Spin transport in inhomogeneous magnetic fields: A proposal for Stern-Gerlach-like experiments with conduction electrons

Jaroslav Fabian[1,2,3] and S. Das Sarma[1]
[1]*Condensed Matter Theory Center, Department of Physics, University of Maryland at College Park, College Park, Maryland 20742-4111*
[2]*Max-Planck Institute for the Physics of Complex Systems, 01187 Dresden, Germany*
[3]*Institute for Theoretical Physics, Karl-Franzens University, 8010 Graz, Austria*
(Received 7 May 2002; published 26 July 2002)

Spin dynamics in spatially inhomogeneous magnetic fields is studied within the framework of Boltzmann theory. Stern-Gerlach-like separation of spin up and spin down electrons occurs in ballistic and diffusive regimes, before spin relaxation sets in. Transient dynamics and spectral response to time-dependent inhomogeneous magnetic fields are investigated, and possible experimental observations of our findings are discussed.

DOI: 10.1103/PhysRevB.66.024436 PACS number(s): 72.25.Ba, 72.25.Dc, 72.25.Rb

Spin of mobile carriers (electrons and holes) plays an active central role in the current spintronics efforts,[1,2] where electronic properties are determined, to a great degree, by the presence of nonequilibrium spin. The spin response to temporal and spatial changes of the magnetic environment determines various properties of such devices. In the recently proposed magnetic diode,[3–5] for example, an inhomogeneous magnetic environment results from inhomogeneous magnetic doping. How a magnetic diode adjusts to the switching of an external magnetic field and to changes in the applied bias depends on the way the mobile carriers reach equilibrium. Since inhomogeneous magnetic fields are ubiquitous in spintronic devices (mostly due to the presence of magnetic/nonmagnetic interfaces), it is important to understand nonequilibrium spin evolution in such fields. In this paper we investigate in detail the transient behavior of conduction electron spins, within a Boltzmann equation model. A unique feature of the model is that it is exactly soluble, allowing a detailed study of the transition from the ballistic to the diffusive regime. We show that in inhomogeneous magnetic fields a spatial separation between spins, an analog of the Stern-Gerlach effect, occurs before spin relaxation begins, but spin current vanishes much sooner, at times of the order of transit times.

The model we consider is a degenerate electron gas (a metal or semiconductor) in a magnetic field with the largest component in the $\hat{\mathbf{z}}$ direction, and with a gradient in that direction. The field has also transverse components (as required by $\nabla\cdot\mathbf{B}=0$), which are essential in rendering SG with electron beams useless,[6] but which do not hinder an effective spin separation of conduction electrons (see below). We show that an effective spatial spin separation, along with a flow of spin, is possible within ballistic and diffusive dynamics, demonstrating a Stern-Gerlach-like (SG) effect with conduction electrons. The formalism we use, linear response theory within the Boltzmann equation, has been applied earlier in various forms in transport in general,[7,8] and more specifically for spin transport in the framework of conduction electron spin resonance[9,10] and giant magnetoresistance.[11] Here we apply this formalism to a special case of spin dynamics in an inhomogeneous magnetic field, and solve it exactly for specific boundary conditions.

In the presence of a uniform electric field $\mathbf{E}$ and inhomogeneous magnetic field $\mathbf{B}$, semiclassical dynamics of electrons in (nonmagnetic) solids is governed by the Boltzmann equation

$$\frac{\partial f_{\mathbf{k}\lambda}}{\partial t}+\mathbf{v}_{\mathbf{k}}\frac{\partial f_{\mathbf{k}\lambda}}{\partial \mathbf{r}}-e\mathbf{E}_{\lambda}\frac{\partial f_{\mathbf{k}\lambda}}{\partial \hbar\mathbf{k}}=-\frac{\delta\overline{f}_{\mathbf{k}\lambda}}{\tau}-\frac{\delta f_{\mathbf{k}\lambda}}{T_1}, \qquad (1)$$

where $f_{\mathbf{k}\lambda}\equiv f_{\mathbf{k}\lambda}(\mathbf{r},t)$ is the distribution function of electrons with lattice momentum $\mathbf{k}$ (band index is suppressed) and spin λ (1 or $\uparrow$ for up and 1 or $\downarrow$ for down), at point $\mathbf{r}$ and time t. The notation for the drift field is simplified as $\mathbf{E}_{\lambda}=E\hat{\mathbf{z}}+\lambda(\mu_B/e)\partial B\hat{\mathbf{z}}/\partial z$, where μ_B is the Bohr magneton and the electron g factor is taken to be 2; the fields are oriented in the $\hat{\mathbf{z}}$ direction. Band velocity $\mathbf{v}_{\mathbf{k}}\equiv\partial\varepsilon_{\mathbf{k}}/\partial\hbar\mathbf{k}$, with $\varepsilon_{\mathbf{k}}$ standing for band energy (we consider systems with inversion symmetry where band energy is spin independent). Two momentum relaxation processes are distinguished in Eq. (1). First, spin-conserving momentum scattering with rate $1/\tau$ (τ is momentum relaxation time), and leading to a quasiequilibrium distribution $\overline{f}_{\mathbf{k}\lambda}$ ($\delta\overline{f}_{\mathbf{k}\lambda}\equiv f_{\mathbf{k}\lambda}-\overline{f}_{\mathbf{k}\lambda}$), in which spin up and down electrons have different chemical potentials. Second, spin-flip momentum relaxation with rate $1/T_1$ (T_1 is spin-relaxation time), and leading to complete (momentum and spin) equilibrium at the local and instantaneous $\mathbf{B}$ field: $f^0_{\mathbf{k}\lambda}=f_0[\varepsilon_{\mathbf{k}}+\lambda\mu_B B(z,t)]$, where $f_0(\varepsilon)=1/[\exp(\varepsilon-\mu)/k_BT+1]$ is the Fermi-Dirac distribution function with chemical potential μ, temperature T, and Boltzmann constant k_B ($\delta f_{\mathbf{k}\lambda}=f_{\mathbf{k}\lambda}-f^0_{\mathbf{k}\lambda}$). In writing the Boltzmann equation as Eq. (1), we neglect the Lorentz force as unimportant, as the largest part of the magnetic field is oriented along the same direction as the drift velocity itself (see below for the reasons why we also neglect the orbital effects of the small transverse magnetic fields). The dynamics of the transverse spin components (x and y) is also not considered, as it is masked by their fast precession about $\mathbf{B}$. Finally, the contribution of the electronic magnetization to the magnetic field is neglected for our nonmagnetic systems. The relaxation time approximation used in Eq. (1) is good for all practical purposes, but some caution is needed especially at low temperatures, as shown in Appendix A. Generalization of Eq. (1) to

