

Self-induced and induced transparencies of two-dimensional and three-dimensional superlattices

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(Received 14 August 2001; revised manuscript received 25 February 2002; published 24 July 2002)

The phenomenon of transparency in two-dimensional and three-dimensional superlattices is analyzed on the basis of the Boltzmann equation with a collision term encompassing three distinct scattering mechanisms (elastic, inelastic, and electron–electron) in terms of three corresponding distinct relaxation times. On this basis, we show that electron heating in the plane perpendicular to the current direction drastically changes the conditions for the occurrence of self-induced transparency in the superlattice. In particular, it leads to an additional modulation of the current amplitudes excited by an applied biharmonic electric field with harmonic components polarized in orthogonal directions. Furthermore, we show that self-induced transparency and dynamic localization are different phenomena with different physical origins, displaced in time from each other, and, in general, they arise at different electric fields.

DOI: 10.1103/PhysRevB.66.045319

PACS number(s): 72.20.Ht, 72.15.Rn, 73.21.–b

I. INTRODUCTION

Semiconductor superlattices have been at the focus of attention for several decades, due to their unique electronic properties. The additional spatial periodicity of the superlattice leads to the formation of narrow Brillouin minizones and energy minibands.^{1–3} Bloch oscillations⁴ and Wannier–Stark levels⁵ can be observed in superlattices due to the narrowness of these minibands even in relatively weak static electric fields (10^2 – 10^4 V/cm). The Bloch oscillations are due to Bragg reflections by the periodic superlattice potential and are characterized by the frequency $\Omega_c = eE_c d/\hbar$ and amplitude $Z_c = \Delta/2eE_c$, where E_c is the constant electric field applied along the axis of the superlattice of period d and miniband width Δ . In the case of an applied harmonic ac field, the Bragg reflections do not generate a new type of oscillation beyond that of the static field, but they do modulate electron motion during the field period. This modulation is described by oscillatory dependencies of the amplitudes of electron velocity harmonics on the amplitude, E_1 , and/or the frequency, ω_1 , of the applied harmonic field.^{6,7} Manifestations of this modulation can be found in various nonlinear macroscopic effects and, in particular, in *superlattice transparency*.^{7–11} The zeroth harmonic of electron nonlinear oscillations responsible for dc current is of special interest. Its vanishing corresponds to electron spatial localization and is called *dynamic localization*.⁶ This dynamic localization occurs only for electrons having a sinelike dispersion law and for specific ratios of amplitude and frequency of the applied field, such that $J_0(eE_1 d/\hbar \omega_1) = 0$ [where $J_0(x)$ is the zeroth order Bessel function]. In the case of deviation from a sinusoidal dispersion law, dynamic localization can arise only at multifrequency fields.⁷ In the literature (see, for example, Ref. 12), dynamic localization is very often erroneously identified with self-induced superlattice transparency, predicted in Ref. 9 and verified experimentally in Ref. 10. It was shown there that the macroscopic polarization of the electron gas can vanish and the superlattice behaves al-

most like a linear dielectric having the permittivity of the crystal lattice in the absence of mobile electrons, with small nonlinear absorption due to electron oscillations. The conditions for this effect within the single τ -approximation for a one-dimensional superlattice sample are the same as for dynamic localization, but they have different physical origins. The error of their identification was shown in Refs. 7,13 for a three-dimensional sample with a one-dimensional superlattice and it will be further confirmed below for two- and three-dimensional superlattices.

The single τ -approximation has been reasonably successful in describing the cases of a one-dimensional superlattice and also a one-dimensional model of a three-dimensional superlattice *without* redistribution of energy and momentum among the various degrees of freedom due to electron scattering. However, as was shown in Refs. 14,15, such energy-momentum redistribution resulting from scattering can substantially effect superlattice properties and, in particular, the current–voltage characteristics can change due to transverse heating. To overcome the deficiency inherent in the single τ -approximation, we develop a new method based on the Boltzmann equation with a collision term encompassing three distinct relaxation times. The three relaxation times include (a) a time for redistribution of energy and momentum supplied by an electric field to a given electron among its various degrees of freedom, (b) a time for redistribution of energy and momentum among all electrons by inelastic electron–electron interactions, and (c) a time for transfer of the excess energy to the crystal lattice. In this, we employ a separation of the relaxation processes into elastic, inelastic and electron–electron, which is commonly recognized in the study of nonlinear properties of semiconductors at high fields (see, for example, Refs. 16,17). The resulting balance equations which we obtain can be solved analytically for systems having high symmetry (two- and three-dimensional superlattices). However, the qualitative results obtained here are also valid for bulk semiconductors having a one-dimensional superlattice, i.e., for structures with minibands in the growth

direction and free electron motion in the lateral plane, which can be studied experimentally currently. Moreover, at the present time, three-dimensional cluster lattices are actually grown¹⁸ and technological progress^{19–21} offers hope that two-dimensional and three-dimensional superlattices will be fabricated in the near future using quantum dots, relating directly to our studies here. Furthermore, a simple three-dimensional Kronig–Penney model was proposed in order to describe such quantum dots superlattices theoretically.²² On the basis of the three relaxation time description, we show that electron heating in the plane perpendicular to the current drastically changes the self-induced transparency of the superlattice. In particular, it leads to an additional modulation of the current amplitudes excited by an applied biharmonic electric field with harmonic components polarized in orthogonal directions. We obtain analytical results in the weak scattering approximation ($\omega\tau \gg 1$) and extend the analysis numerically for stronger scattering.

This article is structured as follows: In Sec. II, starting from a Boltzmann equation, we derive balance equations for average electron velocity (current) and electron energy by means of the new collision term accounting for momentum and energy redistribution among the various degrees of freedom. On the basis of these balance equations the theory of self-induced transparency and its mechanisms is developed in Sec. III for one-, two-, and three-dimensional superlattices in the presence of a high-frequency (hf) harmonic field. In Sec. IV we analyze the amplitude modulation of hf current by an orthogonal hf field of a different frequency. The main results of this work and comparison to previous studies are presented in Sec. V.

II. GENERAL RELATIONS

There are several prominent sources of nonlinear electron response in superlattices. Electron dynamics in narrow minibands features phenomena manifested as Bloch oscillations, static electron localization and the Wannier–Stark ladder^{4,5} in a static electric field; also, in a harmonic electric field there are nonlinear oscillations with amplitudes modulated by Bragg reflections and dynamic electron localization^{6,7} with the collapse of the electron’s minibands.¹³ In the case of interminiband transitions, nonlinear phenomenology includes interminiband tunneling in a static electric field, with or without photon-assistance,²³ also interminiband tunneling in a harmonic electric field and Rabi oscillations.²⁴ Of course, there are also nonlinear electron response properties involved in the relaxation processes that redistribute energy supplied by the electric field among the various degrees of freedom, controlling the anisotropic heating of the electron gas,^{14,15} upon which our attention is focused in this paper.

