Effect of magnetic field on excitons in bulk and heterostructure semiconductors containing disorder

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A theory of exciton formation is developed for double-layer semiconductor systems in which electrons and holes are spatially separated by a potential barrier in the presence of a magnetic field. The effect of disorder due to interface roughness of the double-layer structures is included. Use is made of a lattice-gas model to calculate electron, hole, and exciton densities. Kinetic processes are neglected because they are negligible when strong disorder is present in the system. The theory is applied to type-II AlAs/GaAs quantum wells and to bulk GaAs in which electrons and holes are spatially separated. It is predicted that the formation of excitons in spatially separated electron-hole systems is enhanced by the presence of a magnetic field.

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I. INTRODUCTION

Electron-hole coupling in double-layer (DL) systems has been studied with much interest in semiconductor quantum wells (QW's) in which quasi-two-dimensional electrons and holes are spatially separated by a potential barrier.[1–6] Magnetoexciton spectra have been studied by several research groups.[6,7] Lerner and Lozovik[8] first calculated the excitation spectrum and the correlation energy of a two-dimensional electron-hole system with a strong transverse magnetic field. Lozovik and Rubinskii[9] also calculated the exciton spectrum in a strong magnetic field for different Landau levels for arbitrary QW separations. Butov *et al.* studied the indirect exciton luminescence in type-II AlAs/GaAs QW's, and showed that the probability of generating excitons is increased by applying a magnetic field.[6] A theory of magnetoexcitons taking into account valence-band mixing effects in QW's was developed by Bauer and Ando.[10] Dzyubenko and Bauer[5] calculated the low-temperature transport of a dilute exciton gas in type-II AlAs/GaAs QW's. Experimental evidence[6] was reported for a stable excitonic ground state in a strong magnetic field which favors the stability of the excitonic phase.[8,11] The search for a stable excitonic phase in these systems is motivated by the possibility of Bose-Einstein condensation of excitons in QW's.[2] The critical conditions for exciton condensation are improved by applying a magnetic field.[6]

The aim of the present paper is to study the effect of a magnetic field on the formation of electron-hole pairs (excitons) in DL systems with disorder. We consider the case of strong disorder in which the maximum value of the random potential is greater than the exciton binding energy. The question addressed in this paper relates to the influence of a magnetic field on these systems. The quantum-mechanical problem of charged particles in a strong random disordered potential is very complicated even for noninteracting particles. To study such complex systems in the presence of an interaction which forms bound states, we used an effective lattice-gas model where the electron-hole system is divided into unit cells.[12] The lattice-gas Hamiltonian includes the interaction of carriers within each unit cell and between the unit cells. We have neglected kinetic processes such as hopping or tunneling of particles to other sites, because kinetic processes have negligible effects in the presence of strong disorder. We restrict our formulation to magnetic fields where the linear Zeeman terms are important. Therefore, nonlinear terms (B^2) are neglected in the present calculations. We have applied our theory to type-II AlAs/GaAs QW's. It is found that the exciton density increases with magnetic field for excitons with total spin projection $S_z=0$, whereas it decreases for $S_z=\pm 1$ excitons. However, the total exciton density increases with increasing magnetic field. There is a qualitative agreement between our theory and experimental results. Numerical calculations are also performed for bulk GaAs' where electrons and holes can be spatially separated for example by the application of an external electric field. In this case, the density of $S_z=0$ excitons decreases whereas the density of $S_z=1$ excitons increases with magnetic field. The total exciton density once again increases with magnetic field.

II. THEORY

The random potential barrier in QW's may come from disorder which is usually due to interface roughness and thickness variations of the QW's unavoidable in the course of fabrication. In a strong disorder, excitons are localized. The separation in space of the electron and hole systems greatly enhances their recombination lifetime. We use a lattice-gas model in which the lattice-gas within the system is divided into N unit cells, and the lth unit cell contains $n_e^{\pm}(l)$ electrons and $n_h^{\pm}(l)$ holes. The signs "+" and "−" correspond to spin-up and spin-down, respectively. Here we consider the case where each unit cell can have no more than one electron and one hole. In the presence of a magnetic field, the lattice-gas model Hamiltonian is written as