 ©2002 The American Physical Society

k-dependent τ, T_1 (which can vary more wildly over the Fermi surface than τ—see Ref. 12), and the g factor is straightforward.

We search for the solution of Eq. (1) in the form $f_{\mathbf{k}\lambda} = f^0_{\mathbf{k}\lambda} - (\partial f_0/\partial\varepsilon_{\mathbf{k}})\phi_{\mathbf{k}\lambda}$, and write the quasiequilibrium distribution function as $\overline{f}_{\mathbf{k}\lambda} = f^0_{\mathbf{k}\lambda} - (\partial f_0/\partial\varepsilon_{\mathbf{k}})\mu_\lambda$, where the nonequilibrium chemical potential $\mu_\lambda = \langle\phi_{\mathbf{k}\lambda}\rangle$ is obtained self-consistently from the integral relation for spin conservation $\Sigma_{\mathbf{k}}\delta\overline{f}_{\mathbf{k}\lambda} = 0$. The angular brackets introduce the Fermi-surface averaging: $\langle\cdots\rangle \equiv \Sigma_{\mathbf{k}}\cdots\partial f_0/\partial\varepsilon_{\mathbf{k}}/\Sigma_{\mathbf{k}}\partial f_0/\partial\varepsilon_{\mathbf{k}}$. After linearizing Eq. (1) in terms of ϕ and E_λ, we obtain

$$-\lambda\mu_B\frac{\partial B}{\partial t} + \frac{\partial\phi_{\mathbf{k}\lambda}}{\partial t} + v_{z\mathbf{k}}\frac{\partial\phi_{\mathbf{k}\lambda}}{\partial z} + eEv_{z\mathbf{k}} = I(\phi_{\mathbf{k}\lambda}), \quad (2)$$

with the collision integral

$$I(\phi_{\mathbf{k}\lambda}) = -\frac{\phi_{\mathbf{k}\lambda} - \langle\phi_{\mathbf{k}\lambda}\rangle}{\tau} - \frac{\phi_{\mathbf{k}\lambda}}{T_1}. \quad (3)$$

Particle number conservation requires that $\langle\phi_{\mathbf{k}\uparrow} + \phi_{\mathbf{k}\downarrow}\rangle$ vanishes. The total spin density is

$$S = -g_F\mu_B B + (1/2)g_F\langle\phi_{\mathbf{k}\uparrow} - \phi_{\mathbf{k}\downarrow}\rangle, \quad (4)$$

where $g_F = -2\Sigma_{\mathbf{k}}\partial f_0/\partial\varepsilon_{\mathbf{k}}$ is the density of states, per unit volume, at the Fermi level. The first term on the RHS of Eq. (4) is the equilibrium spin value, yielding the electron gas paramagnetic susceptibility of $\mu_B^2 g_F$, while the second part δS represents the nonequilibrium contribution to spin density. The spin current density is

$$J_s = (1/2)g_F\langle v_{z\mathbf{k}}(\phi_{\mathbf{k}\uparrow} - \phi_{\mathbf{k}\downarrow})\rangle, \quad (5)$$

and is connected to S via the continuity equation derived from Eq. (2),

$$\partial S/\partial t + \partial J_s/\partial z = -\delta S/T_1, \quad (6)$$

which, together with the linear response equation (see Ref. 13 for a systematic treatment of linear spin transport)

$$J_s = -D\partial\delta S/\partial z, \quad (7)$$

where $D = \langle v_{z\mathbf{k}}^2\rangle\tau$ is the electron diffusivity constant, gives the diffusion formula for investigating diffusive spin transport

$$\frac{\partial\delta S}{\partial t} - D\frac{\partial^2\delta S}{\partial z^2} = -\frac{\delta S}{T_1} - \frac{\partial S_0}{\partial t}. \quad (8)$$

