Effect of disorder on superconductivity in the boson-fermion model

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We study how a randomness of either boson or fermion site energies affects the superconducting phase of the boson fermion model. We find that, contrary to what is expected for *s*-wave superconductors, the nonmagnetic disorder is detrimental to the *s*-wave superconductivity. However, depending on which subsystem the disorder is located, we can observe different channels being affected. Weak disorder of the fermion subsystem is mainly responsible for the renormalization of the single-particle density of states, while disorder in the boson subsystem leads directly to fluctuations of the strength of the effective pairing between fermions.

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I. INTRODUCTION

The boson fermion (BF) model is an example of a microscopic theory of nonconventional superconductivity. It describes a mixture of itinerant electrons or holes (fermions) which interact via charge exchange with a system of immobile local pairs (hard-core bosons). Due to this interaction, bosons acquire finite mass, and under proper circumstances might undergo a Bose condensation transition while fermions simultaneously start to form a broken symmetry superconducting phase.

This model was introduced *ad hoc* almost two decades ago[1] to describe the electron system coupled to the lattice vibrations in a crossover regime, between the adiabatic and antiadiabatic limits. Later it was formally derived from the Hamiltonian of wide band electrons hybridized to the strongly correlated narrow band electron system.[2] Very recently[3] the same effective BF model was derived purely from the two dimensional Hubbard model in the strong interaction limit using the contractor renormalization method of Morningstar and Weinstein.[4]

Some authors have proposed it as a possible scenario for description of high temperature superconductivity (HTSC). The unconventional way of inducing the superconducting phase in the BF model has been independently investigated in a number of papers.[5–11] Moreover, this model reveals also several unusual properties of the normal phase (for $T>T_c$) with an appearance of the pseudogap being the most transparent among them.[12–14] Apart from the eventual relevance of this model to HTSC there are attempts to apply the same type of picture for a description of the magnetically trapped atoms of alkali metals.[15]

The important question which we want to address in this paper is the following: what is the influence of disorder on superconductivity of the BF model? The conventional *s*-wave symmetry BCS-type superconductors are known to be rather weakly affected by paramagnetic impurities[16]—a fact which is known as the "Anderson theorem." Nonmagnetic impurities have a remarkable detrimental effect on superconductors with the anisotropic order parameters. Magnetic impurities lead to pair-breaking effects which result in a strong reduction of T_c even in *s*-wave superconductors. Studying the effect of impurities on the superconductors has always been an established tool for an investigation of the internal structure of the Cooper pairs.

Due to the nonconventional pairing mechanism (i.e., the exchange of the hard-core bosons between fermion pairs) it is of a fundamental importance to see how nonmagnetic impurities (disorder) affect the isotropic superconducting phase of the BF model. Previously, such a study was carried out by Robaszkiewicz and Pawłowski,[17] who considered disorder only in the boson subsystem. Using a method of configurational averaging for the free energy, the authors showed a strong detrimental effect of disorder on superconductivity. Apart from a reduction of the transition temperature T_c, they also reported a remarkable change of a relative ratio $\Delta(T=0)/k_BTc$. In this paper we analyze the effect of disorder present in both fermion and boson subsystems using a different method of the coherent potential approximation.

II. MODEL AND APPROACH

A. Hamiltonian of the disordered BF model

We consider the following Hamiltonian of the disordered BF model:

$$
\begin{aligned}
H^{BF} = & \sum_{i,j,\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i (\varepsilon_i - \mu) c_{i\sigma}^\dagger c_{i\sigma} \\
& + \sum_i (\Delta_B + E_i - 2\mu) b_i^\dagger b_i \\
& + v \sum_i (b_i^\dagger c_{i\downarrow} c_{i\uparrow} + b_i c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger). \qquad (1)
\end{aligned}
$$

We use the standard notations for annihilation (creation) operators of fermion $c_{i,\sigma}$ ($c_{i,\sigma}^\dagger$) with spin σ and of the hard core boson b_i ($b_i^\dagger$) at site i. Fermions interact with bosons via the charge-exchange interaction v, which is assumed to be local. There are two ways in which disorder enters into the consideration. Either (a) fermions are affected by it and this is expressed by the random site energies ε_i, or (b) hard-core bosons via their random site energies E_i cause the disorder.