$$H=\sum_{l,\pm}\left[V_e(l)\pm\frac{1}{2}g_e\mu_B B\right]n_e^{\pm}(l)+\sum_{l,\pm}\left[V_h(l)\mp\frac{1}{2}g_h\mu_B B\right]n_h^{\pm}(l)-\sum_{l;\alpha,\beta=\pm}E_0^{\alpha\beta}n_e^{\alpha}(l)n_h^{\beta}(l)+\sum_{l,l';a=e,h;\alpha,\beta=\pm}E_r^{\alpha\beta}n_a^{\alpha}(l)n_a^{\beta}(l')-\sum_{l,l';\alpha,\beta=\pm}E_b^{\alpha\beta}[n_e^{\alpha}(l)n_h^{\beta}(l')+n_h^{\beta}(l)n_e^{\alpha}(l')], \quad (1)$$

where $n_a^{\pm}(l)$ takes values 0 or 1. Here $a=e$ for electrons and $a=h$ for holes. $V_a(l)$ is an effective random potential at the unit cell l. $E_0^{\alpha\beta}$ is the binding energy for an electron-hole pair occupying the same unit cell. $E_b^{\alpha\beta}$ is the Coulomb binding energy between an electron (hole) in the unit cell l and a hole (electron) in the unit cell $l+1$. $E_r^{\alpha\beta}$ is the Coulomb repulsion energy between an electron (hole) in the unit cell l and an electron (hole) in the unit cell $l+1$. g_a are the Lande g factors, μ_B is the Bohr magneton, and B is the magnetic field. Here we consider the Zeeman splitting of the electron-hole energy levels and we restrict our formulation to magnetic fields where nonlinear terms (B^2) can be neglected. The binding, attractive, and repulsive energies between electrons and holes are taken to be independent of the applied magnetic field. We consider the temperature T of the system to be small compared to $E_0^{\alpha\beta}$. This is a necessary condition to form excitons. The energies are measured from the minimum value of the disorder potential which is taken as the origin of the energy scale.

Following the method of Ref. 12 and using Eq. (1) we have calculated the average free carrier (electrons or holes) density $n_e(B)$ and the average exciton density $n_{ex}^{\alpha\beta}(B)$ formed from an electron with spin α and a hole with spin β as

$$n_e(B)=\frac{1}{\beta^2 V_0^e V_0^h}\int_0^1\int_0^1 \frac{dx_1 dx_2}{x_1 x_2}\,\frac{(z^2-F)\Phi_e+2z\Theta_e}{z(z^2+F)}, \quad (2)$$

$$n_{ex}^{\alpha\beta}(B)=\frac{1}{\beta^2 V_0^e V_0^h}\int_0^1\int_0^1 \frac{dx_1 dx_2}{x_1 x_2}\,\frac{(z^2-F)\Phi_{ex}^{\alpha\beta}+2z\Theta_{ex}^{\alpha\beta}}{z(z^2+F)}, \quad (3)$$

where $F=\Sigma_{\alpha,\beta=\pm}F^{\alpha\beta}$, $\Theta_e=\Sigma_{\alpha,\beta=\pm}\Theta_e^{\alpha\beta}$, $y=e^{\beta\mu}$, $\varepsilon_0^{\alpha\beta}=e^{\beta E_0^{\alpha\beta}}$, $\varepsilon_b^{\alpha\beta}=e^{\beta E_b^{\alpha\beta}}$, $\varepsilon_r^{\alpha\beta}=e^{-\beta E_r^{\alpha\beta}}$, $x_1^{\pm}=\exp(\mp\beta[\frac{1}{2}g_e\mu_B B])x_1$, $x_2^{\pm}=\exp(\pm\beta[\frac{1}{2}g_h\mu_B B])x_2$, $x_1=e^{-\beta V_e}$, $x_2=e^{-\beta V_h}$, $\beta=1/(k_B T)$, and μ is the chemical potential. V_0^e and V_0^h are the maximum values of disorder potential for electrons and holes respectively. $z(V_e,V_h)=\Sigma_{\alpha,\beta=\pm}\frac{1}{4}[1+(x_1^{\alpha}+x_2^{\beta})y+(x_1^{\alpha}x_2^{\beta})y^2\varepsilon_0^{\alpha\beta}]$, $\Phi_e=\Sigma_{\alpha,\beta=\pm}\frac{1}{4}[x_1^{\alpha}y+\varepsilon_0^{\alpha\beta}x_1^{\alpha}x_2^{\beta}y^2]$, $\Phi_{ex}^{\alpha\beta}=\frac{1}{4}\varepsilon_0^{\alpha\beta}x_1^{\alpha}x_2^{\beta}y^2$, and

