# Landau orbital description of the vortex state in a two-dimensional extremely type-II superconductor

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We extend the application of the recently developed Landau orbital approach to the fluctuating vortex lattice in two-dimensional (2D) extremely type-II superconductors at high magnetic fields to the region above the melting point. We find that the abrupt incomplete melting of the quasicrystalline vortex phase at $T=T_m \ll T_c$ is followed by a broad crossover into an intermediate state with a nematic liquid-crystalline-like order without 2D positional order. A special type of amplitude fluctuations, which can be described as classical transverse vibrations of vortex chains (while vortices are accumulated and depleted alternately in individual chains), controls this crossover. Due to the 1D nature of these vibrations, the structure factor suffers thermal broadening of the diffraction peaks, which remains effective at temperatures well below the mean field $T_c$. The complete melting into an isotropic vortex fluid is shown to occur only far above $T_m$, since the energy cost of the fluctuations which break the chain structure is of the order of the superconducting condensation energy.

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## I. INTRODUCTION

Melting of the lattice of the magnetic flux lines in extremely type-II, layered superconductors has been investigated extensively during the last decade. Clear evidence for a first-order melting phase transition has been found in the high-$T_c$ compounds $YBa_2Cu_3O_7$ (Ref. 1) and $Bi_2Sr_2CaCu_2O_8$ (Ref. 2) in the high-temperature low-magnetic-field region of the phase diagram. Similar phenomena have been studied in low-$T_c$ superconductors based on the organic molecule BEDT-TTF,[3] which share similar features with the high-$T_c$ cuprates. These materials offer a unique possibility to investigate vortex lattice dynamics in the low-temperature high-magnetic-field region of the phase diagram due to their nearly two-dimensional (2D) electronic structure.

In such a quasi-2D superconductor the transition to zero resistivity in the high-magnetic-field region is dramatically broadened with respect to the zero-field transition,[3] whereas the characteristic jump of the specific heat at the transition in the absence of a magnetic field is completely smeared by applying a moderate magnetic field.[4] It is believed that thermal fluctuations are responsible for these closely related phenomena. Fluctuations of the superconducting (SC) order parameter are, indeed, expected to be enhanced significantly at high magnetic fields when the Landau quantization of the Cooper-pair energy leads to an effective dimensionality reduction of 2 (Ref. 5).

Theoretically speaking it is well known that phase fluctuations play a crucial role in controlling the stability of the vortex lattice and its possible melting processes. In 2D systems the energy scale of these fluctuations is much smaller than the SC condensation energy, implying a melting temperature $T_m$ well below the mean field $T_c$.[6] Indeed, the magnetization irreversibility line, which follows the boundary between the vortex solid and the vortex liquid, appears in the quasi-2D organics at temperatures far below the mean field $T_c$,[3,7] where magnetoquantum oscillations are detectable.[8,7] In accordance with the resistivity and specific heat data mentioned above, the SC-induced damping of the de Haas–van Alphen (dHvA) signal in the region around the mean field $H_{c2}$ was found to be a very smooth function of the magnetic field.[9] This behavior is consistent with the broad crossover between the SC and normal states predicted theoretically for 2D superconductors.[6]

The nature of the vortex lattice melting transition in 2D superconductors has been debated in the literature for many years.[9] Early proposals,[10,11] based on the similarity to the Kosterlitz-Thouless-Halperin-Nelson-Young (KTHNY) theory of melting in 2D solids,[12] have led to the conclusion that the melting transition is continuous. A weak first-order melting transition was predicted more recently, however, by several Monte Carlo simulations of the Ginzburg-Landau (GL) model,[13–16] as well as of the frustrated $XY$ model.[17,18] In approaches based on the Ginzburg-Landau theory an arbitrary configuration of $N$ vortices is constructed by a superposition of $N$ Landau gauge orbitals of Cooper pairs restricted to the lowest Landau level (LLL) in a magnetic field whose total flux through the sample amounts to $N$ flux quanta. Due to the short-range repulsive interaction between vortex cores, the totality of $N$ Landau orbitals collapses in the zero-temperature limit into $\sqrt{N}$ orbitals, each of which is characterized by a common projection of $\sqrt{N}$ guiding centers, to form the most uniform superfluid density allowed by the condition of flux quantization—the Abrikosov triangular lattice. It has been shown recently[6] that shear motions of Bragg chains along the principal crystallographic axes of the vortex lattice cost a very small fraction of the SC condensation energy and are responsible for the low-temperature vortex lattice melting. These fluctuations preserve the area of any elementary cell in the fluctuating lattice, so that the corresponding vortex network is incompressible everywhere in the sample.

The great simplicity associated with this Bragg chain picture is lost, however, at temperatures significantly larger than the melting temperature $T_m$, where thermal excitations have

 

enough energy to overcome the repulsion between vortex cores. Under these circumstances it is not allowed anymore to describe the fluctuating vortex network in terms of collectively moving rigid chains of vortices, since individual vortices tend to fluctuate independently as the temperature is raised.