To proceed, we first apply the mean-field decoupling for the boson fermion interaction

$$
b_i^\dagger c_{i\downarrow} c_{i\uparrow} \simeq \langle b_i \rangle^* c_{i\downarrow} c_{i\uparrow} + b_i^\dagger \langle c_{i\downarrow} c_{i\uparrow} \rangle, \qquad (2)
$$

which is justified until v is small enough in comparison to the kinetic energy of fermions. After decoupling [Eq. (2)] we have to deal with the effective Hamiltonian composed of the separate fermion and boson contributions $H \simeq H^F + H^B$,[2,9]

$$H^F = \sum_{i,j,\sigma} [t_{ij} + \delta_{ij}(\varepsilon_i - \mu)] c_{i\sigma}^\dagger c_{j\sigma} + \sum_i (\rho_i^* c_{i\downarrow} c_{i\uparrow} + \rho_i c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger), \tag{3}$$

$$H^B = \sum_i [(\Delta_B + E_i - 2\mu) b_i^\dagger b_i + x_i b_i^\dagger + x_i^* b_i], \tag{4}$$

where $x_i = v\langle c_{i\downarrow} c_{i\uparrow}\rangle$ and $\rho_i = v\langle b_i\rangle$. The site dependence of ρ_i and x_i indicates the disorder-induced amplitude fluctuations of the order parameters.

B. Boson part

For a given configuration of disorder we can exactly find the eigenvectors and eigenvalues corresponding to the lattice site i using a suitable unitary transformation. Statistical expectation values of the number operator $b_i^\dagger b_i$ and the parameter ρ_i are given by[2,9]

$$\langle b_i^\dagger b_i\rangle = \frac{1}{2} - \frac{\Delta_B + E_i - 2\mu}{4\gamma_i} \tanh\left(\frac{\gamma_i}{k_B T}\right), \tag{5}$$

$$\rho_i = -\frac{v x_i}{2\gamma_i} \tanh\left(\frac{\gamma_i}{k_B T}\right) \tag{6}$$

where $\gamma_i = \frac{1}{2}\sqrt{(\Delta_B + E_i - 2\mu)^2 + 4|x_i|^2}$ and k_B is the Boltzmann constant. Note that the site dependent fermion order parameter x_i enters the expression for the boson number operator (5) and the parameter ρ_i [Eq. (6)]. Disorder of any subsystem is thus automatically transfered onto the other one.

C. Fermion part

An analysis of the fermion part [Eq. (3)] is more cumbersome. To study it we use the Nambu representation $\Psi_i^\dagger = (c_{i\uparrow}^\dagger, c_{i\downarrow})$, $\Psi_i = (\Psi_i^\dagger)^\dagger$ and define the matrix Green's function $\boldsymbol{G}(i,j;\omega) = \langle\langle \Psi_i ; \Psi_j^\dagger \rangle\rangle_\omega$. The equation of motion for this function reads

$$\sum_l \begin{bmatrix} (\omega - \varepsilon_l + \mu)\delta_{il} - t_{il} & -\rho_i^* \delta_{il} \\ -\rho_i \delta_{il} & (\omega + \varepsilon_l - \mu)\delta_{il} + t_{il} \end{bmatrix} \boldsymbol{G}(l,j;\omega) = \mathbf{1}\delta_{ij}. \tag{7}$$

Using the matrix Green's function $\boldsymbol{G}^0(i,j;\omega)$ of a clean system,

$$[\boldsymbol{G}^0(\mathbf{k};\omega)]^{-1} = \begin{pmatrix} \omega - \varepsilon_{\mathbf{k}} + \mu & -\rho^* \\ -\rho & \omega + \varepsilon_{\mathbf{k}} - \mu \end{pmatrix}, \tag{8}$$

where $\rho = v(1/N)\Sigma_i\langle\rho_i\rangle$ is a global order parameter (which plays the role of the effective gap in the superconducting fermion subsystem), and defining the single site impurity potential $\boldsymbol{V}_l$ as

$$\boldsymbol{V}_l = \begin{pmatrix} \varepsilon_l & -\rho_l^* \\ -\rho_l & -\varepsilon_l \end{pmatrix}, \tag{9}$$

one can write down the following Dyson equation for the Green's function $\boldsymbol{G}(i,j;\omega)$:

$$\boldsymbol{G}(i,j;\omega) = \boldsymbol{G}^0(i,j;\omega) + \sum_l \boldsymbol{G}^0(i,l;\omega)\boldsymbol{V}_l \boldsymbol{G}(l,j;\omega). \tag{10}$$

This Green's function depends on the specific disorder configuration. In order to pass through one usually averages it over the all possible configurations.

