Néel transition, spin fluctuations, and pseudogap in underdoped cuprates by a Lorentz invariant four-fermion model in 2+1 dimensions

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We show that the Néel transition and spin fluctuations near the Néel transition in planar cuprates can be described by an SU(2) invariant relativistic four-fermion model in 2+1 dimensions. Features of the pseudogap phenomenon are naturally described by the appearance of an anomalous dimension for the spinon propagator.

The two dimensional one-band repulsive Hubbard model[1] is considered as one of the best candidates for describing microscopically the planar cuprate high-T_c superconductors. The strong-coupling limit of the Hubbard model is equivalent to the t-J model.[2] In the t-J model at half-filling, on each site of a square lattice, on average a single electron interacts antiferromagnetically with its nearest neighbors and the system is an antiferromagnetic (AF) insulator (a Mott insulator) described by Néel ordering. By introducing doped holes, thus charge carriers, on this lattice, the AF interaction is frustrated and a transition from the Néel ordered to the disordered, so-called spin-gapped phase or normal state occurs. For the Hubbard-Heisenberg model at and near half-filling Affleck and Marston showed, using a leading-order $1/N$ expansion, that the ground state is the π-flux phase for appropriate values of the hopping amplitude t, doping δ, and AF interaction J.[3] The number N is the generalization of the physical up-down spins, $N=2$, to N types. In the π-flux phase the spinon spectrum has the dispersion

$$E_k \simeq 2|\chi| a \sqrt{\cos^2 ak_1 + \cos^2 ak_2},$$

where $|\chi|$ is the absolute value of the π-flux phase order parameter. This spectrum is gapless at the two Fermi vectors $\vec{f}p = (\pi/2a, \pm \pi/2a)$ in the reduced Brillioun zone of the even and odd lattices with lattice spacing a. The linearization around these Fermi points gives a continuum (2+1)-dimensional massless Dirac theory describing N flavors of four-component Dirac spinors having a global U($2N$) symmetry.[3,4] At half-filling, the Dirac spectrum is isotropic and the flux-phase order parameter is equal to the so-called d-wave pairing order parameter $|\Delta| = |\chi|$.[5]

Recently Kim and Lee addressed the question, how the spin gapped phase is connected to the Néel ordered phase at zero doping.[6] In their work the mean-field π-flux phase of Affleck and Marston is taken as the reference state for describing the spin fluctuations around the AF Fermi points. By introducing gauge-field fluctuations, enhancing AF correlations around the π-flux phase solution, Kim and Lee propose, along the lines of Ref. 7 that Néel ordering is described by dynamical symmetry breaking (DSB) and mass generation in QED$_3$. Néel ordering corresponds to the dynamically broken phase, which is characterized by a ''mass gap'' for the spinon spectrum and Nambu-Goldstone bosons as bound states of spinons and antispinons. These Nambu-Goldstone bosons are the massless AF spin waves. The disordered spin-gapped phase is equivalent to the subcritical symmetric phase and is characterized by the existence of massless spinons and unstable bound states as broad resonances. These resonances supposedly correspond to the spin excitations observed in the normal state and superconducting state of underdoped and optimally doped cuprates.[8]

Although the physical picture sketched by Kim and Lee is plausible, it was pointed out that QED$_3$ is not an appropriate model for describing unstable bound states.[9] The main problem being that DSB in QED$_3$ is not a phase transition of the second-order type, but a so-called conformal phase transition which does not allow light unstable bound states in the symmetric phase.[10] As an alternative for the gauge interactions, we propose that relevant, Lorentz-invariant four-fermion or four-Fermi (4F) interactions with an ultraviolet stable fixed point for the four-fermion coupling drive the AF ordering. The AF lattice Heisenberg interaction $H = J\Sigma_{\langle x,y \rangle} \vec{S}_x \cdot \vec{S}_y$, expanded around the two Fermi points, gives rise to SU(2) invariant attractive 4F terms in the action of the model. At sufficient strong AF coupling DSB occurs, giving rise to the Néel state. Despite the fact that the real temperature is not necessarily zero, the time-dependent quantum fluctuations and ordering are given by a zero-temperature 4F model. The mean-field π-flux phase local order parameter χ describes the thermodynamic equilibrium state and is therefore time independent. In addition, there is no need for a chemical potential in the proposed model, since the (nearly) half-filling constraint has already been taken into account via the mean-field equilibrium real bosonic Lagrange field.[3] The present idea is partly inspired by Ref. 4, where it was suggested, by analyzing various lattice 4F operators, that the only relevant operators are those which are Lorentz invariant in the continuum.