In the present paper we extend the application of the Landau orbital approach to the vortex state above the melting point in a 2D superconductor under high perpendicular magnetic field. We investigate in detail certain types of deviations from the Bragg chain model, which characterize different stages of the crossover from the Abrikosov lattice state at low temperatures to the isotropic vortex liquid state at high temperatures. We find that the energy cost of the fluctuations leading to the breakdown of the vortex-chain structure is of the same order of magnitude as the SC condensation energy, so that in a broad region between the melting point and the SC transition point, predicted by mean-field theory, the vortex liquid state can be described as a collection of weakly interacting vortex chains.

## II. BRAGG CHAIN APPROXIMATION

The extremely smooth crossover between the normal and the SC phases characterizing 2D superconductors at high magnetic fields implies that in a relatively broad region of the phase diagram around the mean-field SC transition the thermodynamic potential may be obtained by using the well-known GL expansion of the free energy functional in the small SC order parameter, which in the LLL approximation reduces to the simple form.[19,13]

$$F_{GL}=\int_{\pi N} dxdy\left(-\alpha|\Delta(x,y)|^2+\frac{1}{2}\beta|\Delta(x,y)|^4\right), \quad (1)$$

where $\alpha(T)$, $\beta$ are phenomenological parameters and $\pi N$ is the volume of the 2D system ($N\to\infty$). All lengths are measured in units of the magnetic length, $a_H=\sqrt{c\hbar/eH}$.

The corresponding SC order parameter in the Landau gauge can be written in the most general form as a coherent superposition of Landau orbitals,

$$\Delta(x,y)=\sum_{n,m} c_q \exp[iqx-(y+q/2)^2], \quad (2)$$

where $c_q$ are arbitrary complex numbers defined on the quasicontinuous lattice $q=(2\pi/a_x)(n+m/\sqrt{N})$, $n,m=-\sqrt{N}/2+1,\ldots,\sqrt{N}/2$, which determines the projections of the orbital guiding centers on the $y$ axis. In the case when all the coefficients $c_q$ are different from zero there is a one-to-one correspondence between all $N$ guiding centers and their projections on the $y$ axis. As will be shown below, the energy cost of this quasicontinuous configuration is a large fraction of the total SC condensation energy and is therefore unfavorable at temperatures well below $T_c$.

Expressing the GL functional, Eq. (1), in terms of the order parameter, Eq. (2), as

$$F_{GL}(\tilde{\Delta}^2;\{c_q,c_q^*\})=\pi N\left[-\alpha\tilde{\Delta}^2+\frac{1}{2}\beta\beta_a(\{c_q,c_q^*\})\tilde{\Delta}^4\right], \quad (3)$$

with $\tilde{\Delta}^2\equiv\overline{\Delta^2}\equiv(1/\pi N)\int dxdy|\Delta(x,y)|^2$ and

$$\beta_a(\Delta)=\beta_a(\{c_q,c_q^*\})=\overline{|\Delta(x,y)|^4}/\overline{|\Delta(x,y)|^2}^2, \quad (4)$$

the partition function can be written in the form

$$Z=\int_0^\infty d(\tilde{\Delta}^2)\int\prod_{n,m}{}' dc_q dc_q^* \exp[-F_{GL}/k_BT], \quad (5)$$

where the prime on the product symbol indicates that the integration over the global amplitude is excluded.

Evidently, in the thermodynamic limit $N\to\infty$, the integral over the global amplitude $\tilde{\Delta}^2$ can be evaluated by the steepest descent method. The corresponding stationary point is $\tilde{\Delta}^2=(\alpha/\beta\beta_a)$, and the partition function takes the form

$$Z=\int\prod_{n,m}{}' dc_q dc_q^* \exp\left[\pi N\left(\frac{\alpha^2}{2\beta\beta_a}\right)\Big/ k_BT\right], \quad (6)$$

where the new SC free energy functional $F_{GL}=-\pi N\alpha^2/2\beta\beta_a$ is similar to the well-known mean-field expression. Thus the fluctuations of the global amplitude $\tilde{\Delta}^2$ vanish and the SC free energy is determined solely by the configuration (Abrikosov) functional, Eq. (4).

The order parameter which minimizes this GL free energy functional corresponds to a discrete configuration where the totality of $N$ guiding centers collapses into $\sqrt{N}$ groups, of $\sqrt{N}$ centers each, with the same projection $q_n/2=(\pi/a_x)n$, $n=-\sqrt{N}/2+1,\ldots,\sqrt{N}/2$, on the $y$ axis. Thus the entire set of guiding centers constitutes a family of parallel Bragg chains in a 2D periodic lattice. The corresponding order parameter, written as a linear combination of Landau orbitals $\phi_n(x,y)=\exp[iq_nx-(y+q_n/2)^2]$, representing the different chains, is given by

