Interlayer Josephson coupling for a gas of pancake vortices

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(Received 2 January 2002; revised manuscript received 22 March 2002; published 2 August 2002)

The dependence of the Josephson interlayer coupling in layered superconductors on the magnetic field H is studied numerically in the limit of complete disorder of the positions of pancake vortices (pancake gas). We find that the spatial average $\langle \cos \varphi(\vec{r}) \rangle$ is proportional to $1/H^{1/2}$, where φ is the gauge-invariant phase difference between two layers. The implication of this result for the interpretation of the magnetoabsorption resonances observed in layered superconductors is discussed.

DOI: 10.1103/PhysRevB.66.064505 PACS number(s): 74.50.+r, 74.25.Nf, 74.60.Ge

Interlayer Josephson coupling is a very important characteristics of layered superconductors, which determines many of their static and dynamic properties. It is widely believed that magnetoabsorption microwave resonances, which have been observed in Bi compounds and some other layered superconductors, provide an excellent experimental probe of the interlayer Josephson coupling and its dependence on the magnetic field normal to the superconducting layers (see Refs. 1–6, and references therein). This belief is based on the assumption that *any* magetoabsorption resonance is related to the Josephson plasma mode (JPR) with the frequency

$$\omega_0^2 = \omega_p^2 \langle \cos \varphi(\vec{r}) \rangle, \tag{1}$$

where ω_p is the Josephson-plasma frequency at zero magnetic field, $\vec{r}=(x,y)$ is the in-plane coordinate, $\varphi(\vec{r})$ is the stationary gauge-invariant phase difference between neighboring layers, and $\langle \cdots \rangle$ denotes spatial averaging. If the magnetic field generates ideally straight vortices normal to the layers [Fig. 1(a)] one has $\varphi(\vec{r})=0$, and the resonance frequency does not depend on the magnetic field. But the phase $\varphi(\vec{r})$ is nonzero if there is a misalignment of the pancake vortices in neighboring layers due to thermal fluctuations and disorder, Fig. 1(b). This misalignment of pancakes was used to explain the power-law decrease of the resonance frequency with increasing magnetic field (anticyclotronic behavior).

The connection of magnetoabsorption resonances with the Josephson plasma mode is well justified at low magnetic fields normal to the superconducting layers. But at high normal magnetic fields (above the phase-transition line of the vortex matter) the JPR interpretation of magnetoabsorption resonances is a controversial issue and was challenged by a competing interpretation in terms of a vortex mode governed by pinning (see discussion in Refs. 7–11). A characteristic feature of the observed magnetoabsorption resonances at high magnetic fields is the anticyclotronic behavior: the resonance frequency squared decreases with increasing magnetic field as $1/H^{\alpha}$, where α was reported to be between 0.7 and 1. The questions to the JPR interpretation of the magnetoabsorption resonances are (i) can one use the expression Eq. (1) for the resonance frequency at high magnetic fields and (ii) if one can, how to calculate $\langle \cos \varphi(\vec{r}) \rangle$ in this expression, namely, whether a correct calculation really predicts the anticyclotronic behavior observed experimentally. The present work is an attempt to answer the second question.

Let us remind, how one can derive the anticyclotronic behavior from Eq. (1). It was assumed that the phase $\varphi(\vec{r})$ is a Gaussian random variable and therefore[12]

$$\langle \cos \varphi(\vec{r}) \rangle = \exp\left(-\frac{\langle \varphi(\vec{r})^2 \rangle}{2} \right). \tag{2}$$

