Combined effect of Zeeman splitting and spin-orbit interaction on the Josephson current in a superconductor–two-dimensional electron gas–superconductor structure

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(Received 21 March 2002; published 12 August 2002)

We analyze spin effects in the current-carrying state of a superconductor–two-dimensional–electron gas–superconductor (S-2DEG-S) device with spin-polarized nuclei in the 2DEG region. The hyperfine interaction of 2D electrons with nuclear spins, described by the effective magnetic field $\boldsymbol{B}$, produces Zeeman splitting of Andreev levels without orbital effects, which leads to an interference pattern of supercurrent oscillations over B. The spin-orbit effects in 2DEG cause a strongly anisotropic dependence of the Josephson current on the direction of $\boldsymbol{B}$, which may be used as a probe for the spin-orbit interaction intensity. Under certain conditions, the system reveals the properties of π junctions.

DOI: 10.1103/PhysRevB.66.052508 PACS number(s): 74.80.Fp, 31.30.Gs, 71.70.Ej

The spin-orbit (SO) and hyperfine (HF) interactions in GaAs heterojunctions and similar two-dimensional (2D) quantum Hall systems have attracted permanent theoretical and experimental attention. The hyperfine field of the nuclear spin subsystem acting upon the spins of charge carriers may reach 10^4 G.[1] At low temperatures, the nuclear spin relaxation time can be macroscopically long,[2] so the nonequilibrium spin population in heterojunctions, once created, is conserved during hundreds of seconds.[3] Zeeman splitting combined with a strong spin-orbit coupling in GaAs/AlGaAs 2DEG gives rise to a novel class of coherent phenomena, e.g., the spontaneous Aharonov-Bohm effect.[4]

In this paper we discuss mesoscopic spin-orbit effects in Josephson current flowing across the superconductor-2D-electro-gas–superconductor (S-2DEG-S) structure (Fig. 1) with polarized nuclei in the 2DEG region. The transfer of the Josephson current through the normal conducting layer is provided by the Andreev reflection of electrons and holes at the normal-metal–superconductor (NS) interfaces, which convert normal electron excitations into Cooper pairs in the superconducting banks. In a pure system with length d smaller than the electron scattering length, the interference between coherent electron states and retro-reflected hole states produces the set of spin-degenerate Andreev energy levels $E_\lambda(\Phi)$, which depend on the quantum numbers λ and on the difference Φ of the order parameter phases in superconducting electrodes.[6] In short structures $d \ll \xi_0$ (ξ_0 is the coherence length in the superconductor), the Josephson current can be presented as the sum of currents transferred by individual Andreev bound states (see, e.g., Ref. 7 and references therein),

$$I(\Phi) = -\frac{2e}{\hbar}\sum_\lambda \frac{\partial E_\lambda(\Phi)}{\partial \Phi}\tanh\frac{E_\lambda(\Phi)}{2T}, \tag{1}$$

and must be sensitive to the HF and SO interaction which eliminate spin degeneracy of the Andreev levels.

The contact HF interaction in a semiconductor is described by the Hamiltonian[8]

$$\hat{H}_{hf} = (8\pi/3)\mu_B\gamma_h \sum_i \boldsymbol{I}_i \boldsymbol{\sigma}\delta(\boldsymbol{r}-\boldsymbol{R}_i). \tag{2}$$

Here μ_B is the Bohr magneton, γ_h is the nuclear magneton, and $\boldsymbol{I}_i$, $\boldsymbol{\sigma}$, $\boldsymbol{R}_i$, and $\boldsymbol{r}$ are nuclear and charge carrier spins and positions, respectively. It follows from Eq. (2) that if the nuclear spins are polarized, $\langle \Sigma_i \boldsymbol{I}_i \rangle \neq 0$, the charge carrier spins feel the effective HF field $\boldsymbol{B}$ which may cause spin splitting in the 2DEG of the order of one-tenth of the Fermi energy E_F.[1,3] The influence of the Zeeman splitting solely on a supercurrent was studied first in Ref. 9 for the SFS junction (F denotes ferromagnetic metal). It was shown that spin splitting suppresses the critical current and produces its oscillations over the intrinsic magnetic field localized within the F layer.

The SO interaction of a charge carrier with the interface potential in GaAs/AlGaAs heterojunctions is modeled by the Bychkov-Rashba term[10,11]

$$\hat{H}_{\rm so} = (\alpha/\hbar)[\boldsymbol{\sigma p}]\boldsymbol{n}, \tag{3}$$

where $\alpha = 0.6\times 10^{-9}$ eV cm for holes[10,12] and $\alpha = 0.25\times 10^{-9}$ eV cm for electrons,[10,13] $\boldsymbol{\sigma}_i$ and $\boldsymbol{p}_i$ are the charge carrier spin and momentum, and $\boldsymbol{n}$ is the normal to the interface directed towards the Al-doped layer.