$$\Delta(x,y)=\sum_{n=-\sqrt{N}/2+1}^{\sqrt{N}/2} c_n\phi_n(x,y), \quad (7)$$

where the phase $\varphi_n$ of the coefficient $c_n=|c_n|e^{i\varphi_n}$ determines the relative ''horizontal'' position $x_n=-\varphi_n/q_n$ of the $n$th chain along the $x$ axis. Shear fluctuations along the $x$ axis are therefore described by the fluctuating phases $\varphi_n$. For a regular lattice of an arbitrary geometry, $\varphi_n=\gamma n^2$ and $c_n=c_n^{(0)}=c_0e^{i\gamma n^2}$, where $c_0\equiv(2\pi/a_x^2)^{1/4}\tilde{\Delta}$. The parameter $a_x$ is the lattice constant in the $x$ direction. The additional minimization with respect to the variational parameters $\gamma$ and $a_x$ leads to the preferred geometry of the Abrikosov triangular lattice with $\gamma=\pi/2$, $a_x^2=2\pi/\sqrt{3}$. Other lattice states with rhombic elementary cell, which can be obtained by varying $a_x$, have a slightly higher free energy (within 2%) since all the differences between them arise from small phase-dependent terms in the quartic part of $F_{GL}$ [Eq. (1)].

The GL free energy, Eq. (1), is invariant under continuous rotations in the $(x,y)$ plane and, therefore, the orientation of the $x$ axis in Eq. (2) can be selected arbitrarily. In addition

the ground lattice state is invariant under discrete rotation by angle $\pi/3$, leading to the existence of completly equivalent chain directions described by the same values of the parameters $a_x$ and $\gamma$. In real samples this degeneracy is removed due to various mechanisms of vortex line pinning, e.g., by atomic lattice, the sample boundaries, the presence of defects, etc. However, in the present model we neglect vortex line pinning, assuming that results obtained for a given chain direction should be subsequently averaged over all other equivalent chains. Such averaging, which does not change the thermodynamical properties of the system, eliminates the linear chain structure of the thermal fluctuation, and restores the original sixfold symmetry. Nevertheless, as will be shown below, some indirect 1D properties persist regardless of the chain orientation averaging.

It should be stressed here that a Landau wave function for a given chain $\phi_n(x,y)$ is extended in the $x$ direction but localized in the perpendicular direction. The great advantage of this chain representation of the vortex state, as will be shown below, is that due to the localized nature of the Landau orbitals in the transverse direction, the chains interact only weakly with each other, as long as their internal structure is not destroyed. It is therefore clear that the Landau gauge employed here is the best choice for dealing with such a chain structure. On the other hand, the opposite extreme situation of randomly distributed vortices is more appropriately described in terms of the symmetric gauge Laughlin-Jastrow wave function,[20] where the positions of the vortex cores (i.e., the zeros of the order parameter) are directly specified. Furthermore, the delocalized nature of the Landau orbitals along the $x$ direction makes this representation more sensitive to the finite size of the superconductor than the Laughlin-Jastrow representation. In a system with a finite size $L_x$ along the $x$ direction the $\delta$ distribution of the coefficients $c_n$ in $q$ space around the ideal discrete array is smeared into finite peaks with a characteristic width of the order of $1/L_x$. Thus, the SC order parameter of a spatially small system includes significant deviations from the Bragg chains approximation.

In the ground state the chains representation, Eq. (7), is not unique since it can be selected along any direction determined by each of the infinitely many families of Bragg chains. In Ref. 6 it was shown that the chains along Bragg directions with small Miller indices interact weakly with each other, so that the shear stiffness of the vortex lattice along these axes is quite small. In contrast, the coupling between chains within a Bragg family having larger Miller indices become comparable to the intrachain energy quite quickly as the indices increase. Along the infinitely many directions of this type the shear stiffness of the lattice is therefore large and nearly isotropic. This phenomenon is nicely illustrated in Fig. 1, where the dependence of the free energy [through the Abrikosov parameter $\beta_A \equiv \beta_a(\{c_n^{(0)}, c_n^{(0)*}\})$ ] on the collective tilt angle $\gamma$ is shown for different discrete values of the parameter

$$s = (\pi/a_x)^2, \tag{8}$$

which correspond to certain Bragg families (determined by the intrachain distance $a_x$). A small value of $s$ (i.e., a large $a_x$) corresponds to a set of large Miller indices. The minimum (or minima) for each Bragg family corresponds to the value (or values) of $\gamma$ characterizing the Abrikosov triangular lattice. The very weak shear stiffness along the principal axis and its quick increase with increasing Miller indices are apparent.

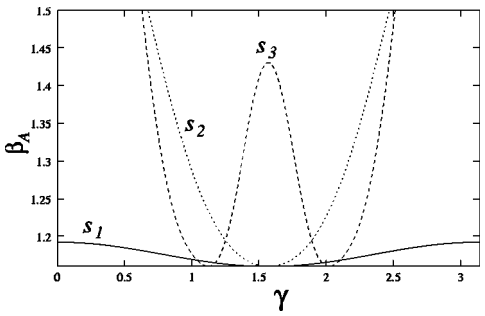


FIG. 1. The dependence of the Abrikosov parameter on the collective tilt angle $\gamma$ for different values of the parameter $s$: $s_1 = \pi\sqrt{3}/2$, $s_2 = \pi/2\sqrt{3}$, and $s_3 = \pi\sqrt{3}/26$ corresponding to increasing values of the Miller's index in the triangular lattice. $s_1$ and $s_2$ correspond to the principal axes.

