

Sharp increase of the effective mass near the critical density in a metallic two-dimensional electron system

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We find that at intermediate temperatures, the metallic temperature dependence of the conductivity $\sigma(T)$ of two-dimensional electrons in silicon is described well by a recent interaction-based theory of Zala *et al.* [Phys. Rev. B **64**, 214204 (2001)]. The tendency of the slope $\sigma^{-1}d\sigma/dT$ to diverge near the critical electron density is in agreement with the previously suggested ferromagnetic instability in this electron system. Comparing theory and experiment, we arrive at a conclusion that the instability, unexpectedly, originates from the sharp enhancement of the effective mass, while the effective Landé g factor remains nearly constant and close to its value in bulk silicon.

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In dilute two-dimensional (2D) electron systems, the energy of electron-electron interactions dominates the kinetic energy making the system strongly correlated.¹ The interaction strength is characterized by the Wigner-Seitz radius $r_s = 1/(\pi n_s)^{1/2} a_B$ (where n_s is the electron density and a_B is the effective Bohr radius in semiconductor), which is equal in the single-valley case to the ratio, r'_s , of the Coulomb and the Fermi energies assuming that the effective electron mass m is equal to the band mass m_b . According to the Fermi liquid theory² (whose applicability to dilute 2D electron systems is discussed in Ref. 3), the electron-electron interactions should give rise to a renormalization of the system parameters including the effective mass and the effective g factor. A sharp enhancement of the product gm with decreasing electron density—possibly, a precursor of the ferromagnetic instability—has been observed in recent studies of the parallel field magnetoresistance of a metallic 2D electron system in high-mobility silicon metal-oxide-semiconductor field-effect transistors (MOSFET's).⁴ This agrees well with the gm value obtained from the beating pattern of Shubnikov–de Haas oscillations in tilted magnetic fields.^{5–8}

Although physics behind the metal-insulator transition observed in strongly interacting 2D systems still remains illusive,¹ significant progress has been recently made in understanding of the nature of the metallic conductivity deeper in the metallic state. Zala *et al.*⁹ have calculated temperature-dependent corrections to conductivity due to electron-electron interactions based on the Fermi liquid approach. In contrast to pre-existing theories (see, e.g., Refs. 10,11), the new theory incorporates strongly interacting 2D electron systems with electron densities down to the vicinity of the metal-insulator transition (provided that the conductivity $\sigma \gg e^2/h$). For sufficiently strong interactions, it predicts a metallic temperature dependence of conductivity in the entire temperature range. At very low temperatures, in the “diffu-

sive” regime ($T \ll \hbar/k_B\tau$, where τ is the elastic relaxation time), this is Finkelstein's weakly metallic (logarithmic) conductivity.^{12,13} At intermediate temperatures, in the “ballistic” regime ($T \gtrsim \hbar/k_B\tau$; $T > 0.2-0.5$ K under the conditions of our experiments), the predicted $\sigma(T)$ is similar to the Gold-Dolgoplov linear dependence¹⁰

$$\frac{\sigma(T)}{\sigma_0} = 1 - Ak_B T, \quad (1)$$

where the slope A is determined by the interaction-related parameters: the Fermi liquid constants F_0^a and F_1^s . These parameters are responsible for the renormalization of the g factor and the effective mass³

$$\frac{g}{g_0} = \frac{1}{1 + F_0^a}, \quad \frac{m}{m_b} = 1 + F_1^s \quad (2)$$

and can be determined experimentally.¹⁴ The slope A is predicted to rise as the interaction strength increases and the 2D electron system is driven toward the ferromagnetic instability. The last is expected to occur, in the simplest case, at $F_0^a = -1$, which corresponds to the diverging effective g factor.