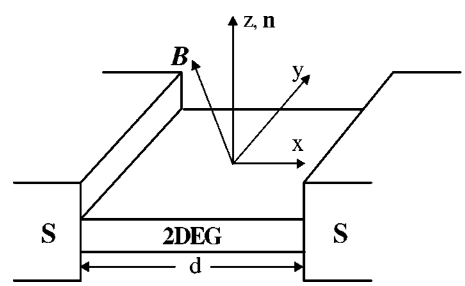

FIG. 1. A model of the superconductor-2DEG-superconductor device based on the GaAs/AlGaAs heterojunction (Ref. 5). The normal $\boldsymbol{n}$ is directed towards the Al-doped layer.

The configuration proposed in Fig. 1 has the following characteristic features.

(i) The nuclear field $\boldsymbol{B}$ is localized outside the superconductors and does not influence the pairing mechanism.

(ii) It affects only the electron spins and does not modify the space motion of charge carriers, whereas the usual magnetic field causes strong orbital effects and transforms Andreev levels into Landau bands.[14]

(iii) Since the SO interaction in Eq. (3) is spatially uniform along the 2DEG plane, it does not suppress the supercurrent, in contrast to the SO scattering at nonmagnetic impurities which acts as a depairing factor.[15] The situation changes in the presence of a nuclear field.

To calculate the Josephson current in Eq. (1), it is enough to find discrete eigenvalues of the BCS Hamiltonian supplied by the HF and SO interaction terms,

$$\mathcal{H}=\int dV[\psi^{+}(E(\hat{\boldsymbol{p}})+\boldsymbol{\sigma}\hat{\boldsymbol{\Lambda}}(\boldsymbol{r}))\psi+\Delta(\boldsymbol{r})\psi_{\uparrow}^{+}\psi_{\downarrow}^{+}+\text{H.c.}] \tag{4}$$

where $\boldsymbol{\sigma}$ is the vector of Pauli matrices, the spinor ψ is composed from the annihilation operators $\psi_{\uparrow}$, $\psi_{\downarrow}$ of the electron with a given spin, and $E(\boldsymbol{p})=\boldsymbol{p}^2/2m-E_F$ is the energy of a normal electron excitation. For simplicity, we accept a stepwise model for the order parameter $\Delta(\boldsymbol{r})$,

$$\Delta(x)=\begin{cases}\Delta\exp(i\Phi\,\text{sgn}\,x/2), & |x|>d/2,\\ 0, & |x|<d/2.\end{cases} \tag{5}$$

The vector $\hat{\boldsymbol{\Lambda}}(\boldsymbol{r})$ in Eq. (4) describes the HF and SO interactions in accordance with Eqs. (2) and (3),

$$\hat{\boldsymbol{\Lambda}}(x)=\begin{cases}0, & |x|>d/2,\\ \boldsymbol{h}+\hat{\boldsymbol{w}}, & |x|<d/2,\end{cases}\quad \boldsymbol{h}=\mu_B\boldsymbol{B},\quad \hat{\boldsymbol{w}}=\frac{\alpha}{\hbar}[\hat{\boldsymbol{p}}\boldsymbol{n}]. \tag{6}$$

In order to consider uniformly the spin and electron-hole states of quasiparticles, it is convenient to express the Hamiltonian in Eq. (4) in terms of the four-spinor field,

$$\varphi=\begin{pmatrix}\psi_{\uparrow}\\ \psi_{\downarrow}\\ \psi_{\downarrow}^{+}\\ \psi_{\uparrow}^{+}\end{pmatrix},\quad \varphi^{+}=(\psi_{\uparrow}^{+},\psi_{\downarrow}^{+},\psi_{\downarrow},\psi_{\uparrow}), \tag{7}$$

$$\mathcal{H}=\frac{1}{2}\int dV\varphi^{+}(\boldsymbol{r})\hat{H}\varphi(\boldsymbol{r}), \tag{8}$$

$$\hat{H}=\tau_z E(\hat{\boldsymbol{p}})+\begin{cases}\sigma_z[\Delta(x)\tau_{+}+\Delta^{*}(x)\tau_{-}], & |x|>d/2,\\ \hat{\boldsymbol{w}}\boldsymbol{\sigma}+h_z\sigma_z+\tau_z\boldsymbol{h}_{\parallel}\boldsymbol{\sigma}, & |x|<d/2.\end{cases} \tag{9}$$