$$F^{\alpha\beta}=\frac{1}{4}[(\varepsilon_b^{\alpha\beta}\varepsilon_r^{\alpha\beta})^4-1](\varepsilon_0^{\alpha\beta}x_1^{\alpha}x_2^{\beta})^2y^4 + \frac{1}{4}[(\varepsilon_r^{\alpha\beta})^2-1][(x_1^{\alpha})^2+(x_2^{\beta})^2]y^2 + \frac{2}{4}[(\varepsilon_b^{\alpha\beta})^2-1]x_1^{\alpha}x_2^{\beta}y^2+\tfrac{2}{4}[(\varepsilon_b^{\alpha\beta}\varepsilon_r^{\alpha\beta})^2-1] \times[(x_1^{\alpha})^2x_2^{\beta}+x_1^{\alpha}(x_2^{\beta})^2]\varepsilon_0^{\alpha\beta}y^3,$$

$$\Theta_e^{\alpha\beta}=\frac{1}{4}[(\varepsilon_b^{\alpha\beta}\varepsilon_r^{\alpha\beta})^4-1](\varepsilon_0^{\alpha\beta}x_1^{\alpha}x_2^{\beta})^2y^4 + \frac{1}{4}[(\varepsilon_r^{\alpha\beta})^2-1](x_1^{\alpha})^2y^2+\frac{1}{4}[(\varepsilon_b^{\alpha\beta})^2-1]x_1^{\alpha}x_2^{\beta}y^2 + \frac{1}{4}[(\varepsilon_b^{\alpha\beta}\varepsilon_r^{\alpha\beta})^2-1][2(x_1^{\alpha})^2x_2^{\beta}+x_1^{\alpha}(x_2^{\beta})^2]\varepsilon_0^{\alpha\beta}y^3,$$

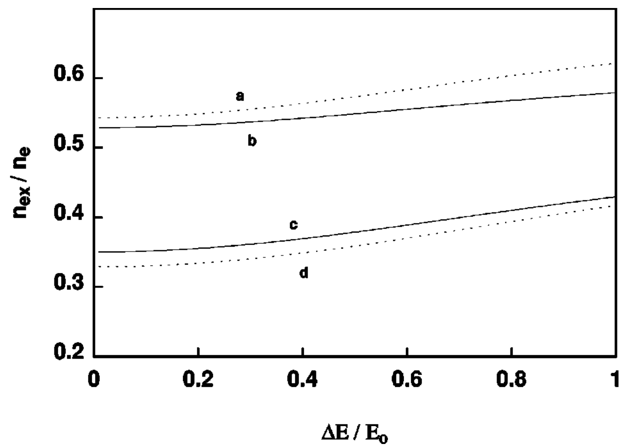

FIG. 1. Relative exciton density n_{ex}/n_e as a function of $\Delta E/E_0$ for type-II AlAs/GaAs QWs. The meaning of the labels a,b,c, and d is discussed in the text.

$$\Theta_{ex}^{\alpha\beta}=\frac{1}{4}[(\varepsilon_b^{\alpha\beta}\varepsilon_r^{\alpha\beta})^4-1](\varepsilon_0^{\alpha\beta}x_1^{\alpha}x_2^{\beta})^2y^4+\frac{1}{4}[(\varepsilon_b^{\alpha\beta}\varepsilon_r^{\alpha\beta})^2-1] \times[(x_1^{\alpha})^2x_2^{\beta}+x_1^{\alpha}(x_2^{\beta})^2]\varepsilon_0^{\alpha\beta}y^3.$$