In this paper, we perform precision measurements of the temperature-dependent conductivity in a metallic 2D electron system in silicon over a wide range of electron densities above the critical electron density n_c for the metal-insulator transition. The theory of Zala *et al.*⁹ is found to be successful in interpreting the experimental data in the ballistic regime. Knowing the product gm from independent measurements, we determine both g and m as a function of n_s from the slope of the temperature dependence of the conductivity. The tendency of the slope to diverge near the critical density is consistent with the suggested ferromagnetic instability in this electron system.^{4,15} However, unlike in the simplest scenario

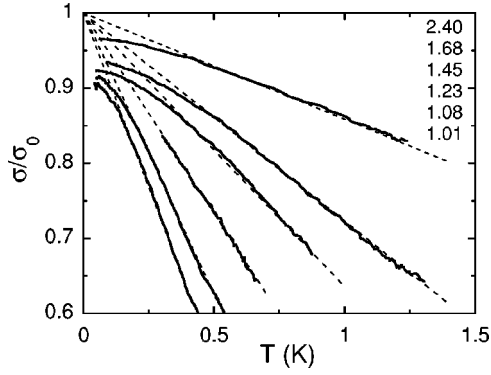


FIG. 1. The temperature dependence of the normalized conductivity at different electron densities (indicated in units of 10^{11} cm^{-2}) above the critical electron density for the metal-insulator transition. The dashed lines are fits of the linear interval of the dependence.

for the ferromagnetic instability, comparison between experiment and theory shows that it is the value of the effective mass that becomes strongly enhanced with decreasing electron density, while the g factor remains nearly constant $g \approx g_0 = 2$ in bulk silicon.

Measurements were made in an Oxford dilution refrigerator with a base temperature of ≈ 30 mK on high-mobility (100)-silicon samples similar to those previously used in Ref. 16. The resistance was measured by a standard four-terminal low-frequency technique. Excitation current was kept low enough to ensure that measurements were taken in the linear regime of response. Contact resistances in our samples were minimized by using a split-gate technique that allows one to maintain a high electron density in the vicinity of the contacts regardless of its value in the main part of the sample. In this paper we show results obtained on a sample with a peak mobility close to $3 \text{ m}^2/\text{Vs}$ at 0.1 K.

Typical dependences of the normalized conductivity on temperature $\sigma(T)/\sigma_0$ are displayed in Fig. 1 at different electron densities above the critical electron density for the metal-insulator transition which in this sample occurs at $n_c = 8 \times 10^{10} \text{ cm}^{-2}$; ¹⁷ the value σ_0 , which has been used to normalize σ , was obtained by extrapolating the linear interval of the $\sigma(T)$ dependence to $T=0$. As long as the deviation $|\sigma/\sigma_0 - 1|$ is sufficiently small, the conductivity σ increases linearly with decreasing T in agreement with Eq. (1), until it saturates at the lowest temperatures. ¹⁸ As seen from the figure, the linear interval of the dependence is wide enough to make a reliable fit.

In Fig. 2, we show the n_s dependence of the inverse slope $1/A$ extracted from the $\sigma(T)$ data. Also shown for comparison is the magnetic energy $\mu_B B_c = \pi \hbar^2 n_s / gm$ (where μ_B is the Bohr magneton and B_c is the parallel field of the magnetoresistance saturation), corresponding to the onset of full spin polarization in this electron system, which is governed by the (enhanced) product gm . ⁴ Over a wide range of electron densities, the values $1/A$ and $\mu_B B_c$ turn out to be close to each other. The low density data for $1/A$ are approximated well by a linear dependence which extrapolates to the critical electron density n_c in a similar way to the behavior of the

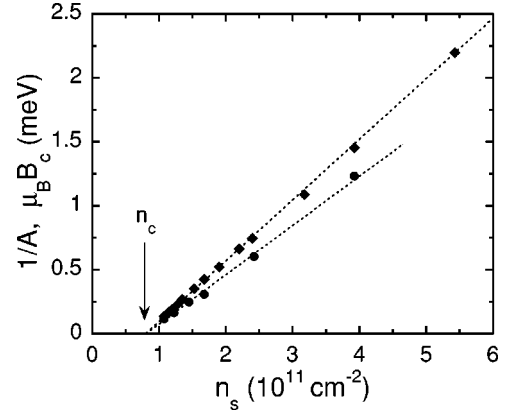


FIG. 2. Comparison of the inverse slope $1/A$ (dots) and the data for the polarization field B_c (diamonds) as a function of electron density. The dashed lines are linear fits which extrapolate to the critical electron density for the metal-insulator transition.

polarization field B_c . We emphasize that the density of the delocalized electrons in the metallic phase is practically coincident with n_s in our samples, as inferred from the above agreement between the gm data obtained by different measurement methods. ⁷

As has already been mentioned, the coefficient A in the linear-in- T correction to conductivity of Eq. (1) is determined by the Fermi liquid parameters ^{9,19}

$$A = - \frac{(1 + \alpha F_0^a) gm}{\pi \hbar^2 n_s}. \quad (3)$$

The factor α is equal to 8 in our case. ²⁰ This theoretical relation allows us to determine the many-body enhanced g factor and mass m separately using the data for the slope A and the product gm as a function of n_s (the latter is known from independent measurements similar to those described in Ref. 4).

