Pseudomomentum of a dipole in a two-dimensional system

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The energy of a two-dimensional neutral particle with a nonzero dipole moment aligned perpendicular to the two-dimensional plane is not controlled by its two-dimensional pseudomomentum, but is a function of a different vector, whereas the particle can gain only two-dimensional pseudomomentum through interaction with the radiation field. A resonance has been found, whose existence derives directly from the above statement.

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The total momentum of a neutral two-particle system whose particles interact through a translationally invariant potential is a constant of motion, and its components commute with each other. Under a static uniform magnetic field, the system is invariant under the group of magnetic translations, whose generators are the components of the pseudomomentum

$$\mathbf{P}_{\mathbf{3D}} = -i(\boldsymbol{\nabla}_1 + \boldsymbol{\nabla}_2) + \frac{e}{c}(\mathbf{A}_1 - \mathbf{A}_2) - \frac{e}{c}[(\mathbf{r}_2 - \mathbf{r}_1)\times\mathbf{B}] \quad (1)$$

($\hbar = 1$, and indices 1 and 2 denote the negatively and positively charged particles), all three components of which commute with each other.[1,2] The pseudomomentum of a system in a magnetic field plays the same role as the momentum does in the absence of magnetic field. A remarkable example is absorption or emission of a photon by a system in a magnetic field. The interaction with the radiation field conserves the sum of the photon momentum and pseudomomentum.[3]

In a space of two dimensions, a neutral two-particle system under a static magnetic field has a similar constant of motion, namely, the two-dimensional (2D) pseudomomentum ($\mathbf{P}$), which can be presented in the form $\mathbf{P}_{\mathbf{3D}}$, with the operators $\mathbf{r}_{1,2}$ and $\boldsymbol{\nabla}_{1,2}$ projected on the two-dimensional space.[4] Examples of such systems are the two-dimensional hydrogen atom and the Mott exciton confined in a narrow semiconductor quantum well (QW) under a magnetic field. Other examples that are less obvious, but important for physical applications, are magnetic excitons or magnetoplasmon modes, bound states of a hole in a filled Landau level, and one electron in an otherwise empty level, which are collective magnetoexcitations of an electronic system confined in a QW.[5] Owing to the finite extension of the electron wave function in the direction perpendicular to the 2D plane, quasiparticles in QW's are, in fact, not two dimensional but quasi-two-dimensional. This, however, does not affect their properties as long as the electron and hole forming the quasiparticle move in the same 2D plane, and the gaps between dimensionally quantized QW subbands are very large as compared with the electron-hole binding energy. Hereafter, we shall always disregard the coupling between dimensionally quantized QW subbands.

If the electron and hole move in two spatially separated planes the system can be described in terms of a 2D dipole, i.e., a neutral 2D quasiparticle which has a constant nonzero dipole moment ($\mathbf{d}$) along the axis of separation between the 2D planes (Fig. 1). Such are Mott excitons in asymmetric single and double QW's, intersubband electron excitations in asymmetrically doped single and double QW's, etc. Similarly to the case of a 3D dipole,[6,7] one can prove that, under an external magnetic field, the vector

$$\boldsymbol{\Pi} = \mathbf{P} - \frac{1}{c}[\mathbf{d}\times\mathbf{B}] \quad (2)$$

should be treated as a 2D dipole pseudomomentum.[8,9]

Thus, the total 2D dipole pseudomomentum is composed of two parts: the 2D pseudomomentum $\mathbf{P}$ and the part determined by the dipole moment of the 2D dipole, $[\mathbf{d}\times\mathbf{B}]$. Even so, owing to the lack of translational symmetry along the direction perpendicular to the 2D plane, the radiation field can transfer to a 2D dipole only the 2D pseudomomentum,[10] but *not the total 2D dipole pseudomomentum* $\boldsymbol{\Pi}$. Therefore, one can increase or decrease the 2D dipole pseudomomentum with respect to the radiationally transferred 2D pseudomomentum by applying an appropriately oriented magnetic field. If the resonance condition

$$\mathbf{P} = \frac{1}{c}[\mathbf{d}\times\mathbf{B}] \quad (3)$$

is met, the complete cancellation of the 2D dipole pseudomomentum should be observed. In a way, the 2D dipole becomes "frozen," and this effect has no analogs in the translationally invariant 3D and purely 2D cases. The situation when the external magnetic field is attuned at a fixed 2D pseudomomentum (or the other way round, the 2D pseudomomentum is attuned at a fixed magnetic field) to satisfy Eq. (3) can be considered as a new resonance, which can be used in determining the dipole moment of a 2D dipole.

