Interplay between quasiperiodicity and disorder in quantum spin chains in a magnetic field

M. Arlego
Departamento de Física, Universidad Nacional de La Plata, C.C. 67, (1900) La Plata, Argentina
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We study the interplay between disorder and a quasiperiodic coupling array in an external magnetic field in a spin-$\frac{1}{2}$ XXZ chain. A simple real space decimation argument is used to estimate the magnetization values where plateaux show up. The latter are in good agreement with exact diagonalization results on fairly long XX chains. Spontaneous susceptibility properties are also studied, finding a logarithmic behavior similar to the homogeneously disordered case.

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Since their discovery in 1984,[1] the properties of quasicrystals have been a source of sustained interest. Many theoretical efforts on Ising models in Penrose lattices[2] and XY Fibonacci spin chains[3–5] have revealed interesting magnetic orderings associated to the quasiperiodicity of these structures. Such kind of spin arrays have been found in recently synthesized rare-earth (R) ZnMg-R quasicrystals (see, e.g., Ref. 6) whose R elements have well localized $4f$ magnetic moments. The study of quasiperiodic 1D chains has recently received renewed attention[7,8] and interesting properties have been elucidated. In Ref. 7 a system of spinless fermions in a quasiperiodic lattice potential was studied within perturbation theory, where it was shown that its behavior is different from both the periodic and the disordered cases: While in the case of a periodic potential one may have a metal-insulator transition only if the potential is commensurate, in the disordered case, the potential is relevant irrespective of the position of the Fermi level. In the quasiperiodic case, two different situations arise, depending on whether the Fermi level coincides with one of the main frequencies of the Fourier spectrum of the quasiperiodic potential or not. In the first case, the situation turns out to be similar to the periodic case while in the second, at a perturbative level, the metal-insulator transition point is strongly modified. These predictions have been also verified numerically in Ref. 8. Motivated by these studies we have recently analyzed the effect of an external magnetic field in a quasiperiodic spin chain, and found that the magnetization curve has a very interesting nature that could be predicted using a decimation procedure and Abelian bosonization.[9] For the Fibonacci case, in particular, one can reproduce the main plateaux within the bosonization approach by approximating the quasiperiodic modulation by considering a subset of the main Fourier frequencies. From the experimental point of view, interest of quasiperiodic systems arise from artificially grown quasiperiodic heterostructures,[13] quantum dot crystals,[14] and magnetic multilayers.[15]

In this short paper we go one step further to analyze the interplay between a quasiperiodic array of couplings and disorder in a XXZ spin chain in the presence of an external magnetic field. Using a simple decimation procedure we predict the appearance of plateaux in the magnetization curve at values of M which depend on both the quasiperiodicity and the strength of the disorder. As in the case of p-merized chains and within the same working hypothesis, i.e., for a hierarchical set of couplings well separated in strength, the presence of binary disorder results in a shift on the plateaux positions as a function its strength, while for the Gaussian case the plateaux are generically wiped out. We have also studied the effects of a binary disorder on the double frequency XX chain studied in Ref. 9, where it turns out that the plateaux structure of the pure case disappears. We have checked this behavior by studying numerically fairly long XX systems. We also predict a logarithmic behavior of the susceptibility at low fields by extending the arguments in Ref. 16 and have also verified this behavior numerically. Other authors have also studied random spin systems recently.[10–12]

Let us consider the antiferromagnetic system

$$H=\sum_n J_n(S_n^xS_{n+1}^x+S_n^yS_{n+1}^y+\Delta S_n^zS_{n+1}^z)-h\sum_n S_n^z, \quad (1)$$

where S^x,S^y,S^z denote the standard spin-$\frac{1}{2}$ matrices, in a magnetic field h applied along the anisotropy direction ($|\Delta|\leqslant 1$). For the pure quasiperiodic chain, the coupling modulation is parametrized as $J_n=J(1+\epsilon_n)$ with $\epsilon_n=\Sigma_\nu \delta_\nu \cos(2\pi\omega_\nu n)$, so quasiperiodicity arises upon choosing an irrational subset of frequencies ω_ν with amplitudes δ_ν.