Before presenting data for g and m , we would like to emphasize that possible uncertainties in the coefficients of Eq. (3) do not affect the main result of the paper: a sharp increase of m and approximate constancy of g as $n_s \rightarrow n_c$. This result directly follows from the fact that the experimentally observed dependence of $1/A$ on n_s is *linear* and, therefore, F_0^a is n_s independent, according to the functional form of Eq. (3). In addition, the extracted g and m values are only weakly sensitive to the coefficients in Eq. (3) because g turns out to be close to g_0 .

In Fig. 3, we show the so-determined values g/g_0 and m/m_b as a function of the electron density (the band mass m_b is equal to $0.19m_e$, where m_e is the free electron mass). Note that in the range of n_s studied here, the low-temperature conductivity $\sigma > 8e^2/h$. The behavior of g and m at electron densities below $n_s = 3 \times 10^{11} \text{ cm}^{-2}$ (corresponding to $r_s \approx 4.8$) turns out to be very different from that at electron densities above this value. In the high n_s region (lower r_s), the enhancement of both g and m is relatively small, both values slightly increasing with decreasing electron density in agreement with earlier data. ²¹ Also, the renormalization of the g factor is dominant compared to that of the effective

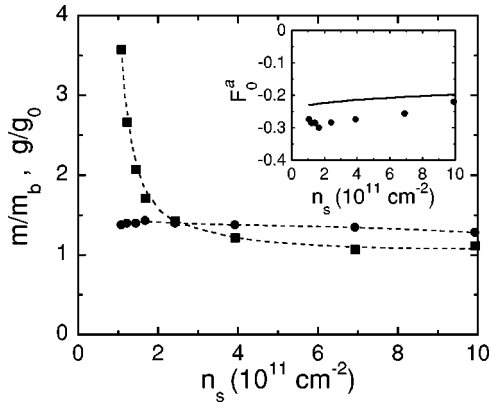


FIG. 3. Renormalization of the effective mass (squares) and g factor (dots) as a function of electron density. The dashed lines are guides to the eye. The inset compares the theoretical dependence of the renormalization parameter F_0^a on n_s (solid line) with the data (dots) calculated using Eq. (2) from our g values.

mass, which is consistent with theoretical studies.²² The dependence $g(n_s)$ is described reasonably well by the theory: the inset of Fig. 3 compares the theoretical renormalization parameter $F_0^a = -r_s/2(2r_s + \sqrt{2})$ (Ref. 9) to that calculated using Eq. (2) and the data for $g(n_s)$.

In contrast, the renormalization in the low n_s (critical) region, where $r_s \gg 1$, is much more striking. As the electron density is decreased, Fig. 3 shows that the renormalization of the effective mass overshoots abruptly while that of the g factor remains relatively small $g \approx g_0$, without tending to increase. Hence, the current analysis indicates that it is the effective mass that is responsible for the drastically enhanced gm value near the metal-insulator transition. The present results for the effective mass and g factor in the critical region are consistent with both the evaluation of $m(n_s)$ and $g(n_s)$ obtained by analysis of the Shubnikov–de Haas oscillations in high-mobility Si MOSFET's (Ref. 8) and the data for the spin and the cyclotron gaps obtained by magnetocapacitance spectroscopy.²³ This gives support to our procedure and conclusions.

One can already see from Fig. 2 that the data points span close enough to the critical electron density. In the inset to Fig. 4, this is additionally checked for the m data plotted against $r_s' = 2(m/m_b)r_s$ (the factor of 2 comes from the valley degeneracy). As the electron density is decreased, the dependence of m on r_s' approaches the linear dependence with the slope $1/2r_s(n_c)$ determined by n_c . At the lowest n_s , the effective mass increases approximately proportionally with r_s' and, therefore, m changes sharply with r_s (and n_s) in the reached vicinity of the critical electron density.