For carrying out the configurational averaging we use a method of the coherent potential approximation (CPA). The main idea of the CPA is to replace the random potential $\boldsymbol{V}_l$ by some uniform coherent potential $\boldsymbol{\Sigma}(\omega)$. Formally, the Green's function which satisfies Eq. (10), with $\boldsymbol{V}_l$ replaced by $\boldsymbol{\Sigma}(\omega)$, is then given (in the momentum coordinates) by

$$[\boldsymbol{G}^{CPA}(\mathbf{k};\omega)]^{-1} = [\boldsymbol{G}^0(\mathbf{k};\omega)]^{-1} - \boldsymbol{\Sigma}(\omega). \tag{11}$$

Configuration at site i is defined by values of the random energies ε_i, E_i—we shall symbolically denote it by $\alpha \equiv \{\varepsilon_i, E_i\}$. Any of possible configurations α can occur with some probability $P(\{\varepsilon_i, E_i\}) \equiv c^{(\alpha)}$, and of course these probabilities are normalized as $\Sigma_\alpha c^{(\alpha)} = 1$.

A particle propagating through the medium characterized by the coherent potential $\boldsymbol{\Sigma}(\omega)$ is thus, at site i, scattered with probability $c^{(\alpha)}$ by the potential $\boldsymbol{V}_i^{(\alpha)} - \boldsymbol{\Sigma}(\omega)$. For a chosen configuration α of the site i the conditionally averaged local Green's function is given by

$$[\boldsymbol{G}^{(\alpha)}(i,i;\omega)]^{-1} = [\boldsymbol{G}^{CPA}(i,i;\omega)]^{-1} - [\boldsymbol{V}_i^{(\alpha)} - \boldsymbol{\Sigma}(\omega)]. \tag{12}$$

This Green's function $\boldsymbol{G}^{(\alpha)}(i,i;\omega)$ describes the system in which all sites, except one indicated by i, are described by the coherent potential $\boldsymbol{\Sigma}(\omega)$. In the CPA one requires that the average of the local Green's function is the same as the Green's function of the averaged system. This CPA condition is identical to the following equation:[18]

$$\sum_\alpha c^{(\alpha)} \boldsymbol{G}^{(\alpha)}(i,i;\omega) = \boldsymbol{G}^{CPA}(i,i;\omega). \tag{13}$$

Equations (11)–(13) have to be solved self-consistently to yield the coherent potential $\boldsymbol{\Sigma}(\omega)$. Physical quantities such as the fermion concentration $n^F \equiv (1/N)\Sigma_{i,\sigma}\langle c_{i\sigma}^\dagger c_{i\sigma}\rangle$ and the superconducting order parameter $x \equiv (1/N)\Sigma_i x_i$ are to be calculated from

$$n^F = -\frac{2}{\pi N}\int_{-\infty}^{\infty} \frac{d\omega}{e^{\beta\omega}+1} \mathrm{Im}\{\boldsymbol{G}_{11}^{CPA}(i,i;\omega + i0^+)\}, \tag{14}$$

$$x=-v\frac{1}{\pi N}\int_{-\infty}^{\infty}\frac{d\omega}{e^{\beta\omega}+1}\mathrm{Im}\{\boldsymbol{G}_{21}^{CPA}(i,i;\omega+i0^+)\},\quad (15)$$

where $\beta=1/k_BT$.

In Sec. III we discuss the changes of the superconducting transition temperature T_c caused by disorder.

III. DISORDER IN A FERMION SUBSYSTEM

It is instructive to investigate the disorder separately for fermion and boson subsystems. Let us start with the fermion disorder ε_i. We set $E_i=0$ for all lattice sites. For the random fermion energies we choose $\varepsilon_i=\varepsilon_0$ with probability c and $\varepsilon_i=0$ with probability $1-c$. It is a bimodal-type disorder:

$$P(\{\varepsilon_i\})=c\,\delta(\varepsilon_i-\varepsilon_0)+(1-c)\,\delta(\varepsilon_i-0).\quad (16)$$

Here we shall be mainly interested in the superconducting transition temperature T_c. In this limit[19] the diagonal disorder affects mainly a diagonal part of the matrix Green's function $\boldsymbol{G}$. In fact, even for no disorder acting directly on a bosonic subsystem the boson order parameter ρ in Eq. (6) depends on the site index via fermion order parameter x_i. However, we expect this *induced* disorder to be weak and neglect it. This allows us to show how disorder, in a fermionic subsystem only, affects T_c.