We revisit the analysis of electron dynamics in a single miniband from a different perspective than that of earlier studies.^{6,7,13} In this, we will examine electron dynamics in two-dimensional ($\mu=2$) and three-dimensional ($\mu=3$) superlattices in the presence of an oscillatory electric field having μ frequency components,

$$\mathbf{E}(t) = \sum_{\alpha=1}^{\mu} \mathbf{x}_{\alpha} E_{\alpha} \cos(\omega_{\alpha} t - \delta_{\alpha}), \quad \mu=2,3, \quad (1)$$

where \mathbf{x}_{α} are the unit lattice vectors of the crystal, δ_{α} are the initial phases of the fields, and the frequencies ω_{α} are different in general. We take the electron energy dispersion relation in the tight-binding approximation as

$$\varepsilon(\mathbf{k}) = \sum_{\alpha=1}^{\mu} \varepsilon_{\alpha}(k_{\alpha}), \quad \varepsilon_{\alpha}(k_{\alpha}) = \frac{\Delta}{2}(1 - \cos(k_{\alpha}d)), \quad (2)$$

where Δ is the miniband width, ε_{α} and k_{α} are the energy and wave number along the α -axis, respectively.

Under the influence of the electric field of Eq. (1), an electron in a μ -dimensional superlattice executes nonlinear oscillations (with a different period in each direction) having the velocity given by (no scattering)

$$V_{\alpha}(k_{\alpha}^{(0)}, t_0, t) = V_m \sum_{n=-\infty}^{\infty} J_n(g_{\alpha}) \sin[n(\omega_{\alpha} t - \delta_{\alpha}) + k_{\alpha}^{(0)} d - g_{\alpha} \sin(\omega_{\alpha} t_0)], \quad (3)$$

where $V_m = \Delta d / 2\hbar$ is the maximum electron velocity, $k_{\alpha}^{(0)}$ is the electron wave vector at initial time t_0 , and $g_{\alpha} = \Omega_{\alpha}^{(0)} / \omega_{\alpha} = eE_{\alpha}d / \hbar\omega_{\alpha}$. One can obtain a similar expression for electron energy by integrating the relation $V_{\alpha}(k_{\alpha}) = \hbar^{-1} \partial \varepsilon_{\alpha}(k_{\alpha}) / \partial k_{\alpha}$.

It is clear from Eq. (3) that the velocity harmonic amplitudes are oscillatory functions of the field amplitude. They sequentially vanish at the zeros of the $J_n(g_{\alpha})$ -functions independently of the initial electron momentum. In particular, the time averaged values of electron velocity and energy (zero harmonics) are given by

$$\overline{V_{\alpha}}(k_{\alpha}^{(0)}, t_0) = V_m J_0(g_{\alpha}) \sin(k_{\alpha}^{(0)} d - g_{\alpha} \sin(\omega_{\alpha} t_0)), \quad (4)$$

and

$$\overline{\varepsilon_{\alpha}}(k_{\alpha}^{(0)}, t_0) = \frac{\Delta}{2} [1 - J_0(g_{\alpha}) \cos(k_{\alpha}^{(0)} d - g_{\alpha} \sin(\omega_{\alpha} t_0))]. \quad (5)$$

At the specific values of g_{α} for which $J_0(g_{\alpha}) = 0$, we have

$$\overline{V_{\alpha}}(k_{\alpha}^{(0)}, t_0) = 0, \quad \overline{\varepsilon_{\alpha}}(k_{\alpha}^{(0)}, t_0) = \frac{\Delta}{2}, \quad (6)$$

i.e., electron motion along the α -axis has no net translation independently of its initial momentum, and its average energy takes the value at the middle of the one-dimensional miniband. This phenomenon is known as *dynamic electron localization*. The electron has a discrete energy spectrum due to the finiteness of the motion and, therefore, dynamic electron localization corresponds to the collapse of its quasienergy minibands¹³ (described by the precollapse relation),

$$\tilde{\varepsilon}_\alpha(k_\alpha) = \frac{\Delta}{2} [1 - J_0(g_\alpha) \cos(k_\alpha d)] + n_\alpha \hbar \omega_\alpha, \quad (7)$$

$$n_\alpha = 0, \pm 1, \pm 2, \dots]$$

Depending on the values of g_α , dynamic electron localization and miniband collapse can be one-dimensional, two-dimensional, or three-dimensional (in which case the localization and collapse are complete).

The dynamical peculiarities of superlattice electrons are evident in nonlinear conduction. However, even a qualitative analysis requires the use of a correct model of the relaxation processes, which may be simplified to the specifications of a particular problem. In the present work we endeavor to take account of the μ -dimensionality of the electron scattering processes and the separation of elastic and inelastic scattering. The single τ -approximation, useful for the one-dimensional model, is not adequate for our purposes, as discussed above. A two relaxation time model was proposed in Ref. 25 as well as in Ref. 8 by one of the authors of the present work. However, this model is, in fact, one-dimensional and, moreover, it produces the *illusion* of a separation of elastic and inelastic electron scattering processes and associated scattering times, which we now understand to be incorrect. In some sense this model is even worse than usual single τ -approximation [but, unfortunately, is still in use (Ref. 26)] because its identification of the two relaxation times from experimental data is erroneous. The balance equation method, developed in Ref. 27, would be useful for our goals, if it would be generalized by replacing the isotropic electron temperature by an anisotropic one (Ref. 15), but this generalization presents a considerable challenge and has not been done yet. The approach proposed in Ref. 28 takes into consideration the interplay between different degrees of freedom and separates phonon and impurity scatterings on a microscopic basis. However, this method is essentially single-particle in nature and it is primarily applicable for a superlattice with low carrier concentration when electron-electron scattering does not play a significant role. To overcome these limitations, we start from a three-dimensional model of a superlattice having the novel phenomenological collision term proposed in Ref. 15. This collision term describes scattering in terms of an improved three-channel electron relaxation process. We employ the commonly understood^{16,17} separation of the relaxation processes into elastic, inelastic and electron-electron with characteristic times specified for each of the three channels working in parallel. In the first channel (usually the fastest one) an electron is subject to redistribution of the additional energy and momentum supplied by the applied electric field among its degrees of freedom by means of elastic scattering during a characteristic time τ_1 . The kinetic energy of each electron is conserved during this scattering to isoenergetic surfaces, but the *direction* of momentum is randomized (with consequent reduction of the drift velocity). In the longer-lasting second channel, the energy supplied by the external electric field is redistributed among all electrons due to inelastic electron-electron scattering, including Umklapp processes. As a result of the Umklapp processes the Fermi distribution becomes

*undrifted*¹⁷ and, furthermore, the redistribution of energy establishes an effective electron temperature T_e during a time τ_{ee} by electron-electron scattering. The total energy of all electrons is conserved during the redistribution process, in spite of their momentum relaxation. Finally, in the third channel, electrons transmit energy to the lattice over a time τ_ε and their distribution relaxes to a Fermi function at the lattice temperature T_0 .