We first study transient phenomena that describe evolution of S and J_s towards equilibrium. Consider an unpolarized sample (whose band structure is assumed isotropic, $v_{\mathbf{k}} = \hbar\mathbf{k}/m$, where m is electron band mass), stretching from 0 to L along the z axis, with no charge current ($E = 0$). At $t = 0$, magnetic field $B(z) = B_0 + B_1 z$ is applied. Our goal is to find, by solving Eq. (2), $\langle\phi\rangle \equiv \langle\phi_\uparrow\rangle = -\langle\phi_\downarrow\rangle$ [so that $S = -g_F\mu_B B + g_F\langle\phi\rangle$], subject to the initial condition $\langle\phi(z,0)\rangle = \mu_B[B(z)]$, where $[B(z)] = B_0 + B_1[z]$ is the even periodic extension of $B(z)$ from interval $(0,L)$ to the whole z axis; thus formulated initial condition guarantees that spin current vanishes at the boundary–the assumption well justified in nonmagnetic interfaces with negligible spin-flip scattering. The spin profile can be written as $S = S_h + S_{in}$, where the homogeneous S_h and inhomogeneous S_{in} spin components are

$$S_h(t) = -g_F\mu_B\left(B_0 + \frac{1}{2}B_1 L\right)[1 - K(0,t)], \quad (9)$$

$$S_{in}(z,t) = g_F\mu_B B_1 4L\sum_{n\geqslant 0}\frac{\cos(q_n z)}{q_n^2 L^2}[1 - K(q_n,t)], \quad (10)$$

with $q_n \equiv (2n+1)\pi/L$. The sum comes from the Fourier expansion of $[B(z)]$. Kernel $K(q,t)$ describes the time evolution of the Fourier q components of the nonequilibrium spin

$$K(q,t) \equiv \delta S(q,t)/\delta S(q,0) = \langle\phi(q,t)\rangle/\langle\phi(q,0)\rangle. \quad (11)$$

Having the spin, the spin current can be calculated from the continuity equation as

$$J_s(z,t) = -\int_0^z dz'(\partial/\partial t + 1/T_1)\delta S(z',t). \quad (12)$$

Equation (2) gives also an exact *quasilocal* relationship between spin current and spin, valid at all times, and expressed in terms of the Fourier coefficients as

$$J_s(q,t) = iv_F\delta S(q,0)R_1(q,t) + iv_F\frac{1}{\tau}[\delta S * R_1](q,t). \quad (13)$$

Here v_F is the Fermi velocity,

$$R_1(q,t) = \exp(-t/\tau_m)\frac{d[\sin(x)/x]}{dx}, \quad (14)$$

with $x = qv_F t$ and $1/\tau_m = 1/\tau + 1/T_1$ the total momentum scattering rate, and the asterisks denote temporal convolution:

$$[f_1 * f_2](q,t) \equiv \int_0^t dt' f_1(q,t-t')f_2(q,t'). \quad (15)$$

In real space Eq. (13) expresses J_s in terms of derivatives (in principle of all odd orders—that is why the term quasilocal) of δS. In the diffusive regime, at $t \gg \tau$, the memory of the initial condition is lost, and Eq. (13) reduces to Eq. (7). An exact generalization of the diffusion Eq. (8) is obtained by substituting J_s from Eq. (13) to the continuity equation. The result is

$$K(q,t) = R_0(q,t) + \frac{1}{\tau}[K * R_0](q,t), \quad (16)$$

where now

$$R_0(q,t) = \exp(-t/\tau_m)\sin(qv_F t)/(qv_F t). \quad (17)$$

At $t > \tau$, Eq. (16) is equivalent to Eq. (8).

Equation (2) can be solved exactly with the help of Laplace transform. The solution is provided in Appendix B. The result is

$$K(q,t)=(qv_F\tau)^2 e^{-t/\tau_m}\left|\frac{\exp[qv_F t\cot(qv_F\tau)]}{\sin(qv_F\tau)^2}\right|^{\text{sing}}_{qv_F\tau}, \tag{18}$$

where the vertical bars denote the singular (principal) part of the Laurent series in terms of $qv_F\tau$, of the expression inside. An alternative formulation for the Kernel is

$$K(q,t)=-(\tau/t)e^{-t/\tau_m}\sum_{n=-\infty}^{-1} nF_n(qv_F t)(qv_F\tau)^n, \tag{19}$$

where functions $F_n(x)$ are described in Appendix B.

Let us consider the limiting behavior of $K(q,t)$ for the ballistic and diffusive regimes. For ballistic transport, $t\ll\tau$, the evolution kernel, Eq. (18), reduces to

$$K_{\text{ball}}(q,t)=\frac{\sin(qv_F t)}{qv_F t}. \tag{20}$$

This is the solution of Eq. (2) in the absence of scattering. A finite $S(z,t)$ in the ballistic case is solely due to SG effect of semiclassical separation of spins. At the left boundary,

$$S_{\text{ball}}(0)=-g_F\mu_B B_1 L(v_F t/2L), \tag{21}$$

in the middle $S_{\text{ball}}(L/2)=0$, and at the right boundary $S_{\text{ball}}(L)=-S_{\text{ball}}(0)$. The spin separation $S(L)-S(0)$ grows linearly with time, reaching its maximum of about $-g_F\mu_B B_1\ell$ at $t=\tau$. For the diffusive transport, $t\gg\tau$, Eq. (2) gives $K\approx K_{\text{diff}}$, where

$$K_{\text{diff}}(q,t)=\exp(-q^2Dt-t/T_1), \tag{22}$$

which is also a solution of Eq. (8). A new time scale $t_T=L^2/D\pi^2$ appears, for the transit time of a diffusing electron crossing the sample ($\pi^2 t_T$ is called the Thouless time). We consider L smaller than the spin diffusion length (which can be as large as a millimeter[14]), so that $t_T<T_1$. For $t<t_T$ the spin density grows diffusively,

$$S_{\text{diff}}(0)=g_F\mu_B B_1 L 2(Dt/L^2\pi)^{0.5}, \tag{23}$$

$S_{\text{diff}}(L)=-S_{\text{diff}}(0)$, while $S_{\text{diff}}(L/2)=0$; a large spin current flows in the middle of the sample. While the spin current vanishes at greater times, $t>t_T$ (when drift is being balanced by diffusion), an effective spin separation remains almost stationary until $t=T_1$, when spin relaxation establishes equilibrium. Note that the homogeneous component of spin S_h, evolves towards equilibrium with spin-flip processes only, since $K(0,t)=\exp(-t/T_1)$ at all times. In the following we will use normalized spin $\tilde{S}$ and spin current $\tilde{J}_s$ defined as $S=-g_F\mu_B B_1 L\tilde{S}$ and $J_s=-g_F\mu_B B_1 D\tilde{J}_s$, respectively.