Here $\boldsymbol{h}_{\parallel}$ is the component of Zeeman field in the 2DEG plane; the Pauli matrices σ act upon the spin states, while the τ matrices operate in electron-hole space, e.g.,

$$\sigma_z\tau_z=\begin{pmatrix}\sigma_z & 0\\ 0 & -\sigma_z\end{pmatrix}. \tag{10}$$

The problem of a single-particle spectrum of the Hamiltonian $\mathcal{H}$ in Eq. (8) is equivalent to solution of the Bogolyubov–de Gennes (BdG) equation for the four-component wave function $\Psi_{\lambda}(\boldsymbol{r})$:

$$\hat{H}\Psi_{\lambda}(\boldsymbol{r})=E_{\lambda}\Psi_{\lambda}(\boldsymbol{r}). \tag{11}$$

Assuming all characteristic energies Δ, h, and w to be much smaller than E_F, we use the quasiclassical representation of $\Psi_{\lambda}(\boldsymbol{r})$ as the product of a rapidly oscillating exponent over the slowly varying envelope $u(x)$:

$$\Psi_{\lambda}(\boldsymbol{r})=\exp(ispx+ip_y y)u_s(x),\quad p=\sqrt{p_F^2-p_y^2}, \tag{12}$$

where $s=\pm1$ indicates two signs of the x component of the electron momentum. The spinor function $u(x)$ obeys Eq. (11) with a reduced Hamiltonian, in which the quasiclassical approximation for the kinetic energy and SO operators is used,

$$E(\hat{\boldsymbol{p}})\approx sv\hat{p}_x,\quad \hat{\boldsymbol{w}}\approx\frac{\alpha}{\hbar}[\boldsymbol{p}n],\quad \boldsymbol{p}=(sp,p_y,0),\quad v=\frac{p}{m}. \tag{13}$$

The matching of the solutions of Eq. (11) at the NS interfaces, which are assumed to be completely transparent, yields a dispersion relation

$$\frac{Ed}{\hbar v}=\pi n+\arccos\frac{E}{\Delta}+s\frac{\Phi}{2}+\sigma\gamma,\quad n=0,\pm1,\pm2\ \ldots, \tag{14}$$

$$\gamma(\boldsymbol{h},\boldsymbol{w})=\arcsin\left[\sum_{k=\pm1}\frac{1+k\boldsymbol{n}_{+}\boldsymbol{n}_{-}}{2}\sin^2\frac{A_{+}+kA_{-}}{2}\right]^{1/2}, \tag{15}$$

$$A_{\pm}=(d/\hbar v)|\boldsymbol{h}\pm\boldsymbol{w}|,\quad \boldsymbol{n}_{\pm}=(\boldsymbol{h}\pm\boldsymbol{w})/|\boldsymbol{h}\pm\boldsymbol{w}|, \tag{16}$$

which has the standard structure of the equation for Andreev levels[16] in current-carrying SNS junctions.[6] An additional term $\sigma\gamma$, where $\sigma=\pm1$ indicates the spin direction, describes Zeeman splitting of the Andreev levels renormalized by the SO interaction. In terms of the BdG wave mechanics, the spin effects change phase relations between the wave functions of the incident and retro-reflected quasiparticles. This produces oscillations of the Andreev levels with the change of the interaction parameters h, w which enter the oscillating phase shift $\sigma\gamma$ in Eq. (14). As a result, the Josephson current in Eq. (1) also reveals oscillations with h, w as a manifestation of the complicated interference between partial supercurrents carried by individual Andreev bands. In this sense, the effect represents a spin-induced analog of the Fraunhofer pattern[17] in a planar Josephson junction in an external magnetic field. Note that in the absence of a Zeeman field $(h=0)$, all spin effects disappear: $\gamma(0,\boldsymbol{w})=0$.

The most striking manifestation of the SO interaction itself is the anisotropy of Andreev levels with respect to the direction of the Zeeman field $\boldsymbol{h}$; in the absence of the SO interaction $(\boldsymbol{w}=0)$, $E_n(\Phi)$ depends only on the modulus of $\boldsymbol{h}$:

$$\gamma(\boldsymbol{h},0)=\arcsin[\sin(hd/\hbar v)]. \tag{17}$$

It is helpful to consider this anisotropy for a single-electron state with a fixed direction of the SO vector $\boldsymbol{w}$. As follows from Eqs. (14)–(16), the energy of the Andreev level depends only on the angle between $\boldsymbol{h}$ and $\boldsymbol{w}$ and their moduli, being insensitive to the rotation of $\boldsymbol{h}$ around $\boldsymbol{w}$. At $\boldsymbol{h}\|\boldsymbol{w}$, the effects of the SO interaction vanish, as in Eq. (17). These conclusions can be extended to the angle dependence of the Josephson current in Eq. (1) in a narrow 2DEG channel which holds a single-electron mode ($p=p_F$, $p_y=0$). In this extreme case, the vectors $\boldsymbol{w}$ of all electrons are directed along the y axis and create a fixed reference frame for the Zeeman vector $\boldsymbol{h}$.

