

Nonexistence of a strong coupling two-channel Kondo fixed point for microscopic models of tunneling centers

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We consider the problem of a heavy particle in a double well potential (DWP) interacting with an electron bath. Under general assumptions, we map the problem to a three-color logarithmic gas model, where the size of the core of the charged particles is proportional to the tunneling time, τ_{tun} , of the heavy particle between the two wells. For times larger than τ_{tun} this model is equivalent to the anisotropic two-channel Kondo (2CK) model in a transverse field. This allows us to establish a relationship between the microscopic parameters of DWP and the 2CK problem. We show that the strong coupling fixed point of the 2CK model can never be reached for the DWP problem, in agreement with the results of Kagan and Prokof'ev [Sov. Phys. JETP **69**, 836 (1989)] and Aleiner *et al.* [Phys. Rev. Lett. **86**, 2629 (2001)].

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I. INTRODUCTION

The possibility to observe the two channel Kondo effect in real physical systems has attracted a lot of interest since the pioneering work by Nozières and Blandin,¹ but at present no experimental realization has been conclusively demonstrated. The difficulty lies in the fact that the non-Fermi liquid (NFL) fixed point of the two channel Kondo model² is unstable to various symmetry breakings^{1,3} which turn out to be important for various experimental situations. In particular, channel anisotropy is a relevant perturbation and the exact channel symmetry is required for the NFL physics to be observed. In a conventional magnetic realization of the Kondo effect this would require the exact degeneracy between atomic orbitals that cannot be obtained in any real system. In the search for the two-channel Kondo (2CK) effect, systems which are less directly related to the conventional Kondo physics were considered. It was suggested that non magnetic impurity tunneling between two sites and interacting with an electron bath could be modeled as a two channel Kondo system in which the spin plays the role of the channel index⁴⁻⁸ (see also Refs. 9,10 for a review). Following this suggestion, the NFL behavior of the 2CK fixed point was used in Ref. 11 to interpret the low temperature transport data for narrow metallic constrictions. Indeed, in the limit $k_F a \ll 1$, where k_F is the Fermi wave vector and $2a$ is the distance between the two minima of the double well potential (DWP), only electrons with two spherical harmonics ($l=0$ and $l=1$, $m=0$) strongly interact with the heavy particle. Usually the mapping of the DWP into a two level system (TLS), which behaves like a localized spin, is achieved by restricting the Hilbert space of the atom to the two lowest energy states associated with the two minima. (As we will discuss in the following this approach is not well justified because it fails to capture the connection between the coupling constants of the effective theory.) Then the orbital degrees of freedom play the role of a pseudo-spin while the real

electron spin represent the channel index, and, in absence of magnetic field, the channel degeneracy was guaranteed by construction. A disadvantage of this realization for the 2CK effect is that other relevant terms appear which are no longer forbidden by symmetry, the most important being the spontaneous tunneling of the impurity between the two minima. This corresponds to magnetic field in the conventional Kondo problem and then the resulting Kondo model has the form,

$$H_{\text{Kondo}} = H_k^0 + H_I + hS^x + \Delta_z S^z. \quad (1)$$

Here

$$H_k^0 = \int dq \epsilon_q \psi_{\alpha,i}^\dagger(q) \psi_{\alpha,i}(q) \quad (2)$$

represents the propagation of free band electrons, $\alpha=1,2$ is the pseudo-spin index and $i=1,2,\dots,k$ is the channel index. In the specific case $k=2$. The sum over repeated indices is implied. The second term in (1) describes the interaction between the localized impurity, \mathbf{S} , and the electrons

$$H_I = \frac{1}{\nu} \sum_{a=x,y,z} \lambda_a \psi_{\alpha,i}^\dagger(r) \sigma_{\alpha\beta}^a \psi_{\beta,i}(r) S^a |_{r=0}, \quad (3)$$

where $\sigma_{\alpha\beta}^a$ are the Pauli matrices in the pseudo-spin space, the fermionic operators $\psi_{\alpha,i}(r)$ are the counterparts of $\psi_{\alpha,i}(k)$ in the coordinate representation, and ν is the density of electronic states at the Fermi energy per one channel, which is introduced here to make the coupling constants dimensionless. There are two terms in the Hamiltonian (1) that break the rotational symmetry in the pseudo-spin space. The term proportional to Δ_z is related to the original asymmetry of the DWP, and may be greatly enhanced by the disordered potential acting on electrons.¹² However, it is still possible to imagine, that the original DWP is symmetric, and all the other impurities are remote from the DWP, so the value of Δ_z

is negotiable and may be even set to zero in some particular cases. Nevertheless, the effective magnetic field, h , is related to the tunneling of the impurity in the DWP, and so is the constant λ_x . The relation between them requires microscopic consideration and one is not allowed to neglect h but keep λ_x finite.

The model (1) is known to scale to the strong coupling fixed point,^{1,13} where it has a NFL behavior.² This regime is achieved if both temperature, T , and

$$\Delta = \sqrt{\Delta_z^2 + h^2} \quad (4)$$

do not exceed the Kondo temperature, T_K . Since the interaction in highly anisotropic, λ_y can be chosen to be equal to zero and $\lambda_x \ll \lambda_z$, the Kondo temperature is given by (see, for instance, Ref. 14),

$$T_K = D(\lambda_x \lambda_z)^{1/2} \left(\frac{\lambda_x}{2\lambda_z} \right)^{1/2\lambda_z}, \quad (5)$$

where D is a high energy cutoff. In order to check whether the 2CK effect can be observed in a DWP system one then has to compare T_k with Δ . This problem has recently attracted a lot of attention.^{6-8,15-20} Some alarming results already appeared in Ref. 8 and were confirmed in Ref. 18. The crucial point consists in determining the various coupling constants in (1) and also high-energy cutoff D in Eq. (5) starting from a general microscopic description of the problem.

As first pointed out by Kagan and Prokof'ev,^{7,8} the TLS is not a good starting point since the high energy degrees of freedom of the impurity cannot be taken into account and they turn out to be essential to establish the correct mapping to the Kondo problem.^{7,8,18} In particular it was shown by model calculation of Ref. 18 that D is of the order of the energy distance to the third excited level of the atomic system which is several orders of magnitude smaller than the Fermi energy. Together with the restrictions to the coupling constant it leads to $h > T_K$ and thus to the impossibility to reach the strong coupling limit. However, a question arises, whether those conclusions are model-dependent or there is a general principle for wide class of microscopic models preventing the existence of the strong coupling regime.

In this paper we use a general nonperturbative approach to the problem that takes into account all the states of the DWP and that, we believe, provides a conclusive answer to the question whether it is possible to observe the 2CK effect in the problem of a tunneling impurity. We start from a microscopic description of the moving heavy particle interacting with electrons in a metal. The potential in which the impurity moves is chosen to have the most general form with two minima (the situation of a potential that presents three minima²¹ is not considered here). The interaction of the heavy atom with the electrons is chosen to be an instantaneous density-density interaction, which, we believe, contains all the essential physics; effects of retardation due to the electron screening or the interband excitations are small either as inverse band gap or plasma frequency and will not be taken into account.

Using a semiclassical approximation to describe the dynamics of the impurity and the condition $k_F a \ll 1$ we map the problem to a one-dimensional logarithmically interacting gas model (LGM). Since a similar mapping can be also obtained for the Kondo model,^{22,23} this allows us to establish a general relationship between the microscopic parameters of the DWP and the coupling constants of the Kondo model. Our results are in agreement with the predictions of Refs. 7,8,18 and show that the mapping of the DWP to the 2CK model can be done only for energies below $\hbar \tau_{\text{tun}}^{-1}$, where τ_{tun} is the tunneling time of the heavy particle between the two wells. It is the existence of this small energy scale, together with the relationship between h and λ_x , that makes the Kondo temperature too small for the strong coupling fixed point to be achieved in such systems: T_K turns out to be always smaller than Δ , which makes the strong coupling limit nonaccessible.

The paper is organized as follow: In the next section we describe the model and we map it to a logarithmic gas model in Sec. III. In Sec. IV we obtain the relation between our microscopic model and the 2CK model and, in the last section, we summarize the results and draw some general conclusions. The problem of the effect of the electron-hole asymmetry on our results is discussed in the Appendix.

II. THE MODEL

The partition function describing an heavy particle moving in a DWP and interacting with an electron bath can be written in the following way:

$$\mathcal{Z} = \int D[q(\tau)] D[\bar{\psi}(\mathbf{x}, \tau)] D[\psi(\mathbf{x}, \tau)] e^{-S_E[q, \bar{\psi}, \psi]}, \quad (6)$$

where S_E is the Euclidean action

$$S_E = \int d\tau \mathcal{L} \quad (7)$$

with

$$\mathcal{L} = \mathcal{L}_{\text{at}} + \mathcal{L}_{\text{el}} + \delta\mathcal{L}. \quad (8)$$

Through this section we will consider only $T=0$, an extension to finite temperature is trivial and will be discussed at the end of Sec. III.

In Eq. (8) \mathcal{L}_{at} describes the dynamics of the heavy particle and is given by

$$\mathcal{L}_{\text{at}} = \frac{M}{2} \left(\frac{dq}{d\tau} \right)^2 + V(q), \quad (9)$$

where $q(\tau)$ represent the position of the atom and $V(q)$ is the confining potential, which we assume to have two minima like in Fig. 1, and assume that the level anisotropy is absent, $\Delta_z = 0$, or more precisely,

$$V(q) = V(-q). \quad (10)$$

Note that here and in the following the potential enters the Lagrangian with a minus sign with respect to the usual definition of the Lagrangian since we work in Euclidean space.

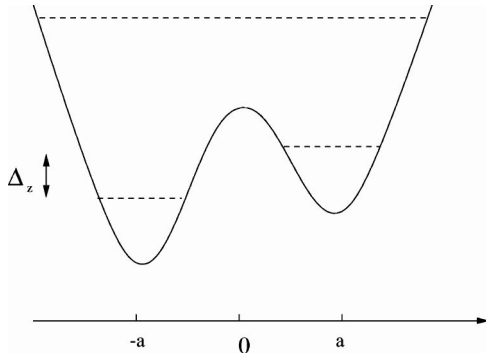


FIG. 1. Schematic representation of a DWP with two minima. The dotted lines represent the lower energy levels and Δ_z is the level anisotropy.