An example of a transient evolution of spin and spin current is shown in Figs. 1 and 2. We take a model sample of size $L=1\,\mu$m, with realistic electronic parameters $\tau=0.1$ ps, $D=0.01$ m^2 s^{-1}, $v_F=(3D/\tau)^{0.5}\approx 5.5\times10^5$ m s^{-1}, $t_T=L^2/D\pi^2\approx 10$ ps, and $T_1=10$ ns. Magnetic field is normalized to $B_0=B_1 L$. We evaluate our exact solution, Eq. (18), numerically to obtain spin, and then calculate the spin current from the continuity equation. The physics that emerges from our calculation, and which can be seen on the model example in Figs. 1 and 2, is the following. There are four time scales to consider (Fig. 2). (i) In the ballistic regime ($t<\tau$), electron spin density at the edges begins to grow as $\sim t$, as electrons with one spin direction after bouncing off the boundary decelerate and stay close, while the electrons with the opposite spin accelerate in the other direction. Spin current, which is always largest in the middle of the sample, rapidly increases to reach its maximum value at $t=\tau$ (see Fig. 2). Note that positive $\tilde{J}_s$ means negative J_s, and largely a drift spin flow (spin diffusion acts in the other direction). (ii) The diffusive regime ($\tau<t<t_T$) is characterized by a further build-up of spin density at the

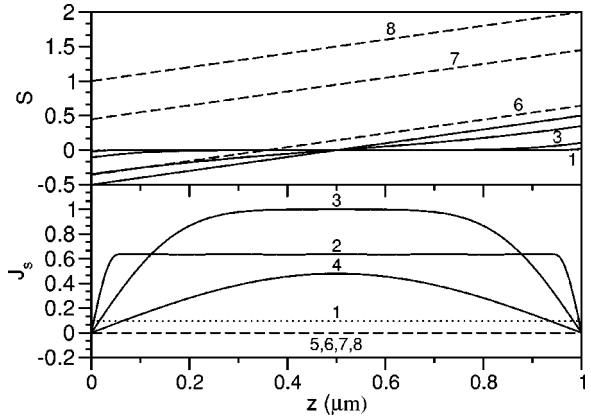

FIG. 1. Calculated normalized spin density $\tilde{S}$ (top) and spin current $\tilde{J}_s$ (bottom) for a model sample defined in the text. The curves represent profiles at times $t=10^{-14}$–10^{-7} s (increasing by a decade), and are denoted by a corresponding number 1–8 (except for a few cases at the top, where the trend is clear). The dotted lines represent the initial ballistic transport at $t=10^{-14}$ s, while the long-dashed lines are for the longest times.

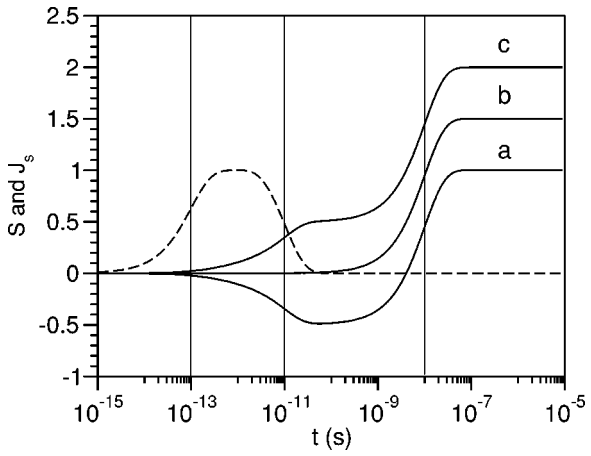

FIG. 2. Calculated time evolution of normalized spin density $\tilde{S}$ at $z=0$ (a), $z=L/2$ (b), and $z=L$ (c). The dashed line is the time evolution of normalized spin current $\tilde{J}_s$ at $z=L/2$. The three vertical lines separate the ballistic ($t<\tau=10^{-13}$ s), diffusive ($\tau<t<t_T\approx10^{-11}$ s), quasiequilibrium ($t_T<t<T_1=10^{-8}$ s), and spin relaxational ($t>T_1$) regimes.

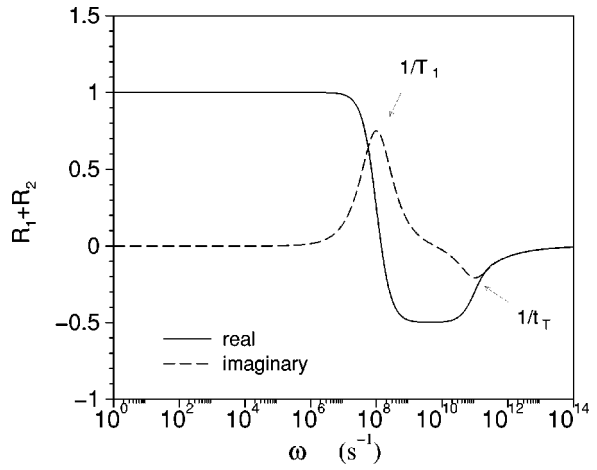

FIG. 3. Calculated dimensionless spectral response $R_1(\omega)+R_2(z,\omega)$ at $z=0$ for the model sample. The two shoulders in the real part of the response (and the corresponding peaks in the imaginary part) correspond to spin relaxation ($\omega=1/T_1=10^8$ s^{-1}) and diffusion ($\omega=1/t_T\approx 10^{11}$ s^{-1}).

edges of the sample (by diffusion) at the rate $\sim t^{0.5}$. This is accompanied by a decay of spin current, as the initial drift is now being balanced by diffusion. We call this diffusive SG effect. (iii) The quasiequilibrium regime ($t_T<t<T_1$), where momenta are in equilibrium characterized by a finite difference in the chemical potentials of spin up and down electrons, δS is spatially uniform so that no spin currents flow, and spin densities remain almost constant in time. (iv) Finally, in the spin relaxation regime ($t>T_1$), the uniform nonequilibrium spin density vanishes and complete equilibrium is established.