## III. INCOMPRESSIBLE VORTEX LIQUID

The order parameter in the LLL subspace, given by Eq. (2), reflects a fundamentally important property of the vortex state in extremely type-II superconductors at high magnetic fields; that is, the vortex matter under these conditions is incompressible. This feature is associated with the fact that the total number of zeros of $\Delta(\vec{r})$ within a fixed macroscopic area $\pi N$ is equal to the total number of guiding centers, $N$, used in constructing the wave function in Eq. (2), which is also equal to the total number of flux lines threading this area (see, e.g., Ref. 16). In the general case this is only a global constraint which can be violated locally by sufficiently energetic fluctuations (see the next sections). In the low-temperature region around the melting point, where the Bragg chain approximation, Eq. (7), is valid, this constraint is satisfied everywhere in the superconductor since all fluctuations are restricted to shear motions along parallel Bragg chains, for which the area of each unit cell is conserved (and equals $\pi$).

In the present section we discuss the salient features of this absolutely incompressible vortex phase. We start with the GL free energy density per chain, measured in units of $k_B T$:

$$f_{GL} \equiv \frac{F_{GL}}{\pi k_B T \sqrt{N}} = -\bar{\alpha} F_2 + \frac{\bar{\beta}}{2} F_4,$$

$$F_2 = \sum_n |c_n|^2,$$

$$F_4 = \sum_{n_i, n_1+n_2=n_3+n_4} \exp\left\{ -\frac{1}{4}\left[ \sum q_{n_i}^2 - \frac{1}{4}\left( \sum q_{n_i} \right)^2 \right] \right\} \times c_{n_1}^{\star} c_{n_2}^{\star} c_{n_3} c_{n_4}, \tag{9}$$

where $\bar{\alpha}=\alpha a_x/\sqrt{2\pi}k_BT$, $\bar{\beta}=\beta a_x/\sqrt{4\pi}k_BT$, and $\sqrt{N}a_x$ is the size of the system in the $x$ direction. The partition function in this representation is written as $Z=\varsigma^{\sqrt{N}}$, where

$$\varsigma=\int\prod_n dc_n dc_n^*\exp(-\pi f_{GL}). \tag{10}$$

A direct consequence of the representation, Eq. (9), is that the quadratic terms, as well as the quartic ones with $n_1=n_2=n_3=n_4=n$, which can be regarded as intrachain energies, do not depend on the positions (vertical, $q_n$, as well as horizontal, $-\varphi_n/q_n$) of the chains. They constitute the dominant part of the SC condensation energy, but do not influence the vortex structure of the condensate. The rest of the terms, which describe interactions between chains, depend on the distance between neighboring chains, $\pi/a_x$. Since this dependence on $\pi/a_x$ is determined by the overlap between the LLL Gaussian wave functions localized on neighboring chains, the selection of the Bragg family with the largest interchain distance (i.e., with chains along the short diagonal of the rhombic unit cell) leads to interaction terms much smaller than the intrachain energy. The characteristic small coupling parameter $\lambda=\exp[-(\pi/a_x)^2]$ for this selection is $\exp[-\sqrt{3}\pi/2]\approx 0.066$.

Expanding the quartic term in $\lambda$ up to second order one finds that

$$F_4\approx\sum_n\,[|c_n|^4+4\lambda|c_n|^2|c_{n+1}|^2 +4\lambda^2|c_n|^2|c_{n+1}||c_{n-1}|\cos\chi_n], \tag{11}$$

where $\chi_n=\varphi_{n+1}+\varphi_{n-1}-2\varphi_n$, which shows two types of interactions: repulsive phase-independent two-chain interactions of the order $\lambda$ and phase-dependent three-chain interactions of the order $\lambda^2$, which are attractive for chain configurations near the triangular Abrikosov lattice geometry $\cos(\chi_n)=-1$. Thus the fluctuations with the lowest energies are shear motions of the chains along the ''main crystallographic axis'' of the triangular Abrikosov lattice, which are described by fluctuations in the phases $\varphi_n$. It should be stressed here that in the case when the long diagonal of the unit cell is selected as the chain direction the phase-dependent terms are still small, since $\lambda(a_{x'})^2=\exp[-\sqrt{3}\pi/3]\approx 0.16$.