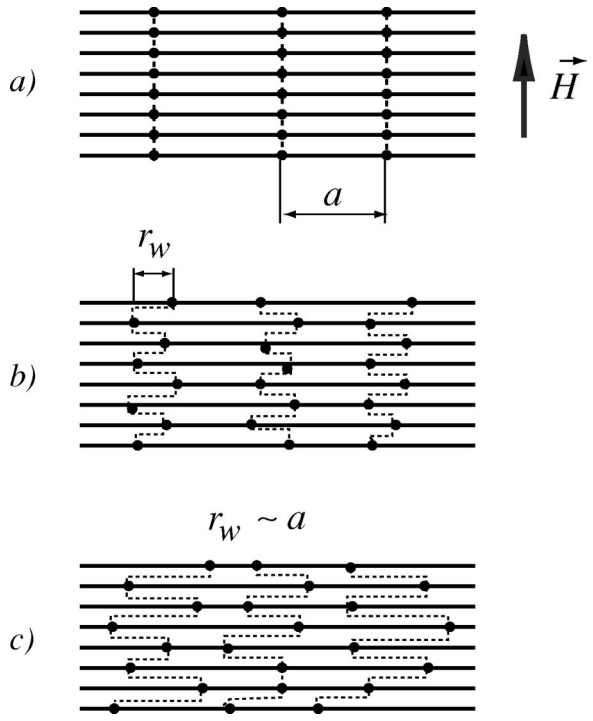

FIG. 1. Vortices in a magnetic field normal to the layers. The small black circles indicate pancake cores. The dashed lines are Josephson strings. (a) Ideally straight vortex lines. (b) Vortex lines are bent due to thermal fluctuations and disorder. (c) The wandering distance r_w is so large that the vortex lines are decoupled, and in fact the shown Josephson strings are not well defined, since one may connect the pancakes in neighboring layers in many different ways.

According to Koshelev *et al.*,[13] the disorder in the positions of pancakes (wandering of vortex lines) results in phase fluctuations with

$$\langle \varphi(\vec{r})^2 \rangle \approx \frac{r_w^2}{a^2} \ln \frac{\lambda_J}{r_w}, \tag{3}$$

where $a = \sqrt{\Phi_0/B}$ is the intervortex distance, r_w is the average wandering length (see Fig. 1), and λ_J is the Josephson length. If the vortex line is disintegrated and $r_w \sim a$, then $\langle \varphi(\vec{r})^2 \rangle \sim \ln(\lambda_J/a)$, and Eq. (2) yields the anticyclotronic behavior, though one cannot find the value of the exponent α within this simple approach. Also it is not clear whether one can use Eq. (2) when $\langle \varphi(\vec{r})^2 \rangle$ is of the order of unity or much larger, since $\cos\varphi$ depends only on φ modulo 2π, thus the definition of any $|\varphi| > \pi$ is not unique.

The aim of the present paper is to calculate numerically $\langle \cos\varphi(\vec{r}) \rangle$ for a gas of pancake vortices. We define a ''gas''of pancakes as the state of vortex matter without any correlation between pancake positions, both along the magnetic field normal to the layers and in the plane of the layers. The question where a pancake gas exists on the phase diagram of the vortex matter is beyond the scope of our article. Here we address this gas state because one can derive the anticyclotronic behavior from Eq. (1) only for this state. Indeed, the condition that $r_w \sim a$ in Eq. (3) means that there is no correlation between pancakes in neighboring layers (decoupling or disintegration of vortex lines). And even when the pancakes form regular lattices in the layers [two-dimensional (2D) pancake solid] which are randomly shifted with respect to each other, the derivation of the anticyclotronic behavior also fails. Namely, in the case of a 2D pancake solid Eq. (3) has to be modified. As was shown in Ref. 11 (see also Ref. 14), the Josephson length λ_J should be replaced by the intervortex distance a. This does no longer lead to a power-law dependence of $\langle \cos\varphi(\vec{r}) \rangle$ on the magnetic field. Thus, the anticyclotronic behavior is expected only for a pancake gas as defined above. Then, comparison of our calculation for the pancake gas with the observed power-law dependence of magnetoabsorption resonances at high normal magnetic fields may check the reliability of the widely accepted interpretation of these resonances based on the Josephson-plasma mode.

Strictly speaking, in order to calculate the phase fluctuations produced by disorder in pancake positions, we should solve the system of stationary sine-Gordon equations for the phase differences across all interlayer spacings. However, one may expect to receive a correct physical picture of the phenomenon by solving the simpler problem of a double layer with only one fluctuating phase difference $\varphi(x,y)$, which is described by the sine-Gordon equation

$$\frac{1}{\lambda_J^2} \sin\varphi - \nabla^2 \varphi = 0. \tag{4}$$

Formally, the pancakes in the upper and lower layer may be considered as vortices and antivortices, respectively, projected unto the x,y plane, with $\varphi(x,y)$ now being the phase of this 2D arrangement of vortex-antivortex pairs. For example, in the case of perfect alignment, the vortex and antivortex positions coincide, thus all vortices cancel and one has $\varphi = 0$ everywhere, as it should be.