We now ask the question of how the electron spin system responds to a time-varying inhomogeneous magnetic field. Suppose $B(z,t)=(B_0+B_1z)\exp(-i\omega t)$. We show the results for diffusive dynamics, and solve Eq. (8) with $-\partial S_0/\partial t$ as the source term. Linear-response theory for magnetic susceptibility for diffusive transport is well known;[10] here we illustrate it for the specific boundary conditions of our model of spin separation, where various time scales discussed above will be manifest on the frequency domain. In response to the oscillating field, spin density changes as $\tilde{S}=[R_1(\omega)+R_2(z,\omega)]\exp(-i\omega t)$, where

$$R_1=\left(\frac{B_0}{B_1L}+\frac{1}{2}\right)\frac{1}{-i\omega T_1+1}, \tag{24}$$

$$R_2=-4\sum_{n\geqslant 0}\frac{\cos(q_nz)}{q_n^2L^2}\,\frac{Dq_n^2T_1+1}{-i\omega T_1+Dq_n^2T_1+1}. \tag{25}$$

Spin relaxation is primarily taken over by R_1, which measures the response of the uniform ($q=0$) components of the spin density. On the other hand, R_2 collects the terms responsible for diffusion, as diffusion modes ($q>0$) are the first ones to achieve equilibrium ($t_T\ll T_1$). The total response, calculated for our model system, is displayed in Fig. 3. At small frequencies spins can adiabatically (in equilibrium) follow the local and instantaneous $B(z,t)$: at low ω, $\tilde{S}\approx(B_0/B_1L+z/L)\exp(-i\omega t)$. At greater frequencies, first the spin-relaxation peak (in the $\mathrm{Im}\,R_1$) and shoulder (in $\mathrm{Re}\,R_1$) appear, while at $\omega\approx 1/t_T$, a second peak and shoulder (now due to R_2) appear, as the time scale of the diffusive regime is reached. The second peak signals the SG effects, where dissipation is due to drift spin currents; $\tilde{S}$ reaches negative values of $-1/2$, showing spin separation (compare with Fig. 2). Spectral response of the spin current is $\tilde{J}_s(t)=(1-\partial R_2/\partial z)\exp(-i\omega t)$, and shows a structure only around $\omega\approx 1/t_T$, as spin current relaxes during the transit time (and *not* on T_1 scale).

Finally, we discuss some issues related to a possible experimental observation of our findings. What we call a SG-like effect is an effective spin (not particle) separation of electrons in metals and semiconductors. Let us summarize the time scales involved. Ballistic transport lasts for femtoseconds up to a picosecond, diffusive transit across a micron sample can take from a picosecond to a nanosecond, and spin relaxation times can be between a fraction of a nanosecond to a microsecond.[15] Ordinary SG fails to work with electrons because transverse magnetic fields (say, $B_y=-B_1y$) give rise to the Lorentz force which makes, say, moving to the left spin up electrons turn around and move to the right, smearing out spin separation[6] (see, however, Ref. 16). In our case the time scale of the Larmor precession, $t_L=m/eB$, is large enough to be neglected. Indeed, for such a large B_1 as 10 T/cm, the transverse field would be of order 10 G for a micron sample, turning an electron around in $t_L\approx 5$ ns, long after spin separation sets in. In addition, orbital effects are inhibited due to momentum scattering. One can still go to a one- or two-dimensional sample to study SG with ballistically propagating electrons,[17] if Larmor precession is faster than momentum relaxation.

A SG-like spin separation should be observable in both metals and semiconductors. (We are not aware of any experimental method of measuring directly the spin current, in our case the diffusive regime, although theoretical proposals exist—see Ref. 18). One way of measuring a nonequilibrium spin in metals is the Silsbee-Johnson method of spin-charge coupling.[14] One can either switch an external inhomogeneous magnetic field, or inject nonequilibrium spin into a metal in a static field, to measure the time evolution of the spin. This can be accomplished, for example, by placing a ferromagnetic electrode on the top of a sample's edge, and measure the voltage across the interface, which is proportional to the nonequilibrium spin[14] (the spin in the ferromagnetic electrode can be considered to be in equilibrium, since it relaxes much faster than in the nonmagnetic sample). The voltage would be present even when the spin current vanishes (that is, in the quasiequilibrium regime), and thus can be monitored with the sub-T_1 (not t_T) resolution. Gradient B_1 can create spin at the sample edges of about μ_BB_1L/E_F spins per electron (E_F is the Fermi energy), which, for typical values of, say, $B_1\approx 1$ T/cm, $L=10$ μm, and $E_F\approx 10$ meV gives about 1 spin per 10^8 electrons (note that L must be smaller than the spin diffusion length to observe the separation). For comparison, in the Johnson-Silsbee spin injection experiment 1 spin per 10^{11} electrons was detected.[14]

In semiconductors like GaAs, the traditional tool to observe spin polarization of the carriers has been photoluminescence polarization detection.[19] A degenerate semiconductor with $E_F \approx 1$ meV (and otherwise the same conditions as above) would be polarized to about 0.01% (for more sensitivity a greater B_1 or L, or a material with a larger g factor could be used), emitting light with circular polarization of the same order. If the sample is n doped, for example, and the edges form the interfaces with a p-doped material, the spin polarization of the light emitted at the edges would be opposite in the quasiequilibrium regime, demonstrating the SG separation. Pico-to-micro-second resolved pump and probe photoluminescence measurements in an inhomogeneous but *static* magnetic field could follow the evolution from ballistic regime to full equilibrium of a semiconductor spin system, yielding information not only about spin, but also about charge transport, as seen from our calculation. In addition to the optical technique and the Johnson-Silsbee method, one could also in principle observe our predicted effect using the magnetic resonance force microscopy.