Furthermore, the couplings J_n are randomly distributed. Specifically, we consider a binary distribution of strength p ($p=0$ corresponds to the pure quasiperiodic case while $p=1$ corresponds to the uniform chain)

$$P(J_n)=p\,\delta(J_n-U)+(1-p)\,\delta[J_n-J(1+\epsilon_n)], \quad (2)$$

with ϵ_n defined as above, along with a Gaussian disorder $P(J_n)\propto \exp-(J_n-\overline{J_n})^2/2\sigma_n^2$. These distributions, taken with same mean and variance, are built to enforce quasiperiodicity. Thus, on average ϵ_n is a measure of the couplings quasiperiodicity. In what follows we assume that U is well separated from the other couplings. These assumptions are important for our decimation procedure to be valid. More U-general regimes would require further investigations which are out of the scope of the present article.

We will follow the decimation procedure as described in Ref. 16 to obtain the value of the magnetization for the main plateaux. In our problem (which is at $T=0$) the energy scale is provided by the magnetic field, and in order to compute the magnetization, decimation has to be stopped at an energy scale of the order of the magnetic field. We assume that all spins coupled by bonds stronger than the magnetic field form singlets and do not contribute to the magnetization, whereas spins coupled by weaker bonds are completely polarized. The magnetization is thus proportional to the fraction of remaining spins at the step where we stop decimation. This simple argument happens to apply well to the binary distribution, provided the energy scales of the involved exchanges are well separated.

In studying irrational frequencies or other quasiperiodic modulations, it is natural to analyze the case of the Fibonacci chain, a coupling array $J_A = J(1+\delta)$, $J_B = J(1-\delta)$ generated by iterating the substitution rules $B \rightarrow A$ and $A \rightarrow AB$,[5,7,9,17] with the distribution $P(J_n) = p\,\delta(J_n - U) + (1-p)\,\delta(J_n - J_{A,B})$.

(i) Decimation procedure. We evaluate by decimation the magnetization of the widest plateaux in the *strong* coupling limit ($\delta \rightarrow \pm 1$). There are two different cases to consider, according to $\delta \approx -1$, i.e., $J_B \gg J_A$, and the opposite situation for $\delta \approx 1$.

Starting from saturation, in the first case the magnetic field is lowered until it reaches the value $h_c \approx J_B$ at which the type-B bonds experience a transition from the state of maximum polarization to the singlet state. The magnetization at this plateau is then obtained by decimating the B bonds, which yields

$$\langle M \rangle = 1 - 2\frac{N_B}{N_T} = 1 - 2(1-p)\frac{1}{\gamma^2}, \tag{3}$$

where $N_T = N_U + N_A + N_B$ denotes the total number of bonds, $N_{A,B}$ the number of A and B bonds, respectively, and $\gamma^2 = (N_A + N_B)/N_B$. For a large iteration number of the rules referred to above ($N_T \rightarrow \infty$), N_A/N_B approaches the golden mean $\gamma = [(1+\sqrt{5})/2]$. In the $p=0$ limit, we recover the results in Ref. 9 and a nonvanishing p results in a shift of the position of the plateau.

In the second case $J_A \gg J_B$ we have to distinguish two different unit cells since type-A bonds can appear either in pairs (forming trimers) or isolated (forming dimers). It can be readily checked that when lowering the magnetic field from saturation the first spins to be decimated correspond to those forming trimers. We then have a plateau (the nearest to saturation) at

$$\langle M \rangle_1 = 1 - 2\frac{N_{AA}}{N_T} = 1 - 2(1-p)^2\frac{1}{\gamma^3}, \tag{4}$$

where N_{AA} refers to A pairs. The second plateau is obtained after decimation the type-A bonds, and then we must consider all the sequences J_A between the others bonds. That gives for the second plateau

