Amplification of ultrasonic waves by a moving vortex structure

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The interaction of a longitudinal ultrasonic wave with a moving vortex structure is considered for high-T_c superconductors. It is found that this interaction contributes to the attenuation coefficient and the velocity of this wave. Expressions relating these contributions to the vortex structure velocity are derived. As the velocity of the vortex structure exceeds the wave velocity, the contribution to the attenuation coefficient is found to change its sign, and, hence, the wave can be amplified. This effect can be observed relatively easy in a periodic superconducting film structure when the electric current is passed through the film in the presence of the surface acoustic waves. Values of the current required to observe the amplification are estimated for selected ultrasonic frequencies.

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It is well known that ultrasonic waves propagating in a semiconductor can be amplified by applying dc electric field.[1] This effect results from the interaction of mobile charge carriers with the ultrasonic wave. The dc electric field causes the interacting charge carriers to drift in the direction of wave propagation faster than the sound velocity. There is a rationale to suppose that a similar effect can occur in the high-T_c superconductors (HTS). This is based on the following specific features of HTS: (i) These materials have a relatively high vortex mobility, which is seen in an observable giant flux creep. This high mobility is traced, on the one hand, to a short coherence length, which leads to a small activation energy for pinned vortices and, on the other hand, to high values of the superconducting transition temperature T_c. (ii) A direct current can drag the vortex structure with a required velocity along the ultrasonic wave propagating in the materials.

A purpose of this work is to show that the above effect can appear in the type-II superconductors placed in an external magnetic field. The conditions for this effect to appear in the superconductors are determined by two factors: the mobility of a vortex structure and the strength of interaction between an ultrasonic wave and a moving vortex structure (sound-vortex interaction).

Previously, the sound-vortex interaction was considered for the case, when the vortex structure does not move as a whole.[2–19] Meanwhile it has been shown[2–6] that an ultrasonic wave can drag the vortex structure in the superconductors. This phenomenon has also been observed experimentally.[20–22] In the present paper, we discuss an additional phenomenon related to the mobility of the vortex structure in HTS. Namely, we consider the interaction of a moving vortex structure with a longitudinal ultrasonic wave, and show that the moving vortex structure can amplify the wave. This phenomenon is a result of specific interaction of the longitudinal waves with the vortex structure moving as a whole. To the best of our knowledge, such sound-vortex interaction has not yet been considered in the literature.

It is important to note that the proposed effect does not depend on the particular nature of the interaction between the vortex cores and the ionic lattice of the superconductor, all kinds of such interactions (e.g., centers of pining, deformation interactions, etc.) are taken into account using the phenomenological coefficient of viscosity η. The viscosity coefficient η may depend on a speed of the vortex structure motion (see, for example, Refs. 23,24). In the further consideration, η is not assumed to be independent of the speed, and, hence, all derived expressions are valid, even if η is speed dependent. The proposed theory may thus be used for experimental measurements of the dependence of the viscosity coefficient on the vortex speed.

In order to describe the motion of the vortex structure in a superconductor, we derive a system of equations describing the interaction between a vortex structure and an elastic continuum.[6] All equations are written here in the laboratory reference frame. In this case, the first London‘s equation has the form[25]

$$\Lambda \frac{\partial \mathbf{j}_s}{\partial t} = \mathbf{E} - \mathbf{B}_v \times \dot{\mathbf{W}}, \tag{1}$$

where $\Lambda = m/n_s q^2 = \lambda_L^2 \mu_0$; m,q, and n_s are the mass, charge, and density of the superconducting charge carriers, respectively; λ_L is London‘s penetration depth, E is the electric-field strength in the laboratory reference frame; $\dot{\mathbf{W}}$ denotes the local velocity of the vortex structure, $\mathbf{B}_\nu$ is the magnetic induction produced by the vortex structure in the superconductor. The superconducting current can be expressed in terms of the total electric current in the system and the ionic current

$$\mathbf{j}_s = \mathbf{j} + q n_s \dot{\mathbf{U}}, \tag{2}$$

where $\mathbf{U}$ is the deformation vector of the ion lattice of the superconductor. Here we neglect the normal component of the current. Substituting expression (2) into Eq. (1), applying the operator $\nabla\times$ to both sides of Eq. (1), and using Maxwell equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{3}$$

$$\nabla \times \mathbf{H} = \mathbf{j} \tag{4}$$

after simple manipulations we obtain the expression

$$\frac{\partial}{\partial t}\left(-\lambda_L^2\nabla^2\mathbf{B}+\mathbf{B}+\frac{m}{q}\nabla\times\dot{\mathbf{U}}\right)=\nabla\times(\dot{\mathbf{W}}\times\mathbf{B}_v), \quad (5)$$

which, in the harmonic approximation, gets the form

$$\mathbf{B}-\lambda_L^2\nabla^2\mathbf{B}=-\frac{m}{q}\nabla\times\dot{\mathbf{U}}+\nabla\times(\mathbf{W}\times\mathbf{B}_v). \quad (6)$$