It is instructive to relate the total pseudomomentum of a 2D dipole, $\boldsymbol{\Pi}$, to the motion energy of the dipole, $E(\boldsymbol{\Pi})$. Since $\boldsymbol{\Pi}$ is a vector sum of $\mathbf{P}$ and $(1/c)[\mathbf{d}\times\mathbf{B}]$ the motion energy should depend in a nontrivial way on the external magnetic field through the term $(1/c)[\mathbf{d}\times\mathbf{B}]$. Consider a 2D dipole moving with a 2D pseudomomentum $\mathbf{P}$. The energy of the 2D dipole is $E(\mathbf{P})$. If the external magnetic field is applied in such a way that the vector $(1/c)[\mathbf{d}\times\mathbf{B}]$, is directed

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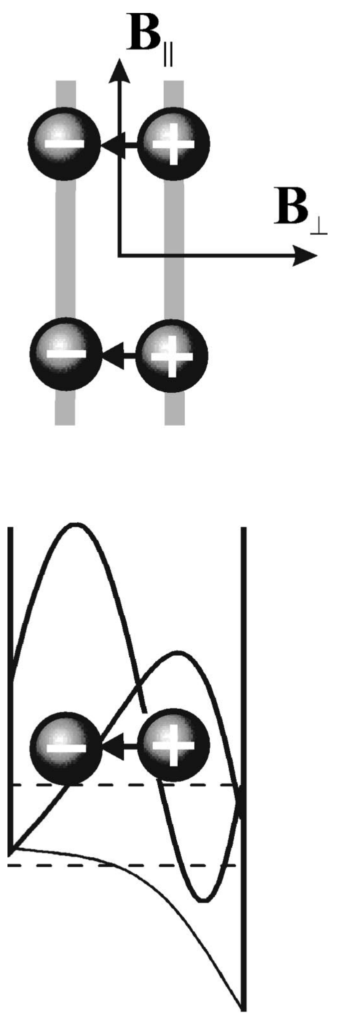

FIG. 1. A diagram illustrating the 2D dipoles under an external magnetic field: ideal dipoles (top) and an intersubband dipole in a quantum well (bottom).

along **P**, one would observe a decrease in the 2D-dipole energy to the value $E(|\mathbf{P}|-(1/c)|[\mathbf{d}\times\mathbf{B}]|)$; conversely, if $(1/c)[\mathbf{d}\times\mathbf{B}]$ is directed opposite to **P**, one would observe an increase in the energy to $E(|\mathbf{P}|+(1/c)|[\mathbf{d}\times\mathbf{B}]|)$. In general, the variations of the motion energy of a 2D dipole under an external magnetic field should follow the variations in the absolute value of the total 2D dipole pseudomomentum $\mathbf{\Pi}$:

$$E(\mathbf{\Pi})=E\left(\left|\mathbf{P}-\frac{1}{c}[\mathbf{d}\times\mathbf{B}]\right|\right). \quad (4)$$

It follows from Eq. (4) that for an arbitrary **P** there must be a magnetic field at which the motion energy of the 2D dipole is exactly zero, and this magnetic field satisfies Eq. (3). Here we present an experimental realization of a 2D dipole, and demonstrate that, under an external magnetic field, the motion energy of the 2D dipole behaves in accordance with Eq. (4).

The system studied in the experiment was an intersubband electron charge-density excitation in an asymmetrically doped single quantum well (Fig. 1). The intersubband excitations consist of an electron in an otherwise empty excited

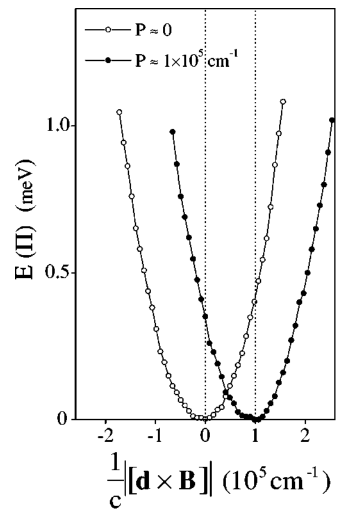

FIG. 2. Motion energy of an intersubband charge density excitation at $B_\perp=0$ as a function of $(1/c)[\mathbf{d}\times\mathbf{B}]$ directed along the 2D pseudomomentum **P** for two values of $|\mathbf{P}|$ measured in the sample with $n_s=3.5\times10^{11}$ cm^{-2}.

subband interacting with a hole left behind in the electron Fermi sea of the lower subband. The electron and hole are spatially separated in the direction perpendicular to the quantum well owing to the asymmetry of the confining potential. Since the energy of interaction between the electron and hole is usually much smaller than the intersubband energy gap, the excitation can be considered as a well-defined 2D dipole.[11]