We adopt the spin liquid ansatz of Refs. 3 and 6; the spin liquid is described by the mean-field large-N π-flux-phase for low doping. The flux-phase order parameter $|\chi|$ depends on temperature and doping. The AF Heisenberg interaction is reinstated for this spin liquid. The fluctuations of the holes are ignored, and their effect is only included via their mean-field effect on reducing the AF exchange J to $J_e = J(1-\delta)^2$.[11] Therefore, on a lattice with spacing $a' = a/\sqrt{2}$, we consider the action

$$S = \int dt \left[\sum_{\langle x,y \rangle} c_\alpha^\dagger(x,t)(i\partial_t - \chi_{yx}) c_\alpha(y,t) - H_I \right], \quad (1)$$

with

$$H_I = \sum_{\langle x,y \rangle} J_e \vec{S}_x \cdot \vec{S}_y, \quad (2)$$

and where $\langle x,y \rangle$ denotes nearest neighbors on an isotropic cubic lattice. The index $\alpha = \uparrow, \downarrow$ labels the spin components and the spin operator is

$$\vec{S}_x = c_\alpha^\dagger(x,t) \vec{\sigma}_{\alpha\beta} c_\beta(x,t)/2,$$

where σ are the Pauli matrices and $c, c^\dagger$ are the spinon, antispinon operators. A particular representation for this Hermitian π-flux-phase hopping parameter χ_{yx} is[3]

$$\chi_{x \pm a_1, x} = i|\chi|, \qquad \chi_{x \pm a_2, x} = 1|\chi|,$$

with the nearest-neighbor vectors $\vec{a}_1 = (a', 0)$ and $\vec{a}_2 = (0, a')$. The low-energy behavior of the kinetic term in Eq. (1) is known to be equivalent to a two-flavor massless Dirac theory with the action

$$S_k = \int dt \int_{k \leqslant \Lambda} d^2k \, \bar{\psi}_\alpha [i\partial_t \gamma^0 + c(k_1 \gamma^1 + k_2 \gamma^2)] \psi_\alpha, \quad (3)$$

where $c = 2|\chi|a$ is the "speed of light" and $\bar{\psi} = \psi^\dagger \gamma^0$. The momentum cutoff Λ is naturally related to the lattice spacing via $\Lambda \simeq \pi/2a$. The fields $\psi, \psi^\dagger$ are four-component spinors,

$$\psi_\alpha = \begin{pmatrix} c_{e1\alpha} \\ c_{o1\alpha} \\ c_{o2\alpha} \\ c_{e2\alpha} \end{pmatrix}, \qquad \psi_\alpha^\dagger = (c_{e1\alpha}^\dagger \quad c_{o1\alpha}^\dagger \quad c_{o2\alpha}^\dagger \quad c_{e2\alpha}^\dagger), \quad (4)$$

where 1,2 labels the Fermi point and e, o label fields on the even and odd lattices, respectively. The 4×4 γ matrices satisfy a Clifford algebra corresponding to the Minkowskian metric $g^{\mu\nu} = \text{diag}(1, -1, -1)$. The following representation for the γ matrices has been chosen:

$$\gamma^0 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}, \qquad \gamma^1 = \begin{pmatrix} i\sigma_1 & 0 \\ 0 & -i\sigma_1 \end{pmatrix},$$

$$\gamma^2 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix},$$

where σ_i are Pauli matrices, acting on the even and odd site fermion operators.