The off-diagonal elements of the coherent potential vanish. Due to the general symmetry $\Sigma_{22}(i\omega)=-\Sigma_{11}(-i\omega)$[19] we can simplify the self-energy matrix to

$$\boldsymbol{\Sigma}(i\omega)=\begin{pmatrix}\Sigma_{11}(i\omega) & 0\\ 0 & -\Sigma_{11}(-i\omega)\end{pmatrix}.\quad (17)$$

$\Sigma_{11}(\omega)$ can be found from the CPA equation (13) which, for a normal phase, takes a well-known form[18]

$$\frac{1-c}{[\Sigma_{11}(\omega)]^{-1}+F(\omega)}+\frac{c}{[\Sigma_{11}(\omega)-\varepsilon_0]^{-1}+F(\omega)}=0,\quad (18)$$

with $F(\omega)=(1/N)\Sigma_{\mathbf{k}}\boldsymbol{G}_{11}^{CPA}(\mathbf{k},\omega)$. Equation (18) should be solved subject to a given dispersion relation $\varepsilon_{\mathbf{k}}$ and parameters c, ε_0.

Finally having calculated $\Sigma_{11}(\omega)$, we can find n^F and x [Eqs. (14) and (15)] as well as n^B, ρ [Eqs. (5) and (6)]. In particular, the critical temperature $T_c=(k_B\beta_c)^{-1}$ is given via

$$1=v^2\frac{\tanh[\beta_c(\Delta_B-2\mu)/2]}{\Delta_B-2\mu}\sum_{\mathbf{k}}\int_{-\infty}^{\infty}d\omega_1\int_{-\infty}^{\infty}d\omega_2\times A(\mathbf{k},\omega_1)A(\mathbf{k},\omega_2)\frac{\tanh[\beta_c\omega_1/2]+\tanh[\beta_c\omega_2/2]}{2(\omega_1+\omega_2)},\quad (19)$$

where $A(\mathbf{k},\omega)=(-1/\pi)\mathrm{Im}\{\boldsymbol{G}_{11}^{CPA}(\mathbf{k},\omega+i0^+)\}$ denotes the spectral function of the normal phase.

We choose for our study a case of weak boson fermion interaction $v=0.1$ (in units of the initial fermion bandwidth) and a total concentration of charge carriers $n_{tot}\equiv 2n_B+n_F=1$. Figure 1 shows how T_c of a clean system depends on the position of the boson level. There are three distinguishable regimes[2,8] of relative occupancy by bosons and fermions. Superconducting correlations are of course most visible when the chemical potential is close to $\Delta_B/2$. We choose the value $\Delta_B/2=-0.3$ to be close to the optimal value of transition temperature and to have comparable amount of fermions and bosons. For computations we use the two-dimensional square lattice dispersion—the van Hove singularity is safely distant from the Fermi energy for the above parameters.

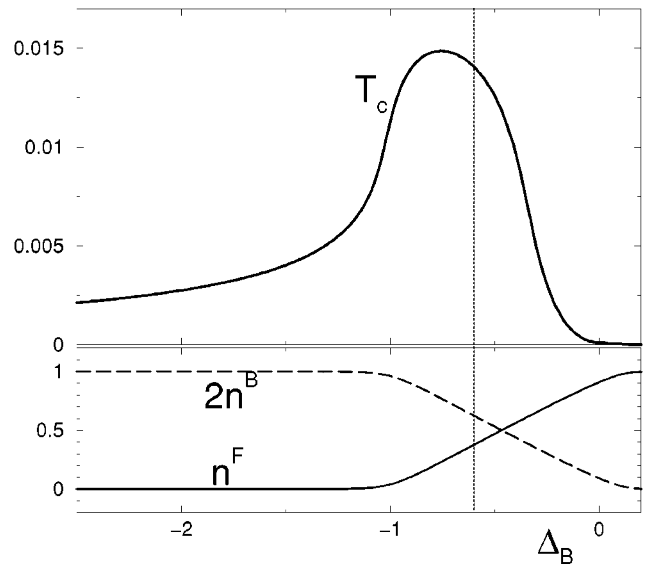

FIG. 1. Variation of T_c with respect to the boson energy Δ_B for a clean system with $n_{tot}=1$. The bottom panel illustrates the concentrations of fermions (n^F) and bosons (n^B) at $T=T_c$. Note the three distinct regimes of: predominantly local pairs $2n^B\sim n_{tot}$, coexisting pairs and fermions $n^F\sim 2n^B$, and predominantly fermions $n^F\sim n_{tot}$ (the so-called BCS limit).