The Boltzmann equation with this model collision term has the form,

$$\frac{\partial f(\mathbf{k}, t)}{\partial t} + \frac{e\mathbf{E}(t)}{\hbar} \frac{\partial f(\mathbf{k}, t)}{\partial k} = - \left(\frac{\partial f}{\partial t} \right)_{\text{coll}}, \quad (8)$$

where

$$\left(\frac{\partial f}{\partial t} \right)_{\text{coll}} = \frac{f(\mathbf{k}, t) - f_S(\varepsilon, t)}{\tau_1} + \frac{f(\mathbf{k}, t) - f_0(\varepsilon, T_e)}{\tau_{ee}} + \frac{f(\mathbf{k}, t) - f_0(\varepsilon, T_0)}{\tau_\varepsilon}, \quad (9)$$

with the isoenergetic distribution function, $f_S(\varepsilon, t)$, expressed as an integral average over the equipotential surface S_ε ,

$$f_S(\varepsilon, t) = \frac{\int_{S_\varepsilon} f(\mathbf{k}, t) \frac{dS}{|\nabla_k \varepsilon|}}{\int_{S_\varepsilon} \frac{dS}{|\nabla_k \varepsilon|}}, \quad (10)$$

and

$$\langle \varepsilon \rangle = \langle \varepsilon \rangle_S = \langle \varepsilon \rangle_e, \quad (11)$$

where we use the notation $\varepsilon = \varepsilon(\mathbf{k})$ and $f(\mathbf{k}, t)$ is the non-equilibrium distribution function driven by the electric field. $f_0(\varepsilon, T_e)$ is the equilibrium Fermi distribution at the elevated electron temperature T_e and $f_0(\varepsilon, T_0)$ is the equilibrium Fermi distribution at lattice temperature T_0 . $\langle \varepsilon \rangle$, $\langle \varepsilon \rangle_S$, $\langle \varepsilon \rangle_e$, and $\langle \varepsilon \rangle_0$ are the energies averaged over the corresponding distribution functions. The effective electron temperature T_e is determined by Eq. (11). It is important to note that the anisotropic function $f_S(\varepsilon, t)$ plays the role of the “isotropic” distribution function of Ref. 16, but not the symmetric one, i.e., $f_S(\varepsilon, t) \neq (f(\mathbf{k}, t) + f(-\mathbf{k}, t))/2$. A symmetric form of $f_S(\varepsilon, t)$ would occur if there were *no* energy and momentum redistribution among all degrees of freedom and it would correspond to the one-dimensional model of a superlattice used in Refs. 8,25.

Let us summarize our generalizations and simplifications of the three relaxation processes:

(1) As in Ref. 16, the quasielastic electron scattering leading to the “isotropization” of the electron distribution function is considered to be dominant. In our case, the electrons are distributed onto corresponding (nonspherical!) isoenergetic surfaces and the anisotropic function $f_S(\varepsilon, t)$ plays the role of the “isotropic” distribution function of Ref. 16.

(2) The Brillouin minizones are narrow for superlattices, lending importance to Umklapp processes and the establish-

ment of an *undrifted* Fermi distribution with an effective temperature T_e .¹⁷ This underscores the difference of our present description from that of Ref. 27.

(3) The dynamical development of the deviation of the electron distribution function from the “isotropic” one is described by

$$\frac{\partial}{\partial t}(f(k,t) - f_s(\varepsilon,t))_{st} = -\frac{f(k,t) - f_s(\varepsilon,t)}{\tau}, \quad (12)$$

i.e., by the effective relaxation time τ , which is, in general, dependent not only on energy but also on the electron momentum direction. This relation is the same as the one commonly used for the first term of the distribution function expansion in Legendre polynomials.¹⁶ Thus, in this respect, our description is the same as that of Ref. 16 up to this point. In accordance with our classification of the three relaxation processes, we have

$$\frac{1}{\tau} = \frac{1}{\tau_1} + \frac{1}{\tau_{ee}} + \frac{1}{\tau_\varepsilon}. \quad (13)$$

(4) We describe the collision dynamics of the “isotropic” distribution function $f_s(\varepsilon,t)$ approximately by two relaxation times, τ_{ee} and τ_ε , i.e., by the relation

$$\left(\frac{\partial f_s(\varepsilon,t)}{\partial t}\right)_{st} = -\frac{f_s(\varepsilon,t) - f_0(\varepsilon, T_e)}{\tau_{ee}} - \frac{f_s(\varepsilon,t) - f_0(\varepsilon, T_0)}{\tau_\varepsilon}. \quad (14)$$

While this is not an exact equation for $f_s(\varepsilon,t)$, it is acceptable for a qualitative description. Moreover, our interest is not in the details of the distribution functions, but only in the current and in the average energy. Furthermore, it is easier to incorporate necessary corrections directly in balance equations to be derived below.

The balance equations can be obtained from Eqs. (8) to (11). For sake of simplicity we take the relaxation times τ_1 , τ_{ee} , and τ_ε to be energy and momentum independent. Multiplying Eq. (8) sequentially by $\partial \varepsilon_\alpha(k_\alpha)/\partial k_\alpha$ and by $\varepsilon_\alpha(k_\alpha)$ and integrating over first Brillouin minizone, we obtain the following equations for the current components ($j_\alpha(t) = ne\hbar^{-1}\langle \partial \varepsilon_\alpha(k_\alpha)/\partial k_\alpha \rangle$) and the average energies $\langle \varepsilon_\alpha \rangle$:

$$\frac{\partial j_\alpha(t)}{\partial t} - ne^2 \langle m_\alpha^{-1}(\varepsilon) \rangle E_\alpha(t) = -\frac{j_\alpha(t)}{\tau_p}; \quad (15)$$

$$\begin{aligned} \frac{d}{dt} \langle \varepsilon_\alpha \rangle - \frac{1}{n} E_\alpha(t) j_\alpha(t) \\ = -\frac{\langle \varepsilon_\alpha \rangle - \langle \varepsilon \rangle_S}{\tau_1} - \frac{\langle \varepsilon_\alpha \rangle - \langle \varepsilon \rangle_e}{\tau_{ee}} - \frac{\langle \varepsilon_\alpha \rangle - \langle \varepsilon \rangle_0}{\tau_\varepsilon}; \end{aligned} \quad (16)$$

and

$$\sum_{\alpha=1}^{\mu} \langle \varepsilon_\alpha \rangle_S = \sum_{\alpha=1}^{\mu} \langle \varepsilon_\alpha \rangle_e = \langle \varepsilon \rangle, \quad (17)$$

where

$$\langle m_\alpha^{-1}(\varepsilon) \rangle = \left\langle \frac{1}{\hbar^2} \frac{\partial^2 \varepsilon(k_\alpha)}{\partial k_\alpha^2} \right\rangle, \quad (18)$$

n is the three-dimensional electron density and $\tau_p^{-1} = \tau_1^{-1} + \tau_{ee}^{-1} + \tau_\varepsilon^{-1}$ is the overall inverse electron relaxation time.