The bare electronic Lagrangian has the form,

$$\mathcal{L}_{\text{el}} = \int d^3\mathbf{x} \bar{\psi}(\mathbf{x}, \tau) \partial_\tau \psi(\mathbf{x}, \tau) + H_0 \quad (11)$$

with

$$H_0 = \int d^3k \epsilon_{\mathbf{k}} \bar{\psi}(\mathbf{k}) \psi(\mathbf{k}), \quad (12)$$

where $\psi(\mathbf{x})$ is the electron field,

$$\psi(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \psi(\mathbf{k}). \quad (13)$$

For the moment we do not include spin, as it is only a spectator. It will be reintroduced at the end where it will play the role of the channel index. We chose a general density–density interaction between the electrons and the atom of the form,

$$\delta\mathcal{L} = \delta H = \lambda \int d^3x \bar{\psi}(\mathbf{x}, \tau) \psi(\mathbf{x}, \tau) \delta(\mathbf{x} - \mathbf{q}(\tau)), \quad (14)$$

which, as explained in the introduction, we believe contains all the essential physics.

Given the symmetry of the problem it is convenient to expand the electron field in partial waves

$$\psi(\mathbf{r}) = \sum_{l,m} \psi_{l,m} = \sum_{l,m} \int_0^\infty \frac{dk}{2\pi} R_{k,l}(r) Y_{l,m}\left(\frac{\mathbf{r}}{r}\right) \psi_{k,l,m}, \quad (15)$$

where $Y_{l,m}$ are spherical harmonics and $R_{k,l} = 2kj_l(kr)$, with $j_l(x) = \sqrt{\pi/2x} J_{l+1/2}(x)$ being Bessel functions. From the condition;

$$k_F a \ll 1, \quad (16)$$

where a is the distance between two minima of the potential, it follows that only the first two harmonics, $l=0$, and $l=1$, $m=0$, strongly interact with the heavy particle. Then we can approximate the electron field as

$$\begin{aligned} \psi(\mathbf{r}) \simeq & Y_{0,0}\left(\frac{\mathbf{r}}{r}\right) \int_0^\infty \frac{dp}{2\pi} \psi_+(p) R_{p,0}(r) \\ & + Y_{1,0}\left(\frac{\mathbf{r}}{r}\right) \int_0^\infty \frac{dp}{2\pi} \psi_-(p) R_{p,1}(r), \end{aligned} \quad (17)$$

where we have introduced $\psi_+(k) = \psi_{k,0,0}$ and $\psi_-(k) = \psi_{k,1,0}$. The low energy properties are determined by electron states close to the Fermi surface, $p \ll k_F$, so that we can introduce fields smooth on the scale $1/k_F$,

$$\psi_\pm(x) = \int_{p \ll k_F} \frac{dp}{2\pi} e^{ipx} \psi_\pm(p). \quad (18)$$

These fields determine the asymptotic behavior of the three dimensional field at $rk_F \gg 1$, thus specifying the scattering matrix for the states close to the Fermi energy shell. In Eq. (17) we suppressed all the higher angular harmonics not scattered by the heavy particle. Then, using the asymptotic behavior of the Bessel functions for $rk_F \gg 1$ and Eq. (18), we can rewrite Eq. (17) as

$$\begin{aligned} \psi(\mathbf{r}) \simeq & \frac{1}{r} Y_{0,0}\left(\frac{\mathbf{r}}{r}\right) [\psi_+(r) e^{ik_F r} - \psi_+(-r) e^{-ik_F r}] \\ & + \frac{1}{r} Y_{1,0}\left(\frac{\mathbf{r}}{r}\right) [\psi_-(r) e^{ik_F r} + \psi_-(-r) e^{-ik_F r}] \\ = & \frac{1}{i\sqrt{4\pi}r} [\psi_+(r) e^{ik_F r} - \psi_+(-r) e^{-ik_F r}] \\ & + \frac{\sqrt{3}z}{\sqrt{4\pi}r^2} [\psi_-(r) e^{ik_F r} + \psi_-(-r) e^{-ik_F r}], \end{aligned} \quad (19)$$

where we have chosen the direction of the motion of the heavy particle restricted along the z axis. This leaves us with an effective one-dimensional problem with two species of electrons. These are left movers on the entire axis as in Eq. (19) or, equivalently, one can introduce left and right movers on the half line. We have chosen the former approach for technical convenience. With the same accuracy we can linearize the spectrum in the vicinity of the Fermi surface, $\epsilon_k = v_F(k - k_F)$ and $v_F|k - k_F| \ll D$, where D is the energy scale smaller than the Fermi energy.

Before we proceed, let us note that there have been some suggestions¹⁹ that the electron–hole asymmetry of the original electronic band can lead to an enhancement of the Kondo temperature. These effects will not be taken into account in this section, and the corresponding discussion is relegated to the Appendix. As we will explicitly show there, they can be introduced perturbatively and do *not* change qualitatively our results.

For the linearized spectrum, the bare part of the fermionic Hamiltonian takes the standard form,

$$H_0 = iv_F \int dx [\bar{\psi}_+ \partial_x \psi_+ + \bar{\psi}_- \partial_x \psi_-]. \quad (20)$$

Substitution of Eq. (17) into Eq. (14) yields

$$\begin{aligned} \delta H = & \frac{\lambda}{4\pi} \int \frac{dp_1}{2\pi} \frac{dp_2}{2\pi} [\bar{\psi}_+(p_1) R_{k_F+p_1,0}[q(\tau)] \\ & + \sqrt{3} \bar{\psi}_-(p_1) R_{k_F+p_1,1}[q(\tau)]] \\ & \times [\psi_+(p_2) R_{k_F+p_2,0}[q(\tau)] \\ & + \sqrt{3} \psi_-(p_2) R_{k_F+p_2,1}[q(\tau)]]. \end{aligned} \quad (21)$$

Now we neglect $p_{1,2}$ in comparison with the Fermi momentum and use the condition (16) to expand Eq. (21) in $k_F q(\tau)$ up to the second order. The smallness of this parameter is guaranteed by Eq. (16) together with the fact that the dynamics of the heavy particle is dominated by $q(\tau) \approx \pm a$. We obtain

$$\delta H = \delta H_0 + \delta H_q,$$

$$\delta H_0 = \frac{\lambda k_F^2}{\pi} \left[\left(1 - \frac{k_F^2 a^2}{3} \right) \bar{\psi}_+ \psi_+ + \frac{k_F^2 a^2}{3} \bar{\psi}_- \psi_- \right], \quad (22a)$$

$$\begin{aligned} \delta H_q = & \frac{\lambda k_F^2}{\pi} \left[\frac{k_F q(\tau)}{\sqrt{3}} (\bar{\psi}_+ \psi_- + \bar{\psi}_- \psi_+) \right. \\ & \left. + \frac{k_F^2 [a^2 - q^2(\tau)]}{3} (\bar{\psi}_+ \psi_+ - \bar{\psi}_- \psi_-) \right], \end{aligned} \quad (22b)$$

from which it is clear that mixing of higher harmonics would have been of order $\mathcal{O}((k_F a)^3)$. Here and in the following we use the short hand notation,

$$\psi \equiv \frac{\psi(+0) + \psi(-0)}{2},$$

whenever there is no spatial integration. The Hamiltonian (22a) describes the electron scattering on a static potential created by the heavy particle smeared between potential minima. This term does not have any dynamics and may be eliminated by a unitary transformation. On the other hand, the term (22b) describes the excitations of the electron system by the moving heavy particle, and should be treated carefully.

To get rid of the Hamiltonian (22a), we perform the transformation

$$\begin{aligned} \psi_{\pm}(x) & \rightarrow \psi_{\pm}(x) e^{i \operatorname{sgn}(x) \delta_{\pm}}, \quad \bar{\psi}_{\pm}(x) \rightarrow \bar{\psi}_{\pm}(x) e^{-i \operatorname{sgn}(x) \delta_{\pm}}, \\ \tan \delta_+ & = \frac{\lambda k_F^2}{\pi v_F} \left(1 - \frac{k_F^2 a^2}{3} \right), \quad \tan \delta_- = \frac{\lambda k_F^4 a^2}{3 \pi v_F}, \end{aligned} \quad (23)$$

in Eqs. (20) and (22). After this transformation the term $H_0 + \delta H_0$ acquires the form (20), and Eq. (22b) becomes

$$\begin{aligned} \frac{\delta H_q}{v_F} = & 2\pi \Lambda_z \frac{q(\tau)}{4a} [\bar{\psi}_+ \psi_- + \bar{\psi}_- \psi_+] \\ & + 2\pi \Lambda_x \frac{a^2 - q^2(\tau)}{2a^2} [\bar{\psi}_+ \psi_+ - \bar{\psi}_- \psi_-] \\ & + 2\pi \Lambda_{\rho} \frac{a^2 - q^2(\tau)}{2a^2} [\bar{\psi}_+ \psi_+ + \bar{\psi}_- \psi_-], \end{aligned} \quad (24)$$

where the dimensionless constants $\Lambda_{x,z,\rho}$ are

$$\begin{aligned} \Lambda_z & = \frac{2k_F a}{\pi \sqrt{3}} \sin(\delta_+ + \delta_-), \\ \Lambda_x & = \frac{1}{2\pi} \left(\frac{k_F a}{\sqrt{3}} \right)^2 \sin(\delta_+ + \delta_-) \frac{\cos^2 \delta_+ + \cos^2 \delta_-}{\cos \delta_+ \cos \delta_-}, \\ \Lambda_{\rho} & = \frac{1}{2\pi} \left(\frac{k_F a}{\sqrt{3}} \right)^2 \sin(\delta_+ + \delta_-) \frac{\cos^2 \delta_+ - \cos^2 \delta_-}{\cos \delta_+ \cos \delta_-}. \end{aligned} \quad (25)$$

It follows from the condition (16) that

$$\Lambda_x, \quad \Lambda_{\rho} \ll \Lambda_z \ll 1, \quad (26)$$

independently of the value of the coupling constant λ . Similar conclusions about the values of the coupling constants were reached in Ref. 8.