The total number of excitons is written as $n_{ex}(B)=\Sigma_{\alpha,\beta=\pm}n_{ex}^{\alpha\beta}(B)$. Note that $n_{ex}^{+-}+n_{ex}^{-+}$ gives excitons with total spin projection $S_z=0$, n_{ex}^{2+} gives excitons with spin $S_z=1$, and n_{ex}^{2-} gives excitons with spin $S_z=-1$. To obtain the above expressions, we have assumed that the interaction between carriers within each lattice cell is stronger than that between the cells (i.e., $E_0^{\alpha\beta}\gg E_a^{\alpha\beta}$, and $E_0^{\alpha\beta}\gg E_r^{\alpha\beta}$). The correlation effect due to disorder has been neglected. Therefore, the correlation length is smaller than the lattice spacing.

III. RESULTS AND DISCUSSIONS

Equation (2) is used to calculate the chemical potential for a given carrier concentration $n_e=n_h$. This chemical potential is then used in Eq. (3) to calculate the relative exciton density (n_{ex}/n_e) as a function of $\Delta E/E_0$ [$\Delta E=(|g_e|+|g_h|)\mu_B B$] for a type-II AlAs/GaAs QW. The numerical results are presented in Fig. 1. The parameters are taken as $g_e=1.9$, $g_h=2.3$,[4] $E_0^{\alpha\beta}=E_0$, $E_b^{\alpha\beta}=E_r^{\alpha\beta}=0.05E_0$, and $V_0^e=V_0^h=V_0$. A typical exciton binding energy E_0 is 2 meV for type-II heterostructures.[3] The temperature is taken as $T=E_0/5.8$, which corresponds approximately to 4 K. The maximum value $\Delta E=E_0$ corresponds to $B=8.2$ T. The upper curves a and b correspond to $V_0=5E_0$ while the lower curves c and d correspond to $V_0=10E_0$. The solid curves correspond to $n_e=0.02a_B^{-2}$ and the dotted curves correspond to $n_e=0.05a_B^{-2}$. Here a_B is the exciton Bohr radius with a typical value $a_B=100$ Å. Therefore, $n_e=0.02a_B^{-2}$ corresponds to $n_e=2\times10^{10}$ cm^{-2}. Note from this figure that the relative exciton density increases with the magnetic field. This is consistent with the experimental finding of Butov *et al.*[6] These experiments were performed for samples with a negligible random potential, but the overlap between electron and hole wave functions was controlled by an external gate voltage.

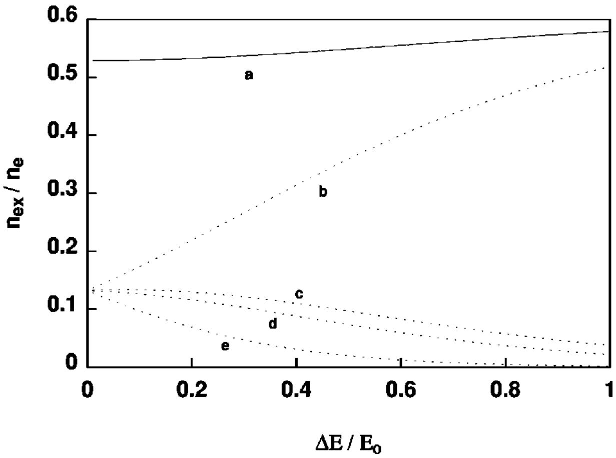

FIG. 2. Relative exciton density with $S_z=0$ and $S_z=\pm 1$ as a function of $\Delta E/E_0$ for type-II AlAs/GaAs QW's. The meaning of the labels a,b,c,d, and e is discussed in the text.