The balance equations [Eqs. (15)–(17)] are valid both for homogeneous semiconductors (without superlattice) and for semiconductor superlattices of any dimensions. All terms in these equations have clear physical meaning and allow some generalizations, for example, the replacement of scalar relaxation times by a relaxation tensor (for symmetric structures we consider it unnecessary). It should be noted that the relaxation times, τ_p and τ_ε , can be taken from independent calculations using the actual scattering mechanisms. Such a calculation for one-dimensional GaAs-based superlattices was done in Ref. 29, where, in particular, it was shown that the relaxation times, τ_p and τ_ε , can be taken to be independent of energy, if the miniband width is less than optical phonon energy.

In general, the set of Eqs. (15)–(17) is not closed because of coupling to higher order moments of the distribution function $f(\mathbf{k},t)$. However, there are simplifications for one-, two-, and three-dimensional superlattices with a sinusoidal dispersion law, Eq. (2). In these cases, symmetry dictates that

$$\langle \varepsilon_\alpha \rangle_S = \langle \varepsilon_\alpha \rangle_e = \frac{1}{\mu} \langle \varepsilon \rangle. \quad (19)$$

Furthermore, the sinusoidal dispersion law provides the linear proportionality between the effective electron mass and its energy,

$$\langle m_\alpha^{-1}(\varepsilon) \rangle = \left(\frac{\Delta}{2} - \langle \varepsilon_\alpha \rangle \right) \frac{d^2}{\hbar^2}. \quad (20)$$

Accordingly, Eqs. (15), (16), taken jointly with Eqs. (19), (20), form a closed set of equations. It is convenient to write this set in complex form, introducing a dimensionless complex μ -component “vector” with components defined by

$$\Phi_\alpha(t) = \frac{\Delta/2 - \langle \varepsilon_\alpha \rangle}{\Delta/2 - \langle \varepsilon_\alpha \rangle_0} - i \frac{j_\alpha(t)}{j_{0\alpha}}, \quad (21)$$

where $j_{0\alpha} = (end/\hbar)(\Delta/2 - \langle \varepsilon_\alpha \rangle_0)$. The balance equation for $\Phi_\alpha(t)$, equivalent to Eqs. (15), (16), (19), (20), is given by

$$\begin{aligned} \frac{d\Phi_\alpha(t)}{dt} + (\tau_p^{-1} + i\Omega_\alpha(t))\Phi_\alpha(t) \\ = \tau_\varepsilon^{-1} + \frac{1}{\mu} (\tau_p^{-1} - \tau_\varepsilon^{-1}) \sum_{\beta=1}^{\mu} \text{Re} \Phi_\beta(t), \end{aligned} \quad (22)$$

where $\Omega_\alpha(t) = edE_\alpha(t)/\hbar$. The last term on the right-hand side of Eq. (22) describes the redistribution of electron energy and momentum among all degrees of freedom. This feature is absent in the single relaxation time description and is of crucial importance for our present considerations. For a one-dimensional superlattice, ($\mu=1$), Eq. (22) is identical to the balance equations obtained in Ref. 8. It should be noted that the only significant feature of the distribution functions,

$f_S(\varepsilon, t)$ and $f_0(\varepsilon, T_e)$, is that given by Eq. (19) in regard to the derivation of Eq. (22) for two-, and three-dimensional superlattices, because of the high symmetry of the electron dispersion relation. The specific forms of $f_S(\varepsilon, t)$ and $f_0(\varepsilon, T_e)$, beyond Eq. (19), are not pertinent. A further consequence of this symmetry is that the inverse relaxation times τ_1^{-1} and τ_{ee}^{-1} are involved in Eq. (22) only in the form of their sum. For bulk semiconductors having a one-dimensional superlattice with free motion in the lateral plane, one obtains an integrodifferential equation (allowing only numerical solution) instead of Eq. (22). Such an integrodifferential equation was analyzed for the case of a static electric field in Ref. 15 in detail.

It is easily shown that the complex "vector" with components $\Phi_\alpha(t)$ introduced above is constituted by the first Fourier-components of distribution function $f(\mathbf{k}, t)$. Using periodicity in \mathbf{k} -space, this distribution function may be expanded in a μ -dimensional Fourier series,

$$f(\mathbf{k}, t) = \sum_{\nu_\alpha} F_\nu \exp\{i \nu \cdot \mathbf{k}d\} \Phi_\nu(t), \quad (23)$$

with Fourier coefficients given by

$$F_\nu = \left(\frac{d}{2\pi}\right)^\mu \int_{BZ} d^\mu k f_0(\mathbf{k}) \exp\{-i \nu \cdot \mathbf{k}d\}, \quad (24)$$

where $\nu \rightarrow (\nu_1)$ for one-dimensional, $\nu \rightarrow (\nu_1, \nu_2)$ for two-dimensional, $\nu \rightarrow (\nu_1, \nu_2, \nu_3)$ for three-dimensional superlattices, and the integration is taken over the first Brillouin zone (BZ). Only the first momentum harmonics (in any direction) of the distribution function,

$$\Phi_\alpha(t) = \frac{\int d^\mu k f(\mathbf{k}, t) \exp\{-ik_\alpha d\}}{\int d^\mu k f_0(\mathbf{k}) \exp\{-ik_\alpha d\}} \quad (25)$$

contribute to the current density $\mathbf{j}(t)$ and the electron energy $\langle \varepsilon_\alpha \rangle$ for the miniband case of Eq. (2). One can see that they are the same as those given by Eq. (21).

For arbitrary time-dependence of the electric field $\mathbf{E}(t)$, it is useful to write $\Phi_\alpha(t)$ in a form that is convenient for the representation of Bloch oscillations, as

$$\Phi_\alpha(t) = a_\alpha(t) \Psi_\alpha(t), \quad (26)$$

where

$$\Psi_\alpha(t) = \exp\left\{-i \int_0^t \Omega_\alpha(t_1) dt_1\right\} \quad (27)$$

is a solution of the homogeneous counterpart of Eq. (22) associated with Bloch oscillations in the absence of scattering terms. This function describes the dynamic modulation of the electron distribution function by the applied electric field without scattering. In particular, for a simple harmonic field, $E_\alpha(t) = E_\alpha^{(0)} \cos \omega t$, we have

$$\Psi_\alpha(t) = \exp\{-i g_\alpha \sin \omega t\} = \sum_{\nu=-\infty}^{\infty} J_\nu(g_\alpha) e^{-i \nu \omega t}, \quad (28)$$

where $g_\alpha = \Omega_\alpha^{(0)}/\omega$ is the projection of the dimensionless field amplitude onto the x_α -axis. The functions $a_\alpha(t)$ represent dissipative processes, describing changes in the amplitude of $\Phi_\alpha(t)$ (deviation from the Bloch oscillation solution) due to scattering. They obey the following equations, based on Eq. (22):

$$\begin{aligned} \dot{a}_\alpha(t) + \tau_p^{-1} a_\alpha(t) \\ = \tau_\varepsilon^{-1} \Psi_\alpha^*(t) + \frac{1}{\mu} (\tau_p^{-1} - \tau_\varepsilon^{-1}) \times \text{Re} \left(\sum_{\beta=1}^{\mu} a_\beta \Psi_\beta \right) \Psi_\alpha^*(t). \end{aligned} \quad (29)$$