At arbitrary lengths of the 2DEG region, Eq. (14) can be solved only numerically. Below we consider an analytically solvable case of the 2DEG channel much shorter than the coherence length ξ_0, when the left-hand side of Eq. (14) is negligibly small,

$$E_\lambda(\Phi)=s\Delta\cos(\Phi/2+\sigma\gamma), \quad (s,\sigma)=\pm 1. \tag{18}$$

At $\gamma=0$, Eq. (18) describes two spin-degenerated Andreev levels in a superconducting constriction[18] which traverse across the whole energy gap with the change of Φ and intersect each other at $\Phi=\pi$.[19] The spin effects split each level into two spin-dependent terms and, in addition, expand them into four energy bands which transfer the Josephson current:

$$I(\Phi)=\frac{e\Delta}{\hbar}\int_{-p_F}^{p_F}\frac{dp_y}{2\pi\hbar}\sum_{\sigma=\pm 1}\tanh\left[\frac{\Delta}{2T}\cos\left(\frac{\Phi}{2}+\sigma\gamma\right)\right]\times\sin(\Phi/2+\sigma\gamma). \tag{19}$$

At $h=0$, Eq. (19) is reduced to a 2D analog of the current-phase relationship[20] for pure constriction.

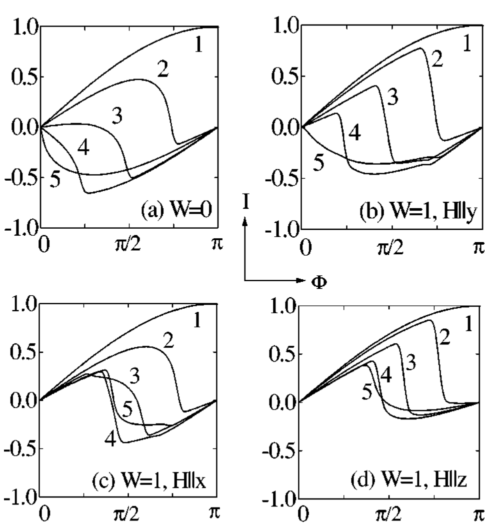

FIG. 2. Current-phase relationships $I(\Phi)$ at $T=0$ normalized on the critical current at $H=0$ for various directions and magnitudes of the Zeeman field and intensities of the SO interaction: $H=0$ (1), 0.4 (2), 0.8 (3), 1.2 (4), and 1.6 (5).

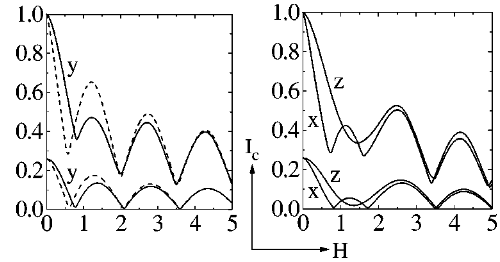

FIG. 3. The set of dependences of the critical current $I_c(\boldsymbol{H},T)/I_c(0,0)$ on the dimensionless magnetic field H, at $W=1$, $\boldsymbol{H}\|\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{z}$ (solid curves) and $W=0$, $\boldsymbol{H}$ directed arbitrarily (dashed curves). Upper pairs of curves were calculated for $T=0$, lower pairs for $T=0.9T_c$.

The set of curves $I(\Phi)$ calculated numerically at $T=0$ for various directions and magnitudes of Zeeman field combined with the SO interaction is presented in Figs. 2(b)–2(d) in dimensionless variables,

$$\boldsymbol{H}=(\hbar v_F/d)\boldsymbol{h}, \quad \boldsymbol{W}=(\hbar v_F/d)\boldsymbol{w}, \tag{20}$$

in comparison with those plotted for $W=0$ in Fig. 2(a). The common features of these dependencies are drastic variations of the shape of $I(\Phi)$ at $H\sim 1$ and the rapid change of sign of the derivative $dI/d\Phi$ at $\Phi=\pi$ in small field H. It is interesting that the curves at $W=0$ and at $W=1$, $\boldsymbol{H}\|\boldsymbol{y}$ [Figs. 2(a) and 2(b)] are similar each to other, as well as the curves for $W=1$, $\boldsymbol{H}\|\boldsymbol{x}$ and $W=1$, $\boldsymbol{H}\|\boldsymbol{z}$ [Figs. 2(c) and 2(d)]. This reflects the results of our analysis of the anisotropy of Andreev levels in the 1D case which appears to be qualitatively applicable for the 2D system: the SO effects are relatively small at $\boldsymbol{H}\|\boldsymbol{y}$ and approximately isotropic under rotation of the Zeeman field around the y axis.

