Spins, charges, and currents at domain walls in a quantum Hall Ising ferromagnet

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We study spin textures in a quantum Hall Ising ferromagnet. Domain walls between ferromagnetic and unpolarized states at $\nu=2$ are analyzed with a functional theory supported by a microscopic calculation. In a neutral wall, Hartree repulsion prevents the appearance of a fan phase provoked by a negative stiffness. For a charged system, electrons become trapped as solitons at the domain wall. The size and energy of the solitons are determined by both Hartree and spin-orbit interactions. Finally, we discuss how electrical transport takes place through the domain wall.

Recently there is a great interest in the study of spin properties of quantum Hall states. For some filling factors ν in the integer quantum Hall effect (QHE), a transition from a ferromagnetic (F) to an unpolarized (U) ground state (GS) can be achieved by changing the ratio between the cyclotron ($\hbar\omega_c$) and the Zeeman (E_Z) energies.[1] Experimental evidence of this transition has been addressed recently.[2–5] In the fractional QHE, transition between F and U states at filling factors $\nu=2/3$ and 2/5 can be tuned by varying E_z with respect to the electron-electron interaction energy. Experimental indication of this transition has been reported[6–13] although the order of the transition is not clear.[14–16]

In this work we study a domain wall (DW) separating the U state from the F state, at $\nu=2$. At this filling factor, the F state has electrons occupying the Landau levels $n=0$, 1 with spin up. In the U state, the electrons occupy the Landau level $n=0$ with the two spins orientations, i.e., the U state coincides with the singlet (S) state.

At integer ν, the phase transition between the uniform GS's can be described by the functional $\mathcal{F}=\alpha m_z+\beta m_z^2$, where $\mathbf{m}=(\mathbf{m}_\perp,m_z)$ is a unitary vector field, parallel to an isospin variable that points to the positive (negative) z direction when the GS is F (U).[17] The phase transition occurs when $\alpha=0$. For ν being an integer greater than or equal to 2, β is negative, indicating a first-order phase transition between the $m_z=1$ and the $m_z=-1$ states: for this reason these systems are called quantum Hall Ising ferromagnets. The existence of hysteresis in transport experiments is the smoke signal of the occurrence of a first-order phase transition.[2–5]

In this work we present the following result

(i) We obtain a functional for describing isospin textures in the system at $\nu=2$. Due to the odd parity of the product of the Landau levels wave functions participating in the F and S phases, we find a negative stiffness for distortions of $\mathbf{m}_\perp$. This stiffness is not able to change the order of the phase transition at $\nu=2$.

(ii) By integrating out the transverse coordinate, we obtain a one-dimensional functional for describing spin textures at the DW. The adequacy of the functional is established by microscopic calculations. This functional also has a negative stiffness, which could produce a fan spin texture in the trans-

verse isospin component along the DW but, for GaAs quantum wells having widths of a few hundred angstroms and electron densities from 10^{11} to 10^{12} cm^{-2}, Hartree repulsion prevents the formation of such topological structure.

(iii) When the system is charged, extra electrons get trapped at the DW as topological excitations, solitons, with size and energy controlled by both the Hartree and spin-orbit (SO) interactions. The energy of the soliton controls the transport properties through the DW, the conductance being nonzero only for finite SO coupling. This result solves the problem of the spin conservation; transport through a DW implies a carrier spin flip, something that can occur in the presence of SO interaction.

Energy functional for isospin textures. The electron states of a two-dimensional electron gas confined in the x-y plane and a magnetic field applied in the z direction, are characterized by the Landau level index n, the degeneracy index X and the spin σ. In the Landau gauge, X is the momentum in the y direction as well as the orbit center of the x part of the wave function. In this work, the magnetic length l and the interaction $e^2/\epsilon l$ are the units of length and energy, respectively. In both the F and S states, all the $|n=0,X,\sigma=\uparrow\rangle$ states are occupied and we consider them as electrically inert, being included in the vacuum. The $\nu=2$ states are described by

$$\Psi = \prod_X \left[\cos\theta(X) c^\dagger_{X,\Uparrow} + \sin\theta(X) e^{i\psi(X)} c^\dagger_{X-G,\Downarrow} \right]|0\rangle, \quad (1)$$