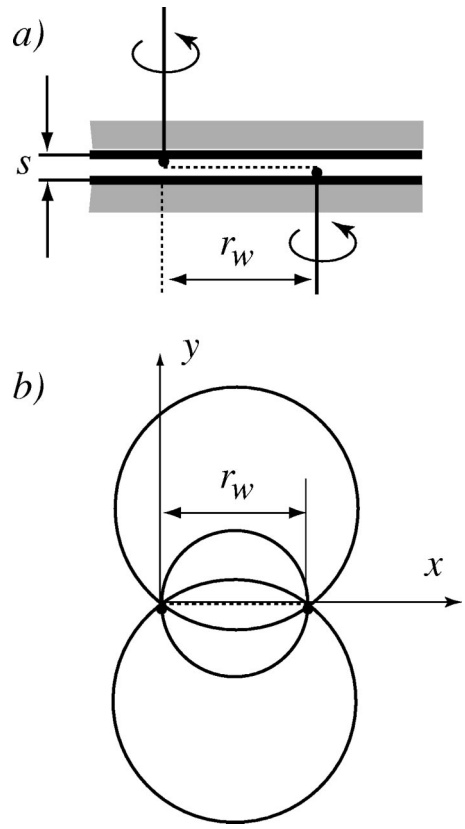

FIG. 2. Josephson string in a single large planar Josephson junction. (a) Side view on the junction. (b) Lines of constant phase difference $\varphi(x,y)$ in the junction plane for a short Josephson string $r_w \ll \lambda_J$. The dotted line on the x axis is a cut, where the phase jumps by 2π.

At high fields the intervortex distance $a = \sqrt{\Phi_0/B}$ is much less than the Josephson length λ_J. If one neglects the Josephson-coupling term $\propto 1/\lambda_J^2$ in Eq. (4), two pancakes in the neighboring layers, positioned at the points (0,0) and $(0,r_w)$, generate the phase φ shown in Fig. 2:

$$\varphi_0(\vec{r}) = \arctan\frac{y}{x} - \arctan\frac{y}{x - r_w}. \tag{5}$$

At large distances $r = \sqrt{x^2+y^2} \gg r_w$ (but not larger than λ_J) the phase, Eq. (5), is $\varphi_0(\vec{r}) = -r_w y/r^2$. At distances $r \gg \lambda_J$ the phase $\varphi_0(\vec{r})$ is exponentially small. This phase distribution corresponds to a short Josephson string.

Thus at large r, $\varphi_0(r)^2$ decreases as $1/r^2$, and the contribution of one Josephson string to $\langle \varphi_0(r)^2 \rangle$ is logarithmically divergent with the system size. This means that we cannot neglect the Josephson coupling even in the limit $\lambda_J \gg a$. It is, however, reasonable to expect that the main outcome of a more elaborate calculation based on the sine-Gordon equation would be a proper upper cutoff for the logarithmic divergence. Thus one may hope that the effect of finite λ_J can be simulated by choosing a finite size $L \approx \lambda_J$ of the consid-

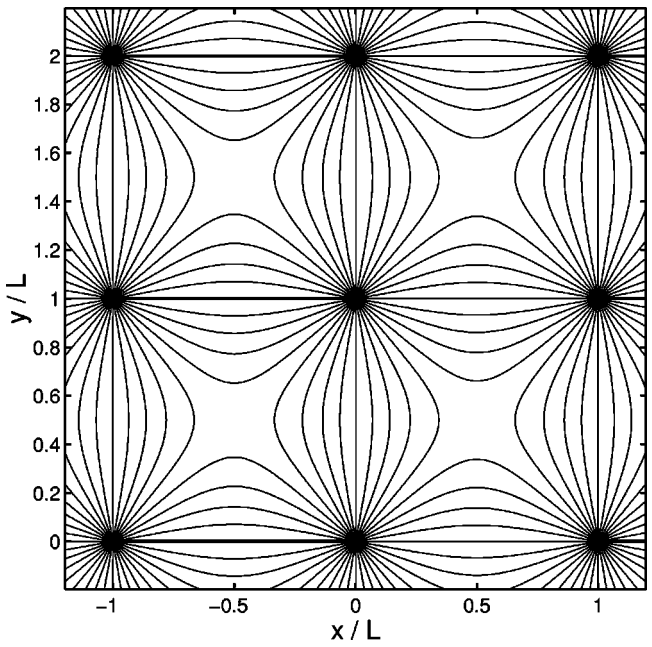

FIG. 3. Lines of equal phase for an infinite square superlattice of vortex-antivortex pairs. The spacing of the contours is $\pi/18 \equiv 10°$. At the three bold lines the phase jumps from $-\pi$ to $+\pi$.