Introducing the pseudo-spin notation,

$$\boldsymbol{\psi} = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}, \quad \bar{\boldsymbol{\psi}} = (\bar{\psi}_+, \bar{\psi}_-), \quad (27)$$

and choosing the Pauli matrices in pseudo-spin space in the following representation:

$$\tau_z = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \quad \tau_x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (28)$$

the expressions (20) and (24) can be rewritten in the more compact form,

$$\begin{aligned} \frac{H_0 + \delta H_q}{v_F} = & i \int dx \bar{\boldsymbol{\psi}} \partial_x \boldsymbol{\psi} + 2\pi \Lambda_z Z(\tau) \bar{\boldsymbol{\psi}} \tau_z \boldsymbol{\psi} \\ & + 2\pi \Lambda_x X(\tau) \bar{\boldsymbol{\psi}} \tau_x \boldsymbol{\psi} + \pi \Lambda_{\rho} X(\tau) \bar{\boldsymbol{\psi}} \boldsymbol{\psi}, \end{aligned} \quad (29)$$

where we introduced the short hand notation,

$$Z(\tau) = \frac{q(\tau)}{2a}; \quad X(\tau) = \frac{a^2 - q^2(\tau)}{a^2}. \quad (30)$$

One can see from the Hamiltonian (29) that v_F can be removed by the proper rescaling of the time coordinates. In what follows, we will set $v_F = 1$.

To calculate the partition function it is convenient to introduce the U(1) (charge) and SU(2) (pseudo-spin) currents

$$J = \bar{\psi}\psi, \quad \mathbf{J} = \frac{\bar{\psi}\boldsymbol{\tau}\psi}{2} \quad (31)$$

that satisfy the commutation relations

$$[J(x), J(0)] = i \frac{\delta'(x)}{\pi}; \quad (32a)$$

$$[J_j(x), J_k(0)] = i \frac{\delta_{jk} \delta'(x)}{4\pi} + i \epsilon^{jkl} \delta(x) J_l(x);$$

$$[J(x), J_k(y)] = 0. \quad (32b)$$

The pseudo-spin current commutation relations (32b) define the SU(2) Kac–Moody algebra at level 1 (SU(2)₁). In terms of the currents (31) the Hamiltonian (29) takes the Sugawara form,

$$H_0 = \int dx \left\{ \frac{\pi}{2} J^2 + \frac{2\pi}{3} \mathbf{J}^2 \right\}, \quad (33a)$$

$$\delta H = 2\pi \Lambda_z Z(\tau) J_z + 2\pi \Lambda_x X(\tau) J_x + \pi \Lambda_\rho X(\tau) J. \quad (33b)$$

Pseudo-spin and charge degrees of freedom are separated and in the bare part, H_0 , one recognizes the free charge boson model and the SU(2)₁ Wess–Zumino–Novikov–Witten (WZNW) model. For a future use, we generalize Eq. (33b) by introducing one more coupling Λ_y ,

$$\frac{\delta H}{2\pi} = \Lambda_z Z(\tau) J_z + \Lambda_x X(\tau) J_x + \Lambda_y Y(\tau) J_y + \frac{\Lambda_\rho X(\tau) J}{2},$$

$$Y(\tau) = -i \frac{\tau_{\text{tun}}}{2a} \frac{dq}{d\tau}, \quad (33c)$$

where i is introduced in front of the first time derivative because we work with the imaginary time. The parameter τ_{tun} has the dimensionality of time and is introduced to make Λ_y dimensionless. It is defined rigorously in the next section, however, its exact meaning is not important here. In the original model at high energies $\Lambda_y = 0$, however, in our analysis, this coupling will be generated in the higher order perturbation theory, as shown in the next section.

As first step in the process of mapping our problem to the LGM we need to integrate out the fermionic degrees of freedom. In the framework of the formalism that we have chosen here, this requires to construct the action corresponding to the Hamiltonian (33). A general procedure to do it is provided by non-Abelian bosonization and leads to the WZNW

action (see, for instance, Ref. 25). For simplicity we chose a different approach that uses Abelian bosonization. Despite the fact that the SU(2) is a non-Abelian group, the use of Abelian bosonization in this case is made possible by the fact that the SU(2)₁ WZNW model has central charge equal to one. Introducing two bosonic fields for the charge and pseudo-spin degrees of freedom in the standard way,

$$J = \frac{1}{\sqrt{\pi}} \partial_x \phi_c; \quad (34a)$$

$$J_z = \frac{1}{\sqrt{4\pi}} \partial_x \phi_s;$$

$$J_+ \equiv J_x + iJ_y = \frac{D}{2\pi} e^{-i\sqrt{8\pi}\phi_s};$$

$$J_- \equiv J_x - iJ_y = \frac{D}{2\pi} e^{i\sqrt{8\pi}\phi_s}; \quad (34b)$$

we can write the electronic Lagrangian density in the bosonic form,

$$\mathcal{L}_b = \mathcal{L}_b^0 + \delta H, \quad (35)$$

where

$$\mathcal{L}_b^0 = \frac{1}{2} \sum_{\rho=c,s} [i\partial_\tau \phi_\rho \partial_x \phi_\rho + (\partial_x \phi_\rho)^2], \quad (36)$$

and δH can be easily obtained substituting Eqs. (34) into (33c). In the bosonization formulas (34b), D is an arbitrary cutoff required to make the correlation functions in the bosonic theory finite. It can be fixed by requiring that the correlation function of vertex operators in this theory is given by

$$\langle \exp(i\sqrt{4\pi}\phi_s(x)) \exp(i\sqrt{4\pi}\phi_s(0)) \rangle = \frac{1}{Dx}. \quad (37)$$

After bosonizing we can integrate out the electronic degrees of freedom in (6) and rewrite it as

$$\mathcal{Z} = \int D[q(\tau)] e^{-\int d\tau \mathcal{L}_{\text{at}} \mathcal{Z}_\rho[q(\tau)] \mathcal{Z}_{xzy}[q(\tau)]}, \quad (38)$$

where \mathcal{L}_{at} is defined in Eq. (9) and the product $\mathcal{Z}_\rho[q(\tau)] \mathcal{Z}_{xzy}[q(\tau)]$ is nothing but the electronic determinant for the given path of heavy particle $q(\tau)$. It has the following form:

$$\mathcal{Z}_\rho = \left\langle \exp \left(-\pi \int d\tau \Lambda_\rho X(\tau) J \right) \right\rangle = \exp \left(\frac{\Lambda_\rho^2}{2} \int d\tau_1 d\tau_2 \frac{X(\tau_1) X(\tau_2)}{(\tau_1 - \tau_2)^2} \right); \quad (39a)$$

$$\begin{aligned}
\mathcal{Z}_{xzy} &= \sum_m \left(\frac{1}{m!} \right)^2 (2\pi)^{2m} \int \left[\prod_{j=1}^m d\tau_j^+ d\tau_j^- R_+(\tau_j^+) R_-(\tau_j^-) \right] \left\langle \prod_{j=1}^m J_+(\tau_j^+) J_-(\tau_j^-) \exp \left(-4\pi \int d\tau \Lambda_z Z(\tau) J_z \right) \right\rangle \\
&= \sum_m \left(\frac{1}{m!} \right)^2 (2\pi)^{2m} \int \left[\prod_{j=1}^m d\tau_j^+ d\tau_j^- R_+(\tau_j^+) R_-(\tau_j^-) \right] \exp \left[\Lambda_z \int d\tau Z(\tau) \sum_{j=1}^m \left(\frac{1}{\tau - \tau_j^+} - \frac{1}{\tau - \tau_j^-} \right) \right] \\
&\quad \times \mathcal{Z}_z \left\langle \prod_{j=1}^m J_+(\tau_j^+) J_-(\tau_j^-) \right\rangle \\
&= \sum_m \left(\frac{1}{m!} \right)^2 \int \left[\prod_{j=1}^m d\tau_j^+ d\tau_j^- R_+(\tau_j^+) R_-(\tau_j^-) \right] \exp \left[\Lambda_z \int d\tau Z(\tau) \sum_{j=1}^m \left(\frac{1}{\tau - \tau_j^+} - \frac{1}{\tau - \tau_j^-} \right) \right] \\
&\quad \times \mathcal{Z}_z \prod_{i,j}^m \frac{1}{(\tau_i^+ - \tau_j^-)^2} \prod_{i>j}^m (\tau_i^+ - \tau_j^+)^2 (\tau_i^- - \tau_j^-)^2, \tag{39b}
\end{aligned}$$

where

$$R_{\pm}(\tau) \equiv \frac{\Lambda_x}{2} X(\tau) \pm \frac{\Lambda_y}{2} Y(\tau),$$

functions $Z(\tau)$, $X(\tau)$, and $Y(\tau)$ are defined in Eqs. (30), (33c), and

$$\begin{aligned}
\mathcal{Z}_z &= \left\langle \exp \left(-2\pi \int d\tau \Lambda_z Z(\tau) J_z \right) \right\rangle \\
&= \exp \left(\frac{\Lambda_z^2}{2} \int d\tau_1 d\tau_2 \frac{Z(\tau_1) Z(\tau_2)}{(\tau_1 - \tau_2)^2} \right). \tag{39c}
\end{aligned}$$

In Eqs. (39) averaging over the bosonic fields is defined as

$$\langle \dots \rangle = \int D[\phi(x, \tau)] e^{\int d\tau \mathcal{L}_b^0} \dots$$

In the next section we will use a semiclassical approximation to map (38) into the partition function of a one-dimensional gas of logarithmically interacting particles. This will enable us to establish a nonperturbative relationship between our original problem and the Kondo model.

III. MAPPING TO A LOGARITHMIC GAS MODEL

The main goal of this section is to map the model of the previous section to the logarithmic gas model. A similar approach for the TLS was done in Refs. 24,17. As usual, for such kind of mapping, the first step is to identify the logarithmically interacting objects. We will show in Sec. III A that, for the present problem, they are instanton-antiinstanton configurations of the tunneling problem.²⁶ In order to find the upper cutoff for the logarithmic gas problem \mathcal{D} which plays the role of the prefactor in the Kondo temperature, we calculate in Sec. III B the interaction between the instantons. As we will see, it turns out that there is an intrinsic short distance cutoff, that makes the theory regular, and is given by

the size of the instanton. It is analogous to the lattice constant for 2D melting problem, or to the vortex core size for superfluid films. Then a rough estimate for \mathcal{D} is given by this quantity. To define the cutoff with the higher accuracy we find the coupling constants corresponding to the chosen value of \mathcal{D} by microscopic calculation of the energy of the logarithmic gas model. The relation between the scale \mathcal{D} and the coupling constants will be shown to have the form of

$$\begin{aligned}
h(\mathcal{D}) &= h[1 + \Lambda_z^2 (\ln \mathcal{D} \tau_{\text{tun}} + \mathcal{O}(1))]; \\
\Lambda_y(\mathcal{D}) &= -\Lambda_x \Lambda_z (\ln \mathcal{D} \tau_{\text{tun}} + \mathcal{O}(1)). \tag{40}
\end{aligned}$$

On the other hand, Eq. (40), can be interpreted as the result of the first iteration of the renormalization group equations for the Kondo problem with $\Lambda_y \ll \Lambda_x, \Lambda_z$,

$$\frac{dh(\mathcal{D})}{d \ln \mathcal{D}} = h \Lambda_z^2; \quad \frac{d\Lambda_y(\mathcal{D})}{d \ln \mathcal{D}} = -\Lambda_x \Lambda_z. \tag{41}$$

It means that the cutoff \mathcal{D} could be chosen arbitrary, provided that the coupling constants are adjusted accordingly, as it is well-known for any logarithmic problem. Accordingly, we will choose the cutoff \mathcal{D} to have the Hamiltonian of the most simple form,

$$\Lambda_y(\mathcal{D}) = 0. \tag{42}$$

This equation exclude any ambiguity in the definition of \mathcal{D} and will allow us to determine the cutoff even with numerical coefficient for a wide class of the potentials $V(q)$.