In the low-temperature regime studied here the reduced partition function, Eq. (10), can be obtained by exact integrations over the phases $\varphi_n$ and steepest descent integrations over the amplitudes $|c_n|$. The resulting SC free energy has a form similar to the mean-field free energy, $-\pi N\alpha^2/2\beta\beta_a$, with an effective Abrikosov parameter given by[6]

$$\beta_a=\sqrt{\frac{s}{\pi}}\left(1+4\lambda-4\lambda^2\frac{I_1(\tau)}{I_0(\tau)}\right), \tag{12}$$

where $I_0(\tau), I_1(\tau)$ are the modified Bessel functions of orders 0,1 respectively, $\tau=T_{cm}/T$ and

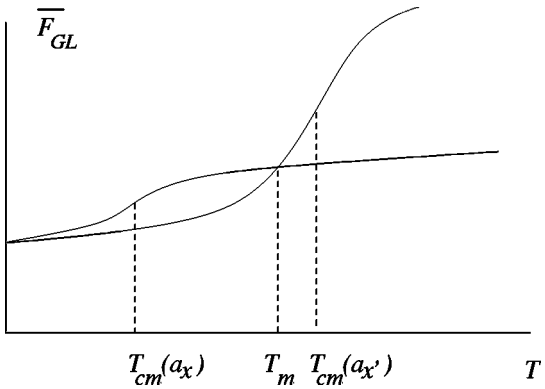


FIG. 2. Schematic temperature dependence of the SC free energy for two different selections of the Bragg family of chains, i.e., along the two principal axes of the Abrikosov lattice.

$$k_BT_{cm}=\frac{4\lambda^2}{(1+4\lambda)}\,\frac{\pi\alpha^2}{2\beta\beta_A(\lambda)},$$

with $\beta_A(\lambda)\approx 1.159$. The integration over the phase variables, which is equivalent to averaging the free energy functional, Eqs. (9) and (11), over the phase fluctuations, tends to suppress the weak three-chain attractive interactions due to their oscillatory behavior, while keeping the repulsive two-chain interaction unchanged. This suppression takes place continuously but rather sharply as a function of temperature within each family of chains (see Fig. 2). The family of fluctuating chains along the large diagonal ($a_{x'}^2=2\sqrt{3}\pi$) is more stable at low temperature than that along the small diagonal ($a_x^2=2\pi/\sqrt{3}$) due to the relatively large value of the coupling constant $\lambda(a_{x'})$. However, the crossover temperature $T_{c.m.}$ around which the free energy rises sharply and the magnitude of the rise itself are higher for the former family than for the latter, and the two curves intersect at some intermediate temperature $T_m$. At the crossing point (see Fig. 2) the chains of the latter family move almost independently so that the shear modulus drops abruptly down to a small nonzero value and tends to zero gradually with further temperature increase.

The first-order nature of the melting transition can be illuminated by plotting the dependence of $\beta_a$ on the parameter $s$, defined by Eq. (8), at different temperatures (see Fig. 3). In

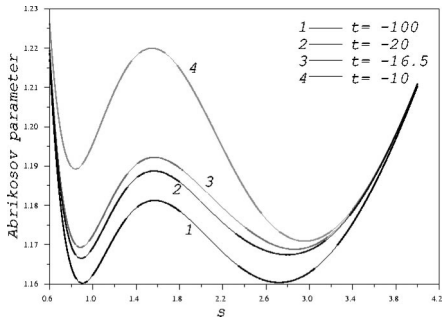


FIG. 3. Dependence of the Abrikosov parameter $\beta_a$ on $s=(\pi/a_x)^2$ at different values of the effective ''temperature'' $t=-\sqrt{2\pi\alpha^2/k_BT\beta}$

this plot both minima [referred to as $x'$ (left) and $x$ (right) in what follows], which coincide at $T=0$, correspond to the same triangular Abrikosov lattice in two different orientations: In the first orientation the long diagonal is parallel to the $x$ axis while in the second one the short diagonal is parallel to this axis. At finite temperatures this degeneracy is removed due to the different coupling between chains corresponding to different Bragg families, as explained above. It should be emphasized, however, that unlike the situation in Hamiltonian quantum mechanics, where the linearity of the Hamiltonian operator ensures that any linear combination of the two degenerate states has the same energy, in the nonlinear GL theory such a superposition always corresponds to a larger free energy. *This feature ensures that the minimum free energy always corresponds to a single Bragg family of sliding chains*.

The double-minimum structure of the free energy shown in Fig. 3 is typical to a system undergoing a first-order phase transition, with the abscissa $s$ playing the role of an order parameter.[21] At the melting point, where the two minima coincide again (curve 3 in Fig. 3) the vortex system can exist in two different phases: an ''ordered'' phase in which there are weak shear fluctuations along the $x'$ chains and a ''disordered'' phase consisting of strongly fluctuating $x$ chains. The transition from the ordered phase to the disordered one is associated with a small jump of entropy. It is remarkable that in both phases the mean positions of the Bragg chains constitute an exactly periodic (but no longer rhombic) lattice. The jump of the order parameter $s$ at the transition corresponds to a small discontinuous change in the average lattice constant $a_x$.

The discontinuous nature of the vortex lattice melting transition is found in contrast to the melting (as described by KTHNY theory[12]) in 2D solids. In KTHNY theory the melting mechanism is associated with the thermally activated proliferation of dislocations, which reduce the elastic shear modulus to zero in a continuous fashion. In the 2D Abrikosov lattice the discontinuous transition is due to the localized (Gaussian) nature of the ground-state Landau orbitals, which considerably reduces the shear stiffness along the principal Bragg families with respect to the nearly isotropic shear stiffness characterizing Bragg families with high Miller indices.