In the absence of scattering, $a_\alpha(t) \equiv 1$. The transfer from the description in terms of the functions $\Phi_\alpha(t)$ to a description in terms of functions $a_\alpha(t)$ corresponds to a transformation to a new system of coordinates, K_0 , oscillating in momentum space together with the unscattered electron. In the system K_0 , each electron is at a fixed point \mathbf{k}_0 . Only scattering changes the distribution of these points. In the case of rare collisions ($\omega \tau \gg 1$), such changes are small during the period of the applied field, but they can accumulate and become important over the time of a few collisions. Otherwise, the equilibrium distribution functions (to which the electrons relax) in the system K_0 are modulated by the field and become rapid functions of time, as

$$f_0(\varepsilon, T_0) \rightarrow f_0(k_3) = f_0\left(\mathbf{k}_0 - \frac{1}{d} \int_0^t \boldsymbol{\Omega}(t_1) dt_1\right). \quad (30)$$

This feature is embodied in the structure of Eq. (29): the "scattering-out" term (second term on the left-hand side) has the usual relaxation form with the overall inverse relaxation time due to all scattering mechanisms, and the "scattering-in" term (first term on the right-hand side) is the dynamically modulated equilibrium distribution function with inverse relaxation time τ_ε^{-1} . The last term on the right-hand side of Eq. (29), describing the redistribution of energy and momentum over the various degrees of freedom, is modulated by the electric field twofold; once in connection with the transformation to the system K_0 , and, secondly, because the corresponding equilibrium distribution functions are determined by the average normalized electron energy [Eqs. (21) and (26)] which, in turn, depends on the field and time. This average energy is involved in the last term through the relation,

$$\text{Re} \left(\sum_{\beta=1}^{\mu} a_\beta \Psi_\beta \right) = \text{Re} \left(\sum_{\beta=1}^{\mu} \Phi_\beta(t) \right) = \sum_{\beta=1}^{\mu} \frac{\Delta/2 - \langle \varepsilon_\beta \rangle}{\Delta/2 - \langle \varepsilon_\beta \rangle}.$$

III. SELF-INDUCED TRANSPARENCY

In this section, we analyze superlattice response to a high frequency ($\omega \tau_p \gg 1$) monochromatic field directed along the

x_1 -axis. Considering a_α to be slowly varying functions of time, we average Eq. (29) over an interval Δt given by $2\pi/\omega \ll \Delta t \ll \tau_p$, obtaining an algebraic system of equations, for which the stationary solution is

$$a_1 = \frac{\lambda \mu^2 J_0(g_1)}{\mu[1 + (\mu - 1)\lambda] \cdot B(g_1) - (\mu - 1)(1 - \lambda)^2 J_0^2(g_1)}, \quad (31)$$

and

$$a_\alpha = \lambda \frac{\mu B(g_1) + (1 - \lambda) J_0^2(g_1)}{[1 + (\mu - 1)\lambda] \cdot B(g_1) - (\mu - 1)(1 - \lambda)^2 J_0^2(g_1)}, \quad (32)$$

$\alpha \neq 1,$

where $\lambda = \tau_p / \tau_\varepsilon$ and

$$B(g) = 1 - \frac{1 - \lambda}{2\mu} [1 + J_0(2g)]. \quad (33)$$

In the derivation of Eqs. (31) and (32), we used the relations,

$$\overline{\Psi_\alpha(t)} = J_0(g_\alpha), \quad \overline{(\text{Re}(\Psi_\alpha(t)))^2} = \frac{1}{2} [1 + J_0(2g_\alpha)], \quad (34)$$

where the overhead bar indicates averaging over the period of the impressed electric field.

According to Eqs. (21),(26),(28),(31), and (32), the current, energy dissipation rate, Q , and ratio between the transverse and longitudinal electron heating, δ , are given by

$$\tilde{j}_1 \equiv j_1 / j_{01} = a_1 \sin(g_1 \sin(\omega t)) + O\left(\frac{1}{\omega \tau}\right), \quad (35)$$

$$Q \equiv n \frac{\overline{\langle \varepsilon \rangle} - \langle \varepsilon \rangle_0}{\tau_\varepsilon} = Q_0 (1 - a_1 J_0(g_1)),$$

$$Q_0 = \frac{\mu(\Delta/2 - \langle \varepsilon_\alpha \rangle_0) n}{\tau_\varepsilon + (\mu - 1)\tau_p}, \quad (36)$$

and

$$\delta \equiv \frac{(\mu - 1)(\overline{\langle \varepsilon_\alpha \rangle} - \langle \varepsilon_\alpha \rangle_0)}{\overline{\langle \varepsilon_1 \rangle} - \langle \varepsilon_1 \rangle_0} = \frac{(1 - \lambda)(\mu - 1)}{1 + \lambda(\mu - 1)}. \quad (37)$$

Figure 1 depicts the function a_1 involved in the expression for the current, and also shows Q/Q_0 , as functions of g for a three-dimensional superlattice ($\mu = 3$) with $\lambda = 1, 0.1, 0.01$. In the case $\tau_p = \tau_\varepsilon$ ($\lambda = 1$), we obtain $a_1 = J_0(g_1)$ and expressions (35) and (36) coincide with those obtained for

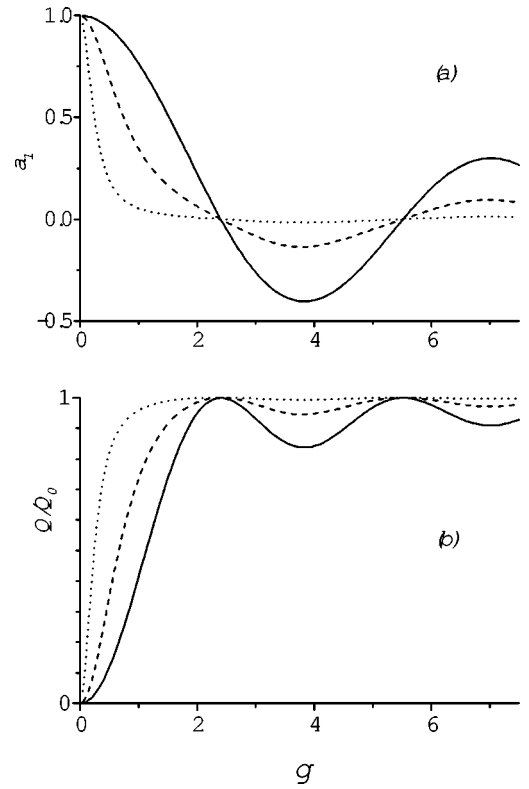


FIG. 1. Normalized (a) current and (b) energy dissipation rate as functions of $g = eEd/\hbar\omega$.

the one-dimensional model of a superlattice.⁸ With increasing τ_p while holding τ_ε fixed, the current in the superlattice increases, the dissipation decreases, and the amplitude of their modulation increases. As in the case of the usual single τ -approximation,⁷⁻⁹ the three-dimensional superlattice current vanishes at fields such that $J_0(g_1) = 0$ [with an accuracy of $(\omega\tau_p)^{-1}$], where dissipation is maximal. When elastic collisions dominate over inelastic scattering ($\tau_p \ll \tau_\varepsilon$) there is a rapid redistribution of energy among the electron degrees of freedom and, consequently, the current decreases sharply for $g_1 \gg 1$. This is caused by a strong expansion of the distribution function in momentum space, due to both longitudinal and transverse heating.

