beta\pi^2 \rho_{0d}^2} \ln \frac{E}{\gamma} \quad (E/2 < \gamma < E) \\ &= -\frac{2s^2(1-f)}{\beta\pi^2 \rho_{0d}^2} \ln \frac{E}{4\gamma} \quad (\gamma < E/2), \quad (8) \end{aligned}$$

where f is the parameter characterizing the BR's: $f=1/4$ at half-filling and $f=0$ far from commensurate points. Accord-

ing to Eq. (8) the Dyson repulsion of levels for energies $E/2 < \gamma < E$ turns to an attraction of levels for large energy distances $E > 2\gamma$.

For the diffusive limit, when $E \gg E_c$, summing over \mathbf{q} in Eq. (4) can be replaced by integration. As a result we get the following expressions for the DOS variance $\langle [\delta\rho_{\epsilon_0=0}(E)]^2 \rangle$:

$$\begin{aligned} \langle [\delta\rho_{\epsilon_0=0}(E)]^2 \rangle_{dif} &= \frac{(\sqrt{2}-1)(1-f)s^2}{6\pi^3 \beta \rho_{03}^2} \left(\frac{E}{E_c} \right)^{3/2} \quad (d=3) \\ &= -\frac{(1-f)s^2}{4\pi^3 \beta \rho_{02}^2} (E\tau_0) \left(\frac{E}{E_c} \right) \quad (d=2). \quad (9) \end{aligned}$$

In the $d=2$ case linear contributions in E to $\langle [\delta\rho_{\epsilon_0=0}(E)]^2 \rangle$ in Eq. (9) are completely canceled and the fluctuations are not as strong as in 3D systems. This seems to be connected with the localized character of levels in 2D systems.

Far from half-filling when the Fermi surface is approximately spheric the variance of fluctuations in static conductivity is designated by the diagrams given, e.g., in Fig. 4 of

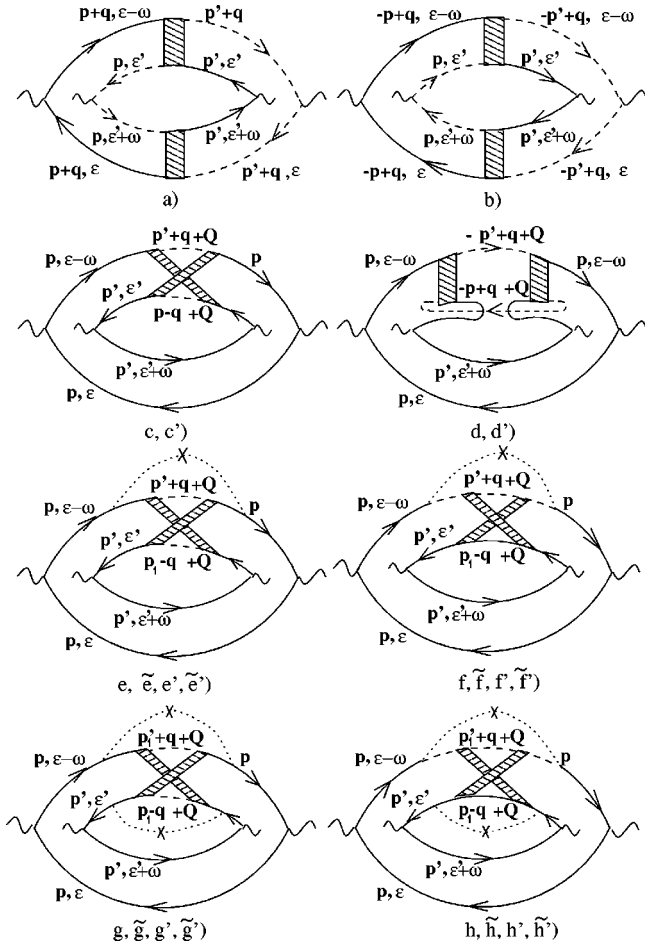


FIG. 3. The diagrams which contribute to the conductivity variance due to BR's. The diagrams denoted by primes differ from the presented ones through the direction of the electron lines. (e), (f) are symmetric to (e), (f) with respect to the single-impurity line, and (g), (h) are obtained from (g), (h) by interchanging the straight and dashed lines under single-impurity lines. Diagrams similar to (e)–(h') exist also in the Cooper channel which are produced from (d, d').