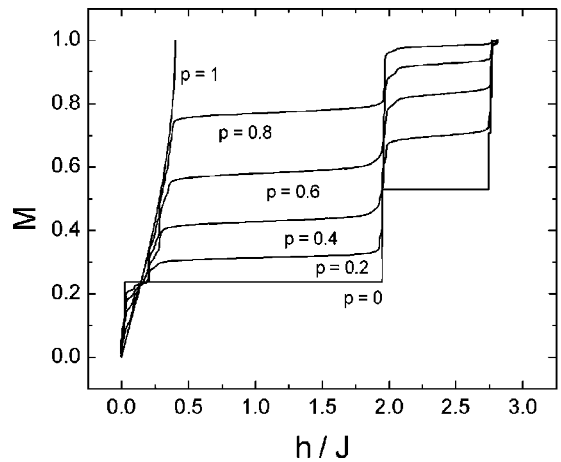

FIG. 1. Magnetization curves of modulated XX Fibonacci spin chains, immersed in disordered binary backgrounds of strength p after averaging over 5×10^4 samples with $f(18)=2584$ sites, $\delta=0.95$, $U=0.2$ and $p=0$, 0.2, 0.4, 0.6, 0.8, 1 in ascending order. The left and rightmost lines denote, respectively, the pure uniform and pure Fibonacci cases.

$$\langle M \rangle_2 = \langle M \rangle_1 - 2\left[(1-p)^3\frac{1}{\gamma^4} + 2p(1-p)^2\frac{1}{\gamma^2} + p^2(1-p)\frac{1}{\gamma}\right]. \tag{5}$$

Again, we recover our results in Ref. 9 for $p=0$.

With this simple technique, one can predict the presence and position of the plateaux, provided that there is a finite difference between the highest values of the couplings in the inequivalent sites.

Since the decimation procedure applies for generic XXZ chains,[16,18] we conclude that the emergence of these strong coupling plateaux is a generic feature, at least with an antiferromagnetic anisotropy parameter $0<\Delta<1$, and within the range of couplings discussed above.

(ii) Exact diagonalization. To enable an independent check of these assertions, we turn to a numerical diagonalization of the Hamiltonian (1) contenting ourselves with the analysis of the particular case $\Delta=0$. This allows us to explore rather long chains using a fair number of disorder realizations (whose magnetization properties on the other hand, are self-averaging). In Fig. 1 we show, respectively, the whole magnetization curves obtained for various disorder concentrations $p=0$, 0.2, 0.4, 0.6, 0.8 and 1 averaging on 5×10^4 samples of $L=f(18)=2584$ sites under the exchange disorder (2), with $\delta=0.95$ and $U=0.2$. It can be readily verified that a set of robust plateaux emerges quite precisely at the critical magnetizations given by Eq. (4) for the plateau closest to saturation and by Eq. (5) for the second one.

It is important to stress that the derivation of our results for the quantization conditions Eqs. (3)–(5) rely strongly on the discreteness of the probability distribution and would not to be applicable to an arbitrary continuous exchange disorder. In accordance to this observation, for sufficiently strong Gaussian disorder it turns out that no traces of plateaux can

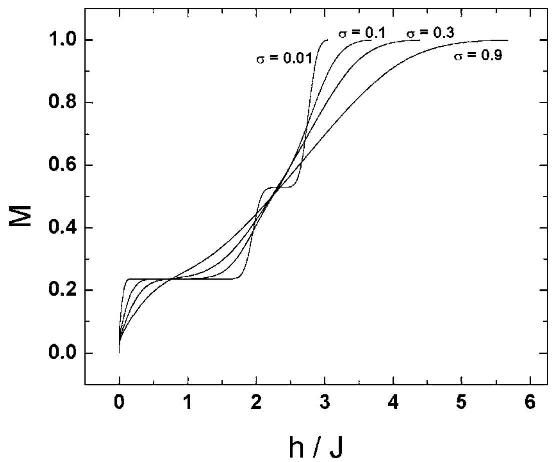

FIG. 2. Magnetization curves of modulated XX Fibonacci spin chains, immersed in Gaussian exchange distributions, after averaging over 4×10^4 samples with $f(18)=2584$ sites, $\delta=0.95$ and increasing standard deviation from left to right (note that the leftmost is practically the pure Fibonacci case).