This picture is reminiscent of the Cooper-pair charge-density-wave state described by Sasik, Stroud, and Tesanovic,[22] which shows, at a given configuration of $\Delta(x,y)$, considerable positional disorder, while the average $\langle|\Delta(x,y)|^2\rangle$, taken over the full ensemble of such configurations, exhibits a perfectly periodic triangular modulation. It should be stressed, however, that in the Bragg chain model the average superfluid density $\langle|\Delta(x,y)|^2\rangle$ is not a perfect periodic function of $x,y$; only the positions of the vortices form a perfectly periodic lattice (see Fig. 4). Also note that, in contrast to the average superfluid density which exhibits long-range positional order perpendicular to the chain axis, the SC order parameter decays to zero over a distance of a few chains only [see the plot of $|\langle\Delta(x,y)\rangle|^2$ in Fig. 4]. This picture is similar to the 2D flux-line-lattice phase described by Kato and Nagaosa,[14] which has a strong positional correlation and nonvanishing shear modulus without long-range superconducting order.

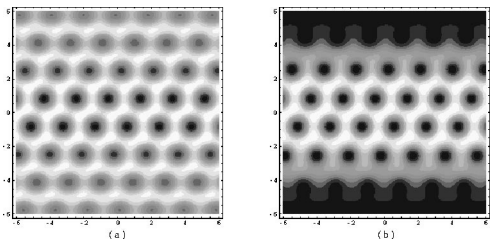

FIG. 4. Contours of $\langle|\Delta(x,y)|^2\rangle$ (a) and $|\langle\Delta(x,y)\rangle|^2$ (b), calculated within the framework of the Bragg chain model at $\tau\equiv T_m/T=2$ (i.e., below the melting point). Note the perfect periodic lattice created by the minima of these contours, which is, however, distorted with respect to the triangular Abrikosov lattice.

This unusual incompressible ''Bragg liquid'' exists as long as $T\sim T_{cm}\sim\lambda^2 T_c(H)$. At higher temperatures, where thermal excitations can overcome the repulsion between vortex cores, the Bragg chain structure is distorted and the corresponding network of vortices becomes locally compressible. In the next sections we will discuss various types of deviations from the Bragg chain approximation, which characterize different stages of the broad crossover from the low-temperature incompressible vortex liquid phase to the high-temperature isotropic vortex fluid.

## IV. INTERMEDIATE "NEMATIC" VORTEX STATE

The energy cost of the phase fluctuations which control the Bragg liquid phase discussed in the previous sections is a very small ($\sim\lambda^2$) fraction of the total SC condensation energy, $\varepsilon_0=\pi\alpha^2/2\beta\beta_A$ ($\beta_A\simeq 1.159$). As will be shown later, the energy cost of the amplitude fluctuations in the Bragg chain free energy functional, Eq. (9), is of the order of $\varepsilon_0$, which is much higher than that of the phase fluctuations. In considering intermediate deviations from the sliding Bragg chain approximation, there is a special type of amplitude fluctuations which break the quasi-long-range translational order but preserve the long-range orientational order characterizing the sliding Bragg chains. This is achieved by preserving the integrity of all chains but allowing their vertical positions to fluctuate.

Let us study this type of fluctuations in some detail. Our formal starting point is thus the partition function, Eq. (5), obtained by integrating over all possible configurations of the SC order parameter in the LLL. As in Sec. II, we pile the total $N$ guiding centers into $\sqrt{N}$ groups of $\sqrt{N}$ centers with the same projection $q_n$ on the $y$ axis, but now allow $q_n$ to deviate slightly from the periodic array $q_n^{(0)}=(2\pi/a_x)n$, namely, take $q_n=q_n^{(0)}+\xi_n$, with $|\xi_n|\ll\pi/a_x$. Note that since $q_n$ must coincide with one of the lattice points $q=(2\pi/a_x)\times(n+m/\sqrt{N})$, the small displacement $\xi_n$ is equal to $2\pi m/a_x\sqrt{N}$ with the integer $m$ varying in a region $|m|\ll\sqrt{N}/2$. Each selection of $\xi_n$ is actually a special case of amplitude fluctuations when the strength of $|c_{q_n^{(0)}}|$ is transferred entirely to $|c_{q_n}|$. Thus, in addition to the phase fluc-

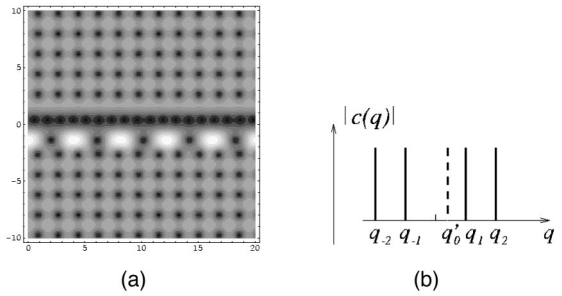


FIG. 5. Superfluid density contour (a) of a square Abrikosov vortex lattice in which a single Landau orbital is displaced vertically from its ideal position $q_0$. The corresponding $c_q$ distribution is shown in (b). Note the destruction of the translational order by this fluctuation and the preservation of the orientational order in the direction of the chains.