A. Dilute instanton gas approximation

The partition function (38) can be calculated using a semiclassical approximation, which amounts to considering only small fluctuations around the stationary points of the action, which correspond to the Euler-Lagrange (EL) equations for the problem. A rough condition for the applicability of the semiclassical approximation is $S_{\text{inst}} \gg 1$, where S_{inst} is the classical action corresponding to the trajectory connecting two extrema of the potential $V(q)$.

To first approximation we can calculate the stationary points using the EL equations for the bare problem,

$$M \frac{d^2 q}{d\tau^2} = V'[q], \quad (43)$$

which corresponds to the equation of motion for a classical particle in the potential minus $V[q]$. This approximation can be improved by taking into account the modification of the optimal trajectory due to the electrons, however, it introduces only parametrically small changes of the coupling constants (because the shape of the classical solution is a rigid mode) and does not change any of our conclusions. Beside the trivial solution $q(\tau) = \pm a$, Eq. (43) also admits solutions of the form,

$$q(\tau) = \pm af(\tau - \tau_i) \quad (44)$$

(kink and antikink, respectively) with $f(\pm\infty) = \pm 1$. These solutions are called instantons because they produce an almost instantaneous blimp in the Lagrangian. The action is invariant under translations of the instanton center τ_i , which reflects the time translation invariance of the original Lagrangian (9). The instanton is characterized by its bare action

$$S_{\text{inst}} = \int d\tau \mathcal{L}_{\text{at}}[af(\tau)], \quad (45)$$

and by the tunneling time,

$$\tau_{\text{tun}} = \int d\tau [1 - f^2(\tau)]. \quad (46)$$

By construction, the integrand is nonzero only within the core of the instanton, so τ_{tun} has the meaning of the size of the core.

In what follows, we will write the explicit results for the model potential,

$$V(q) = g(a^2 - q^2)^2, \quad (47)$$

even though our considerations by no means are restricted for such potential. For the potential (47), one easily finds

$$S_{\text{inst}} = \frac{M^2 \omega^3}{12g}; \quad f(\tau) = \tanh \frac{\omega\tau}{2}; \quad \tau_{\text{tun}} = \frac{4}{\omega}, \quad (48)$$

where $\omega^2 = 8ga^2/M$ is the frequency of the harmonic oscillation in the extrema of the potential.

The partition function must sum over multi-instanton solutions satisfying initial and final conditions. Since the instanton core is small, it is a good approximation to consider multi-instanton solutions (see Fig. 2) as if instanton and anti-instantons were dilute (dilute instanton gas approximation), which requires $|\tau_i - \tau_j| \gg \tau_{\text{tun}}$ [this assumption is completely justified for $S_{\text{inst}} \gg 1$ (Ref. 26)]. Then, in the absence of the interaction of heavy particle with electrons, a general trajectory starting and ending at point $-a$ can be written as

$$q_n(\tau) = -a \left[1 + \sum_{j=1}^{2n} (-1)^j f(\tau - \xi_j) \right], \quad (49)$$

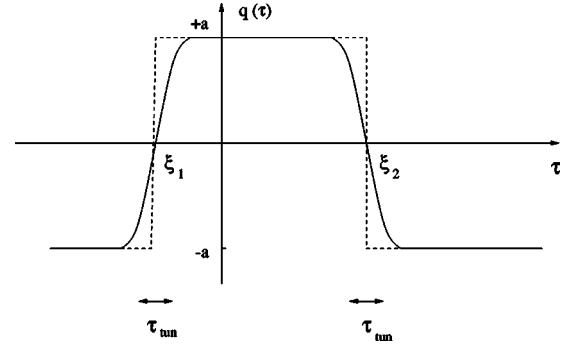


FIG. 2. Schematic form of the instanton solution (44) (solid line). The dotted line represents instantaneous spin flip in the Kondo problem.

where the positions of the kinks ξ_{2j+1} antikinks ξ_{2j} are subject to constraints,

$$\xi_j < \xi_{j+1}. \quad (50)$$

The action corresponding to a configuration with n instanton–anti-instanton pairs with the exponential accuracy is

$$\int d\tau \mathcal{L}_{\text{at}}[q_n(\tau)] = 2n S_{\text{inst}}, \quad (51)$$

i.e., kinks would not interact with each other if the heavy particle were isolated from the electron system.

In the dilute instanton gas approximation, the low-temperature partition function of an isolated heavy particle can be rewritten also in a form,

$$\mathcal{Z}_0 = \sum_{n=0}^{\infty} h^{2n} \prod_{j=1}^{2n} \int_0^{1/T} d\xi_j = \cosh \frac{h}{T}. \quad (52)$$

Hereinafter, integrals over ξ_j are calculated with the constraint (50). The tunneling splitting between two lowest levels of the heavy particle is estimated as

$$h = \kappa \frac{1}{\tau_{\text{tun}}} \sqrt{S_{\text{inst}}} e^{-S_{\text{inst}}} \ll \frac{1}{\tau_{\text{tun}}}. \quad (53)$$

The numerical factor κ comes from the instanton determinant due to the integration around the saddle point.²⁶ For potential (47), one finds $\kappa = \sqrt{24/\pi} \approx 2.76$.

Closing the subsection, we discuss the limits of the applicability of the dilute instanton approximation. If the potential $V(q)$ does not have singular points the main condition of the applicability is $S_{\text{inst}} \gg 1$. We emphasize, that this condition has nothing to do with the number of levels localized in each well. In particular, numerical solution of the Schrödinger equation with the potential (47), shows that the third level in the DWP lies larger than the maximum of the potential already at $S_{\text{inst}} \leq 6.06$, (at this point $h\tau_{\text{tun}} = 10^{-2}$), whereas the dilute instanton approximation breaks down at $S_{\text{inst}} \approx 2.8$ (when the second level crosses the maximum of the potential).

B. Interaction between instantons

The purpose of this subsection is to establish the form of interaction between instantons and anti-instantons due to the dynamics of the electron system. To do so we substitute Eq. (49) into Eq. (38). The functional integration is reduced to the integration around small oscillations about the optimal trajectory (49). The calculation of such instanton determinant is standard²⁶ and may be performed without taking into account the electrons, provided that the condition (26) holds. As the result we find

$$\mathcal{Z} = \sum_{n=0}^{\infty} h^{2n} \prod_{j=1}^{2n} \int d\xi_j \mathcal{Z}_\rho[\{\xi\}_{j=1}^{2n}] \mathcal{Z}_{xy}[\{\xi\}_{j=1}^{2n}], \quad (54)$$

where $\mathcal{Z}_{\rho,xy}[\{\xi\}_{j=1}^{2n}]$ are the integrals (39) calculated over $q(\tau)$ given by Eq. (49). These functions depend on the positions of the kinks and therefore produce interactions between them. In what follows, we analyze this interaction in details.

Substituting Eq. (49) into Eq. (30) and neglecting the exponentially small overlap of the instanton cores one can rewrite $Z(\tau)$ and $X(\tau)$ as

$$Z(\tau) = \frac{1}{2} \left[-1 + \sum_{j=1}^{2n} (-1)^j f(\tau - \xi_j) \right], \quad (55)$$

$$X(\tau) \approx \sum_{j=1}^{2n} \eta_1(\tau - \xi_j), \quad \eta_1 \equiv 1 - f^2.$$

It is clear that the function $X(\tau)$ has peaks at the core of instantons and vanishes exponentially otherwise.

Our strategy now is to substitute Eq. (55) into Eqs. (39) and perform simplifications using the fact that the instanton gas is dilute. This can be easily done for the terms (39a) and (39c) while Eq. (39b) requires more work. For Eq. (39a) we find²⁷

$$\int d\tau_1 d\tau_2 \frac{X(\tau_1)X(\tau_2)}{(\tau_1 - \tau_2)^2} = 2n \delta S_0 + \sum_{i \neq j}^{2n} U_0(\xi_i - \xi_j);$$

$$\delta S_0 \equiv U_0(0); \quad U_0(\tau) = \text{Re} \int \frac{d\tau_1 d\tau_2 \eta_1(\tau_1) \eta_1(\tau_2)}{(\tau_1 - \tau_2 - \tau + i0)^2}. \quad (56)$$

One can see from Eqs. (56) and (55) that $U_0(\tau)$ decays rapidly: $U_0(\tau) \approx (\tau_{\text{tun}}/\tau)^2$ at $\tau > \tau_{\text{tun}}$. This short range interaction between the instantons is not important for the dilute instanton gas and we can neglect it. Thus, for a configuration of n kink–antikink pairs, Eq. (39a) takes the form,

$$\mathcal{Z}_\rho = \exp(2n \Lambda_\rho^2 \delta S_0), \quad (57)$$

which is just an independent renormalization of the action for each kink. Such renormalization is nothing but the polaronic effect for the tunneling.

Next, we substitute Eq. (55) into Eq. (39c) and, omitting an irrelevant constant term, we obtain

$$\int d\tau_1 d\tau_2 \frac{Z(\tau_1)Z(\tau_2)}{(\tau_1 - \tau_2)^2} = 2n \delta S_1 + \sum_{i \neq j}^{2n} (-1)^{i+j} U_1(\xi_i - \xi_j);$$

$$U_1(\tau) = \int d\tau_1 d\tau_2 \eta_2(\tau_1) \eta_2(\tau_2) \ln \mathcal{D} |\tau_1 - \tau_2 - \tau|;$$

$$\eta_2(\tau) \equiv \frac{df}{2d\tau}, \quad \delta S_1 \equiv U_1(0), \quad (58)$$

where energy scale \mathcal{D} is introduced here to make the argument of the logarithmic interaction dimensionless: it does not enter into the expression for the total action. In the following we will see that the natural choice is $\mathcal{D} = 1/\tau_{\text{tun}}$, where the tunneling time is defined in Eq. (46), however, we will keep \mathcal{D} as an independent energy scale for pedagogical reasons.