In Fig. 2 we plot the transition temperature T_c, calculated from Eq. (19) against concentration c for several values of ε_0. With an increase of the concentration c of scattering centers we notice a gradual reduction of the critical temperature. This tendency can be understood by looking at the behavior of the fermion density of states at the Fermi energy $g(\varepsilon_F)$. Disorder is responsible for a renormalization of the low-energy sector and these low-energy states are involved in forming the superconducting-type correlations. As shown in the bottom panel there is an additional effect coming from the rearrangement of occupations n^F and n^B. With an increasing concentration c the fermion band is shifted toward higher energies and the system is then mainly occupied by bosons [the so-called local pair (LP) limit].

For negative values of ε_0 the disorder shows a stronger influence on T_c. On the one hand we again have a direct effect of the renormalized density of states [see $g(\varepsilon_F)$ in the middle panel of Fig. 3]. On the other hand, with an increase of c for any negative value of ε_0 the fermion band and the position of the chemical potential drift toward lower energies. As a consequence the number of fermions increases and the number of bosons decreases. Effectively we thus ap-

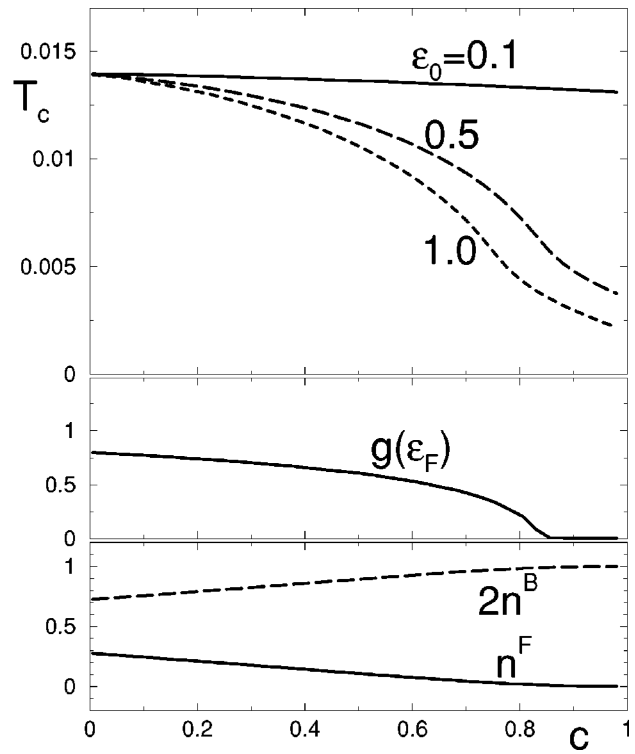

FIG. 2. Transition temperature T_c as a function of concentration c of scattering centers with various positive values of ε_0 (top panel). Density of states at the Fermi energy $g(\varepsilon_F)$ (middle panel) and relative occupations by bosons and fermions (bottom panel) for $\varepsilon_0=0.5$ at $T=T_c$.

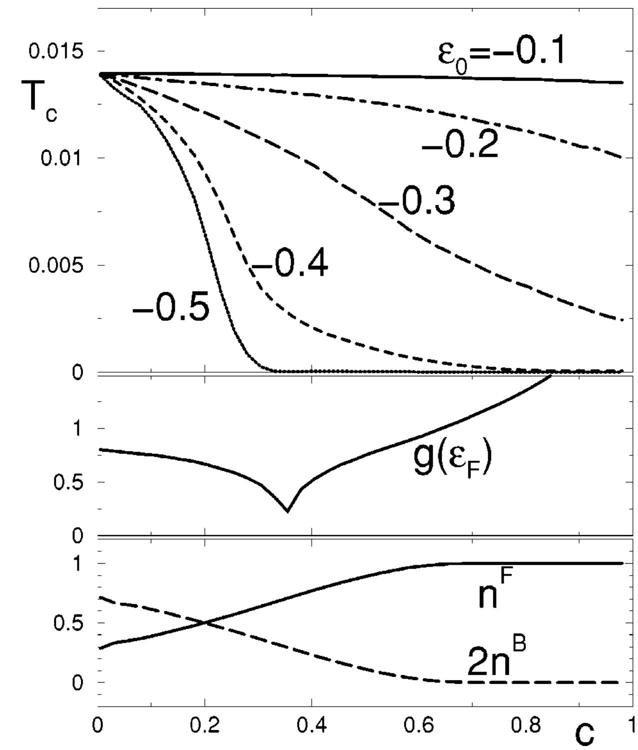

FIG. 3. The same as in Fig. 2 except for negative values of ε_0 (top panel). The middle and bottom panels correspond to $\varepsilon_0=-0.5$.

proach the BCS limit where the transition temperature diminishes very fast if $\Delta_B/2$ goes above μ (check for example the curves for $\varepsilon_0=-0.4$ and -0.5). The strong disorder in fermion subsystem makes the pairing mechanism almost ineffective.