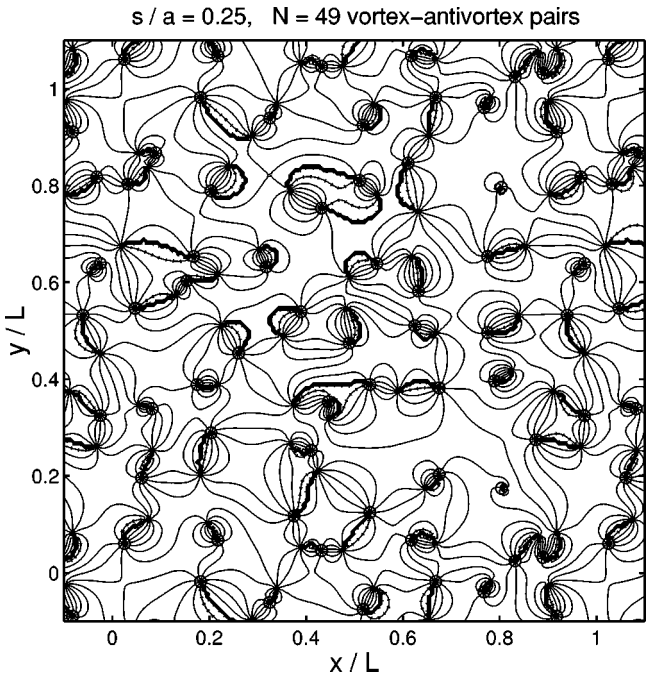

FIG. 4. Lines of equal phase φ for 49 vortex-antivortex pairs shifted from ideal square lattice positions with spacing a by random displacements with variance $s/a=0.25$. The variance of the pair spacing is $\sqrt{2}s$. The contour spacing is $\pi/6$. The jump of the phase from $+\pi$ to $-\pi$ may be chosen at the bold lines; the lines with dots then shown $\varphi=5\pi/6$.

ered area with a large number of vortex-antivortex pairs. Indeed, usually a logarithmic divergence is not sensitive to the method that provides the cutoff (see also below).

The procedure of our numerical calculation is as follows. We choose more or less random positions of N vortices and N antivortices in a quadratic cell of the size $L\times L$, so the average distance of each type is $a=L/\sqrt{N}$. The phase $\varphi(x,y)$ of the wave function of this vortex arrangement is a solution of the Laplace equation, which follows from Eq. (4) in the limit $\lambda_J\to\infty$:

$$\varphi(x,y)+2\pi n=\arg\prod_1^N \frac{z-z_i^v}{z-z_i^a}$$

$$=\sum_1^N\left[\arctan\frac{y-y_i^v}{x-x_i^v}-\arctan\frac{y-y_i^a}{x-x_i^a}\right], \quad (6)$$

where (x_i^v, y_i^v) and (x_i^a, y_i^a) are the positions of the vortices and antivortices, respectively, and $z=x+iy$, etc., are complex numbers. Note that the phase φ is defined only modulo 2π, expressed by the integer n in Eq. (6), since in all physical expressions enter only $\cos\varphi$ and $\sin\varphi$. Below we need the spatial average $\langle\exp[i\varphi(\vec{r})]\rangle$.

To simulate disorder, we start with a perfect square lattice and then shift each vortex and each antivortex by a Gaussian distributed random vector of mean square length s away from this ideal position. When $s=0$ one has $\varphi=0$ since at all ideal lattice positions a vortex and an antivortex cancel each other. With increasing displacement amplitude s the phase fluctuations increase, and when s/a exceeds the value ≈ 0.5, we find saturation to the limit of completely uncorrelated randomly positioned N vortices and N antivortices, which may be called a vortex gas.