To examine the peculiarities of self-induced transparency in the case of a two-dimensional superlattice, we consider an electric field of the form,

$$\mathbf{E} = (E_1 \mathbf{x}_1 + E_2 \mathbf{x}_2) \cos(\omega t). \quad (38)$$

In a manner similar to that of the foregoing analysis, we obtain the components of the dissipative function as

$$a_{1,2} = \frac{4\lambda(J_0(g_{1,2})B(g_{1,2}) + (1 - \lambda)J_0(g_{1,2})(J_0(g_1 + g_2) + J_0(g_1 - g_2)))}{16B(g_1)B(g_2) - (1 - \lambda)^2(J_0(g_1 + g_2) + J_0(g_1 - g_2))^2}. \quad (39)$$

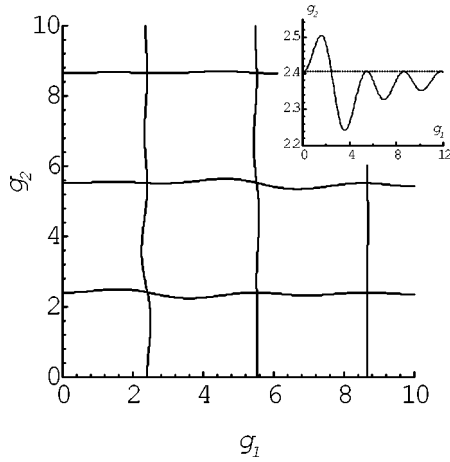


FIG. 2. Loci for one-dimensional self-induced transparencies in directions \mathbf{x}_1 (horizontal curves) and \mathbf{x}_2 (vertical curves). The inset exhibits a magnification of the locus for one-dimensional self-induced transparency in direction \mathbf{x}_1 and the locus for one-dimensional dynamic localization in this direction (horizontal line).

To obtain Eq. (39), we used the relations (34) and the expression

$$\overline{\text{Re } \Psi_1(t) \text{Re } \Psi_2(t)} = \frac{1}{2}(J_0(g_1 + g_2) + J_0(g_1 - g_2)). \quad (40)$$

The corresponding current and energy dissipation rate are given by

$$j_{1,2}(t) = a_{1,2} \sin(g_{1,2} \sin \omega t), \quad (41)$$

and

$$Q = \frac{n(\Delta - \langle \varepsilon \rangle_0)}{\tau_\varepsilon} \left(1 - \frac{1}{2}(a_1 J_0(g_1) + a_2 J_0(g_2)) \right). \quad (42)$$

It is clear from Eqs. (39) and (41) that the current along each superlattice axis depends on all field projections, in contrast to the results of Ref. 30. In general, the current of each excited harmonic is not parallel or antiparallel to the resultant electric field and has its own elliptic polarization. There are exceptions if the electric field is in the lattice directions $[10]$ or $[11]$, in which case the field and excited current are parallel.

Furthermore, it is evident from Eqs. (39) and (41), that both dynamic localization and self-induced transparency can be either one-dimensional or complete for two-dimensional superlattices. In Fig. 2 we show the loci of one-dimensional self-induced transparencies occurring in directions \mathbf{x}_1 (horizontal curves) and \mathbf{x}_2 (vertical curves), respectively, as functions of g_1 and g_2 . At fields corresponding to these curves, the current components vanish [$j_1(t) \approx 0$ and $j_2(t) \approx 0$, respectively]. These dependencies exhibit oscillations around the line $J_0(g_{1,2}) = 0$ (see Fig. 2 inset), which is the condition for dynamic localization to occur. The average energies and the dissipation rate for these values of the fields are given by

$$\begin{aligned} \overline{\langle \varepsilon_{1(2)} \rangle} &= \frac{\Delta}{2}, \quad \overline{\langle \varepsilon_{2(1)} \rangle} = \frac{\Delta}{2} - \left(\frac{\Delta}{2} - \langle \varepsilon_{2(1)} \rangle_0 \right) \\ &\times \frac{4\lambda J_0^2(g_{2(1)})}{4 + (1-\lambda)(1 + J_0(2g_{2(1)}))}, \end{aligned} \quad (43)$$

and

$$Q = \frac{n(\Delta - \langle \varepsilon \rangle_0)}{\tau_\varepsilon} \left(1 - \frac{2\lambda J_0^2(g_{2(1)})}{4 + (1-\lambda)(1 + J_0(2g_{2(1)}))} \right), \quad (44)$$

where the first of the subscripted indices is related to the horizontal curves and the second subscripted index (in parentheses) is related to the vertical curves. At points of intersection, determined by the condition $J_0(g_1) = J_0(g_2) = 0$, there is complete self-induced transparency. In such cases, the average electron energies and the dissipation rate are maximal and are given by

$$\overline{\langle \varepsilon_1 \rangle} = \overline{\langle \varepsilon_2 \rangle} = \frac{\Delta}{2}, \quad (45)$$

and

$$Q = \frac{n(\Delta - \langle \varepsilon \rangle_0)}{\tau_\varepsilon}. \quad (46)$$

The dissipation rate in Eq. (46) is larger than that in Eq. (44), and is also larger than the maximum dissipation rate for the fields oriented strictly along the crystal axes, the latter being given by Eq. (36) as

$$Q = \frac{n(\Delta - \langle \varepsilon \rangle_0)}{\tau_\varepsilon + \tau_p}. \quad (47)$$

It is apparent that, within the three-relaxation-time description presently under consideration, complete self-induced transparency and dynamic localization occur at the same fields, whereas one-dimensional self-induced transparency and dynamic localization arise at different fields. In two-dimensional superlattices complete dynamic localization and self-induced transparency occur at the discrete amplitude values and applied electric field directions determined by the relations,

$$E_{m,n} = \frac{\hbar \omega}{ed} \sqrt{\xi_m^2 + \xi_n^2}, \quad \varphi_{m,n} = \arctan(\xi_m^2 / \xi_n^2), \quad (48)$$

where $\varphi_{m,n}$ is the angle of field orientation with respect to the superlattice crystal axis and ξ_m is the m th-order root of the zeroth-order Bessel function. This can be generalized easily to the three-dimensional case.