In Fig. 4 we plot T_c versus (positive) ε_0 for several concentrations c. Again, T_c roughly follows, variation of the density of states $g(\epsilon_F)$ which is shown in the bottom panel. As discussed above for large values of concentration c and large positive ε_0 the system is mainly filled by bosons (the LP limit) so there is some finite T_c even when $\mu=\Delta_B/2$ is far below the fermion band, this is an artifact of the mean-field approximation.[2,8] The behavior of T_c with respect to negative values of ε_0 can be easily deduced from Fig. 3 so we skip this illustration.

In summary we notice that a change of the transition temperature T_c caused by weak disorder in a fermion system is controlled mainly by modification of the low-lying energy states. This is in accord with the Anderson theorem for spin-singlet s-wave superconductors. However, additional influence comes from redistribution of particle spectrum and their relative occupancy and such effects are dominant for large values of impurity concentration c and for their large scattering strength $|\varepsilon_0|$. In this limit the boson-fermion exchange becomes ineffective.

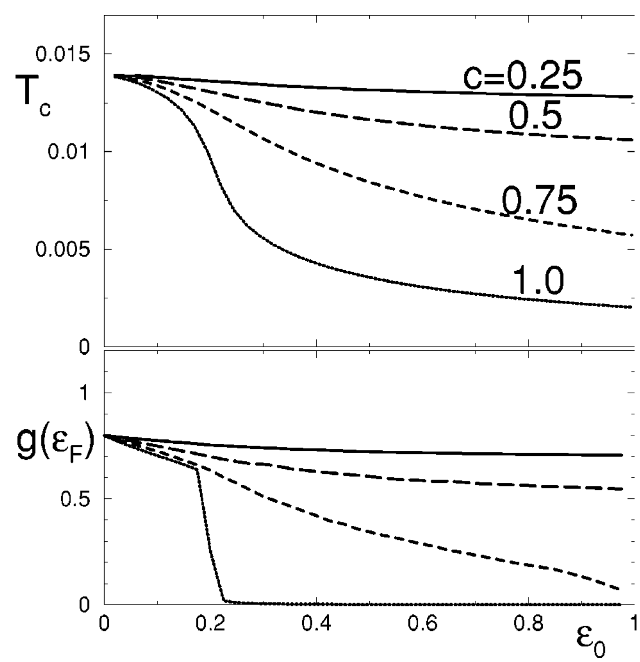

FIG. 4. Transition temperature T_c as a function of the energy ε_0 of the scattering centers whose concentration is c (top). Density of states $g(\varepsilon_F)$ for each of the concentrations c (bottom panel). For $c=1$ and $\varepsilon_0\geqslant 0.2$ the Fermi energy goes below the fermion band. The system is then strictly in the LP limit of the BF model.

IV. DISORDER IN A BOSON SUBSYSTEM

Now we turn attention to a case when boson energies are random $E_i\neq 0$ and, for simplicity, assume no fermion disorder i.e. $\varepsilon_i = 0$ for all the lattice sites. Scattering potential (9) then reduces to

$$\boldsymbol{V}_l = \begin{pmatrix} 0 & -f_l \langle c_{l\uparrow}^\dagger c_{l\downarrow}^\dagger \rangle \\ -f_l \langle c_{l\downarrow} c_{l\uparrow} \rangle & 0 \end{pmatrix}, \tag{20}$$

with

$$f_l = v^2 \frac{\tanh[\beta \gamma_l]}{2\gamma_l} \tag{21}$$

and

$$\gamma_l = \sqrt{\left(\frac{\Delta_B + E_l}{2} - \mu\right)^2 + |v \langle c_{l\downarrow} c_{l\uparrow} \rangle|^2}. \tag{22}$$

This means that the fluctuating boson energy level E_l induces fluctuations of the pairing strengths f_l in the fermion subsystem. To some extent, this situation reminds the negative U Hubbard model[20] for which the random local attraction $U_l < 0$ leads to the following scattering matrix:

$$\boldsymbol{V}_l^{(Hub)} = \begin{pmatrix} U_l \langle n_l \rangle /2 & U_l \langle c_{l\uparrow}^\dagger c_{l\downarrow}^\dagger \rangle \\ U_l \langle c_{l\downarrow} c_{l\uparrow} \rangle & U_l \langle n_l \rangle /2 \end{pmatrix}. \tag{23}$$

We see that in our case the role of a random pairing potential U_l is played by $-f_l$ given in Eq. (21). There are two extreme limits, as far as the effectiveness of the random boson energy $\Delta_B + E_l$ is concerned.