We used two types of boundary conditions at the border of the square $L\times L$.

(a) We continue the basic square periodically but such that the vorticity changes sign in neighboring cells. This means, each vortex in the basic square actually represents an ideal superlattice of vortex-antivortex pairs of side length $\sqrt{2}L$. Similarly, each antivortex in the basic square means such a lattice. This change of sign in neighboring cells cuts off the long-range interaction of vortices and thus simulates the finite Josephson length $\lambda_J\approx L$. The phase of this superlattice is shown in Fig. 3.

(b) We consider only one square and average the phase in this finite area.

We find that both types of boundary conditions yield practically the same results, and the treatment of the vortices near the boundaries is not crucial.This confirms our expectation that the result should not be sensitive to how we cut off the long-range interaction between strings. Therefore, if we equate L to λ_J, our calculation is valid also for an infinite system, in which the cutoff is provided by the Josephson length λ_J.

Figures 4 to 6 show the lines of equal phase of arrangements of 49 vortex-antivortex pairs with such periodic boundary conditions for three values of the displacement amplitude s: $s/a=0.25$ (still well defined pairs), $s/a=0.5$ (nearly uncorrelated random positions of vortices and antivortices), and $s/a=1$ (vortex-antivortex gas or plasma).

In our simulations we calculate the following average. Without Josephson coupling, the phase pattern $\varphi(x,y)$ can be shifted by an arbitrary constant phase φ_0 because the

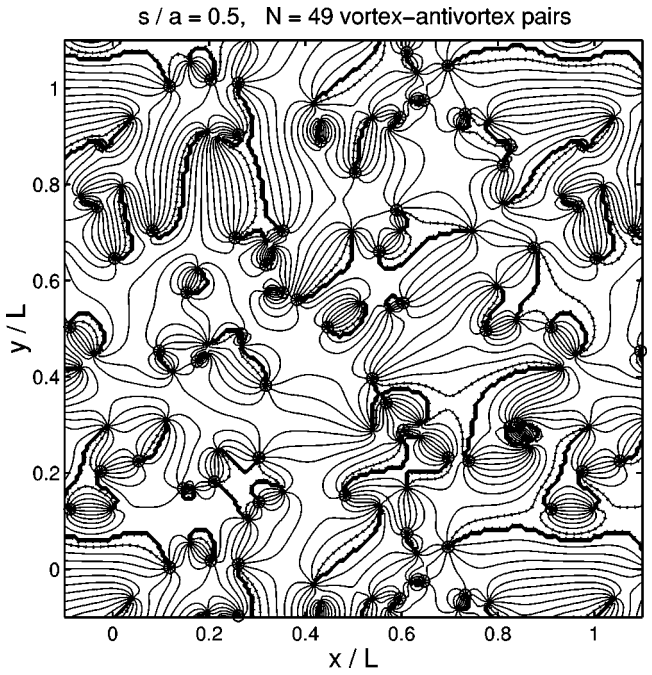

FIG. 5. As in Fig. 4 but for $s/a=0.5$.

energy does not depend on this constant. However, the spatial average $\langle\cos[\varphi(x,y)-\varphi_0]\rangle$ entering Eq. (1), does depend on this constant φ_0. The correct choice of this constant follows from the Josephson coupling term in the free energy, proportional to $-\lambda_J^{-2}\langle\cos[\varphi(x,y)-\varphi_0]\rangle$. This Josephson energy is minimized by choosing φ_0 such as to maximize $\langle\cos[\varphi(x,y)-\varphi_0]\rangle$. With this choice of φ_0 one has $\langle\sin[\varphi(x,y)-\varphi_0]\rangle=0$. The resulting maximum of $\langle\cos[\varphi(x,y)-\varphi_0]\rangle$ is equal to the length of the complex vector $\langle\exp[i\varphi(x,y)]\rangle$. We thus define the factor f which enters in Eq. (1), $\omega_0^2=\omega_p^2 f$, as