tuations $\varphi_n$, describing the longitudinal displacements of the chains, the relevant amplitude fluctuations $|c_q|$ can be represented as fluctuating transverse displacements $\xi_n$, so that the corresponding partition function can be written in the simple chain form

$$\varsigma = \int \prod_n d\varphi_n d\xi_n \exp[-\pi f_{GL}(\{\varphi_n, \xi_n\})]. \tag{13}$$

Obviously these fluctuations do not destroy the periodic structure of individual chains along the $x$ axis. However, they break the global periodicity by changing the period within each fluctuating chain. Indeed, such a fluctuation, e.g., with $\xi_n > 0$, is associated with a vertical displacement $\xi_n$ of the entire ($n$th) chain and a contraction of its lattice constant to $a_x/(1+a_x\xi_n/2\pi)$, which is followed by a rearrangement of the nearby superfluid density. The nature of this rearrangement can be fully appreciated by evaluating the spatial distribution of vortices in the close vicinity of the displaced chain. Considering, for the sake of simplicity, a square lattice and approximating $\Delta(x,y)$ near the upper row of the displaced zeros (see Fig. 5) by the superposition of the Landau orbitals localized on the displaced chain and on the chain just below it, the positions of the zeros are at $x_{k+}=(k+1/2)a_x/(1+a_x\xi_n/2\pi)$, $k=0,\pm1,\pm2,\ldots$, $y_+=(\pi/2a_x)\times(1-a_x\xi_n/2\pi)$. Similarly the hybrid of the orbitals localized on the displaced chain and the chain just above it yields for the lower row of zeros the coordinates $x_{k-}=(k+1/2)a_x/(1-a_x\xi_n/2\pi)$, $y_-=(-\pi/2a_x)(1+a_x\xi_n/2\pi)$. Thus, it is seen that the *accumulation of vortices within the contracted chain is exactly compensated by the depletion of vortices within the neighboring dilated chain, so that the total number of vortices is conserved.* This phenomenon is expected in view of the locality of the Landau basis wave functions and the global constraint imposed on the total number of vortices $N$, as discussed above.

These longitudinal accumulation and depletion of vortices within neighboring chains result in superfluid density fluctuations in the direction transverse to the chain axis. Under these circumstances the quasi-long-range positional order, described in Sec. III, is broken. This takes place, however, without destroying the long-range orientational order characterizing the chain structure. In that sense the macroscopic state of the vortex system described here is similar to the nematic liquid crystal phase intervening between the crystal and isotropic liquid states appearing in 2D solids.[12]

Let us make an estimate of the characteristic temperature at which the fluctuations responsible for this behavior become significant. To start with we may use the GL free energy functional given in Eq. (9) in the partition function introduced in Eq. (13), by allowing $q_n$ to fluctuate around the periodic lattice points, i.e., by letting $q_n = q_n^{(0)} + \xi_n$. The quadratic term $F_2$ determines the leading energy of a chain and does not depend on the internal chain structure. The quartic, phase-independent terms, with $n_1=n_3=n$, $n_2=n_4=n+l$ (or with $n_1=n_4$, $n_2=n_3$), represent interchain pairwise interactions with the Gaussian dependence,

$$\lambda_{n,l} = \exp[-(q_{n+l}-q_n)^2/4],$$

on the interchain distance. Note that the contribution of these transverse fluctuations to the SC free energy is a function of the relative displacements $(\xi_{n+l}-\xi_n)$ only, not of the displacement $\xi_n$ themselves. For the sake of simplicity we assume small displacements, i.e., $|\xi_n| \ll 1$ for all $n$, so that $\lambda_{n,l} \simeq \lambda^{l^2}$, where $\lambda = \exp(-s) \ll 1$ ($s=\pi^2/a_x^2$). For $l=1$ (i.e., corresponding to coupling between nearest-neighboring chains only), the energy cost of these fluctuations is significantly higher than that of the shear fluctuations considered previously, which was shown to scale like $\lambda^2$.

Since we focus here on the high-temperature region, well above the melting transition, $T \gg T_m$, the phase averaging leads to zero mean value of all phase-dependent terms. Furthermore, as the stable family of chains corresponds to $a_x^2 = 2\pi/\sqrt{3}$, the corresponding small value of the interaction constant $\lambda$ justifies the use of the nearest-neighbor approximation, and so the quartic term of the free energy functional reduces to

$$F_4 \simeq \sum_n [|c_n|^4 + 2|c_n|^2(\lambda_{n,1}|c_{n+1}|^2 + \lambda_{n,-1}|c_{n-1}|^2)], \tag{14}$$

where $\lambda_{n,\pm1}$ are functions of $\xi_n$.