One can see, e.g., from Fig. 2, that function $\eta_2(\tau)$ decays rapidly at $\tau > \tau_{\text{tun}}$. Therefore, the interaction between kinks, $U_1(\tau)$, is logarithmic at $\tau \gg \tau_{\text{tun}}$ and it saturates at $\tau \approx \tau_{\text{tun}}$. This is nothing but the manifestation of the usual orthogonality catastrophe,²⁸ where the high-energy cutoff is determined by the dynamics of the heavy particle.

Substituting (58) into Eq. (39c) we find that this part of the partition function is equivalent to that of the classically interacting gas with n positive and n negative particles,

$$\mathcal{Z}_z = \exp(n \Lambda_z^2 \delta S_1) \exp(-\mathcal{H}_1), \quad (59)$$

$$\mathcal{H}_1 = -\frac{\Lambda_z^2}{2} \sum_{i \neq j}^{2n} (-1)^{i+j} U_1(\xi_i - \xi_j).$$

To begin the manipulations with the contribution (39b), we first study the simplest, $m = 1$, term to illustrate the principle, and then switch to the higher order terms. We rewrite the prefactor in Eq. (39b) as

$$\frac{R_+(\tau_+) R_-(\tau_-)}{(\tau_+ - \tau_-)^2}$$

$$= \sum_{j=1}^{2n} \frac{\eta_+^j(\tau_+ - \xi_j) \eta_-^j(\tau_- - \xi_j)}{(\tau_+ - \tau_-)^2}$$

$$+ \sum_{i \neq j}^{2n} \frac{\eta_+^i(\tau_+ - \xi_i) \eta_-^j(\tau_- - \xi_j)}{(\tau_+ - \tau_-)^2}$$

$$\approx \sum_{j=1}^{2n} \frac{\eta_+^j(\tau_+ - \xi_j) \eta_-^j(\tau_- - \xi_j)}{(\tau_+ - \tau_-)^2}$$

$$+ \sum_{i \neq j}^{2n} \frac{\eta_+^i(\tau_+ - \xi_i) \eta_-^j(\tau_- - \xi_j)}{(\xi_i - \xi_j)^2},$$

$$\eta_\pm^j(\tau) = \frac{\Lambda_x}{2} \eta_1(\tau) \mp (-1)^j \frac{\Lambda_y}{2} \tau_{\text{tun}} \eta_2(\tau), \quad (60)$$

where in the last line we used once again the fact that the instanton gas is dilute, and functions $\eta_\pm(\tau)$ decay exponentially outside the core of the instantons. The first term in the last line of Eq. (60) keeps times τ_\pm close to each other, and,

as we will see, will produce the renormalization of the action for a single kink. The second term will give rise to the interaction between kinks.

For the exponent in Eq. (39b) one obtains

$$\begin{aligned} & \int d\tau Z(\tau) \left(\frac{1}{\tau - \tau^+} - \frac{1}{\tau - \tau^-} \right) \\ &= \sum_{j=1}^{2n} (-1)^j [U_2(\xi_j - \tau_+) - U_2(\xi_j - \tau_-)], \\ U_2(\tau) &= \int d\tau_1 \eta_2(\tau_1) \ln \mathcal{D} |\tau_1 - \tau|, \end{aligned} \quad (61)$$

where we introduced the arbitrary cutoff D for the same purpose as in Eq. (58). Similarly to $U_1(\tau)$, the potential $U_2(\tau)$ is logarithmic at $\tau \gg \tau_{\text{tun}}$ and it saturates at $\tau \approx \tau_{\text{tun}}$.

From Eqs. (60) and (61), we can easily rewrite, within the dilute instanton approximation, the $m=1$ contribution to \mathcal{Z}_{xzy} as²⁷

$$\begin{aligned} \frac{\mathcal{Z}_{xzy}^{(m=1)}}{\mathcal{Z}_z} &= 2n \left[I_x \frac{\Lambda_x^2}{4} + I_y \frac{\Lambda_y^2}{4} + \mathcal{O}(\Lambda_x \Lambda_y \Lambda_z) \right] + K_1, \\ I_x &= \text{Re} \int d\tau_1 d\tau_2 \frac{\eta_1(\tau_1) \eta_1(\tau_2)}{(\tau_1 - \tau_2 + i0)^2}, \\ I_y &= \tau_{\text{tun}}^2 \text{Re} \int d\tau_1 d\tau_2 \frac{\eta_2(\tau_1) \eta_2(\tau_2)}{(\tau_1 - \tau_2 + i0)^2}, \\ 4K_1 &= \tau_{\text{tun}}^2 \sum_{i \neq j}^{2n} \frac{[\tilde{\Lambda}_x - (-1)^i \tilde{\Lambda}_y][\tilde{\Lambda}_x + (-1)^j \tilde{\Lambda}_y]}{(\xi_{ij})^2} \\ & \times \exp \left\{ \Lambda_z \sum_{k=1}^{2n'} (-1)^k [U_2(\xi_{ik}) - U_2(\xi_{jk})] \right\}, \end{aligned} \quad (62)$$

where we used the short hand notation

$$\xi_{ij} \equiv \xi_i - \xi_j \quad (63)$$

for the distance between the instantons, and Σ' means that terms involving ξ_{ii} are excluded. Deriving K_1 from Eq. (62), we used the properties $\int d\tau \eta_1(\tau) = \tau_{\text{tun}}$, and $\int d\tau \eta_2(\tau) = 1$. The dimensionless coupling constants $\tilde{\Lambda}_{x,y}$ entering into Eq. (62) are defined as

$$\begin{aligned} \tilde{\Lambda}_x &= \Lambda_x - \Lambda_y \Lambda_z \int d\tau \eta_2(\tau) U_2(\tau) + \mathcal{O}(\Lambda_z^2 \Lambda_x), \\ \tilde{\Lambda}_y &= \Lambda_y - \Lambda_x \Lambda_z \int \frac{d\tau}{\tau_{\text{tun}}} \eta_1(\tau) U_2(\tau) + \mathcal{O}(\Lambda_z^2 \Lambda_y). \end{aligned} \quad (64)$$

The factor K_1 can be also rewritten in terms of the partition function of the classical logarithmic gas. In order to do so, we relate to any j th instanton the additional charge

$$\mu_j = -1, 0, 1. \quad (65)$$

As we will see immediately, the physical meaning of the “neutral” kink, $\mu_j=0$ is the tunneling of the heavy particle without excitations of the electron system, where as “charged” kinks $\mu_j = \pm 1$ describe the electron assisted tunneling. Using the notation introduced in (65) and the fact that asymptotic behavior of the potential $U_2(\tau)$ is logarithmic, we rewrite Eq. (62) as $K_1 = K_{m=1}$, where the more general quantity K_m is given by

$$K_m(\{\xi_j\}_{j=1}^{2n}) = \sum_{\{\mu_j\}_{j=1}^{2n}} \Gamma(\{\mu_j\}) \exp(-\mathcal{H}_2),$$

$$\mathcal{H}_2 = - \sum_{i \neq j}^{2n} U_2(\xi_{ij}) [\mu_j \mu_i + (-1)^i \mu_j \Lambda_z],$$

$$\Gamma(\{\mu_j\}) = \prod_{j=1}^{2n} \left(1 - \mu_j (-1)^j \frac{\tilde{\Lambda}_y}{\tilde{\Lambda}_x} \right) \left(\frac{\tilde{\Lambda}_x \mathcal{D} \tau_{\text{tun}}}{2} \right)^{|\mu_j|}. \quad (66)$$

The summation over all the configurations of $\{\mu_j\}$ is subject to the condition of charge neutrality

$$\sum_{j=1}^{2n} \mu_j = 0 \quad (67)$$

and to the m th order perturbation theory constraint

$$\sum_{j=1}^{2n} |\mu_j| = 2m. \quad (68)$$

Equations (62) and (68) allow for very natural generalization to the higher order terms. For a configuration containing n kinks and n antikinks we wish to keep only terms of the order of $(n\Lambda_x^2)^m$ and neglect the terms which scale like $(n\Lambda_x^4)^m$, etc. It amounts to the neglecting in a product $\prod_{j=1}^m X(\tau_j^+) X(\tau_j^-)$ configurations that contain more than two kinks coinciding. We employ this approximation to extend Eq. (62) for general m . To get rid of the combinatorial factor $1/(m!)^2$, we impose additional constraints on the integration $\tau_j^+ < \tau_{j+1}^+$, and $\tau_j^- < \tau_{j+1}^-$. The result then acquires the form²⁹

$$\frac{\mathcal{Z}_{xzy}^{(m)}}{\mathcal{Z}_z} = \frac{\Lambda_x^{2m}}{4^m} \sum_{k=0}^m \frac{1}{k!} (2nI)^k K_{m-k}(\{\xi_j\}_{j=1}^{2n}), \quad (69)$$

where the combinatorial factor takes care of the ordering of the paired kinks and the factor K_m is given by Eq. (66) with the constraints (67) and (68).

The total contribution for $2n$ instantons is obtained by summation of all the orders of perturbation theory,

$$\begin{aligned}
\frac{\mathcal{Z}_{xzy}}{\mathcal{Z}_z} &= \frac{1}{\mathcal{Z}_z} \sum_{m=0}^{\infty} \mathcal{Z}_{xzy}^{(m)} \\
&= \sum_{m=0}^{\infty} \sum_{k=0}^m \frac{(\Lambda_x^2 n I)^k (\Lambda_x^2/4)^{m-k} K_{m-k}}{k!} \\
&= e^{n\Lambda_x^2 I} \sum_{m=0}^{\infty} (\Lambda_x^2/4)^m K_m.
\end{aligned} \tag{70}$$

We now substitute Eqs. (70), (59), and (57) into Eq. (54). As result one finds

$$\mathcal{Z} = \sum_{n=0}^{\infty} \tilde{h}^{2n} \sum_{\{\mu_j\}} \Gamma(\{\mu_j\}) \int d\xi_j e^{-\mathcal{H}}, \tag{71}$$

where the integration is performed with the constraint (50), $\Gamma(\{\mu_j\})$ are defined in Eq. (66), and the summation over charge configurations $\{\mu_j\}$ is performed with the neutrality condition (68). Level splitting due to the spontaneous tunneling renormalized by the interaction with electrons (compare it with (53)) is

$$\begin{aligned}
\tilde{h} &= h \exp\left(\frac{s\Lambda_\rho^2 U_0(0)}{2} + \frac{s\Lambda_z^2 U_1(0)}{2} + \frac{s\Lambda_x^2 I_x}{2}\right) \\
&\approx h \left(1 + \frac{s\Lambda_\rho^2 U_0(0)}{2} + \frac{s\Lambda_z^2 U_1(0)}{2} + \frac{s\Lambda_x^2 I_x}{2}\right)
\end{aligned} \tag{72}$$

with entries defined in Eqs. (25), (56), (58), and (62), and $s=1$, and we put $\Lambda_y=0$.