Note that, if the vector $\mathbf{U}$ in Eq. (6) is equal to 0 meaning that the ionic lattice does not move, after simple manipulations, Eq. (6) coincides with Eq. (11) Ref. 26.

Now we write the local equation for motion of the vortex structure (neglecting the inertial mass of the vortex), which results from the condition of balance of the forces: $\mathbf{F}_{fr}=\mathbf{F}_L$, where $\mathbf{F}_L=\mathbf{J}_s'\times\mathbf{B}_\nu$ is the Lorentz force and $\mathbf{F}_{fr}=\eta(\dot{\mathbf{W}}-\dot{\mathbf{U}})-\tilde{\eta}(\dot{\mathbf{W}}-\dot{\mathbf{U}})\times\mathbf{B}_\nu$ is the friction force of the vortex structure acting on the crystal lattice of the superconductor, $\mathbf{J}_s'$ is the current density in the local reference frame attached to the vortex structure. Taking into account that $\mathbf{j}_s'=(\mathbf{j}_s-qn_s\dot{\mathbf{W}})$, we obtain the equation of motion of the vortex structure

$$\eta(\dot{\mathbf{W}}-\dot{\mathbf{U}})-\tilde{\eta}(\dot{\mathbf{W}}-\dot{\mathbf{U}})\times\mathbf{B}_\nu=(\mathbf{j}_s-qn_s\dot{\mathbf{W}})\times\mathbf{B}_\nu, \quad (7)$$

where η and $\tilde{\eta}$ are the longitudinal and transverse coefficients of viscosity of the vortex structure, respectively. Furthermore, $\tilde{\eta}=(q/h)\eta'$, η' is the transverse coefficient of viscosity (introduced in Refs. 27,28) for a single vortex, and h denotes the Planck‘s constant. The discussion of the microscopic nature of this coefficient can be found in Ref. 29. For a complete description of the motion of the vortex structure under the action of the ultrasonic wave, it is necessary to take into account that the density of the vortices is identical to the distribution of $\mathbf{B}_v$ in the superconductor, and their local velocities are not independent quantities, but coupled by the continuity equation. The continuity equation for the vortex structure is

$$\frac{\partial\mathbf{B}_v}{\partial t}=-\nabla\times(\mathbf{B}_v\times\dot{\mathbf{W}}). \quad (8)$$

The Eqs. (6) and (8) completely describe the motion of the vortex structure under the action of ultrasonic wave.

To take into account the inverse effect of the vortex structure motion on the crystal lattice, we write down the elasticity theory equation, which describes the ultrasonic wave propagation in a superconductor

$$\rho\ddot{\mathbf{U}}=\rho c_t^2\Delta\mathbf{U}+\rho(c_l^2-c_t^2)\mathrm{graddiv}\mathbf{U}-qn_s\dot{\mathbf{U}}\times\mathbf{B}-qn_s\mathbf{E}+\mathbf{F}_{fr}. \quad (9)$$

Here ρ is the mass density of the superconductor, c_l, c_t are the velocities of the longitudinal and transverse waves in the absence of a vortex structure. The third and fourth terms of this equation describe the effect of electric and magnetic fields on the ionic lattice of superconductor, and the fifth term describes the effect of friction. In Eq. (9), the effect of ultrasonic attenuation in the absence of vortices is not taken into account, because here we are mainly interested in an additional attenuation due to the sound-vortex interaction. Thus to describe the mutual motion of the elastic continuum and the vortex lattice, the system of Eqs. (6)–(9) has to be solved. These equations are valid for ultrasonic wavelengths that are much larger then the intervortex distance.