In summary we have studied transient spin dynamics of itinerant electrons in metals and degenerate semiconductors placed in an inhomogeneous magnetic field. In particular, we have solved exactly a spin dynamics model based on the Boltzmann equation, and demonstrated that the spin evolution proceeds through four distinct modes: ballistic, diffusive, quasiequilibrium, and equilibrium. An effective spin separation is possible in the quasiequilibrium regime, where the spin current vanishes and the spin is in equilibrium only with the inhomogeneous component of the magnetic field.

We thank Igor Žutić and Xuedong Hu for useful discussions. This work was supported by the U.S. ONR and DARPA.

APPENDIX A

To demonstrate the effect of spin-flip scattering on charge and spin transport, consider electrons in a simple metal, scattering elastically off impurities at the rate $W_{\mathbf{k}\lambda,\mathbf{k}'\lambda'}$. Spin-flip events are characterized by $W_{\mathbf{k}\uparrow,\mathbf{k}'\downarrow}$ and result mainly from the spin-orbit interaction. The collision integral is

$$\sum_{\mathbf{k}'\lambda'} [W_{\mathbf{k}'\lambda',\mathbf{k}\lambda} f_{\mathbf{k}'\lambda'}(1-f_{\mathbf{k}\lambda}) - W_{\mathbf{k}\lambda,\mathbf{k}'\lambda'} f_{\mathbf{k}\lambda}(1-f_{\mathbf{k}'\lambda'})]. \tag{A1}$$

The first term describes scattering from $\mathbf{k}'\lambda'$ to $\mathbf{k}\lambda$, which increases $f_{\mathbf{k}\lambda}$, while the second term represents reversed processes, which decrease $f_{\mathbf{k}\lambda}$. Factors such as $f_{\mathbf{k}\lambda}(1-f_{\mathbf{k}'\lambda'})$ ensure that the initial state is occupied and final state empty, in accord with the Pauli principle.

Let the system is driven off equilibrium by an applied electric field $\mathbf{E}$ and an inhomogeneous chemical potential $\lambda\,\delta\mu$. The latter does not disturb the equilibrium electronic density, but maintains an inhomogeneous nonequilbrium spin polarization ($\lambda\,\delta\mu$ is essentially the driving term for spin diffusion caused, for example, by spin injection). We seek the solution to Eq. (1) with the RHS Eq. (A1) in the form $f_{\mathbf{k}\lambda} = f^0_{\mathbf{k}\lambda} - (\partial f_0/\partial\epsilon_{\mathbf{k}})\phi_{\mathbf{k}\lambda}$, where now $f^0_{\mathbf{k}\lambda} = f_0(\epsilon_{\mathbf{k}} - \lambda\,\delta\mu)$. After linearization the Boltzmann equation becomes

$$(e\mathbf{E} + \lambda\boldsymbol{\nabla}\delta\mu)\mathbf{v}_{\mathbf{k}} = \sum_{\mathbf{k}'\lambda'} W_{\mathbf{k}\lambda,\mathbf{k}'\lambda'}(\phi_{\mathbf{k}\lambda} - \phi_{\mathbf{k}'\lambda'}), \tag{A2}$$

where we left out the Lorentz force as unimportant for the present discussion. Equation (A2) has the formal solution

$$\phi_{\mathbf{k}\lambda} = -e\mathbf{E}\cdot\mathbf{a}_{\mathbf{k}} - \lambda\boldsymbol{\nabla}\delta\mu\cdot\mathbf{b}_{\mathbf{k}}, \tag{A3}$$

with $\mathbf{a}_{\mathbf{k}}$ and $\mathbf{b}_{\mathbf{k}}$ satisfying the integral equations

$$\mathbf{v}_{\mathbf{k}} = \sum_{\mathbf{k}'} [W_{\mathbf{k}\uparrow,\mathbf{k}'\uparrow}(\mathbf{a}_{\mathbf{k}'} - \mathbf{a}_{\mathbf{k}}) + W_{\mathbf{k}\uparrow,\mathbf{k}'\downarrow}(\mathbf{a}_{\mathbf{k}'} - \mathbf{a}_{\mathbf{k}})], \tag{A4}$$

and

$$\mathbf{v}_{\mathbf{k}} = \sum_{\mathbf{k}'} [W_{\mathbf{k}\uparrow,\mathbf{k}'\uparrow}(\mathbf{b}_{\mathbf{k}'} - \mathbf{b}_{\mathbf{k}}) - W_{\mathbf{k}\uparrow,\mathbf{k}'\downarrow}(\mathbf{b}_{\mathbf{k}'} + \mathbf{b}_{\mathbf{k}})]. \tag{A5}$$

Vectors $\mathbf{a}_{\mathbf{k}}$ and $\mathbf{b}_{\mathbf{k}}$ have magnitudes of order ℓ, and their knowledge allows to calculate the tensors of charge and spin conductivities $\sigma = 2e^2\Sigma_{\mathbf{k}}(-\partial f^0_{\mathbf{k}}/\partial\epsilon_{\mathbf{k}})\mathbf{v}_{\mathbf{k}}\cdot\mathbf{a}_{\mathbf{k}}$ and $\sigma_S = 2e^2\Sigma_{\mathbf{k}}(-\partial f^0_{\mathbf{k}}/\partial\epsilon_{\mathbf{k}})\mathbf{v}_{\mathbf{k}}\cdot\mathbf{b}_{\mathbf{k}}$. The spin conductivity σ_S is related to the spin diffusivity D_S as $\sigma_S = e^2 g_F D_S$, since the nonequilibrium spin is $\delta S = g_F\delta\mu$. Here we use spin conductivity instead of the more usual spin diffusivity only to stress the contrast with charge conductivity.