The energy of the classical logarithmic gas $\mathcal{H}=\mathcal{H}_1+\mathcal{H}_2$ is found as

$$\mathcal{H} = - \sum_{i \neq j}^{2n} U_2(\xi_{ij}) \left[\mu_j \mu_i + (-1)^i \mu_j \Lambda_z + \frac{(-1)^{i+j} \Lambda_z^2}{2} \right], \tag{73}$$

where we neglected the difference between the potentials U_2 and U_1 from Eqs. (61) and (58) for ξ_{ij} much larger than the size of the core of the instantons.

So far we have considered only spinless electrons. The real spin is trivially included. We notice that there the electron spin commutes with the Hamiltonian, and therefore the fermionic determinant for spin 1/2 electrons is factorized onto product of two fermionic determinants for each spin. It results in the replacement

$$\mathcal{Z}_{\rho,xzy} \rightarrow [\mathcal{Z}_{\rho,xzy}]^2$$

in Eqs. (38) and (54). Such replacement is taken into account by introducing an additional ‘‘spin’’ for the kink, $\sigma^j = \uparrow, \downarrow$. By simple repeating of all of the consideration of this subsection, one finds instead of Eq. (71),

$$\mathcal{Z} = \sum_{n=0}^{\infty} \tilde{h}^{2n} \sum_{\{\mu_j\}, \{\sigma_j\}} \Gamma(\{\mu_j\}) \int d\xi_j e^{-\mathcal{H}}, \tag{74}$$

with the Hamiltonian

$$\begin{aligned}
\mathcal{H} &= - \sum_{i \neq j}^{2n} U_2(\xi_{ij}) \\
&\times \left\{ \delta_{\sigma_i \sigma_j} [\mu_j \mu_i + (-1)^i \mu_j \Lambda_z] + \frac{(-1)^{i+j} \Lambda_z^2}{2} \right\},
\end{aligned} \tag{75}$$

and $\Gamma(\{\mu_j\})$ are defined in Eq. (66). The expression for the action (72) should be used with $s=2$. This constitutes the result of the mapping of our original problem to a one-dimensional classical gas of charged particles interacting via the potential $U_2(r)$ given by Eq. (61).³⁰ The fugacity and the interaction potentials have been calculated starting from a completely microscopic theory.

Energies of the logarithmic gas explicitly depend on the high energy cutoff \mathcal{D} , see Eq. (61), and so do the coupling constants (64) and (72). It is easy to see that the dependence of the coupling constants of cut-off indeed has the form of Eq. (40) [more formal argument that Λ_y is indeed analogous to the corresponding coupling constant for the Kondo problem will be given in the next section after Eq. (84)]. Indeed, with the help of Eq. (61), we rewrite Eqs. (64) and (72) as

$$\tilde{\Lambda}_y = -\Lambda_x \Lambda_z [\ln \mathcal{D} \tau_{\text{tun}} + \alpha_y], \tag{76a}$$

$$\tilde{h} = h \{1 + \Lambda_z^2 [\ln \mathcal{D} \tau_{\text{tun}} + \alpha_h] + \Lambda_\rho^2 U_0(0) + \Lambda_x^2 I_x\}, \tag{76b}$$

$$\alpha_y = \int \frac{d\tau_1 d\tau_2}{\tau_{\text{tun}}} \eta_1(\tau_1) \eta_2(\tau_2) \ln \left| \frac{\tau_1 - \tau}{\tau_{\text{tun}}} \right|, \tag{76c}$$

$$\alpha_h = \int d\tau_2 d\tau_2 \eta_2(\tau_1) \eta_2(\tau_2) \ln \left| \frac{\tau_1 - \tau}{\tau_{\text{tun}}} \right|. \tag{76d}$$

Because $\int d\tau \eta_1(\tau) = \tau_{\text{tun}}$, and $\int d\tau \eta_2(\tau) = 1$, the parameters $\alpha_{h,y}$, I_x , and U_0 can take only numerical values of the order of unity not depending of the cutoff \mathcal{D} . We now define \mathcal{D} in accordance with the rule (42). We obtain

$$\mathcal{D} = \frac{1}{\tau_{\text{tun}}} e^{-\alpha_y}, \tag{77}$$

which together with Eqs. (76c), (44), (46), (55), (58) solves the problem of the relation of the high-energy cutoff with the form of the instanton solution of the heavy-particle dynamics in the double well potential. Parameter h , in its turn acquires the form,

$$\tilde{h} = h [1 + \Lambda_z^2 (\alpha_h - \alpha_y) + \Lambda_\rho^2 U_0(0) + \Lambda_x^2 I_x]. \tag{78}$$

For illustration purposes we calculate the numerical value of the constant for the potential (47). Using explicit form of the instanton solution (48), one immediately finds

$$\begin{aligned}
U_0(0) = I_x &= -\frac{12}{\pi^2} \zeta(3) \approx -1.461, \\
\alpha_y = \alpha_h &= \frac{1}{4} \ln \left(\frac{2e^{c+1}}{\pi} \right) \approx 0.281,
\end{aligned} \tag{79}$$

where $\zeta(3)$ is the Riemann Zeta function, and $C \approx 0.577$ is the Euler constant.

Closing the section, we write down the result for the logarithmic gas model with the cutoff result. For $\Lambda_y = 0$, Eq. (66) for factors $\Gamma(\{\mu_j\})$ are simplified and one obtains from Eqs. (74) and (75),

$$\mathcal{Z} = \sum_{n=0}^{\infty} \tilde{h}^{2n} \sum_{\{\mu_j\}, \{\sigma_j\}} \prod_{j=1}^{2n} \left(\frac{\Lambda_x e^{-\alpha_y}}{2} \right)^{|\mu_j|} \int d\xi_j e^{-\mathcal{H}}, \quad (80)$$

with the Hamiltonian,

$$\begin{aligned} \mathcal{H} = & - \sum_{i \neq j}^{2n} \ln(\mathcal{D}\xi_{ij}) \\ & \times \left\{ \delta_{\sigma_i \sigma_j} [\mu_j \mu_i + (-1)^i \mu_j \Lambda_z] + \frac{(-1)^{i+j} \Lambda_z^2}{2} \right\}. \end{aligned} \quad (81)$$

The results above will enable us in the next section to make connection between the microscopic parameters of the model of tunneling centers and the 2CK model.

IV. RELATION TO THE TWO CHANNEL KONDO MODEL AND IMPOSSIBILITY OF THE STRONG COUPLING LIMIT

A similar procedure can be done for the 2CK model with real spin.^{22,23} In our notation Eq. (1) acquires the form of Eq. (33) with (33c) of the form,

$$\delta H = 4\pi\lambda_z S_z J_z + 4\pi\lambda_x S_x J_x + 4\pi\lambda_y S_y J_y + 2h S_x, \quad (82)$$

where we included constant λ_y for the sake of generality.

We extended the results of Refs. 22 and 23 to include the presence of a transverse magnetic field by the repeating all the steps of the previous section. The only difference is that the spin flips are instantaneous (see Fig. 2), so that the high-energy cutoff is determined by the electronic scale D . We obtained the partition function for the 2CK model (82) in the form (74) as

$$\mathcal{Z} = \sum_{n=0}^{\infty} h^{2n} \sum_{\{\mu_j\}, \{\sigma_j\}} \int d\xi_j e^{-\mathcal{H}_{\text{Kondo}}}, \quad (83)$$

with

$$\Gamma(\{\mu_j\}) = \prod_{j=1}^{2n} \left(1 - \mu_j (-1)^j \frac{\lambda_y}{\lambda_x} \right) \left(\frac{\lambda_x D}{2h} \right)^{|\mu_j|} \quad (84)$$

and

$$\begin{aligned} \mathcal{H}_{\text{Kondo}} = & - \sum_{i \neq j}^{2n} \{ \delta_{\sigma_i \sigma_j} [\mu_j \mu_i + (-1)^i \mu_j 2\lambda_z] \\ & + (-1)^{i+j} 2\lambda_z^2 \} \ln |D \xi_{ij} / \hbar|. \end{aligned} \quad (85)$$

Again the integration is performed with the constraint (50) and the summation over charge configurations $\{\mu_j\}$ is sub-

ject to the neutrality condition (67). The cutoff D is the same cutoff as in Eq. (37), or more precisely

$$\lim_{\tau \rightarrow \infty} \left[\ln D \tau + 2\pi^2 \text{Re} \int_0^\tau d\tau_1 \int_0^\tau d\tau_2 \langle J_x(\tau_1) J_x(\tau_2) \rangle \right] = 0.$$

Let us now compare the logarithmic gas models for tunneling centers (74), (80) and for the Kondo problem (83). First of all, direct comparison of Eq. (84) with Eq. (66) shows that constant Λ_y is exactly equivalent to the coupling constants of Kondo model indeed and Eq. (41) follows. Next, we put $\lambda_y = 0$ in Eq. (84) and compare the result with Eq. (80). We immediately find that two models become equivalent upon the following identification of the parameters:

$$D \leftrightarrow \mathcal{D} = \frac{e^{-\alpha_y}}{\tau_{\text{tun}}},$$

$$h \leftrightarrow \tilde{h} \approx h,$$

$$\lambda_x \leftrightarrow \Lambda_x \left(\frac{h}{e^{\alpha_y} \mathcal{D}} \right) = \Lambda_x(h \tau_{\text{tun}}),$$

$$\lambda_z \leftrightarrow \frac{\Lambda_z}{2}, \quad (86)$$

where the tunneling time is defined in Eq. (46), and numerical constant α_y is defined in Eq. (76c) for the arbitrary DWP and calculated for model (47) in Eq. (79).

We note that the equivalence between the tunneling impurity model and the 2CK model for times larger than τ_{tun} , is nonperturbative in the sense that it is established at any order in perturbation theory. We reiterate, that the mapping has to be performed with account of all of the excited states of the movable atom.