Now we turn to the interaction of a moving vortex structure with a longitudinal ultrasonic wave. For definiteness, we consider a uniform and isotropic superconductor placed in the external magnetic field B_0, directed along the negative z axis, while the longitudinal ultrasonic wave is assumed to propagate in the positive direction along the y axis and takes the form $\mathbf{U}=\mathbf{U}_0\exp(iky-i\omega t)$, where $\mathbf{U}$ is the deformation vector of the ionic lattice of the superconductor, k and ω denote the wave vector and the oscillation frequency, respectively. The vortex structure is supposed to move with the velocity $\mathbf{V}$ along the ultrasonic wave propagation direction. Here we consider the dirty superconductor and assume that the Magnus force is balanced by the transverse viscosity force: $qn_s-\tilde{\eta}=0$. This problem is solved here in the linear approximation taking into account the terms of a first-order infinitesimal in the amplitude of the ultrasonic wave. Taking $\mathbf{B}=\mathbf{B}_0+\Delta\mathbf{B}$ and $\dot{\mathbf{W}}=\mathbf{V}+\dot{\mathbf{W}}'$, where B_0 is the uniform component of the magnetic induction of the superconductor that coincides with the external field and $\Delta\mathbf{B}$ is the magnetic induction oscillation due to the oscillation of the vortex structure, and assuming $B_0\gg\Delta B$, one can modify Eq. (6) with regard to Eq. (8) as follows:

$$\mathbf{B}-\lambda_L^2\nabla^2\mathbf{B}=-\frac{m}{q}\nabla\times\dot{\mathbf{U}}+\nabla\times(\mathbf{W}'\times\mathbf{B}_0)+\frac{1}{-i\omega}\nabla\times(\mathbf{V}\times\Delta\mathbf{B}_\nu). \quad (10)$$

Solving Eq. (10) for $\mathbf{B}$ and using the Maxwell‘s equation (4), we find the total current in the superconductor as

$$\mathbf{j}=\frac{k^2}{\mu_0[1+(\lambda_L k)^2]}\left[\mathbf{W}'\times\mathbf{B}_0+\frac{1}{\omega}\left(\frac{(\mathbf{W}'-\mathbf{U})\mathbf{k}}{1-\frac{V}{c_l}}\right)(\mathbf{V}\times\mathbf{B}_0)\right]. \quad (11)$$

The equation of motion for the vortex structure then takes the form

$$i\omega\eta(\mathbf{W}'-\mathbf{U})=Dk^2\mathbf{W}'+\frac{(Dk^2+i\omega\eta)\mathbf{k}(\mathbf{W}'-\mathbf{U})}{\omega\left(1-\frac{V}{c_l}\right)}\mathbf{V}. \quad (12)$$

Solving Eq. (12) simultaneously with linearized Eq. (9) and Eqs. (2)–(4), we obtain expressions for the relative velocity

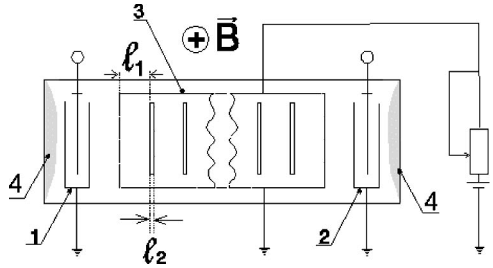

FIG. 1. A scheme of the proposed experimental setup. 1 and 2 are interdigital surface waves transducers, 3 is a complete periodic strip structure, 4 is a SAW absorber, B_0 is an external magnetic field.

shift $\Delta c_l/c_l$ and for the excess attenuation of the longitudinal ultrasonic wave $\Delta\alpha$ due to its interaction with the moving vortex structure

$$\frac{\Delta c_l}{c_l}=\frac{1}{2}\left(1-\frac{V}{c_l}\right)\left(1-2\frac{V}{c_l}\right)\frac{\omega^2}{\rho c_l^2}\frac{D}{\left(1-2\frac{V}{c_l}\right)^2\omega^2+X^2}, \tag{13}$$

$$\Delta\alpha=\frac{1}{2}\frac{\omega^2}{\rho c_l^3}\left(1-\frac{V}{c_l}\right)D\frac{X}{\left(1-2\frac{V}{c_l}\right)^2\omega^2+X^2}, \tag{14}$$

where $X=Dk^2/\eta$, $D=B_0^2/\mu_0(1+\lambda_L^2k^2)$, $D\approx C_{11}$ (C_{11} is the compression modulus of the vortex structure[30]). If $V=0$, Eqs. (13) and (14) coincide with the Pankert's result.[7] We suppose at this point that η coincides with the coefficient Γ^{-1} introduced by Pankert to describe the interaction of ultrasonic waves with vortices because of pinning in the TAFF regime. As seen from Eq. (14), an increase in the vortex structure velocity leads to a decrease in the wave attenuation induced by the interaction with the vortex structure. As the velocity V exceeds the velocity of the ultrasonic wave, the attenuation coefficient changes its sign, and hence, the amplitude of this wave is increased. Note that $\Delta c_l/c_l$ goes to zero at velocity $V=\frac{1}{2}c_l$. This is due to appearance of the new vortex mode having velocity equal to $2V$, which could be easily seen from Eq. (12), if the interaction between the vortex structure and the superconductor lattice is neglected. The details of this effect will be considered elsewhere.