(i) For small (on the scale of fermion-boson interaction v) fluctuations of E_l, the effect of the disorder becomes negligible unless the chemical potential is pinned to the boson level $\mu = \Delta_B/2$, when the amplitude of the pairing potential is controlled by $f_l \sim v^2 \tanh[\beta x_l]/2x_l$ and is usually uniform except at very low temperatures $\beta \to \infty$ when $f_l \sim v^2/x_l$.

(ii) For large fluctuations of E_l one obtains $f_l \sim v^2 \tanh[(\beta/2)(\Delta_B + E_l - 2\mu)]/(\Delta_B + E_l - 2\mu)$.

To analyze effects of the disorder in boson subsystem we use a two-pole distribution $P(\{E_l\}) = \frac{1}{2}[\delta(E_l - E_0) + \delta(E_l + E_0)]$. The boson energy is $\Delta_B \pm E_0$ with an equal probability 0.5. Figure (5) shows a critical temperature T_c, calculated from Eq. (15), as a function of energy E_0 by which the boson energy is split. The strong dependence of T_c on disorder is a combined effect of the density of states, the fluctuating interactions, and the changes in concentration of carriers.

To estimate what influence comes only from the renormalization of the effective pairing, in Fig. 6 we plot the normalized transition temperature denoted by T_c and the normalized averaged $\langle f_l \rangle$ for the parameters given above (left panel), and for a fully symmetric case of the BF model (right panel). The transition temperature $T_c^{(BCS)}$ is the BCS-type estimate of the effect of changes in the effective pairing due to disorder:

$$T_c(E_0) \propto \exp\left(\frac{-1}{g(\varepsilon_F)\langle f_l(E_0)\rangle}\right). \tag{24}$$

A general trend observed in Fig. 6 is that the average effective interaction $\langle f_l(E_0) \rangle$ decreases with increasing disorder,

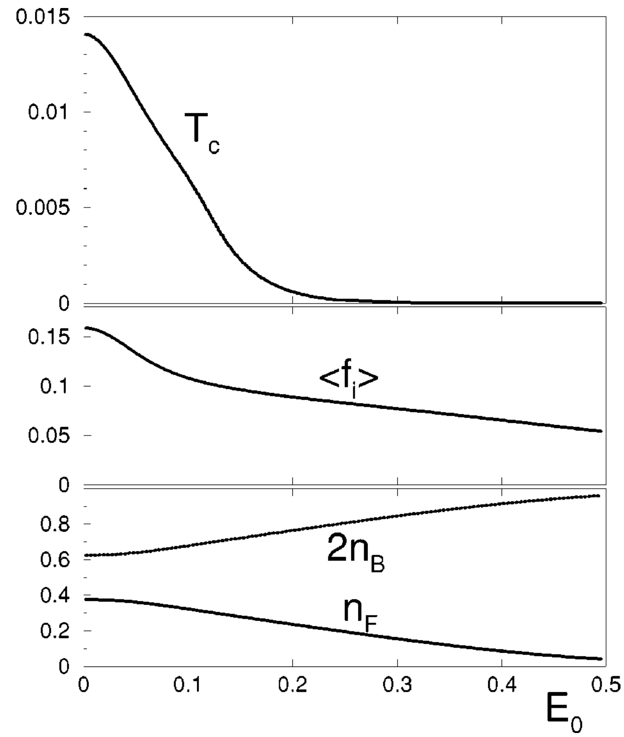

FIG. 5. Transition temperature T_c (top), and the averaged pairing potential $\langle f_l \rangle = \Sigma_{\{E_l\}} P(\{E_l\}) f_l$ (middle), together with the occupation of fermions n_F and bosons n_B at $T = T_c$ (bottom).

even though the bar fermion-boson interaction v remains constant. This decreasing pairing interaction is the only factor responsible for a behavior of T_c versus E_0 in the right panel. We notice the absence of the BCS-like exponential scaling which is due to unconventional pairing in the BF model.