From the relationships (86) it is easy to show that the Kondo temperature is always smaller than h ($\Delta_z = 0$ in our model) and then the 2CK regime can never be reached. Indeed, inserting Eqs. (86) into (5), one has

$$\frac{T_K}{h} = e^{-\alpha_y} \Lambda_x \left(\frac{\Lambda_x}{\Lambda_z} \right)^\gamma (h \tau_{\text{tun}})^\gamma, \quad \gamma = \frac{1}{\Lambda_z} - \frac{1}{2}. \quad (87)$$

From the condition (16), and (25) it follows that

$$\Lambda_x \ll 1, \quad \gamma \gg 1, \quad \frac{\Lambda_x}{\Lambda_z} \ll 1. \quad (88)$$

From Eq. (53), we have usual relation for the tunneling splitting of two levels,

$$h \tau_{\text{tun}} \ll 1.$$

Together with Eq. (87), this implies that

$$T_K / h \ll 1 \quad (89)$$

and then *the strong coupling regime cannot be reached.*

V. DISCUSSION

In this paper we considered a general model describing a tunneling impurity moving in a double well potential and embedded in a metal. The main motivation for this work was to provide a conclusive answer to the question whether it is possible to observe the strong coupling regime of the two-channel Kondo model in such a system. In order to answer this question one has to find the correct relationship between the microscopic parameters of the tunneling impurity problem and the coupling constants of the effective two-channel Kondo model. Previous results showed that the two-level system is not a good starting point because all the excited states of the impurity play an essential role. In order to take into account all the excited states of the double well potential problem we used a different approach. We mapped the tunneling impurity model into a one-dimensional logarithmic gas model using a semiclassical (dilute instanton) approximation to describe the dynamics of the impurity. Since the same mapping can be done for the two-channel Kondo model, we obtain a general relationship between the coupling constants of the two models. This relationship is obtained taking into account all the excited levels of the heavy particle and is valid to any order in perturbation theory. We demonstrated that, in the effective two-channel Kondo model (1), the values of the coupling constant λ_x and the transverse magnetic field h , are intrinsically related, and can be never considered independently. This fact, together with the existence of an intrinsic high-energy cutoff in the theory, $1/\tau_{\text{tun}}$, conspire in such a way that the Kondo temperature is always smaller than the effective magnetic field h . Then the strong coupling fixed point of the two-channel Kondo model can *never* be reached in this system. These results are valid for any form of the double well potential and are robust against specific properties of the electron system.

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APPENDIX:

EFFECT OF THE ELECTRON-HOLE ASYMMETRY

In this Appendix we present the general scheme for the calculation of the parameters of the effective model (74) for a general electron spectrum without involving the linearized approximation (20) from the very beginning. Our motivation for doing so is to provide framework in which further discussion (quite futile to our opinion) of the role of electron-hole asymmetry should be performed.

Integration over the fermionic fields in Eq. (6) gives the formal result,

$$\int D[\bar{\psi}(\mathbf{x}, \tau)] D[\psi(\mathbf{x}, \tau)] e^{-S_E[q, \bar{\psi}, \psi]} = e^{-S_{\text{el}}[q(\tau)] - \int d\tau \mathcal{L}_{\text{at}}(q, \dot{q})}, \quad (\text{A1})$$

where the effect of the electrons on the atom is described by

$$-S_{\text{el}}[q(\tau)] = \text{Tr} \ln \left[-\frac{\partial}{\partial \tau} - \hat{\xi}(\mathbf{p}) - V \left(i \frac{\partial}{\partial \mathbf{p}} + \hat{\Omega}(\mathbf{p}) - \mathbf{q}(\tau) \right) \right], \quad (\text{A2})$$

where \mathbf{p} is the quasimomentum of the electrons, $\hat{\xi}(\mathbf{p}) = \text{diag}[\xi_j(\mathbf{p})]$ is the spectrum of Bloch electrons, and $V(\mathbf{r})$ is the potential of the interaction of the atom with the electrons, which can be of more general form than the local interaction (14). Finally, $\hat{\Omega}(\mathbf{p})$ is the standard³¹ nondiagonal component of the coordinate operator in the basis of the Bloch functions and it describes the interband scattering due to the atomic potential. In Eq. (A2) and thereafter all the energies are counted from the Fermi level.

It will be convenient for us to express all the quantities in terms of the solution of the scattering problem on immobile atom. In order to do so, we perform the unitary transformation in Eq. (A2) as

$$\text{Tr} \ln[\dots] = \text{Tr} \ln[e^{i\mathbf{q}(\tau)\mathbf{p}} \dots e^{-i\mathbf{q}(\tau)\mathbf{p}}]$$

and obtain from Eq. (A2),

$$-S_{\text{el}}[q(\tau)] = \text{Tr} \ln[-\partial_\tau + i\mathbf{p}\dot{\mathbf{q}}(\tau) - \hat{\xi}(\mathbf{p}) - \hat{V}], \quad (\text{A3})$$

$$\hat{V} \equiv V \left(i \frac{\partial}{\partial \mathbf{p}} + \hat{\Omega}(\mathbf{p}) \right).$$

Our goal now is to expand Eq. (A3) in powers of \dot{q} , and relate the expansion coefficients to the coupling constants of the low energy theory (74). To facilitate such an expansion we introduce the Matsubara Green functions and \hat{T} matrix of immobile atom,

$$\hat{G}_0(i\varepsilon_n, \mathbf{p}) = \frac{1}{i\varepsilon_n - \hat{\xi}(\mathbf{p})};$$

$$\hat{G}(i\varepsilon_n, \mathbf{p}_1, \mathbf{p}_2) = \left[\frac{1}{i\varepsilon_n - \hat{\xi}(\mathbf{p}) - \hat{V}} \right]_{\mathbf{p}_1 \mathbf{p}_2}$$

$$= \hat{G}_0(i\varepsilon_n, \mathbf{p}_1) [\delta_{\mathbf{p}_1 \mathbf{p}_2} + \hat{T}(i\varepsilon_n, \mathbf{p}_1, \mathbf{p}_2) \hat{G}_0(i\varepsilon_n, \mathbf{p}_2)];$$

$$\hat{T}(i\varepsilon_n, \mathbf{p}_1, \mathbf{p}_2) = [\hat{V}(1 - \hat{G}_0(i\varepsilon_n)\hat{V})^{-1}]_{\mathbf{p}_1 \mathbf{p}_2}, \quad (\text{A4})$$

with $\varepsilon_n = \pi T(2n+1)$ being the fermionic Matsubara frequency, and their retarded and advanced counterparts

$$\hat{G}_0^{R(A)}(\varepsilon) = \hat{G}_0(\varepsilon \pm i0); \quad \hat{T}^{R(A)}(\varepsilon) = \hat{T}(\varepsilon \pm i0). \quad (\text{A5})$$

We note the identities,

$$\begin{aligned} \hat{G}_0(i\varepsilon_n, \mathbf{p}) - \hat{G}_0(i\varepsilon_n + i\omega_n, \mathbf{p}) \\ = i\omega_n \hat{G}_0(i\varepsilon_n, \mathbf{p}) \hat{G}_0(i\varepsilon_n + i\omega_n, \mathbf{p}); \end{aligned} \quad \mathbf{q}(\omega_n) = \int_0^{1/T} d\tau \mathbf{q}(\tau) e^{i\omega_n \tau}. \quad (\text{A11})$$

With the help of Eq. (A6), one rewrites Eq. (A10),

$$\begin{aligned} \hat{T}(i\varepsilon_n, \mathbf{p}_1, \mathbf{p}_2) - \hat{T}(i\varepsilon_m, \mathbf{p}_1, \mathbf{p}_2) \\ = \int \frac{d^3 p_3}{(2\pi)^3} \hat{T}(i\varepsilon_n, \mathbf{p}_1, \mathbf{p}_3) [\hat{G}(i\varepsilon_n, \mathbf{p}_3) \\ - \hat{G}(i\varepsilon_m, \mathbf{p}_3)] \hat{T}(i\varepsilon_m, \mathbf{p}_3, \mathbf{p}_2), \end{aligned} \quad (\text{A6})$$

where $\omega_m = 2\pi Tm$ is the bosonic Matsubara frequency. Hereafter the momentum integration is performed within the first Brillouin zone.

Using Eq. (A4) we rewrite Eq. (A3) in the form of linked cluster expansion,

$$S_{\text{el}}[q(\tau)] = \sum_{m=1}^{\infty} \frac{S_{\text{el}}^{(m)}}{m}; \quad S_{\text{el}}^{(m)} = \text{Tr}[-i\dot{\mathbf{q}}\mathbf{p}\hat{G}]^m, \quad (\text{A7})$$

where we omitted the term independent of $q(\tau)$ and all of the multiplications should be understood in the matrix sense.

Before we proceed we notice that in the absence of the impurity potential quasimomentum \mathbf{p} is an integral of motion. Therefore, the coupling to \mathbf{p} of the force with nonzero Matsubara frequency has no effect independently of the spectrum $\xi(\mathbf{p})$. Using the fact that

$$\int_0^{1/T} d\tau \dot{\mathbf{q}} = 0, \quad (\text{A8})$$

we find natural result that

$$\text{Tr}[\dot{\mathbf{q}}\mathbf{p}\hat{G}_0]^m = 0. \quad (\text{A9})$$

Now we perform the actual calculation of the expansion (A6). According to Eq. (A7), the first order term vanishes, $S_{\text{el}}^{(1)} = 0$. Using Eqs. (A5) and (A9) one finds for the second order term,

$$\begin{aligned} S_{\text{el}}^{(2)} = T \sum_{\omega_n} \sum_{\alpha\beta=x,y,z} \omega_n^2 q_\alpha(\omega_n) q_\beta(-\omega_n) \Pi^{\alpha\beta}(\omega_n), \\ \Pi^{\alpha\beta} = -T \sum_{\varepsilon_n} \int \frac{d^3 p}{(2\pi)^3} p^\alpha p^\beta \text{Tr}\{\hat{G}_0(i\varepsilon_n, \mathbf{p}) \\ \times \hat{T}(i\varepsilon_n, \mathbf{p}, \mathbf{p}) \hat{G}_0(i\varepsilon_n, \mathbf{p}) \hat{G}_0(i\varepsilon_n + i\omega_n, \mathbf{p}) \\ + \omega_n \rightarrow -\omega_n\} \\ - T \sum_{\varepsilon_n} \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 k}{(2\pi)^3} p^\alpha k^\beta \text{Tr}\{\hat{G}_0(i\varepsilon_n + i\omega_n, \mathbf{p}) \\ \times \hat{T}(i\varepsilon_n + i\omega_n, \mathbf{p}, \mathbf{k}) \hat{G}_0(i\varepsilon_n + i\omega_n, \mathbf{k}) \hat{G}_0(i\varepsilon_n, \mathbf{k}) \\ \times \hat{T}(i\varepsilon_n, \mathbf{k}, \mathbf{p}) \hat{G}_0(i\varepsilon_n, \mathbf{p})\}, \end{aligned} \quad (\text{A10})$$

with

$$\begin{aligned} \Pi^{\alpha\beta} = -\frac{T}{\omega_n^2} \sum_{\varepsilon_n} \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 k}{(2\pi)^3} (p^\alpha p^\beta - p^\alpha k^\beta) \\ \times \text{Tr}\{\hat{G}^*(\mathbf{p}) \hat{T}(i\varepsilon_n + i\omega_n, \mathbf{p}, \mathbf{k}) \hat{G}^*(\mathbf{k}) \hat{T}(i\varepsilon_n, \mathbf{k}, \mathbf{p})\}; \\ \hat{G}^*(\mathbf{p}) \equiv \hat{G}_0(i\varepsilon_n + i\omega_n, \mathbf{p}) - \hat{G}_0(i\varepsilon_n, \mathbf{p}). \end{aligned}$$