To observe the amplification effect experimentally it is necessary to accelerate the vortex structure to the ultrasonic velocity. This can be achieved by passing a strong current through the superconductor. However, this is supposed to produce a large amount of heat that causes the problem of heat removal. In order to simplify the experimental observations, we suggest to use slow harmonics of the periodic structure of elastic medium. In the simplest way, the effect can be observed with surface acoustic waves (SAW). A scheme of the proposed experimental setup is shown in Fig. 1. The superconducting film 1 is fabricated in the form of a periodic structure deposited on a piezoelectric substrate. The structure consists of superconducting strips of width l_1 and uncovered substrate strips of width $l_2=0.1l_1$. We can thus define a unit cell of the structure with the width $d=l_1+l_2$. Two interdigital surface wave transducers 1 and 2 (IDT) are placed on the surface of the substrate. Depending on the direction of the dc current in the periodic structure, a signal can be detected either at IDT 1 or at IDT 2. These IDT's can be also used to measure the attenuation coefficients. In this periodic structure, SAW can be amplified (or generated), if the direct current passing through the sample reaches a particular value. We estimate this value within a one-dimensional model in which the proposed geometry is considered as the one-dimensional periodic structure made up with two types of strips described above. In this case, the dispersion equation takes the form[31]

$$\cos(\tilde{k}d)=A;\quad A=\cos(k_1l_1)\cos(k_2l_2)-\frac{1}{2}\left(\frac{c_2}{c_1}+\frac{c_1}{c_2}\right)\times\sin(k_1l_1)\sin(k_2l_2),$$

where $k_{1,2}=\omega/c_{1,2}$, $d=l_1+l_2$, and c_1, c_2 are the ultrasonic wave velocities in these strips, respectively. Solving this equation for $\tilde{k}$, one can find the velocity of ultrasonic wave harmonics $C^{(n)}$ in this periodic structure

$$C^{(n)}=\frac{\pi}{2}c_1\frac{f}{f_0}\left(1+\frac{l_2}{l_1}\right)\frac{1}{\alpha+2\pi n}.$$

Here n is the number of harmonic, $\alpha=\arccos A$, $f_0=c_1/4l_1$. For the sake of definiteness, we consider a $LiNbO_3$ substrate, because it is a widely used and available material. Using the values $c_1=3.48\times10^3$ m/c and $(c_2-c_1)/c_1=2.41\times10^{-2}$ reported for YZ cut of $LiNbO_3$,[32] and setting the frequency of the generated (attenuated) ultrasonic wave $f=10$ MHz, we obtain the velocities of space harmonic for $l_2l_1=0.1$: $c^{(1)}=93$ m/c, $c^{(2)}=47$ m/c, $c^{(3)}=38$ m/c. Next we estimate the value of the current i which is required to pass through a single strip, to speed up vortices to the first-harmonic velocity. In this case, $i=js$, where s is the area of the strip cross section. The current density has to be $j=(V\eta)/B$, where $\eta=(BB_{c_2})/r_n$,[33] r_n is a resistance in the normal state of the superconductor, B_{c_2} is the second critical field. We set the external field $B_0=0.01$ T, and considering the YBaCuO strips, use the values of $B_{c_2}=2.39$ T and $r_n=65\times10^{-8}$ Ωm. In this case, for the frequency $f_0=100$ MHz, the width of the strips should be 10 μm and, assuming the thickness of the film to be 0.1 μm, the value of the current is 0.1 mA. For the entire strip structure consisting of 100 strips the total current amounts to 10 mA that is a quite reasonable value in terms of the experiment. As follows from above, passing the current of 10 mA through the proposed periodic structure, one can obtain amplification (or generation) of SAW of 10 MHz. It should be noted that this is the maximal magnitude of the current, in the case of higher spatial harmonics, the amplification (or generation) starts at lower current.

Thus, we have shown that a moving vertex structure can amplify or generate the longitudinal ultrasonic wave due to friction on the crystal lattice, if its velocity exceeds the velocity of ultrasonic waves. This effect can be observed without great difficulties with surface acoustic waves by means of the space harmonics of the periodic structure.

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