Expanding the AF Heisenberg interaction (2) around the two Fermi points, we obtain

$$H_I = 4J_e a^2 \int_{p_1, p_2, k_1, k_2 \leqslant \Lambda} (2\pi)^2 \delta(\vec{p}_1 + \vec{k}_1 - \vec{k}_2 - \vec{p}_2)$$

$$\times [c_{e1\alpha}^\dagger(\vec{p}_1, t) c_{e1\beta}(\vec{p}_2, t) + c_{e2\alpha}^\dagger(\vec{p}_1, t) c_{e2\beta}(\vec{p}_2, t)]$$

$$\times [c_{o1\gamma}^\dagger(\vec{k}_1, t) c_{o1\delta}(\vec{k}_2, t) + c_{o2\gamma}^\dagger(\vec{k}_1, t) c_{o2\delta}(\vec{k}_2, t)]$$

$$\times (\delta_{\alpha\delta}\delta_{\gamma\beta} - \delta_{\alpha\beta}\delta_{\gamma\delta}/2), \quad (5)$$

where the SU(2) Fierz identity has been used, $\vec{\sigma}_{\alpha\beta} \cdot \vec{\sigma}_{\gamma\delta} = 2\delta_{\alpha\delta}\delta_{\gamma\beta} - \delta_{\alpha\beta}\delta_{\gamma\beta}$. Using Eq. (4), Eq. (5) can be written as

$$H_I = -J_e a^2 \int_{p_1, p_2, k_1, k_2 \leqslant \Lambda} (2\pi)^2 \delta(\vec{p}_1 + \vec{k}_1 - \vec{k}_2 - \vec{p}_2)$$

$$\times (\bar{\psi}_\alpha \psi_\beta \bar{\psi}_\gamma \psi_\delta - \psi_\alpha^\dagger \psi_\beta \psi_\gamma^\dagger \psi_\delta)(\delta_{\alpha\delta}\delta_{\gamma\beta} - \delta_{\alpha\beta}\delta_{\gamma\delta}/2). \quad (6)$$

Subsequently, it is straightforward to show that Eq. (6) together with Eq. (3) gives rise to the action $S = S_k + S_I$, with

$$S_I = J_e a^2 \int dt \int_{p_1, p_2, k_1, k_2 \leqslant \Lambda} (2\pi)^2 \delta(\vec{p}_1 + \vec{k}_1 - \vec{k}_2 - \vec{p}_2)$$

$$\times \left\{ \sum_{A=1}^{3} [(\bar{\psi}\tau^A \psi)^2 - (\psi^\dagger \tau^A \psi)^2] - \frac{(\bar{\psi}\psi)^2}{4} + \frac{(\psi^\dagger \psi)^2}{4} \right\}, \quad (7)$$

with τ^A the generators of the SU(2) symmetry with $\text{Tr}(\tau^A \tau^B) = \delta^{AB}$ ($\tau^A = \sigma^A/\sqrt{2}$), and $\bar{\psi}\tau^A \psi = \Sigma_{\alpha,\beta} \bar{\psi}_\alpha \tau_{\alpha\beta}^A \psi_\beta$. The action (7) is invariant under global SU(2)×U(1) corresponding to the spin orientation symmetry and total spin conservation. Moreover the action is invariant under the discrete transformations; space reflection, parity, and the combined CT (charge-conjugation and time-reversal) transformations. Naturally, it is invariant under continuous rotations in space. However, the terms of the form $\psi^\dagger \psi$ are not invariant under Lorentz boosts and therefore the action is not relativistic invariant.

In what follows, we shall show that the action (7) lies in the same universality class as that of a Lorentz-covariant SU(2) invariant (2+1)-dimensional 4F model for two massless fermion flavors:

$$S = \int d^3x \left[\bar{\psi} i \hat{\partial} \psi + \frac{cG}{2} \sum_{A=1}^{3} (\bar{\psi}\tau^A \psi)^2 \right], \quad (8)$$

where $x_0 = ct, \hat{\partial} = \gamma^\mu \partial/\partial x^\mu$, and where G is an attractive four-fermion coupling, $cG/2 \simeq J_e a^2$. The universality only holds close to a critical point or ultraviolet stable fixed point G_c of a second-order phase transition of Eq. (8) at which the SU(2) symmetry is dynamically broken to a U(1) symmetry. There is a dynamical generation of a parity-conserving "mass term" m_s connected with the appearance of a nonzero vacuum condensate $\langle \bar{\psi}\tau^3 \psi \rangle$. This Lorentz-invariant condensate describes a staggered spin expectation value giving rise

to AF ordering.[6,7] In the broken phase, two massless pions or spin waves appear as Goldstone bosons.[7]