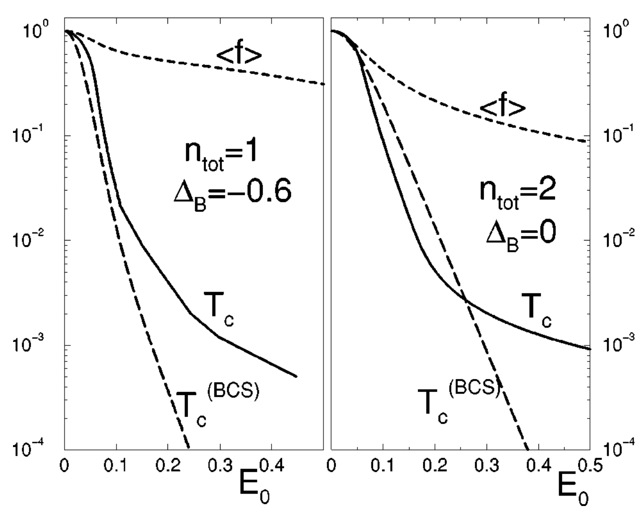

FIG. 6. Normalized critical temperature $T_c/T_c(E_0 = 0)$ and the normalized pairing potential $\langle f_l \rangle / \langle f_l(E_0 = 0) \rangle$ vs energy E_0. $T^{(BCS)}$ shows the BCS-like relation between the critical temperature and pairing potential. The left panel refers to $n_{tot} = 1$, $\Delta_B = -0.6$ discussed above, and the right panel corresponds to the symmetric case of the BF model $\Delta_B = 0$, $n_{tot} = 2$ (with half-filled boson and fermion subsystems).

In the left panel, corresponding to the above studied case $\Delta_B = -0.6$, $n_{tot} = 1$, we notice a larger discrepancy between the pairing amplitude and T_c. With an increase of E_0 the transition temperature is much more strongly reduced than in the symmetric case. This effect has to be assigned to redistributions of particle occupancies. At large values of E_0 we have practically only hard-core boson particles in the system, and they cannot induce superconductivity among fermions whose fraction becomes very small.

In a previous study[17] the authors used the same bimodal distribution of random boson energies. They found a strong reduction of T_c near $E_0 \sim 2v$ which agrees well with our data shown in Fig. 6. Moreover, the authors reported that disorder affects the ratio $\Delta_{sc}(T=0)/k_B T_c$ which changes from 4.2 (for a clean system[2,9]) to the standard BCS result 3.52 at large E_0. A simple explanation of this effect can be offered. The boson energy (which is split by $2E_0$) for sufficiently large E_0 is partly in the LP limit (for $E_i = -E_0$) and partly in the BCS limit (if $E_i = +E_0$). The second limit contributes with the standard BCS value if $|E_0|$ is large enough (see, e.g., Fig. 9 in Ref. 2).

V. CONCLUSION

The randomness of the site energies of both fermions and bosons has a strong effect on superconducting phase of the BF model. Weak disorder in the fermion subsystem affects the superconducting transition temperature mainly via rescaling the low-energy states which are involved in the the formation of the Cooper pairs. Therefore, T_c roughly follows the density of states at the Fermi energy. For sufficiently large disorder ε_0 there appears some critical concentration c at which T_c may eventually drop to zero.

Disorder in the boson subsystem has a much finer influence on superconductivity. The randomness of boson energies is transformed directly into the randomness of the pairing strength. Effectively physics of the disordered BF model becomes similar to that of the random negative-U Hubbard model.[20] Even the relatively small fluctuations of the boson energies show up their strong detrimental effects on superconductivity.

In a simple picture one can envision this situation as a random change between various regimes of superconductivity. Depending on a value of E_l the boson energy $\Delta_B + E_l$ can be either far below the Fermi energy (the LP limit), or far above the Fermi energy (the BCS limit). Each of such random configurations contributes with a different strength of superconducting correlations. On average, the superconducting transition temperature T_c strongly diminishes and practically disappears if the amplitude of the randomly fluctuating boson energies $|E_0|$ is large enough.

In summary, our calculations show that disorder strongly affects the *s*-wave superconducting phase of the BF model. This apparent contradiction with Anderson theorem can be understood because of a change of the effective pairing interaction induced by disorder, and this effect is contrary to the Anderson's main assumption.[16]

To compare our results with experimental data on high temperature superconductors one has to consider the *d*-wave superconducting order parameter. This type of symmetry arises in a natural way according to a recent derivation of the BF model.[4] The effect of disorder on such an anisotropic superconducting phase of the BF model is outside the scope of the present paper and will be discussed elsewhere.

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