where $|0\rangle$ is the vacuum, $c^\dagger$ are creation operators, X runs over all possible states, and the isospins $\Uparrow$ and $\Downarrow$ represent the states $n=1,\sigma=\uparrow$ and $n=0,\sigma=\downarrow$, respectively. In Eq. (1) we only mix two isospins, since we suppose that $\hbar\omega_c$ is large enough for not producing Landau level mixing in the S and F phases.[18,19] Assuming that $\theta(X)$ and $\psi(X)$ change slowly and G is small, the unitary vector field corresponding to the state (1) has the form

$$m_z(x) = \cos 2\theta(x),$$

$$m_x(x,y) + im_y(x,y) = \sin 2\theta(x) e^{i(\psi(x)+Gy)}. \quad (2)$$

By computing the expectation value of the energy for the wave function Eq. (1), we obtain the following energy functional for isospin textures:

$$\mathcal{F}_{2D} = \alpha \int d\mathbf{r} m_z(\mathbf{r}) + \beta \int d\mathbf{r} m_z^2(\mathbf{r}) + \frac{\rho_\parallel}{2} \int d\mathbf{r} [\partial_\mu m_z(\mathbf{r})]^2$$

$$+ \frac{\rho_\perp}{2} \int d\mathbf{r} [\partial_\mu \mathbf{m}_\perp(\mathbf{r})]^2 + \hat{\rho} \int d\mathbf{r} [\partial_\mu^2 \mathbf{m}_\perp(\mathbf{r})]^2 + V_H. \tag{3}$$

The coefficients are,

$$\alpha = \alpha_1 + \left(E_Z - \frac{\hbar \omega_c}{2} \right) \frac{1}{2\pi},$$

$$\alpha_1 = \frac{1}{8\pi} (\Sigma_{0,0,0,0} - \Sigma_{1,1,1,1} - \Sigma_{1,0,1,0})$$

$$\beta = \frac{1}{16\pi} (\Sigma_{0,0,0,0} + \Sigma_{1,1,1,1} - 2\Sigma_{1,1,0,0}),$$

$$\rho_\parallel = \frac{1}{2} (\rho^{0,0} + \rho^{1,1}), \quad \rho_\perp = \rho^{1,0} \tag{4}$$

with

$$\Sigma_{n,n_1,n_2,n_3} = -\frac{1}{S} \sum_{\mathbf{q}} v(\mathbf{q}) F_{n,n_1}(\mathbf{q}) F_{n_2,n_3}(-\mathbf{q}),$$

$$\rho^{n,n_1} = \frac{1}{2\pi L} \sum_{\mathbf{q}} \frac{q^2}{4} v(\mathbf{q}) F_{n,n}(\mathbf{q}) F_{n_1,n_1}(-\mathbf{q}),$$

$$\hat{\rho} = \frac{1}{48\pi L} \sum_{\mathbf{q}} \frac{q^4}{4} v(\mathbf{q}) F_{n,n}(\mathbf{q}) F_{n_1,n_1}(-\mathbf{q}) \tag{5}$$

with S and L being the area and length (along the y direction) of the system and $v(\mathbf{q})$ the Fourier component of Coulomb interaction. For a strictly two-dimensional system, the form factors are $F_{0,0}(\mathbf{q}) = e^{-q^2/4}$, $F_{1,1}(\mathbf{q}) = (1 - q^2/2) e^{-q^2/4}$, and $F_{1,0}(\mathbf{q}) = (-q_y + iq_x) e^{-q^2/4}/\sqrt{2}$. The coefficients become $\alpha_1 = 3/32\sqrt{2\pi}$, $\beta = -3/64\sqrt{2\pi}$, $\rho_\parallel = 11/128\sqrt{2\pi}$, $\rho_\perp = -1/32\sqrt{2\pi}$, and $\hat{\rho} = 0.0035$.

The importance in this functional is the negative value of the transversal stiffness $\rho_\perp$. In order to control the spatial variation of $\mathbf{m}_\perp$, it is necessary to include in the expansion a higher derivative of $\mathbf{m}_\perp$. $\rho_\perp < 0$ due to the different parity of the $n=0$ and the $n=1$ Landau level wave functions; in this way $\rho^{1,0}$ and $\rho^{2,1}$ are negative whereas $\rho^{2,0}$ and $\rho^{3,1}$ are positive. $\rho_\perp < 0$ could produce intermediate helical phases between the F and the S states, however, at $\nu = 2$ the magnitude of $\rho_\perp$ is not big enough for this occurrence.