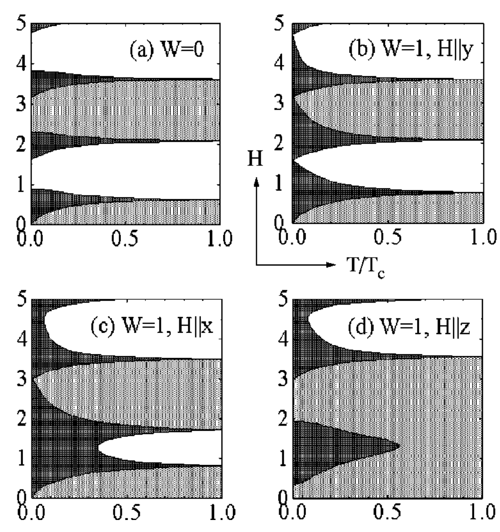

FIG. 4. H-T/T_c diagrams of the stability of S-2DEG-S phase states with $\Phi=0$ and $\Phi=\pi$ for various directions and magnitudes of the Zeeman field and intensities of the SO interaction. Within the gray regions, only the usual 0 state is stable: $dI/d\Phi(0)>0$, $dI/d\Phi(\pi)<0$. Blank regions correspond to the stability of the π state only: $dI/d\Phi(0)<0$, $dI/d\Phi(\pi)>0$. Within the dark regions both derivatives are positive.

In Fig. 3 we present oscillations of the critical current $I_c(H)$ depending on the HF field direction and SO interaction intensity for two values of temperature. At the vicinity of T_c, where $I(\Phi)$ has a single harmonic,

$$I(\Phi)\approx I_m \sin\Phi, \quad I_m=\frac{\pi e\Delta^2}{(2\pi\hbar)^2 T_c}\int_{-p_F}^{p_F} dp_y \cos 2\gamma, \tag{21}$$

the critical current $I_c(W,H)=|I_m|$ turns to zero periodically with H like in SFS systems.[9]

The positive sign of $dI/d\Phi$ at $\Phi=\pi$, which occurs at $H\neq 0$ (Fig. 2), means that this state can be stable and may produce persistent current in the ground state of a superconducting loop with high enough inductance (π junction[21]). On the other hand, the negative sign of $dI/d\Phi$ at $\Phi=0$, which occurs within a certain field range at $W=0$ or at $W\neq 0$, $\boldsymbol{H}\|\boldsymbol{y}$, signifies the instability of the usual ground state with $\Phi=0$ (note that the SO interaction stabilizes this state at $\boldsymbol{H}\perp\boldsymbol{y}$, at least at $T=0$). The results of a numerical analysis of the stability of states $\Phi=0$, π within the whole temperature range are shown in Fig. 4.

In summary, we have shown that the Josephson current in a mesoscopic S-2DEG-S structure is highly sensitive to the combined action of the Zeeman field and spin-orbit interactions. In particular, the critical current reveals oscillations and anisotropy with respect to the Zeeman field $\boldsymbol{B}$, and regions of stability at $\Phi=\pi$ (like in π junctions) emerge. We assumed hyperfine interactions of electrons with polarized nuclei as the source of electron spin polarization, though similar effects should be observed in external magnetic fields lying in the 2DEG plane (to avoid orbital effects). In order to access the regime of strong interactions ($H\sim 1$, $W\sim 1$) in the short 2DEG bridge ($d<\xi_0$) considered here, the interaction energies h, w of the 2DEG should exceed Δ. Since the HF and SO interaction magnitudes in GaAs/AlGaAs heterojunctions reach at most 1 K in temperature scale, the banks of short S-2DEG-S structures should be preferably fabricated from a superconductor with low $T_c\leqslant 1$ K. This restriction can be significantly softened in long ($d\gg\xi_0$) S-2DEG-S junctions where the interaction energies should be comparable with the small distance between Andreev levels: $h,w \sim\hbar v_F/d\ll\Delta$.

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