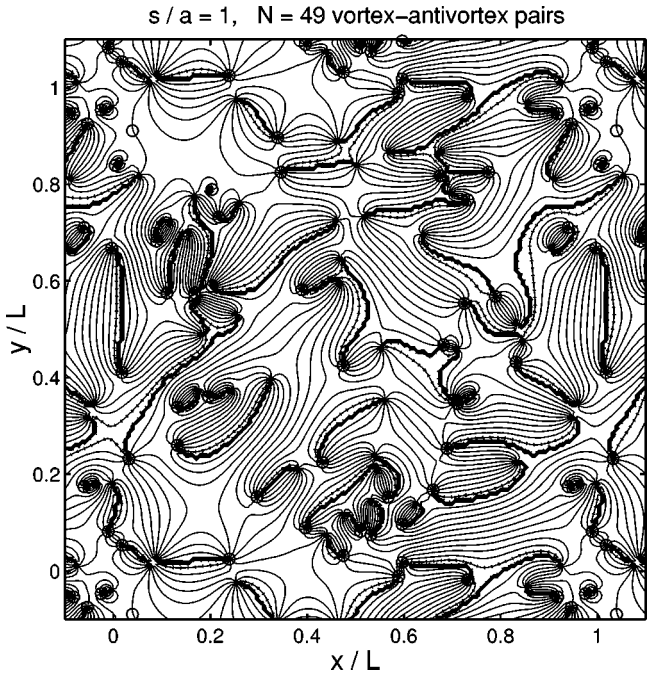

FIG. 6. As in Fig. 4 but for $s/a=1$.

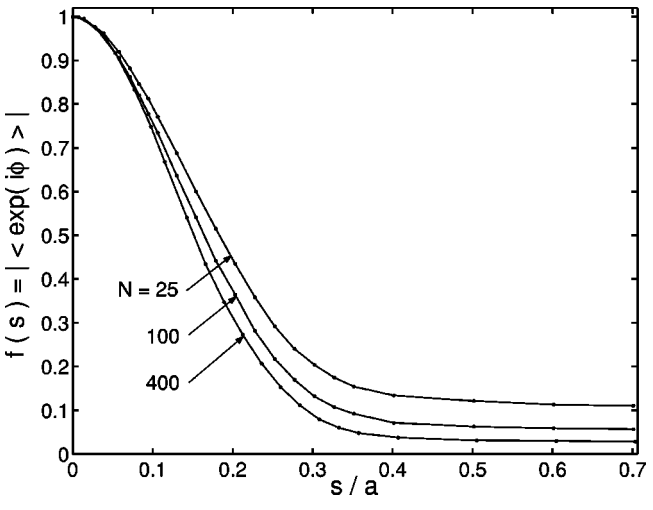

FIG. 7. The spatial average $f(s)=|\langle\exp(i\varphi)\rangle|$, Eq. (7), for lattices of N vortices and N antivortices shifted by random displacements with variance s from ideal square-lattice positions with spacing a, i.e., the rms pair spacing is $\sqrt{2}s$. For $N=25$, 100, and 400. This phase $\varphi(x,y)$ is identical to the phase difference between two layers with N point vortices randomly shifted with variance s away from an ideal lattice.

$$f=\langle\cos[\varphi(x,y)-\varphi_0]\rangle_{\max}=|\langle\exp[i\varphi(x,y)]\rangle|. \qquad (7)$$

In our simulations, this factor depends on the degree of randomness, i.e., on the amplitude s of our random displacements, thus $f=f(s)$.

Figure 7 shows the function $f(s)$ for a system of $N=25$, 100, and 400 vortices and same number of antivortices, averaged over an ensemble of 1000 to 4000 different realizations. Obviously, one has $f(0)=1$, and with increasing s, $f(s)$ decreases monotonically. For $s/a\geq 0.5$ the function $f(s)$ practically saturates to a constant value $f(\infty)$, corresponding to completely uncorrelated vortices and antivortices.

The saturation value $f(\infty)$ depends on the number N of vortex-antivortex pairs in our basic cell of size $L\times L$. Since L has the meaning of the Josephson length λ_J, different N correspond to different ratios $a/\lambda_J\approx a/L=N^{-1/2}$. Figure 8 shows $f(\infty)$ as a function of a/L. We find $f(\infty)=ca/L$ with the constant $c=0.55$.

For small disorder, $s\approx r_w\ll a$, both formulas, Eqs. (2) and (3) should be accurate enough if one replaces λ_J by L in Eq. (3). This is indeed confirmed by our numerical calculations. But more interesting for us is the limit of completely random positions $s\approx r_w\gg a$, when anticyclotronic behavior is expected.