We used high-quality asymmetrically doped $Al_{0.3}Ga_{0.7}As \backslash GaAs$ single QW heterostructures with a QW width of 250 Å, a mobility of 1.5×10^6 $\mathrm{cm^2/(V\,s)}$, and electron concentrations (n_s) of 3.5×10^{11} and 6.8×10^{11} cm^{-2}. The studied QW sample was mounted in an optical cryostat with a horizontal split-coil superconducting solenoid generating a magnetic field of 0 to 6 T at a base temperature of 1.5 K. Through the inelastic light scattering process, the in-plane momentum was transferred to the intersubband excitations, and the motion energy of the intersubband excitations was measured as a function of the in-plane momentum and the magnetic field. An important point is that the intersubband excitations have large dispersions, therefore a relatively small momentum of a photon from the visible spectral band is able to bring about energy shifts which can be measured accurately. A more detailed description of the experimental technique was given elsewhere.[12,13]

First let us consider a simplified experiment with the magnetic field applied along the 2D plane. In this case the 2D pseudomomentum reduces to the 2D momentum. When the 2D momentum is zero, the energy of the studied intersubband excitation increases as a quadratic function of the magnetic field (Fig. 2). If we add a finite 2D momentum of 1×10^5 cm^{-1} directed along the vector $(1/c)[\mathbf{d}\times\mathbf{B}]$, we observe a shift of the dispersion curve in the magnetic field. If the magnetic field axis is expressed as $(1/c)|[\mathbf{d}\times\mathbf{B}]|$, one finds that the shift is equal to the 2D momentum, $|\mathbf{P}|$. Here the dipole moment **d** of the intersubband excitation is

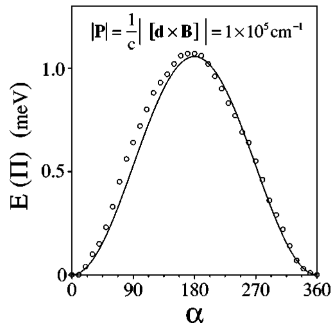

FIG. 3. Motion energy of an intersubband charge density excitation at $B_{\perp}=0$ as a function of the angle between the directions of $\mathbf{P}$ and $(1/c)[\mathbf{d}\times\mathbf{B}]$. The solid line is the curve calculated by Eq. (6).

$$\mathbf{d}=-e|z_{00}-z_{11}|\mathbf{n}, \tag{5}$$

where $\mathbf{n}$ is the normal to the QW plane, and $z_{00}-z_{11}=\int dz\psi_0^*(z)z\psi_0(z)-\int dz\psi_1^*(z)z\psi_1(z)$ is the average distance between the electron and hole (see the diagram in Fig. 1). The function $\psi_n(z)$ is the z component of the electron wave function in the n-th subband obtained as a self-consistent solution of the one-dimensional Schrödinger and Poisson equations. In agreement with Eqs. (3) and (4), the motion energy of the 2D dipole at $\mathbf{P}=(1/c)[\mathbf{d}\times\mathbf{B}]$ is equal to zero, whereas neither $\mathbf{P}$ nor $(1/c)[\mathbf{d}\times\mathbf{B}]$ is zero.

To verify that the excitation energy depends on the vector sum of $\mathbf{P}$ and $(1/c)[\mathbf{d}\times\mathbf{B}]$, the relative orientation between $\mathbf{P}$ and $(1/c)[\mathbf{d}\times\mathbf{B}]$ at $|(1/c)[\mathbf{d}\times\mathbf{B}]|=|\mathbf{P}|=1\times10^5\ \mathrm{cm}^{-1}$ was continuously changed, and the excitation energy was measured as function of the angle α between the directions of $\mathbf{P}$ and $(1/c)[\mathbf{d}\times\mathbf{B}]$ (Fig. 3). The observed angle dependence is accurately described by the equation

$$E(\mathbf{\Pi})=\frac{1}{2m^*}\left(\mathbf{P}-\frac{1}{c}[\mathbf{d}\times\mathbf{B}]\right)^2, \tag{6}$$

where m^* is the effective mass of the excitation derived from the graph in Fig. 2. Thus the motion of a 2D dipole having a quadratic dispersion may be viewed as a motion of an electron with the mass of the dipole in an effective field with a vector potential of $-e^{-1}[\mathbf{d}\times\mathbf{B}]$.