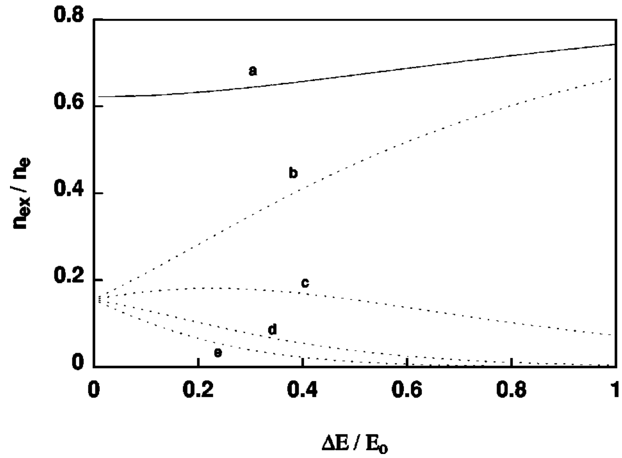

FIG. 3. Relative exciton density with $S_z=0$ and $S_z=\pm 1$ as a function of $\Delta E/E_0$ for GaAs. The meaning of the labels a,b,c,d, and e is discussed in the text.

To understand the physical behavior of Fig. 1, the different populations $n_{ex}^{\alpha\beta}/n_e$ of the relative exciton density have been plotted as a function of $\Delta E/E_0$ in Fig. 2. Curves a, b, c, d, and e correspond to n_{ex}/n_e (total), $n_{ex}^{-+}(S_z=0)/n_e$, $n_{ex}^{2+}(S_z=1)/n_e$, $n_{ex}^{2-}(S_z=-1)/n_e$, and $n_{ex}^{+-}(S_z=0)/n_e$ respectively. Note that $n_{ex}^{-+}(S_z=0)/n_e$ increases with magnetic field and that it is larger than that of any other curves. This can be understood as follows. In a magnetic field, the degenerate electron and hole energy states split in spin-up and spin-down states due to Zeeman effect. In the case of electrons, the state with spin-down has lower energy than the state with spin-up, while in the case of holes the state with spin-up has lower energy than the state with spin-down. According to the Fermi distribution function, there are more electrons occupying the state with spin-down than that of spin-up. Similarly, there are more holes occupying the state with spin-up than that of spin-down. Therefore, the probability to form excitons $n_{ex}^{-+}(S_z=0)$ from electrons with spin-down and holes with spin-up (curve b) is larger than the probability to form excitons from any other combination of spins. The terms $n_{ex}^{2+}(S_z=1)$ (curve c), $n_{ex}^{2-}(S_z=-1)$ (curve d) and $n_{ex}^{+-}(S_z=0)$ (curve e) do not contribute significantly to the total exciton density. As the magnetic field increases, the spin splitting energy also increases. Hence, $n_{ex}^{-+}(S_z=0)/n_e$ increases with magnetic field, whereas the other density terms decrease with magnetic field. It is found that the rate of increase of $n_{ex}^{-+}(S_z=0)/n_e$ with magnetic field is larger than the rate of decrease of the other density terms combined together. Hence the total exciton density increases with an increase of the magnetic field. Note that n_{ex}/n_e decreases when the random potential increases, which is consistent with our previous work.[13] This happens because, in the presence of disorder, it is more likely for an electron and hole to occupy lower-energy positions which may be spatially quite far apart and the number of bound electron-hole pairs decreases.

The present theory is also applied to bulk GaAs materials where electrons and holes are spatially separated. The values $g_e=-0.44$ and $g_h=1.0$ are taken from the work of Snelling *et al.*[14] Datta *et al.*[3] found an exciton binding energy $E_0=2.5$ meV for the spatially separated electron-hole pair in GaAs. The exciton density is calculated in Fig. 3. Curves a,b,c,d, and e correspond to n_{ex}/n_e (total), $n_{ex}^{2+}(S_z=1)/n_e$, $n_{ex}^{-+}(S_z=0)/n_e$, $n_{ex}^{+-}(S_z=0)/n_e$ and $n_{ex}^{2-}(S_z=-1)/n_e$ respectively. Note that only the term $n_{ex}^{2+}(S_z=1)/n_e$ (curve b) increases with magnetic field whereas the other terms decrease with magnetic field. This is the reverse situation of what we obtained for a type-II AlAs/GaAs QW. Because of the negative value of the electron g factor, the electron energy state with spin-up is now lower than that of electron state with spin-down. However, the spin splitting of the hole states remains similar to that of type-II AlAs/GaAs QW's. Therefore, in this case, both electron and hole populations with spin-up increase at the same time so that the $n_{ex}^{2+}(S_z=1)$ exciton population is larger than any of the other one and it increases with magnetic field.

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