The spinon propagator $S_{\alpha\beta}$ can be written as

$$S_{\alpha\beta}(p) = \frac{\hat{p}A(p)\delta_{\alpha\beta} + \sqrt{2}m_s\tau^3_{\alpha\beta}}{A^2(p)p^2 - m_s^2},$$

with Minskowskian momentum $p^2 = p_0^2 - \vec{p}\cdot\vec{p}$ and where $A(p)$ is the fermion wave function. In the Hartree-Fock approximation, the equation for m_s gets contributions only from the tadpole diagram and the fermion wave function $A(p) = 1$. Setting $c = 1$, the gap equation for m_s reads

$$m_s = Gi \int_M \frac{d^3p}{(2\pi)^3} \frac{4m_s}{p^2 - m_s^2},$$

with the subscript M denoting the Minskowskian metric with cutoff Λ. This gap equation gives rise to the familiar critical coupling $g_c^{(s)} \equiv 2G_c\Lambda/\pi^2 = 1$. Above the critical coupling $g = 2G\Lambda/\pi^2 > g_c^{(s)}$, the SU(2) spin symmetry is broken and a $\langle\bar{\psi}\tau^3\psi\rangle$ condensate is formed.

Now let us show that the nonrelativistic interaction terms in Eq. (7) are irrelevant close to $g_c^{(s)}$. We investigate the generation of a mass m_u connected with the uniform spin expectation value $\langle\psi^\dagger\tau^3\psi\rangle$.[6] In the Hartree-Fock approximation, keeping only the Lorentz-noninvariant terms, the spinon propagator is of the form

$$S_{\alpha\beta}(p) = (\hat{p}\delta_{\alpha\beta} - m_u\gamma_0\tau^3_{\alpha\beta})K_\alpha,$$

$$K_\alpha = \{[p_0 - (-1)^{\alpha+1}m_u/\sqrt{2}]^2 - p_1^2 - p_2^2\}^{-1},$$

with $\alpha = 1,2$ ($\alpha = \uparrow,\downarrow$). The gap equation for m_u then reads

$$\frac{m_u}{4Gi} = \int_M \frac{d^3p}{(2\pi)^3}\left[\frac{m_u}{2}(K_1 + K_2) - \frac{p_0}{\sqrt{2}}(K_1 - K_2)\right].$$

The bifurcation equation is

$$\frac{1}{4Gi} = -\int_M \frac{d^3p}{(2\pi)^3}\frac{p_0^2 + p_1^2 + p_2^2}{(p_0^2 - p_1^2 - p_2^2)^2},$$

giving rise to a critical coupling $g_c^{(u)} \equiv 2G_c\Lambda/\pi^2 = 3$. This shows that when g is close to the critical value $g_c^{(s)}$, it is far ($|g - g_c^{(u)}|/g \simeq 2$) from the critical regime of $g^{(u)}$. Upon increasing g it first encounters the critical value for staggered magnetization. Hence, the nonrelativistic terms in Eq. (7) are irrelevant in that area of the coupling constant space.