If there is no spin-flip scattering, $\mathbf{a}_{\mathbf{k}} = \mathbf{b}_{\mathbf{k}}$, the effective mean free paths are the same for both currents, and the conductivities are equal: $\sigma_S = \sigma$. Spin-flip scattering, however, implies $\mathbf{a}_{\mathbf{k}} \neq \mathbf{b}_{\mathbf{k}}$, and so it plays different roles in charge and spin transport. Assume, for a moment, that scattering is isotropic, and energy surfaces spherical. Then Eqs. (A4) and (A5) can be solved exactly[20] by introducing transport relaxation times τ and τ_S: $\mathbf{a}_{\mathbf{k}} = -\tau\mathbf{v}_{\mathbf{k}}$ and $\mathbf{b}_{\mathbf{k}} = -\tau_S\mathbf{v}_{\mathbf{k}}$, and, with θ being the angle between $\mathbf{v}_{\mathbf{k}}$ and $\mathbf{v}'_{\mathbf{k}}$,

$$\frac{1}{\tau} = \sum_{\mathbf{k}'} [W_{\mathbf{k}\uparrow,\mathbf{k}'\uparrow}(1-\cos\theta) + W_{\mathbf{k}\uparrow,\mathbf{k}'\downarrow}(1-\cos\theta)] \tag{A6}$$

and

$$\frac{1}{\tau_S} = \sum_{\mathbf{k}'} [W_{\mathbf{k}\uparrow,\mathbf{k}'\uparrow}(1-\cos\theta) + W_{\mathbf{k}\uparrow,\mathbf{k}'\downarrow}(1+\cos\theta)]. \tag{A7}$$

In charge transport, spin-conserving and spin-flip processes contribute in the same way: they are weighted by the well known $1-\cos\theta$, which suppresses contributions from small-angle scattering as ineffective in degrading charge current. Spin transport is a different story. Here spin-flip processes come with $1+\cos\theta$, and backscattering ($\theta \approx \pi$) is the least effective in degrading spin current, while small-angle events contribute most. There is illuminating physics behind this: Spin up and down electrons move antiparallel to each other, so if any spin flip is accompanied by the velocity reversal, the current does not change. But if the velocity stays the same, the effect is maximal, as if the electron spin does

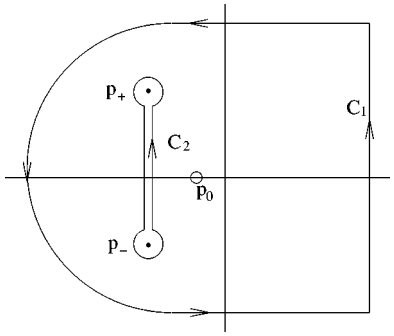

FIG. 4. Integration contour for $K(q,t)$ of Eq. (B1) with the horizontal and vertical axes representing the real and imaginary part of p, respectively. The integral in Eq. (B1), which runs along C_1, is the same as the integral along the path C_2 cutting out the branch line from p_- to p_+, plus the residue at p_0.

not flip, but the velocity reverses. Equations (A6) and (A7) are not valid in more general cases of anisotropic bands and inelastic anisotropic scattering, but, with some caution, they are still a useful approximation (justified by variational analysis[20]) in the interesting case of electrons scattering by thermal fluctuations (lattice or spin) at low T. Such fluctuations will allow only small-angle scattering, suppressing contributions from spin-conserving, but not from spin-flip processes. Spin-flip processes are thus much more important for spin transport than for charge transport. For example, the contribution of the phonon-induced spin-flip scattering to $1/\tau$ falls as T^7, while to $1/\tau_S$ only as T^5 (this follows from Yafet's theory[21] as also confirmed by a numerical calculation[15]). The contribution from the spin-conserving electron-phonon interaction falls as T^5 (the Bloch-Grüneisen law), for both charge and spin currents.

APPENDIX B

The solution of Eq. (2) [or, equivalently, Eq. (16)] can be written with the help of Laplace transform as the integral in the complex plane:

$$K(q,t)=\int_{-\infty+i\sigma}^{\infty+i\sigma}\frac{dp}{2\pi i}\,\frac{e^{pt}\langle(p+iqv+1/\tau_m)^{-1}\rangle}{1-\langle(p+iqv+1/\tau_m)^{-1}\rangle/\tau}, \tag{B1}$$

where $\sigma>0$. For a degenerate system considered in the text,

$$\langle(p+iqv+1/\tau_m)^{-1}\rangle=\frac{1}{2iqv_F}\ln\left[\frac{p+iqv_F+1/\tau_m}{p-iqv_F+1/\tau_m}\right]. \tag{B2}$$

The integral can be evaluated by a suitable contour deformation in the complex plane, as indicated in Fig. 4. The original integral in Eq. (B1), which goes along C_1 is the same as the integral over C_2 plus the residue at p_0. The path C_2 cuts away the branch line extending from $p_-=-1/\tau_m-iqv_F$ to $p_+=-1/\tau_m+iqv_F$, from the complex plane. The residue is evaluated for the pole at $p_0=-1/\tau_m+qv_F\cot(qv_F\tau)$ present for $qv_F\tau\leqslant\pi/2$ (defining the Riemann sheet for arctan to go from $-\pi/2$ to $\pi/2$).