Now consider a 2D dipole under an external magnetic field applied at an arbitrary angle with respect to the 2D plane. The motion energy of the dipole is a complex function of the total 2D pseudomomentum of the 2D dipole, $\mathbf{\Pi}$, due to the perpendicular component of the magnetic field. The perpendicular component $B_{\perp}$ contributes through the vector potential in the vector $\mathbf{P}$; therefore, the dispersion relation $E(\mathbf{P})$ becomes different at different $B_{\perp}$. Here we chose the perpendicular component 1.5 T at which the largest dispersion of the intersubband excitation under study is observed.[13] First let us turn the component $B_{||}$ of the magnetic field in the 2D plane to zero, and analyze the motion energy of the excitation as function of the 2D pseudomomentum, $E(\mathbf{P})$. At the specific field under consideration, $E(\mathbf{P})$ turns out to be a linear function of $|\mathbf{P}|, E(\mathbf{P})=\beta|\mathbf{P}|$ (Fig. 4, open dots). Now let us fix the 2D pseudomomentum $\mathbf{P}$ at $1\times10^5\ \mathrm{cm}^{-1}$ and change the component of the magnetic field in the 2D plane

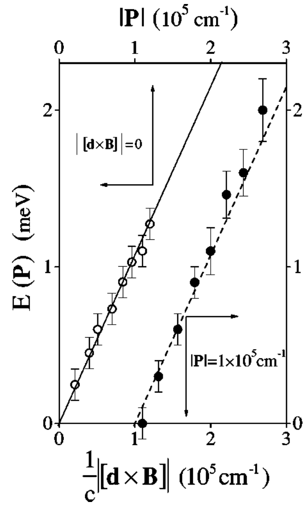

FIG. 4. Motion energy of an intersubband charge density excitation measured at $B_{\perp}=1.5$ T: as a function of P at $|1/c[\mathbf{d}\times\mathbf{B}]|=0$ (open dots, left and top axes), and as a function of $(1/c)[\mathbf{d}\times\mathbf{B}]$ directed along $\mathbf{P}$ at $|\mathbf{P}|=1\times10^5\ \mathrm{cm}^{-1}$ (solid dots, right-hand and lower axes). The solid line is a linear fit of $E(\mathbf{P})$. The dashed line is calculated by Eq. (7) with the same coefficient as in the linear fit of $E(\mathbf{P})$.

in such a way that the vector $(1/c)[\mathbf{d}\times\mathbf{B}]$ is directed along $\mathbf{P}$. This means that we have changed only the term $(1/c)[\mathbf{d}\times\mathbf{B}]$ in the dispersion relation: $E(|\mathbf{P}-(1/c)[\mathbf{d}\times\mathbf{B}]|)$. The function $E(\mathbf{\Pi})$ has the same linear slope as the function $E(\mathbf{P})$, and at the same time it is shifted along the x axis through an increment of the 2D pseudomomentum $|\mathbf{P}|$:

$$E(\mathbf{\Pi})=\beta\left|\left(\mathbf{P}-\frac{1}{c}[\mathbf{d}\times\mathbf{B}]\right)\right|. \tag{7}$$

The motion energy equals zero at $\mathbf{P}=(1/c)[\mathbf{d}\times\mathbf{B}]$, as follows from Eqs. (3) and (4).

We verified the equation (4) for three different intersubband excitations in the case when the magnetic field lies in the 2D plane and for two excitations in the case when the magnetic field is directed at an arbitrary angle with respect to the 2D plane. The studied excitations differed in the quantum numbers characterizing their degrees of freedom associated with internal motion and spin, but had the same dipole moment. In all experiments, the motion energy was found to be in agreement with Eq. (4). Since Eq. (4) directly derives from Eq. (2), we concluded that $\mathbf{\Pi}$ is the principal vector that governs the motion of a 2D dipole under external magnetic field. Equation (3) follows straightforwardly.

From the experimental point of view, we want to emphasize the significance of resonance condition (3). The exemplar 2D dipoles studied here possess a unique property: their motion energy can be directly measured as a function of the total 2D dipole pseudomomentum, $E(\mathbf{\Pi})$. In general, this is not the case. A dipole can acquire a well-defined 2D pseudomomentum $\mathbf{P}$, e.g., through absorption of a photon of known energy and momentum, but neither its energy nor its total pseudomomentum $\mathbf{\Pi}$ can be measured accurately afterwards. The existence of resonance condition (3) implies merely that one can always find an external magnetic field at which the dipole does not move, the condition which can be easily satisfied in an experiment. As soon as such a magnetic field is found, one obtains the dipole momentum of the 2D dipole from the Eq. (3).

To sum up, we have shown that $\mathbf{\Pi}$ is the principal vector which governs the motion of a 2D-dipole under an external magnetic field. One can take the opportunity to transfer separately parts of $\mathbf{\Pi}$ to a 2D dipole with a view to observing a resonance determined by Eq. (3) and measuring experimentally the dipole moment of a 2D dipole.

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