Naturally, the Hartree-Fock approximation ignores 4F fluctuations and therefore gives a rather crude description of the critical behavior. Nevertheless the irrelevance of the non-Lorentz-covariant terms in Eq. (7) to $g_c^{(s)}$ can be demonstrated in more advanced approximations, such as the $1/N$ expansion. In particular, the SU(2) Heisenberg antiferromagnets can be generalized to SU(N) and studied in the $1/N$ expansion.[3] This is analogous to the SU(N) generalization of Eq. (8). Contrary to models of the Gross-Neveu type,[12] such an expansion resembles the topological $1/N$ expansion of 't
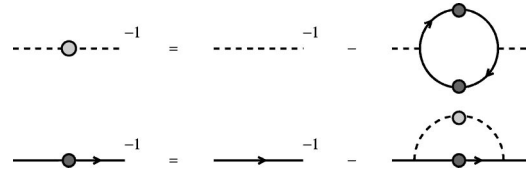

FIG. 1. The leading large-N truncation or ladder approximation for the Schwinger-Dyson equations for the spin-wave propagator $\Delta_\sigma^A(p)$ (dashed line with blob) and the spinon propagator $S(p)$ (solid line with blob).

Hooft,[13] corresponding to an expansion in planar Feynman diagrams instead of Fermion loops.

Recently, the generalization of Eq. (8) to N fermion flavors with a SU(N)$\times$U(1) invariant 4F potential containing $N^2 - 1$ terms has been studied in the planar large-N approximation.[14] After a Hubbard-Stratonovich transformation ($\sigma^A = -G\bar{\psi}\tau^A\psi$), the action (8) can be expressed as

$$S = \int d^3x\left\{\bar{\psi}i\hat{\partial}\psi - \sum_A \bar{\psi}\tau^A\psi\sigma_A - \frac{1}{2G}\sum_A \sigma_A^2\right\}. \quad (9)$$

In the SU(N) case, the spin label A runs from 1 to $N^2 - 1$ and the fermion label from 1 to N. Thus there are $N^2 - 1$ composite operators, giving rise to $N^2 - 1$ spin propagators (connected), $i\Delta_\sigma^A(q) \equiv \langle\sigma_A(q)\sigma_A(-q)\rangle_c$, and N spinon propagators, $iS(p) \equiv \langle\psi(p)\bar{\psi}(-p)\rangle$. This allows for a 't Hooft topological $1/N$ expansion as is conjectured in Ref. 14. Moreover, it was argued in Ref. 14 that the leading large-N or planar approximation reduces to the ladder approximation for the so-called Yukawa vertex [i.e., $\Gamma_\sigma^A(k,p) = \tau^A 1$]. The Yukawa vertex Γ_σ^A is the fully amputated three-point interaction vertex for the action (9). In the ladder approximation, the Schwinger-Dyson equations for the spin propagators Δ_σ^A and the spinon propagators S form a closed set, as depicted in Fig. 1.

For the physically relevant case ($N = 2$), the propagators $\Delta_\sigma^1, \Delta_\sigma^2$ of the auxiliary fields σ_1, σ_2 describe the Goldstone modes, whereas the field σ_3 acquires a nonzero, but Lorentz-covariant, vacuum expectation value in the broken phase ($\langle\sigma_3\rangle \neq 0, g > g_c^{(s)}$), describing the Néel state, corresponding to the symmetry-breaking pattern SU(2)$\rightarrow$U(1). In the subcritical region ($g \leq g_c^{(s)}$) the propagators of all three auxiliary fields describe Goldstone precursor modes that come down in energy as the transition is approached. Hence, the staggered spin fluctuations or spin waves are described by the propagators of the auxiliary fields σ_A, which become light close to the critical point $g = g_c^{(s)}$.

In Ref. 14, the Schwinger-Dyson equations represented in Fig. 1 were solved. It was shown that the fermion or spinon propagator S acquires an anomalous dimension ζ via the fermion wave function $A(p) \sim (\Lambda/p)^\zeta$, so that at the critical coupling $G = G_c$ the fermion (spinon) propagator scales as

$$S^{-1}(p) \simeq \hat{p}(-\Lambda^2/p^2)^{\zeta/2}. \quad (10)$$

The dependence of the anomalous dimension ζ on N is determined, and for $N = 2$, we have $\zeta \approx 0.21$[14]. Moreover the

model described in Ref. 14, with the dimensionless 4F coupling $g=2G\Lambda/\pi^2$, has an ultraviolet stable fixed point at $g=g_c=1+2\zeta$. The appearance of a positive anomalous dimension for S is considered to provide a description of the pseudogap phenomenon.[15–17]