To develop a physical understanding of one-dimensional and complete self-induced transparencies, we examine the evolution of the electron distribution in the K_0 -system. One can see from Eq. (29) and its following discussion that, under the influence of the field and scattering, the number of

electrons entering the current components, $\text{Im} \Phi_{1,2}$ (including redistribution among them), averaged over the field period, $P_{1,2}$, is given by

$$P_{1,2} = \tau_\varepsilon^{-1} \overline{\Psi_{1,2}(t)} + \frac{1}{\mu} (\tau_p^{-1} - \tau_\varepsilon^{-1}) \times (a_{1,2} \overline{(\text{Re} \Psi_{1,2}(t))^2} + a_{2,1} \overline{\text{Re} \Psi_1(t) \text{Re} \Psi_2(t)}). \quad (49)$$

If the dynamic modulation of the equilibrium distribution function is such that $\Psi_1(t) = \Psi_2(t) = 0$, then the average number of electrons entering the current components vanishes (there is only electron redistribution among the components), and, therefore, the current components are eventually completely eliminated from the nonequilibrium distribution function by “scattering-out” over a time of order τ_p . It should be noted that for $\Psi_1(t) = \Psi_2(t) = 0$, the set of Eq. (29) averaged over the field period becomes homogeneous and its steady-state solution is zero. In this case electron heating is maximal due to complete dynamic localization. Thus, after a time of order τ_p , the superlattice becomes transparent, i.e., behaves like a dielectric having the permittivity of the crystal lattice and relatively small, but resonant, absorption. This is to say that we have complete self-induced transparency. It should be emphasized that, at arbitrary fields, the absorption rate stabilizes after time τ_ε , i.e., later than the vanishing of the current.

If dynamic localization takes place in only one of the two crystal directions, for example, in [10], i.e., $\text{Re} \Psi_1(t) = J_0(g_1) = 0$, but $\text{Re} \Psi_2(t) = J_0(g_2) \neq 0$, then the electrons only enter the current component $\Phi_2(t)$ due to dynamic modulation of the equilibrium distribution function. However, because of the redistribution of energy and momentum among the degrees of freedom, electrons also flow into $\Phi_1(t)$. Therefore, even at $J_0(g_1) = 0$ both P_1 and j_1 are nonzero. The current j_1 vanishes only if the components of the flow P_1 caused (a) by direct dynamical modulation of the distribution function, and (b) by redistribution via scattering, compensate each other in this direction. However, this occurs at $J_0(g_1) \neq 0$, i.e., when dynamic localization in this direction is absent, as reflected in Eq. (32) and Fig. 2. Furthermore, it should be noted that the time-averaged one-dimensional energy is $\Delta/2$ for each electron when dynamic localization occurs, but this is valid only for both time and ensemble averaged energy in the case of one-dimensional self-induced transparency.

One can easily see that if the relaxation times τ_p and τ_ε are energy dependent (*not* constant), then the first harmonics of the distribution (23) become coupled to higher harmonics. In this case self-induced transparency does not occur even when there is complete dynamic localization. This was demonstrated in Ref. 7 for a one-dimensional superlattice. However, as mentioned above, for an appropriate set of superlattice parameters, these relaxation times can be considered energy-independent²⁹ and, therefore, even the quantitative results discussed above have a wide range of validity.

It should be emphasized that, although both self-induced transparency and dynamic localization occur in a superlattice due to the narrowness of its Brillouin minizones, the physical origins of these effects are completely different. Dynamic localization arises when the zeroth harmonic of nonlinear electron oscillations, modulated by Bragg reflections, vanishes. In contrast to dynamic localization, self-induced transparency is a result of the joint action of Bragg reflections of miniband electrons and collisions creating strongly modulated electron distributions in which the first harmonics are absent for discrete values of the electric field amplitudes (in the limit $\tau_p \rightarrow \infty$). As a result, dynamic localization appears immediately after turn-on of electric field, and self-induced transparency occurs only after a time of order of τ_p . The conditions for self-induced transparency depend on the scattering mechanisms and, in general, it takes place even without dynamic localization. It can be shown that states of self-induced transparency are not stable with respect to the generation of static and hf fields (having frequencies not equal to ω_1)³¹ and, therefore, experimental studies should be performed at low electron concentrations and with the use of pulsed electric fields.

IV. CURRENT MODULATION BY ORTHOGONAL FIELDS

In this section we examine superlattice behavior in the presence of a high frequency biharmonic electric field given by

$$\mathbf{E} = E_1 \mathbf{x}_1 \cos(\omega_1 t - \delta_1) + E_2 \mathbf{x}_2 \cos(\omega_2 t - \delta_2). \quad (50)$$

In this case the field components have different frequencies and are directed along different crystal axes taken to be orthogonal (the case of parallel fields was analyzed in Ref. 32 in a single τ -approximation). We are interested to explore the occurrence of current amplitude modulation by a high frequency electric field orthogonal to the current direction. This is determined by the redistribution of energy and momentum among the various degrees of freedom. We assume that the frequencies of the most important electric field harmonics are well separated, i.e.,

$$|n_1 \omega_1 - n_2 \omega_2| \tau_p \gg 1, \quad n_{1,2} = 1, 2, \dots \quad (51)$$

Using Eq. (34) and the expression

$$\overline{\text{Re} \Psi_\alpha(t) \text{Re} \Psi_\beta(t)} = J_0(g_\alpha) J_0(g_\beta), \quad \alpha \neq \beta, \quad (52)$$

we obtain the following relations for current components and averaged energies:

$$j_{1,2} = a_{1,2}(g_1, g_2) \sin(g_{1,2} \sin(\omega_{1,2} t - \delta_{1,2})), \quad (53)$$

and

$$\overline{\langle \varepsilon_{1,2} \rangle} - \langle \varepsilon_{1,2} \rangle_0 = \left(\frac{\Delta}{2} - \langle \varepsilon_{1,2} \rangle_0 \right) (1 - a_{1,2}(g_1, g_2) J_0(g_{1,2})), \quad (54)$$

where, in the case of a two-dimensional superlattice,

$$a_{1,2}(g_1, g_2) = \frac{4\lambda J_0(g_{1,2})(2B(g_{2,1}) + (1-\lambda)J_0^2(g_{2,1}))}{4B(g_1)B(g_2) - (1-\lambda)^2 J_0^2(g_1)J_0^2(g_2)}, \quad (55)$$

whereas, for a three-dimensional superlattice we have

$$a_{1,2}(g_1, g_2) = \frac{9\lambda J_0(g_{1,2})(3B(g_{2,1}) + (1-\lambda)J_0^2(g_{2,1}))}{9(2+\lambda)B(g_1)B(g_2) - (1-\lambda)^2(3J_0^2(g_1)B(g_2) + 3J_0^2(g_2)B(g_1) + (4-\lambda)J_0^2(g_1)J_0^2(g_2))}. \quad (56)$$

In Fig. 3(a) we exhibit the functions $\tilde{a}_1^{-1}(0.05, g_2) \equiv a_1(g_1=0.05, g_2=0)/a_1(g_1=0.05, g_2)$ and $\tilde{a}_1^{-1}(g_1, 2.405) \equiv a_1(g_1, g_2=0)/a_1(g_1, g_2=2.405)$ for $\lambda=0.1$. The former function describes the modulation due to the electric field E_2 (orthogonal to E_1) of current driven by a weak electric field E_1 ; whereas the latter function describes the same current modulation in the presence of an arbitrary field E_1 under conditions of dynamic localization in the direction \mathbf{x}_2 ($J_0(g_2)=0$). The functions $a_1(g_1, g_2=0)$ and $a_1(g_1, g_2$

$=2.405$), corresponding to the curve $\tilde{a}_1^{-1}(g_1, 2.405)$ of Fig. 3(a), are shown in Fig. 3(b).