Quantum Hall ferromagnets have the unique property that the topological charge is directly related to the electrical charge.[20,21] Therefore, we include in the functional Eq. (3) a Hartree term V_H, representing the interaction between the charge densities $q(\mathbf{r}) \times \mathbf{m} \cdot (\partial_\mu \mathbf{m} \times \partial_\nu \mathbf{m})/8\pi$ associated with the isospin texture. We use a standard[20,21] expresion for

V_H including the semiconductor dielectric constant and finite width of the quantum well. The F-S degeneracy occurs when $\alpha = 0$, and the negative sign of β indicates the first-order character of the transition. For $E_Z = 0$ the phase transition occurs at $\hbar \omega_c = 0.472$, which corresponds to an electron separation $r_s = 2.12$. This justifies the use, in Eq. (1), of just the $n=0$ and the $n=1$ Landau levels.[18]

Domain wall structure. When $\alpha = 0$, the S and the F states are degenerated, and disorder or finite temperature can produce DW's separating these GS's. For studying the structure of a DW, we assume $\alpha = 0$ and impose to the functional (3) the boundary conditions $m_z = \pm 1$ at $x = \pm \infty$. By doing that we obtain a DW thickness W_X of the order of l. In order to get a functional to describe a DW, we write $\mathbf{m}_\perp = \sin 2\theta(x) \times [\cos \phi(y), \sin \phi(y)]$, and integrate in Eq. (3) over x using a simple model in which $\theta(x)$ varies linearly through the DW, obtaining

$$\Delta \mathcal{F}_{DW}(\phi(y)) = \rho/2 \int dy [\partial_y \phi(y)]^2 + B \int dy [\{\partial_y \phi(y)\}^4$$

$$+ (\partial_y^2 \phi(y))^2] + V_H + \Delta \mathcal{F}_{SO}, \tag{6}$$

where $\Delta \mathcal{F}_{SO}$ is a SO term that will be essential in the discussion below. Using the simple model $\theta(x) = \pi x/2W_X$ for $x < W_X$ and zero otherwise, the parameters in Eq. (6) are $\rho = W_X \rho_\perp/2$ and $B = W_X \hat{\rho}/2$. However, the rapid change of $m_z(x)$ over a magnetic length, raises some doubts on the validity of the functional (3) as a good starting point to obtain Eq. (6). Therefore, we have taken the alternative of performing a microscopic Hartree-Fock (HF) calculation[22] for describing DW's. In Fig. 1 we plot the HF quasiparticle energies as a function of the orbit guiding center. The chemical potential is located at the gap energy. The reduction of the energy gap at the DW is an indication of the loss of coherence of the wave-function. We find that W_X is roughly $2l$ and the energy per magnetic length of the DW is 0.0448. In the inset of Fig. 1 we plot the z component of the unitary vector field, m_z, isospin as a function of the position. At the center of the DW $m_z = 0$ and $m_\perp$ should be the unity. In absence of spin-orbit coupling, the system has $U(1)$ symmetry and the energy of the DW does not depend on global rotations of $\mathbf{m}_\perp$. From the HF results, we find that the functional (6) is adequate for describing textures of $\mathbf{m}_\perp$ along the DW. The coefficients ρ and B for the terms with derivatives can be obtained from a fitting to the HF results. The ratio ρ/B is the same as that for the simple model above, but each coefficient has increased by a factor of 3.7.

Since $\rho < 0$, the first term in Eq. (6) tends to produce a rotation of the isospin along the DW. Although this rotation is limited by the second term, a fan phase could appear if one neglects any Hartree contribution as it is usually done.[23,24] However, a rotation implies the existence of electrical dipoles is associated with oscillations of the topological charge (but with zero total topological charge),