An important point is whether the critical coupling $g_c^{(s)} \simeq 1+2\zeta$ is in agreement with the estimations for the physical parameters. Since $Gc/2 \simeq J_e a^2, c \simeq 2|\chi|a, \Lambda \simeq \pi/2a$, we have that $g \simeq J_e/\pi|\chi|$. With the estimation $g_c^{(s)} \simeq 1.4$, we obtain that a value $|\chi| \approx 0.23 J_e$ would get us close to criticality. This is remarkably close to the mean-field value $|\chi| \approx 0.24 J$ given in Ref. 3.

In the ladder approximation (Fig. 1), the connected propagator of the σ_A field in momentum space reads

$$\Delta_\sigma^{A-1}(q) \equiv -\frac{1}{G} + i \int_M d^3k \operatorname{Tr}[\tau^A S(k+q)\tau^A S(k)],$$

where $S(k)$ is the full spinon propagator, given by Eq. (10), see also Fig. 1. In the subcritical or SU(N) symmetric regime, we can take $S(k)=\hat{k}/[k^2 A(k)]$. The integral can be performed, and $\Delta_\sigma^A(q)$ has the following scaling form for $|q| \ll \Lambda$ $(q^2=q_0^2-q_1^2-q_2^2)$:

$$\Delta_\sigma^A(q) \simeq -\frac{C}{\Lambda} \frac{(-\Lambda^2/q^2)^{\zeta+1/2}}{[1+(-m_\sigma^2/q^2)^{\zeta+1/2}]}, \tag{11}$$

where C is some flavor dependent positive constant.[14] The mass m_σ (spin-wave stiffness) denotes the position of the resonance peak given by the imaginary part of Eq. (11), and plays the role of the inverse correlation length,[9]

$$m_\sigma/\Lambda \sim (g_c^{(s)}-g)^{1/(1+2\zeta)}. \tag{12}$$

From these expressions, it follows that the critical exponents η, ν, and γ are $\eta=1-2\zeta$, $\nu=1/(1+2\zeta)$, and $\gamma=1$. These exponents satisfy the three-dimensional hyperscaling equations.[14] Moreover, in Lorentz-invariant field theories, the scaling of the energy equals the scaling of momentum, and consequently the dynamical scaling exponent $z=1$.

In experiments, the isotropic dynamical susceptibility $\chi''(q)$ is measured,[8] with $q=(\omega,\vec{q})$. In Ref. 8 magnetic resonances were observed in underdoped and optimally doped YBa$_2$Cu$_3$O$_{6+x}$, with the famous 41-meV peak at optimal doping. For lower doping the resonance peak shifts to lower energies. It is tempting to assume that, along the lines of Refs. 6 and 9, these resonances might be described by the Dirac models, see Fig. 2.

However, optimal doping ($\delta \simeq 0.2$) is rather far from Néel doping, which is one order of magnitude less, $\delta_c \simeq 0.02$.[18] Therefore the question is whether we are "sufficiently" close to the scaling region of the Néel transition.

Let us compare a couple of (2+1)-dimensional Lorentz-covariant quantum field models, capable of describing AF fluctuation and Néel transition. For instance, Kwon[19] presented a nodal d-wave spin liquid model for the Néel transition, which has the universality class of the Gross-Neveu model. This model has a single AF order parameter, corresponding to the singlet composite order parameter $\langle \bar{\psi}\psi \rangle$,
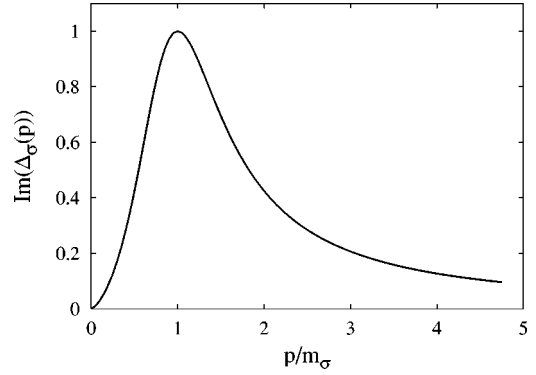