The result of the contour integration can be formally written as

$$K(q,t)=(qv_F\tau)^2e^{-t/\tau_m}\left|\frac{\exp[qv_Ft\cot(qv_F\tau)]}{\sin^2(qv_F\tau)}\right|_{qv_F\tau}^{\text{sing}}, \tag{B3}$$

where the vertical bars denote the singular (principal) part of the Laurent series in terms of $qv_F\tau$, of the expression inside. An alternative formulation is

$$K(q,t)=-(\tau/t)e^{-t/\tau_m}\sum_{n=-\infty}^{-1}nF_n(qv_Ft)(qv_F\tau)^n, \tag{B4}$$

where functions $F_n(x)$ are the coefficients of the Laurent series

$$\exp[x\cot(y)]=\sum_{n=-\infty}^{\infty}F_n(x)y^n, \tag{B5}$$

satisfying the recursion relation

$$F_n(x)+F_n''(x)=-\frac{n+1}{x}F_{n+1}(x), \tag{B6}$$

with the boundary conditions $F_n(0)=0$ and $F_n'(0)=\delta_{n,-1}$ for $n\leqslant-1$. In principle, all the functions $F_n(x)$, $n=-2,\ldots,-\infty$ can thus be generated from

$$F_{-1}(x)=\sin(x), \tag{B7}$$

which is readily obtained from Eq. (B6).

The limiting case for ballistic transport can be obtained from Eq. (B4) by letting $qv_F\tau$ (and thus also qv_FT_1) going to infinity. The result is

$$K(q,t)\approx K_{\text{ball}}(q,t)=\frac{F_{-1}}{qv_Ft}. \tag{B8}$$

On the other hand, the diffusive limit can be obtained by letting $qv_F\tau$ to zero, in which case the vertical bar in Eq. (B3) can be removed (the singular part equals the whole part since the regular part vanishes). By expanding $\cot(qv_F\tau)\approx1/(qv_F\tau)-(qv_F\tau)/3$ and denoting as $D\equiv v_F^2\tau/3$, one obtains

$$K(q,t)\approx K_{\text{diff}}(q,t)=\exp(-t/T_1-q^2Dt). \tag{B9}$$

[1] G. A. Prinz, Science **282**, 1660 (1998).

[2] S. Das Sarma, J. Fabian, X. Hu, and I. Žutić, IEEE Trans. Magn. **36**, 2821 (2000); Superlattice Microst. **27**, 289 (2000); Solid State Commun. **119**, 207 (2001).

[3] I. Žutić, J. Fabian, and S. Das Sarma, Phys. Rev. B **64**, 121201 (2001).

[4] I. Žutić, J. Fabian, and S. Das Sarma, Appl. Phys. Lett. **79**, 1558 (2001).

[5] I. Žutić, J. Fabian, and S. Das Sarma, Phys. Rev. Lett. **88**, 066603 (2002).
[6] N. F. Mott, Proc. R. Soc. London, Ser. A **124**, 425 (1929).
[7] W. A. Harrison, *Solid State Theory* (Dover, New York, 1979).
[8] D. Forster, *Hydrodynamic Fluctuations, Broken Symmetry, and Correlation Functions*, Vol. 47 of Frontiers in Physics (Benjamin/Cummings, New York, 1975).
[9] M. B. Walker, Phys. Rev. B **3**, 30 (1971).
[10] P. M. Platzman and P. A. Wolff, *Waves and Interactions in Solid State Plasmas* (Academic, New York, 1973).
[11] T. Valet and A. Fert, Phys. Rev. B **48**, 7099 (1993).
[12] J. Fabian and S. Das Sarma, Phys. Rev. Lett. **81**, 5624 (1998); **83**, 1211 (1999).
[13] M. Johnson and R. H. Silsbee, Phys. Rev. B **35**, 4959 (1987).
[14] R. H. Silsbee, Bull. Magn. Reson. **2**, 284 (1980); M. Johnson and R. H. Silsbee, Phys. Rev. Lett. **55**, 1790 (1985).
[15] J. Fabian and S. Das Sarma, J. Vac. Sci. Technol. B **17**, 1708 (1999).
[16] H. Batelaan, T. J. Gay, and J. J. Schwendiman, Phys. Rev. Lett. **79**, 4517 (1997); G. A. Gallup, H. Batelaan, and T. J. Gay, *ibid.* **86**, 4508 (2001).
[17] J. Wrobel, T. Dietl, K. Fronc, A. Lusakowski, M. Czeczott, G. Grabecki, R. Hey, and K. H. Ploog, Physica E **10**, 91 (2001).
[18] M. I. Dyakonov and V. I. Perel, Phys. Lett. **35A**, 459 (1971); JETP Lett. **13**, 467 (1971); J. E. Hirsch, Phys. Rev. Lett. **83**, 1834 (1999); R. D. R. Bhat and J. E. Sipe, *ibid.* **85**, 5432 (2000).
[19] *Optical Orientation*, edited by F. Meier and B. P. Zakharchenya (North-Holland, New York, 1984).
[20] J. M. Ziman, *Electrons and Phonons* (Oxford University Press, Oxford, 1960).
[21] Y. Yafet, in *Solid State Physics* edited by F. Seitz and D. Turnbull (Academic, New York, 1963), Vol. 14, p. 1–98.