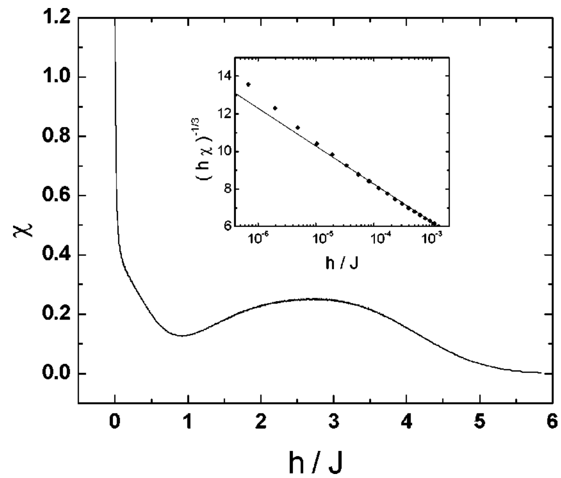

FIG. 4. Magnetic susceptibility of modulated XX Fibonacci spin chains, immersed in a Gaussian exchange distribution $(\sigma=1)$, after averaging over 4×10^4 samples with $f(18)=2584$ sites, $\delta=0.95$. The inset show the susceptibility behavior at low magnetic fields which follows closely the logarithmic regime predicted in the text. The bold line corresponds to the theoretical prediction.

be observed. This is corroborated in Fig. 2, where we see that the plateaux structure is smoothed when the standard deviation is increased. Here we averaged on 4×10^4 samples with $L=f(18)$. However, preliminary computations using larger chains yielded no substantial differences. Our data strongly suggest that there is a threshold value of $\sigma>0$ below which the Fibonacci plateau structure remains basically unaltered. This is particularly noticeable (see Fig. 2), as $\delta\rightarrow1$, i.e., within the regime in which our decimation procedure becomes most reliable. The understanding of this feature still awaits further investigations.

In a previous work,[9] we have observed that the magnetization curve for the Fibonacci chain, could be well approximated by considering a rather small subset of the main frequencies in its Fourier spectrum. Here we study a two-frequency case, for $\omega_1=5/8$ and $\omega_2=7/8$, in the presence of disorder, where it is observed that the plateaux are erased even by small disorder (see Fig. 3). It is interesting to observe that in contrast to the Fibonacci situation studied above, in which the plateaux structure is robust and just shifts with the strength of the disorder, here the plateaux seem to smear out even for a small value of p. We have further checked that a model with a finite subset of the main (now irrational) frequencies of the Fourier spectrum of the Fibonacci, is also unstable to disorder. This is surprising since in the absence of disorder, this approximate model turned out to lead to a good approximation to the magnetization curve of the real Fibonacci chain.[9] From the bosoniza-

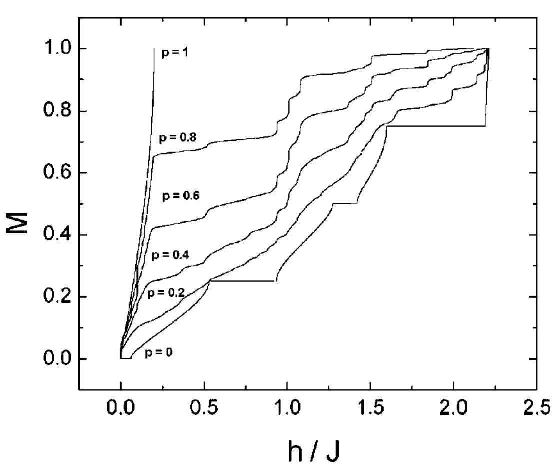

FIG. 3. Double frequency magnetization curves of the XX chain for $\omega_1=5/8$ and $\omega_2=7/8$ with amplitudes $\delta_1=0.2$ and $\delta_2=0.3$, with $U=0.1$ and 10^4 spins over 100 samples, immersed in disordered binary backgrounds of strength $p=0$, 0.2, 0.4, 0.6, 0.8, and 1 in ascending order.