FIG. 2. The spin susceptibility or spin-correlation function $\operatorname{Im}[\Delta_\sigma^A(p)]$ vs p/m_σ, with the peak renormalized at unity, $\zeta=0.21$ ($\eta=0.58$).

with a single two-component Dirac fermion ($N=1$). The universality class of the model presented in this paper [i.e., Eq. (8)] deviates considerably from the Gross-Neveu model.[14] Although the anomalous dimension η for both models is comparable: $\eta \approx 0.58$ for the present model, and $\eta=16/(3\pi^2 N) \approx 0.54$ ($N=1$) for the Gross-Neveu model, the main difference is in the value of the anomalous dimension of the fermion propagator, which for the present model is $\zeta \approx 0.21$, whereas for the Gross-Neveu model it is $\eta=2/(3\pi^2 N) \approx 0.07$ ($N=1$). Another approach was adopted in Ref. 20, where the antiferromagnetic correlations and the Néel transition are described by the nonlinear sigma model in the large-N expansion. In that work, the order parameter is a three-component one, corresponding to the three spin-1 components, and no Dirac fermions are taken into account. The anomalous dimension η for the spin waves turned out to be $\eta=8/(3\pi^2 N) \approx 0.09$ ($N=3$), which is considerably smaller than η for the two above-mentioned 4F Dirac models. In this paper, we have a three-component order parameter (the composite spin degrees) and two flavors ($N=2$) of four-component Dirac fermions. To determine which universality class describes the Néel transition, the low doping region $\delta \sim \delta_c$ needs to be examined in more detail experimentally.

Since in the subcritical phase, all three correlation functions of the staggered spin components are degenerate, the imaginary part of the correlation function $\Delta_\sigma^A(q)$ is directly proportional to the so-called odd acoustic mode of $\chi''(q)$.[18] The spin-correlation function only gets low-energy contributions from the staggered spin operators:

$$\langle S_A(p)S_B(-p)\rangle \propto \langle \sigma_A(p)\sigma_B(-p)\rangle \simeq \delta_{AB} i\Delta_\sigma^A(p).$$

Moreover, since g is proportional to J_e, g reduces when doping is increased.[11] The critical coupling $g_c^{(s)}$ of the Néel transition is a quantum critical point[21] and corresponds to a critical Néel doping $\delta_c \ll 1$. Equation (12) gives the relation between the position of the peak of the magnetic resonance and the doping rate δ. Assuming δ_c is sufficiently close to zero, we obtain that the resonance peak is linearly proportional to doping (for small doping rates with $\delta > \delta_c$). Conse-

quently, the peak position moves to lower energies when doping is reduced; near δ_c the peak height diverges. For doping values $\delta < \delta_c$ spin waves appear.

In summary, we have shown that a $(2+1)$-dimensional Lorentz-invariant 4F model with a global spin SU(2) symmetry describes the low-energy time-dependent "quasiparticle" spin excitations of the t-J model near the AF wave vector. The spin excitations are given in terms of quantum fluctuations around the mean-field π-flux phase. The Néel transition is described as the DSB of SU(2)$\rightarrow$U(1) in the model. The magnitude of the critical coupling $g_c^{(s)}$ of the 4F model turned out to be in good agreement with the input parameters J and $|\chi|$, which define the flux-phase spin liquid. Nevertheless, the question of the precise effects of hole dop-

ing on the AF interaction and the anisotropy of the Dirac spectrum is left open. For the future, it would be interesting to include the contribution of the slave bosons (holons) (e.g., see Refs. 6 and 18) in order to take into account anisotropy and to determine the effective AF coupling J_e. We demonstrated that the unstable spin modes found in experiments might be well described by the Nambu-Goldstone boson precursor modes in the subcritical region. Furthermore, the spinon propagator acquires an anomalous dimension, but remains gapless near the AF wave vector in the normal state. The appearance of a spinon anomalous wave function gives a natural description of the pseudogap phenomenon.

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