It is important to discuss another consequence of the theory:⁹ the slope A of the temperature dependence of the conductivity should increase as the ferromagnetic instability in a dilute 2D electron system is approached. Since renormalization parameters have not been theoretically calculated in the limit $r_s \gg 1$, the simplest scenario of the ferromagnetic instability is prompted by Eq. (2): $F_0^a \rightarrow -1$ causes the effective g factor (and the slope A) to diverge. Experimentally, the slope A tends to diverge near the critical electron density

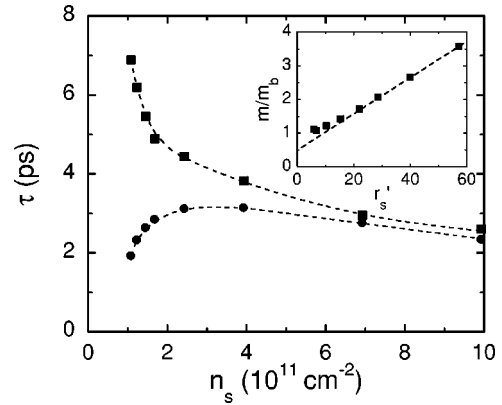


FIG. 4. The elastic relaxation time versus electron density at a temperature of 0.1 K, assuming $m = m_b$ (dots), and taking into account the renormalization of m (squares). The dashed lines are guides to the eye. The inset shows the effective mass data of Fig. 3 as a function of the parameter r_s' . A linear fit (dashed line) of the low n_s data has a slope which is equal to $1/2r_s(n_c)$.

in a way similar to the behavior of the product gm seen in Fig. 2. This is consistent with the conclusion of Refs. 4,15 about the possibility of ferromagnetic instability in this electron system. At the same time, the simplest scenario of a *diverging g factor* is not the case; instead, it is the growing *effective mass* which controls the anomalous behavior of the dilute 2D electron system near the metal-insulator transition.

In principle, g or m or both might be diverging at the occurrence of a ferromagnetic instability. It is important that rather than r_s , the ratio r_s' of the interaction and the kinetic energies is the relevant parameter for ferromagnetic instability. Spontaneous spin polarization is expected to occur at the large r_s' at which the increase in the kinetic energy of a fully spin-polarized electron system is excelled by the decreasing energy of electron-electron interactions. In our case, both the kinetic energy drop and the increase of r_s' with decreasing electron density are controlled by the sharply growing effective mass.

The effective mass enhancement was traditionally considered to be small and, therefore, the value $m \approx m_b$ was used to calculate some of the important system parameters, e.g., the elastic relaxation time τ extracted from mobility.²¹ In Fig. 4, we compare the so-determined τ (circles) with that calculated taking into account the enhancement of the effective mass (squares). As seen from the figure, in the range of electron densities studied, the corrected τ keeps increasing down to the lowest n_s ; note that such a behavior is representative of surface roughness scattering.²¹ Therefore, the mobility drop at low n_s in high-mobility Si MOSFET's (see, e.g., Ref. 24) turns out to originate from the m enhancement rather than from the decrease in τ , although the value τ is still expected to vanish in the insulating phase. The observed behavior of τ is consistent with that of the temperature range corresponding to the ballistic regime (see Fig. 1), which gives additional confidence in our analysis down to the vicinity of the metal-insulator transition. Finally, values of τ much larger than those previously estimated yield appreciably smaller quantum level widths in perpendicular magnetic

fields, which helps to understand why the Shubnikov–de Haas oscillations survive near the metal-insulator transition, as well as the origin of the oscillations of the metal-insulator phase boundary as a function of (perpendicular) magnetic field.²⁵

In summary, we have studied the temperature-dependent conductivity in a wide range of electron densities above the critical electron density for the metal-insulator transition. Using the recent theory of interaction-driven corrections to conductivity,⁹ we extract Fermi-liquid parameters from the experimental data and determine the many-body enhanced g factor and the effective mass. The tendency of the slope A of the temperature dependence of the conductivity to diverge near the critical density is in agreement with the suggested

ferromagnetic instability in this electron system.^{4,15} Unexpectedly, it is found to originate from the growing effective mass rather than the g factor. In addition, the mass enhancement is found to be responsible for the previously underestimated values of elastic scattering time near the metal-insulator transition.

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