$$q(\mathbf{r}) = \frac{1}{4\pi} \partial_y \phi(y) \partial_x (m_z(x)). \tag{7}$$
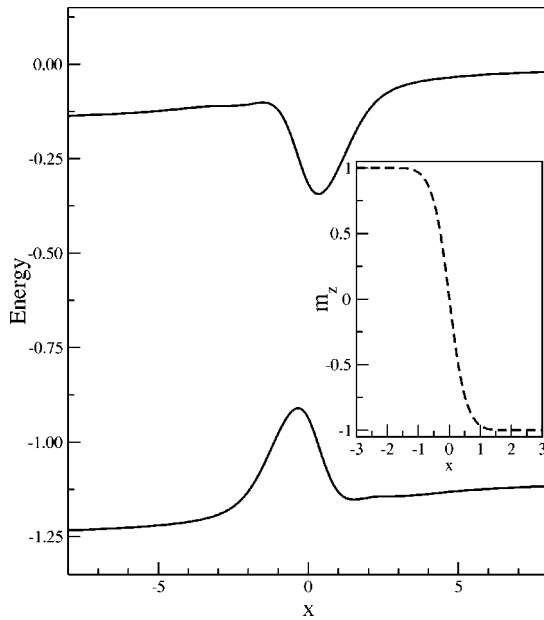
FIG. 1. Dispersion relation (in units of $e^2/\epsilon l$) of the highest occupied and lowest unoccupied states in the region of the DW. The GS to the left is the F state while to the right is the S one. The inset shows the variation of the order parameter $m_z(x)$ at the DW. All the lengths are measured in units of l.
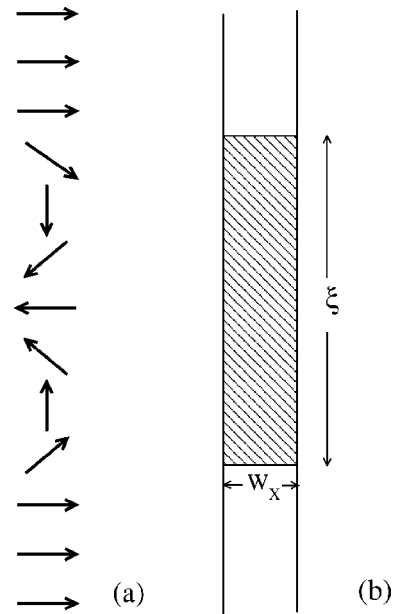

FIG. 2. (a) Schematic behavior of the in-plane component $\mathbf{m}_\perp$ of the spin at the charged DW. (b) Schematic charge density (dashed region) of the soliton at the charged DW.

The V_H prevents the appearance of a fan phase induced by the negative stiffness. The Hartree repulsion of the charge density associated with the texture keeps the spin direction constant along the DW. It must be stressed that the Coulomb term in Eq. (6), has the same dependence on derivatives of the field ϕ as the elastic term has, and therefore it cannot be neglected in the study of DW's.

We have also included in Eq. (6) a SO term $\mathcal{F}_{SO}$. The SO interaction *directly* couples a state $|0,X,\downarrow\rangle$ with a state $|1,X,\uparrow\rangle$ (Ref. 25) producing a Zeeman-like coupling to the isospin and an effective in plane magnetic field. Therefore, in our functional, SO is described by a term,

$$\Delta\mathcal{F}_{SO}=-\lambda_{SO}\int dy(\cos\phi(y)-1)xs, \qquad (8)$$

with $\lambda_{SO}=W_X\beta_{SO}/2^{3/2}\pi^2$ where β_{SO} is the bulk spin-orbit coupling.[25]

Charged domain wall. The solutions of Eq. (6) can be characterized by integers that correspond to the total topological charge Q_T of the solution. Q_T is the increase, in units of 2π, of the phase $\phi(y)$ when going from $-\infty$ to $+\infty$. Hitherto, we have just considered solutions with $Q_T=0$. Let us now consider the solutions for $Q_T>0$.

Solutions of Eq. (6) in the sector $Q_T=1$, are very important since the equivalence between topological and electrical charge allows the isospin textures to be the relevant charged excitations in the system.[20,21] In the presence of domains, charge excitations can be trapped in the walls forming confined isospin textures, which are solitons in the phase $\phi(y)$.[23] Analytical expression for the soliton have been obtained, neglecting the Hartree interaction, in the case of positive stiffness.[23] In our case, the Hartree term is essential and

we have not been able to obtain an analytic solution. For $Q_T=1$ we take a simplified shape for the soliton. In the sector of $Q_T=1$, we look for solitons of size ξ having a simple form $\phi(y)=2\pi y/\xi$, within an interval of length ξ and zero out of that interval. The spin texture and the charge density of this soliton is shown schematically in Fig. 2. The size ξ of the soliton is determined by the competition between the different terms of the functional; the Hartree and the quartic term ($B>0$) tend to make the texture large whereas the SO and the quadratic term ($\rho<0$) try to make it small. Figure 3 shows the energy and size of the charged wall as a function of λ_{SO}. For $\lambda_{SO}=0$, the functional (6) has $U(1)$ symmetry and the soliton has zero energy being extended to the whole wall, i.e., $\xi=L_y$. SO interaction ($\lambda_{SO}\neq0$) reduces the soliton size. ξ takes a value much smaller than L_y and the energy of the excitation becomes finite. The energy of the soliton is the energy cost to add an electron to the DW. Since this value is smaller than the energy gap in the S and F phases, $\sim e^2/\epsilon l$, we expect that extra charges in the system will become located at the DW. The coupling λ_{SO} depends of the system characteristics as the DW width W_X. Typical values[25,26] vary from 2×10^{-4} to 8×10^{-4}. In this range, the energy of the soliton is much smaller than the DW gap obtained in HF calculations (Fig. 1), $\sim e^2/\epsilon l$. The HF gap is dominated by exchange Coulomb interactions, and represents the excitation gap when the isospin order parameter is held fixed. The actual low-energy charge excitations come from fluctuations in the order parameter field. Once again, V_H has been essential in the properties of the (in this case charged) DW.