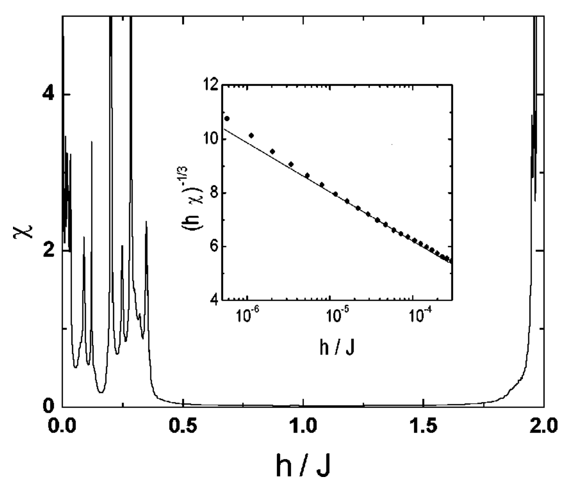

FIG. 5. Magnetic susceptibility of modulated XX Fibonacci spin chains, immersed in a binary exchange disorder of strength $p=0.6$ and $U=0.2$, averaged over 5×10^4 samples with $f(18)=2584$ sites, $\delta=0.95$. The inset show the susceptibility behavior at low magnetic fields that similar to the Gaussian disorder follows closely the logarithmic regime. The bold line corresponds to the theoretical prediction.

tion point of view one should expect no substantial difference between the real Fibonacci chain and the approximate one, so this issue deserves further investigations. However, this is out of the scope of the present paper, and we hope to discuss this issue elsewhere.

(iii) Low field susceptibility. For homogeneously disordered chains, one can use the decimation procedure of Ref. 16 along with the universality of the fixed point, to show that either for discrete or continuous distributions the low field magnetic susceptibility behaves according to

$$\chi_z \propto \frac{1}{h[\ln(h^2)]^3}. \tag{6}$$

Following a simple argument based on random walk motion used in Ref. 19, it can be readily shown that for $\Delta=0$ (or XX chains), these arguments can be extended to the case of a disordered Fibonacci chain. The singularity in Eq. (6), as in the case of a XXZ disordered chain, cannot be explained by simple perturbative arguments (see Ref. 19 for details). It is interesting to note that the effect of the disorder is crucial since it changes the power law behavior of the free Fibonacci chain obtained in Ref. 4 to a logarithmic one. This can be clearly observed in the insets of Figs. 4 and 5 where the validity of these arguments seem to apply over more than two decades. However, care must be taken when analyzing smaller field scales. Notice that the smallest accessible critical fields are given by the level spacings of the Hamiltonian, which in turn depend on the chain length. Thus, deviations from the expected exponent can occur on finite samples. In fact, this is reflected by the slight though progressive departures observed below $h/J \sim 10^{-5}$.

To summarize, we have studied the effect of disorder on the plateaux structure in quasiperiodic XXZ chains under an external magnetic field. By means of a simple real space decimation procedure we found the values of the magnetization for which the main plateaux emerge, Eqs. (3)–(5). This was tested by numerical diagonalizations of large XX chains finding a remarkable agreement with the quantization conditions in a variety of scenarios. Since the decimation scheme applies for generic XXZ chains,[18] we conclude that the appearance of these plateaux is a generic feature, at least with an antiferromagnetic anisotropy parameter $0<\Delta<1$. This issue still awaits numerical confirmation on sufficiently long chains using state of the art methodologies such as density matrix renormalization group.[20] Finally, we have also studied the low magnetic field susceptibility which exhibits a clear logarithmic behavior, Eq. (6). We trust this work will convey a motivation for both experimental and numerical studies.

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