Ref. 2. These diagrams have been calculated also in Refs. 3 and 17,18 for continuum and isotropic systems. Other contributions to the conductivity variance exist in the lattice model under consideration at the commensurate points due to BR's which come from the diagrams given in Fig. 3. The total contributions to the conductance fluctuations due to normal and umklapp scatterings on impurities are

$$\langle \delta G_{\alpha\beta} \delta G_{\gamma\mu} \rangle = G_D^2 \{ \delta_{\alpha\gamma} \delta_{\beta\mu} + \delta_{\alpha\mu} \delta_{\beta\gamma} \} + G_\rho^2 \delta_{\alpha\beta} \delta_{\gamma\mu}. \quad (10)$$

Here we followed the notation in Ref. 24, where the contributions from the diffusion coefficient and the DOS fluctuations to the conductance variance $\langle \delta G_{\alpha\beta} \delta G_{\gamma\mu} \rangle$ were denoted by the temperature-dependent coefficients G_D^2 and G_ρ^2 , respectively. The expressions for G_D^2 and G_ρ^2 can be presented as

$$G_D^2 = \frac{s^2}{\beta} \left(\frac{e^2 E_c}{\hbar \pi} \right)^2 \int \frac{d\epsilon}{2T} f \left(\frac{\epsilon}{2T} \right) \times \sum_{\mathbf{q}} \left\{ \frac{1}{|Dq^2 - i\epsilon|^2} + \frac{\tau_0^2}{\left| (1 - i\epsilon\tau_0)^2 + \frac{2}{d}(ql)^2 - 1 \right|^2} \right\} \quad (11)$$

and

$$G_\rho^2 = \frac{s^2}{\beta} \left(\frac{e^2 E_c}{\hbar \pi} \right)^2 \int \frac{d\epsilon}{2T} f \left(\frac{\epsilon}{2T} \right) \text{Re} \sum_{\mathbf{q}} \frac{1}{(Dq^2 - i\epsilon)^2}, \quad (12)$$

where $f(x) = (x \coth x - 1) / \sinh^2 x$. The second term in the brackets in Eq. (11) comes from the diagrams given in Fig. 3 due to π scatterings. However, BR's give no contribution to G_ρ^2 . According to the Thouless picture,¹⁹ only one-electron states lying in an interval of E_c centered on the Fermi level give contributions to the conductivity. Contributions to G_ρ^2 from the levels correlated on the same side of the Fermi level seem to cancel each other.

At small temperatures $T \ll E_c$ the coefficients G_D^2 and G_ρ^2 do not depend on temperature:

$$G_\rho^2 = (s^2/\beta)(e^2/\pi^3\hbar)^2 b_d, \quad G_D^2 = (1+f)G_\rho^2, \quad (13)$$

where b_d is a constant, which depends on the system dimension. In the case when $T \gg E_c$ the values of G_D^2 and G_ρ^2 strongly differ from each other and depend on temperature:

$$G_\rho^2 = (s^2/\beta)(e^2/2\pi\hbar)^2 a_d (E_c/T)^{(4-d)/2} \quad (14)$$

and

$$G_D^2 = \frac{1}{2} (1+f) G_\rho^2 \quad (d=3) = (1+f)(s^2/\beta) \times (e^2/2\pi\hbar)^2 \frac{E_c}{T} \ln \frac{T}{\max\{E_c, \gamma\}} \quad (d=2), \quad (15)$$

where a_d is some coefficient.² As can be seen from Eqs. (14) and (15) the main contribution to the conductance variance comes from fluctuations of the diffusion coefficient, which are intensified at half-filling.

Multiplication of the variance by the additional prefactor $(1+f)$ means that umklapp scatterings of electrons on impurities change the distribution function of G . In the language of random matrix theory, an insulating phase of a disordered system can be prescribed by an ensemble of $N \times N$ diagonal matrices with random elements. The existence of off-diagonal terms in $N \times N$ matrices due to overlapping states of different energies in diffusive systems transforms the distribution function from a Poisson function to a Wigner-Dyson one. Scattering on the impurities with large momentum transfer gives additional contributions to the off-diagonal matrix elements. Therefore, the change in the distribution function due to umklapp scattering is reasonable. Two-level correlations are sensitive to whether the levels attract or repel each other and to the relative position of these levels, either on the same side or on the opposite sides of the

Fermi surface. However, the conductance fluctuations seem not to be sensitive to the character of the level interactions and level positions; both attraction and repulsion give similar contributions to $\langle \delta G_{\alpha\beta} \delta G_{\gamma\mu} \rangle$. This fact seems to be the reason why Eqs. (8) and (9) for $\langle [\delta \rho_{\epsilon_0}(E)]^2 \rangle$ and Eqs. (13)–(15) contain the factor f coming from BR's in different way.