Transport properties of the DW. Let us analyze the transport through a DW. If the chemical potential of the system, fixed by impurities or edge states, is located at the energy gap of the DW, no current can flow parallel to the DW.[27] The
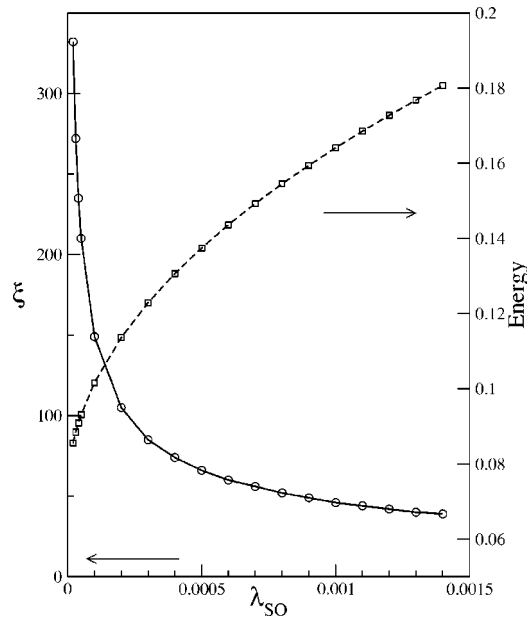
FIG. 3. Size ξ (left axis, in units of l) and energy (right axis, in units of $e^2/\epsilon l$) of the soliton in a charged DW as a function of the SO coupling.

only possibility for the carriers is to pass across the DW. On the contrary, when the chemical potential resides in a band, there is a perfect unity transmission along the direction parallel to the wall and no carriers are passing through the DW. The same argument is valid for the charged excitations gap instead of that of the uncharged DW. The current through a DW separating a F from a U phase is different from zero if and only if the chemical potential lies on the charged excitation gap of the DW.

In the absence of SO coupling, there is not a gap for the charged excitations and, consequently, transport across the DW is not possible. This is in agreement with spin conservation arguments; when $\lambda_{SO}=0$, the spin is a good quantum number and no transport of charge through the DW is possible unless some other scattering mechanism is able to flip

an electron spin. The hyperfine coupling to nuclear spins has been sometimes invoked,[9] but a nonzero SO coupling is much more efficient to flip electron spins. Due to SO, the solution of the functional (6) changes smoothly its isospin when going from one side to the other of the wall. One electron with a given (real) spin can pass across the wall smoothly flipping its spin. The finite, due to SO coupling, energy of the soliton is rather small, which means that very few electrons pass across the wall flipping their spins because a small gap reflects coupling between very few states at the two sides of the barrier.[28] In other words, there is a small current passing across a domain wall with a large resistance. This explains the large resistance observed in different systems where domains exist.[2,3,9,11–13]

A final question to comment on is the role played by nuclear spins. Apart from the possible role played in the process of domain formation, nuclear spins are not needed, in our picture, in the process of carrier transport. However, due to the hyperfine interaction, nuclear spins will suffer a dynamic nuclear spin polarization within the electronic domains. This is very important because, if current is turned off for a while, as done in some experiments,[13,29] the electrons in different domains immediately lose memory of theirs spins. However, nuclear spins relax so slowly in time that they serve as memory reservoirs of spin states and, if electronic current is reestablished after a while, the domains will reappear in exactly the same position they had before.

In summary, we study DW in a quantum Hall Ising ferromagnet at $\nu=2$ by means of a functional theory supported by a HF calculation. In a neutral DW, Hartree repulsion prevents the appearance of a fan phase provoked by a negative stiffness. When the system is charged, electrons are trapped as solitons at the DW. Hartree and SO interactions determine the energy and size of these solitons. Finally, a discussion of transport through the DW is presented.

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