Our computations show that in this case $\langle\cos\varphi(\vec{r})\rangle_{\max}=f(\infty)\propto 1/\sqrt{N}$ (Fig. 8). On the other hand, the number of pancakes $N=L^2/a^2$ in our cell is proportional to the magnetic field $H=\Phi_0/a^2$, where a is the intervortex distance. Thus our numerical calculation yields anticyclotronic power-law behavior $\omega_0^2\propto 1/H^\alpha$ with an exponent $\alpha=1/2$.[15] Note that this law was obtained for two types of boundary conditions. The independence of this law on the boundary conditions indicates that modeling the Josephson length λ_J by a finite system size L is reasonable.

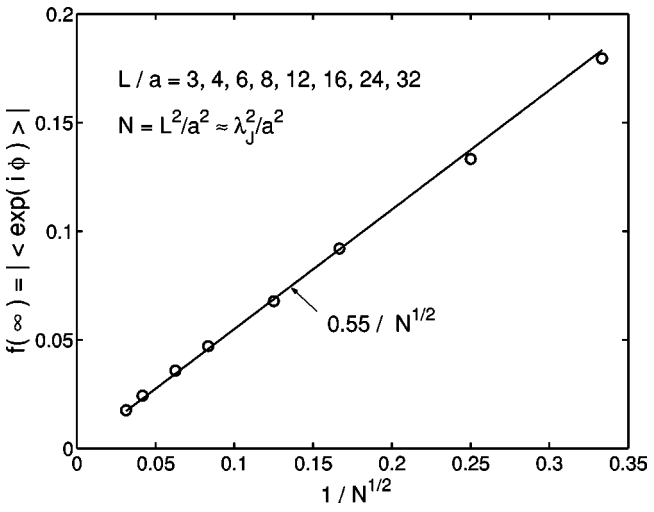

FIG. 8. The spatial average $f(\infty)=|\langle\exp(i\varphi)\rangle|$ for a gas of randomly positioned vortices and antivortices as a function of the number of pairs in the periodicity cell $N=L^2/a^2\approx\lambda_J^2/a^2$. The linear fit shows $f(\infty)=0.55/N^{1/2}$.

However, the obtained power law $f(\infty)\propto N^{-1/2}\propto H^{-1/2}$does *not* result from the phase φ being a Gaussian variable as suggested in Ref. 12. In fact, this law can be obtained from the following heuristic picture. Since $\cos\varphi$ varies between values of order ± 1 over the intervortex distance a, the correlation length for the fluctuating $\cos\varphi$ is a. Therefore, the average of $\cos\varphi$ in the area L^2 is roughly given by the sum of $N=L^2/a^2$ randomly distributed numbers between $+1$ and -1. This average itself is also a random quantity, with ensemble average zero, and with variance of the order of $N^{-1/2}=a/L$. The average value of $\cos\varphi$ vanishes in this picture because there is no preferred phase in this model. However, due to the finite Josephson coupling, $\cos\varphi$ tends to a maximum value, as close to unity as possible. Shifting the phase distribution $\varphi(\vec{r})$ by a constant phase φ_0, we can maximize the value of $\langle\cos\varphi(\vec{r})\rangle$ up to a value of the order of its variance $N^{-1/2}=a/L$. This argument reproduces our numerical result.

The calculated exponent $\alpha=1/2$ in the power-law dependence $\langle\cos\varphi(\vec{r})\rangle\propto 1/H^{\alpha}$ clearly differs from the exponent α deduced from the experiment with help of Eq. (1). Indeed, the earlier experimental papers reported α between 0.7 and 1,[1,2] while in recent papers[5,6] an $\alpha=1$ was observed. This discrepancy with our theoretical exponent $\alpha=1/2$ indicates that the agreement of the theory based on Eq. (1) with the experiment is not so excellent as widely accepted, and the nature of the magnetoabsorption resonances in high magnetic fields normal to the layers requires further investigation. One possible way to resolve this disagreement is related to the discussion whether and when Eq. (1) is valid.[7–11]

ACKNOWLEDGMENTS

E.B.S. acknowledges support by a grant of the Israel Academy of Sciences and Humanities.

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