Performing the standard trick with the replacement of summation over ε_n to the integration, we obtain

$$\begin{aligned} \Pi^{\alpha\beta} = -\frac{1}{\omega_n^2} \int \frac{d\varepsilon}{\pi} \tanh \frac{\varepsilon}{2T} \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 k}{(2\pi)^3} (p^\alpha p^\beta - p^\alpha k^\beta) \\ \times \text{Im}\{\text{Tr}[\hat{G}^+(\mathbf{p}) \hat{T}(\varepsilon + i|\omega_n|, \mathbf{p}, \mathbf{k}) \hat{G}^+(\mathbf{k}) \hat{T}^R(\varepsilon, \mathbf{k}, \mathbf{p})] \\ - \text{Tr}[\hat{G}^-(\mathbf{p}) \hat{T}(\varepsilon + i|\omega_n|, \mathbf{p}, \mathbf{k}) \hat{G}^-(\mathbf{k}) \hat{T}^A(\varepsilon, \mathbf{k}, \mathbf{p})]\}, \end{aligned} \quad (\text{A12})$$

where

$$\begin{aligned} \hat{G}^+(\mathbf{p}) \equiv \hat{G}_0(\varepsilon + i|\omega_n|, \mathbf{p}) - \hat{G}_0^R(\varepsilon, \mathbf{p}), \\ \hat{G}^-(\mathbf{p}) \equiv \hat{G}_0(\varepsilon + i|\omega_n|, \mathbf{p}) - \hat{G}_0^A(\varepsilon, \mathbf{p}). \end{aligned} \quad (\text{A13})$$

At that point it is important to emphasize that $|\mathbf{q}(\omega_n)|$ decays exponentially at large frequencies for the saddle point solution (44), $|\mathbf{q}(\omega_n)| \propto e^{-|\omega_n| \tau_{\text{um}}}$. The fluctuations around the saddle point are also suppressed at large frequencies due to the kinetic energy in Eq. (9). The Green functions in Eq. (A13) are analytic functions of energy except the branch cut at the real axis. It allows one to expand over $|\omega_n|$ in Eq. (A13),

$$\begin{aligned} \Pi^{\alpha\beta} = -2 \int \frac{d\varepsilon}{\pi} \tanh \frac{\varepsilon}{2T} \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 k}{(2\pi)^3} (p^\alpha p^\beta - p^\alpha k^\beta) \\ \times \left\{ \frac{1}{|\omega_n|} \frac{\partial}{\partial \varepsilon} \text{Tr}[\text{Im} \hat{G}_0^R(\varepsilon, \mathbf{p})] \hat{T}^R(\varepsilon, \mathbf{p}, \mathbf{k}) \right. \\ \times [\text{Im} \hat{G}_0^R(\varepsilon, \mathbf{k})] \hat{T}^A(\varepsilon, \mathbf{k}, \mathbf{p}) \\ - \frac{1}{2} \text{Im} \text{Tr} \frac{\partial \hat{G}_0^R(\varepsilon, \mathbf{p})}{\partial \varepsilon} \hat{T}^R(\varepsilon, \mathbf{p}, \mathbf{k}) \frac{\partial \hat{G}_0^R(\varepsilon, \mathbf{k})}{\partial \varepsilon} \hat{T}^R(\varepsilon, \mathbf{k}, \mathbf{p}) \\ + \frac{\partial}{\partial \varepsilon} \text{Re} \text{Tr}[\text{Im} \hat{G}_0^R(\varepsilon, \mathbf{k})] \hat{T}^A(\varepsilon, \mathbf{k}, \mathbf{p}) \\ \times [[\text{Im} \hat{G}_0^R(\varepsilon, \mathbf{p})] \partial_\varepsilon \hat{T}^R(\varepsilon, \mathbf{p}, \mathbf{k}) \\ \left. + [\text{Re} \partial_\varepsilon \hat{G}_0^R(\varepsilon, \mathbf{p})] \hat{T}^R(\varepsilon, \mathbf{p}, \mathbf{k}) \right\}. \end{aligned} \quad (\text{A14})$$

Substituting Eq. (A14) into Eq. (A10) we find

$$S_{\text{el}}^{(2)} = T \sum_{\omega_n} \sum_{\alpha\beta=x,y,z} q_{\alpha}(\omega_n) q_{\beta}^*(\omega_n) (|\omega_n| R^{\alpha\beta} + \omega_n^2 Q^{\alpha\beta}). \quad (\text{A15a})$$

Here

$$R^{\alpha\beta} = 2\pi \int d\varepsilon \frac{d}{d\varepsilon} \tanh \frac{\varepsilon}{2T} \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 k}{(2\pi)^3} (p^{\alpha} p^{\beta} - p^{\alpha} k^{\beta}) \\ \times \text{Tr} \delta[\varepsilon - \hat{\xi}(\mathbf{p})] \hat{T}^R(\varepsilon, \mathbf{p}, \mathbf{k}) \delta[\varepsilon - \hat{\xi}(\mathbf{k})] \hat{T}^A(\varepsilon, \mathbf{k}, \mathbf{p}), \quad (\text{A15b})$$

and the first term in (A15b) is precisely the contribution which describes the effect of the gapless excitations–orthogonality catastrophe. It is present for the constant density of states. For the spherically symmetric case, one can easily recover from Eq. (A15b) the exponential factor in Eq. (39c).

The second term in the expression for the action characterizes the electron–hole asymmetry. It has an explicit expression,

$$Q^{\alpha\beta} = \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 k}{(2\pi)^3} (p^{\alpha} p^{\beta} - p^{\alpha} k^{\beta}) (Q_1 + Q_2), \\ Q_1 = \text{Im} \int \frac{d\varepsilon}{\pi} \tanh \frac{\varepsilon}{2T} \text{Tr} \frac{\partial \hat{G}_0^R(\varepsilon, \mathbf{p})}{\partial \varepsilon} \hat{T}^R(\varepsilon, \mathbf{p}, \mathbf{k}) \\ \times \frac{\partial \hat{G}_0^R(\varepsilon, \mathbf{k})}{\partial \varepsilon} \hat{T}^R(\varepsilon, \mathbf{k}, \mathbf{p}),$$

$$Q_2 = 2\pi \text{Re} \int d\varepsilon \frac{d}{d\varepsilon} \tanh \frac{\varepsilon}{2T} \text{Tr} \left\{ \delta[\varepsilon - \hat{\xi}(\mathbf{k})] \hat{T}^A(\varepsilon, \mathbf{k}, \mathbf{p}) \right. \\ \left. \times \left[\partial_{\varepsilon} \hat{T}^R(\varepsilon, \mathbf{p}, \mathbf{k}) \delta[\varepsilon - \hat{\xi}(\mathbf{p})] \right. \right. \\ \left. \left. + \frac{[\text{Re} \partial_{\varepsilon} \hat{G}_0^R(\varepsilon, \mathbf{p})]}{\pi} \hat{T}^R(\varepsilon, \mathbf{p}, \mathbf{k}) \right] \right\}, \quad (\text{A15c})$$

where the term Q_1 depends on the spectrum only and Q_2 is due to the frequency dependence of the kinetic coefficients. Equation (A15c) vanishes for the constant density of states approximation, but it is not so for the arbitrary band structure. However, we saw already that it generates the contribution proportional to the higher power of ω_n . Thus, this term produces *nonsingular* correction to the leading term. The characteristic value of this correction may be estimated as $\approx 1/(\tau_{\text{tun}} \epsilon^*)$, where ϵ^* is the energy scale governing the electron-hole asymmetry. In principle, it might be estimated from the thermopower measurements or from the first principle band structure calculation of Eq. (A15c).

One can proceed with the similar expansion in the next orders of perturbation theory to recover the constant Λ_x . The structure remains the same, the first nonanalytic contribution in ω_n comes from terms which are kept in the constant density of states approximation, and the next contribution has higher power of ω_n , and thus gives the correction small as $1/(\tau_{\text{tun}} \epsilon^*)$. Thus, the electron–hole asymmetry (if treated systematically) cannot produce anything except parametrically small corrections to the coupling constants in contradiction to claims of Ref. 19.

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- ²⁹Coinciding kinks also produce terms $\propto (\tau_+ - \tau_-)^{-1}$. Such terms vanish in the physical limit $\Lambda_y = 0$.
- ³⁰An extension of these results to finite temperature can be obtained via the conformal transformation, $z \rightarrow z' = (i/2\pi T)\ln z$, with $z = \tau + ix$. This is analytic in the complex plane except the origin and maps the complex z -plane (without the origin) into the strip

of width $1/T$ with periodic boundary conditions, which is the geometry that describes a system at finite temperature. Then the two point function in the strip can be obtained from the one in the plane using the transformation properties of primary fields under a conformal transformation (see, for instance Ref. 25). For example, the correlation functions of vertex operators in a strip geometry, needed in (39) at finite temperature, can be obtained from the one on the whole complex plane via the substitution, $1/\tau \rightarrow \pi T / \sin \pi T \tau$. Then the partition function of the LGM at finite temperature will have the form (74)–(75) with $U_2(\tau) \rightarrow \int_0^{1/T} d\tau_1 \eta_2(\tau_1) \ln |\sin \pi T(\tau_1 - \tau) / \pi T \tau_{\text{tun}}|$.

³¹See for instance, L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Pergamon, Oxford, 1980), Vol. 9.