One can see from Eqs. (55), (56) and Figs. 3(a), 3(b), that it is impossible for the orthogonal field to cause a complete vanishing of polarization, and, therefore, *induced* transparency does not occur. However, the modulation of polarization by the orthogonal field can be significant, especially for small λ . The maximum decrease of polarization comes about with the occurrence of one-dimensional dynamic electron localization in the transverse direction ($J_0(g_2)=0$). In particular, for $E_1 \rightarrow 0$, it decreases by the factor $(1+(\mu-1)\lambda)/\mu\lambda$. The reason for this is that, under condition of dynamic localization, electron heating is maximal and the distribution function widens in all directions in momentum space due to energy and momentum redistribution among all degrees of freedom, which always leads to decreased current.

As in the case of self-induced transparency, for specific ratios of field amplitudes and frequencies determined by the conditions $J_0(g_1)=J_0(g_2)=0$, complete induced superlattice transparency takes place. In this case a two-dimensional superlattice is transparent to an arbitrarily polarized third weak signal with frequency ω_3 (well separated from the frequencies $n_1\omega_1 \pm n_2\omega_2$, where n_1 and n_2 are integers).

Similar to self-induced transparency, current modulation by an orthogonal field and induced transparency occur (vanish) in a time of order of τ_p and they become stationary in a time of order of τ_e after turn-on (turn-off) of the electric field, i.e., they are displaced in time from dynamic localization.

To establish the frequency limitations of the phenomena we have explored, we carried out a numerical analysis of Eq. (22) with finite values of τ_p and τ_e . The results are shown in Fig. 4, where the dashed lines represent the above-described analytical calculations for the functions $\tilde{a}_1^{-1}(0.05, g_2)$ (upper curve) and $\tilde{a}_1^{-1}(g_1, 2.405)$ (lower curve), respectively. The solid lines represent our numerical determinations of the amplitudes of the first current harmonics (thick curves) and the maximal currents (thin curves) at $\omega\tau_p=1$ (maintaining $\omega\tau_e \gg 1$). One can see that the dependencies presented in Figs. 3(a) and 3(b) change only slightly with decreasing τ_p .

V. SUMMARY

In summary, we have applied the Boltzmann equation with an improved three-relaxation-time collision term to the analysis of self-induced and induced transparencies in semiconductor superlattices. The three relaxation times include

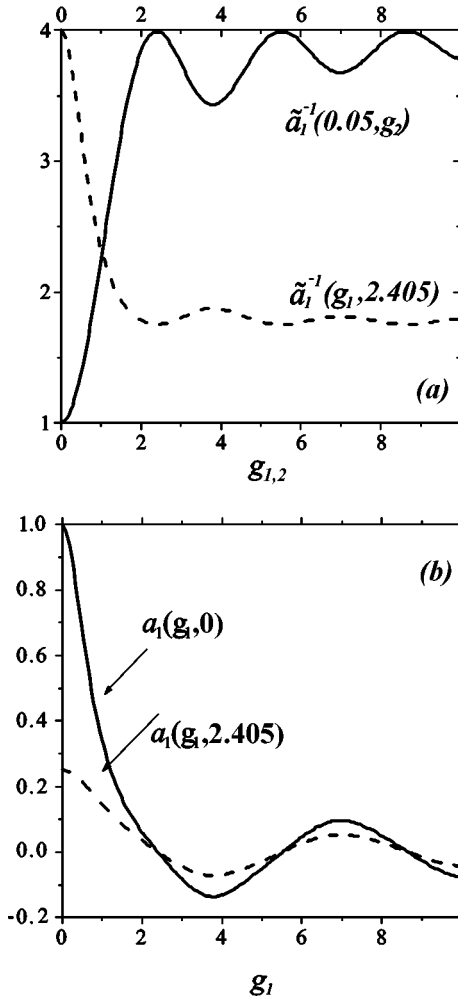


FIG. 3. (a) Current modulation by orthogonal fields as described by $\tilde{a}_1^{-1}(0.05, g_2)$ and $\tilde{a}_1^{-1}(g_1, 2.405)$ (defined in text). (b) The functions $a_1(g_1, g_2=0)$ and $a_1(g_1, g_2=2.405)$ employed in the determination of $\tilde{a}_1^{-1}(g_1, 2.405)$ of Fig. 3(a).

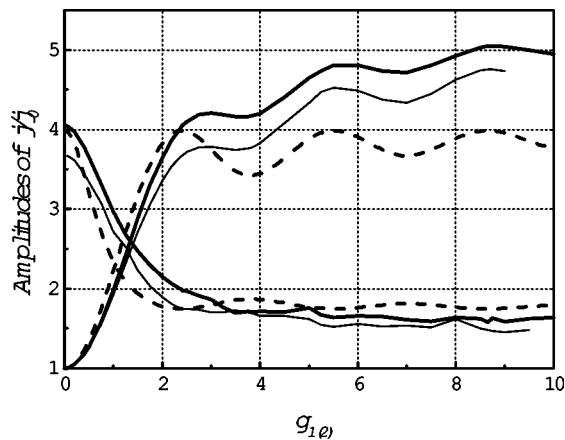


FIG. 4. Amplitude of j/j_0 as a function of field ($g_{1(2)}$).

(a) a time for redistribution of energy and momentum supplied by an electric field to a given electron among its various degrees of freedom, (b) a time for redistribution of energy and momentum among all electrons by inelastic electron–electron interactions, and (c) a time for transfer of the excess energy to the crystal lattice. We have performed analytical calculations for systems having high symmetry

(for one-, two-, and three-dimensional superlattices). However, the results obtained here are valid qualitatively for bulk semiconductors with a one-dimensional superlattice, which are currently available for experimentation. Furthermore, we have shown that self-induced transparency and dynamic localization are different phenomena with different physical origins, displaced in time from each other, and, in general, they arise at different electric fields. Moreover, we have found that the redistribution of energy and momentum among the various degrees of freedom is of crucial importance in two-dimensional and three-dimensional superlattice transport and optical properties. Transverse electron heating drastically changes the conditions for self-induced transparency, and this effect facilitates current modulation by an applied perpendicular high-frequency electric field.

ACKNOWLEDGMENTS

The work of Y.A.R. and J.Y.R. is supported by the Russian Foundation for Basis Research (Grant No. 01-02-16446) and Ministry of Industry, Science, and Technology of the Russian Federation. L.G.M. and N.J.M.H. gratefully acknowledge support from the Department of Defense, DAAD 19-01-1-0592.

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