Starting from the fact that the statistical properties of energy levels in disordered systems are determined by the symmetries of the Hamiltonian, Zirnbauer has recently classified²⁰ all possible universality classes of random matrix ensembles according to Cartan's families of symmetric spaces. He showed in Ref. 20 that, apart from the three existing Wigner-Dyson random matrix classes, there exist three chiral symmetric classes and four Bogolyubov–de Gennes classes. The subdivision in the different classes occurs due to the time-reversal (TR) or spin-rotational (SR) invariances. To clarify to what class of the universal random matrix ensembles our model belongs, we start from the impurity Hamiltonian written by means of second quantization operators $\psi_{\sigma}(\mathbf{r})$ as²¹ $H_{imp} = \sum_{\sigma} \int d\mathbf{r} \psi_{\sigma}^{\dagger}(\mathbf{r}) U_{imp}(\mathbf{r}) \psi_{\sigma}(\mathbf{r})$. Here, $U_{imp}(\mathbf{r}) = \sum_{\mathbf{l}} V_{eff}(\mathbf{r} - \mathbf{r}_{\mathbf{l}}) = \sum_{\mathbf{l}} [V_{imp}(\mathbf{r} - \mathbf{r}_{\mathbf{l}}) - V_a(\mathbf{r} - \mathbf{r}_{\mathbf{l}})]$; i.e., an electron around the impurity located at site \mathbf{l} “feels” the potential $V_{eff}(\mathbf{r} - \mathbf{r}_{\mathbf{l}})$, which is the difference of the impurity potential $V_{imp}(\mathbf{r} - \mathbf{r}_{\mathbf{l}})$ and the host atom potential $V_a(\mathbf{r} - \mathbf{r}_{\mathbf{l}})$. When the doping atoms belong to a chemical element group neighboring the one of the host atoms, $V_{eff}(\mathbf{r} - \mathbf{r}_{\mathbf{l}})$ vanishes at $\mathbf{r} = \mathbf{r}_{\mathbf{l}}$, and it differs from zero on the tail, since a typical case of a perturbing field in a solid is the electrostatic field around a charged impurity. By expanding the ψ operators in H_{imp} over the orthogonal Wannier functions $w(\mathbf{r} - \mathbf{r}_{\mathbf{l}})$ as $\psi_{\sigma}(\mathbf{r}) = \sum_{\mathbf{n}} c_{\mathbf{n},\sigma} w(\mathbf{r} - \mathbf{r}_{\mathbf{l}})$, the impurity Hamiltonian can be written in the “site” representation, $H_{imp} = \sum_{\sigma, \langle \mathbf{n}, \mathbf{m} \rangle} t_{\mathbf{n},\mathbf{m}} c_{\mathbf{n},\sigma}^{\dagger} c_{\mathbf{m},\sigma}$, which contributes to the hopping between nearest-neighbor sites. The last form of the Hamiltonian shows that the disorder studied in the problem corresponds to off-diagonal disorder. It is worthwhile to point out that another model of randomness—namely, diagonal disorder—describes the idealized situation of a binary disordered d -dimensional substitutional alloy with varying values of the chemical potential. Diagonal disorder can be realized also in 1D disordered systems, where the localization length is comparable with the mean free path l and leads to a variation of the chemical potential from site to site due to the absence of the diffusive regime. The exponentially large value of the localization length $\xi_{loc}, \xi_{loc} \sim l \exp(\epsilon_F \tau_0)$, however, allows realization of the metallic regime for $L \ll \xi_{loc}$ in 2D weakly disordered systems. By applying the crossed-

diagram technique²¹ to average over the randomness in a metallic 2D system with a definite value of the chemical potential, we neglect the energy fluctuations at the atomic sites, since an elastic scattering off impurities is considered and the δ -correlated white-noise potential with $\langle V_{eff} \rangle = 0$ is used in the calculations. Following Oppermann and Wegner,^{22,23} the lattice can be subdivided into two sublattices, so that the Hamiltonian contains the matrix elements connecting only different sublattices and becomes purely off-diagonal. The block off-diagonal Hamiltonian anticommutes with the Pauli matrix σ_3 . Such a kind of symmetry is commonly referred to as chiral symmetry.^{20,24} The one-particle Green's functions of the model contain the chiral symmetry at the center of the band, which leads to an unusual behavior of the DOS and the conductivity as the middle of the band is approached. Therefore, the model studied in our paper corresponds to chiral symmetry in the presence of both TR and SR invariances.

Our study of the fluctuations of the DOS and the conductance have been restricted to the case of noninteracting electrons in a random potential. It is easy to show that at $T=0$ interactions do not affect the numerical value of the universal conductance fluctuations and of the DOS fluctuations to leading order in $(p_F l)^{-1}$ in the diffusive regime. This fact has been explicitly confirmed in Ref. 25 for the 2D electron gas model which is identified with our model far from half-filling, where the effects of periodicity can be incorporated into the electron effective mass. At finite temperature, interactions introduce a finite inelastic scattering rate, which cuts off the diffusion poles of the impurity ladder blocks.

The recently fabricated C_{60} -based novel field-effect transistor (FET) devices²⁶ allow one to control the band filling by changing the gate potential. A single crystal of C_{60} is an insulator and has a band gap of approximately 2 eV. The band filling, however, can be changed by either a chemical doping²⁷ or by applying a gate potential in a FET structure.²⁶ Half-filling is reached for three electron dopings per C_{60} molecule. Notice that the recent researches on C_{60} -based FET structure are focused on the superconducting properties of this material, and therefore the single crystal of C_{60} is chosen in Ref. 26 to be ultraclean. The possibility of doping the fullerenes with substitutional impurities, while preserving the periodicity of the Bravais lattice, will allow the observation of the commensurability effects on the mesoscopic fluctuations at half-filling in these devices.

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