We then expand $F_4$ in the small displacements $\xi_n$, up to second order, and replace the amplitudes $|c_n|^2$ with their mean-field value, i.e., $|c_n|^2 \to c_0^2(T) = \bar{\alpha}/\bar{\beta}(1+4\lambda)$, so that the GL free energy functional per vortex chain reduces to

$$f_{GL} = -\sqrt{N}\,\frac{\varepsilon_0}{\pi k_B T} + \frac{1}{2}\tau_{al}\sum_n (\xi_{n+1}-\xi_n)^2 + \cdots, \tag{15}$$

where $\varepsilon_0 = \pi\alpha^2/2\beta\beta_A$ [$\beta_A \approx (\sqrt{\pi/s}(1+4\lambda)$] is the effective condensation energy per vortex in the absence of the fluctuations, and $\tau_{al} = 2s(1-1/2s)\lambda\bar{\beta}c_0^4(T)$. The second term on the right-hand side (RHS) of Eq. (15) determines the correction to the chain energy due to fluctuations. The characteristic temperature of this type of fluctuations is thus

$$T_{al} = T\tau_{al} = s\left(1-\frac{1}{2s}\right)\frac{4\lambda}{1+4\lambda}T_0, \tag{16}$$

where $T_0=\varepsilon_0/k_B\sim T_c$. A simple estimate shows that, for $s=\sqrt{3}\pi/2$, $T_{al}\equiv 0.46T_0$; that is, $T_{al}$ is of the order of $T_c$. In the absence of the transverse fluctuations, discussed above, the vortex state at finite temperature is characterized by short-range SC order, quasi-long-range translational order,[6,9] and long-range orientational order along the principal Bragg chain direction. The transverse fluctuations preserve the orientational order, but destroy the translational order. The large elastic constant found in Eq. (16) for the transverse vibrations of vortex chains means, however, that appreciable deviations from the translational order occur only well above the melting point.

The range of translational order existing in the vortex state at finite temperature can be evaluated by considering the size (i.e., $N$) dependence of the structure factor

$$S(\vec{Q})=\frac{1}{N}\langle|I(\vec{Q})|^2\rangle \tag{17}$$

at the reciprocal Abrikosov-lattice points $\vec{Q}=\vec{G}$, with $G_x=2\pi\nu/a_x$, $G_y=2\pi m/b_y-2\nu b_x$, $b_y=\pi/a_x$, $b_x=\overline{\chi}a_x/2\pi$, where $\nu$ and $m$ are integers. Here $I(\vec{Q})=\int d^2r|\Delta(\vec{r})|^2e^{i(\vec{Q}\cdot\vec{r})}$ and $\overline{\chi}=\langle\chi_n\rangle_{ph}$, where the subscript $ph$ indicates average over phase fluctuations. At zero temperature the long-range order is manifested by the Bragg peaks with $S(\vec{G})\propto N$. At a finite temperature, far below $T_{al}$, where the transverse fluctuations may be neglected, one finds[6] a perfect long-range order along the reciprocal lattice chain $G_x=0$, i.e., $S(G_x=0,G_y)\sim N$, and quasi-long-range order along all other parallel chains, i.e., $S(G_x\neq 0,G_y)\sim N^{1/2}$. The power-law dependence on $N$ for $G_x\neq 0$ reflects the 1D long-range order within the individual vortex chains. The stronger power law for $G_x=0$ is due to the spatial averaging along the chains, which evades the destructive interference effect of the phase fluctuations on positional order perpendicular to the chains.

It should be stressed here that since there are three equivalent ways to select the principal axes in the ideal triangular Abrikosov lattice, the actual direction of the sliding chains depends on extrinsic factors, such as pinning interactions to defects, or the particular selection of the boundary conditions. Thus, there are several different orientations, determined by the local pinning interaction in the macroscopic sample, along which clusters of Bragg chains would prefer to grow. Since the reciprocal lattice points with $G_x=0$ depend on the specific choice of these axes, it is expected that by averaging over all three equivalent orientations, the size dependence of the structure factor will be isotropic, satisfying $S\sim N^{\sigma}$ with $1/2<\sigma<1$. This result is similar to the quasi-long-range positional order predicted by the KTHNY theory of 2D melting,[12] according to which $S\sim N^{\sigma}$ with $\sigma\lesssim 5/6$. Furthermore, in any numerical simulation of the vortex lattice melting, such as that performed by Kato and Nagaosa,[14] the random selections of the coefficients $c_q$ imply that the corresponding Landau orbitals are distributed randomly over the different Bragg families of chains in the Abrikosov lattice, which again correspond to averaging over all possible equivalent orientations.

As mentioned above, the presence of the transverse fluctuations breaks the translational lattice symmetry so that one expects no genuinely long-range positional order at any finite temperature. The effect is expected to be particularly important due to the 1D nature of these fluctuations. Neglecting the coupling between the longitudinal (i.e., phase) fluctuations and the transverse ones, which is of third order in $\lambda$, the corresponding structure factor for an arbitrary 2D wave vector $\vec{Q}=(Q_x,Q_y)$ can be written in the separable form

$$S(\vec{Q})=\frac{|c_0|^4}{N}\sum_{n,n',l,l'}C_4(l',n,l,n')\langle I_{n,n'}I^*_{l,l'}\rangle, \tag{18}$$

where $C_4(l',n,l,n')=\langle e^{i(\varphi_n-\varphi_{n'}-\varphi_l+\varphi_{l'})}\rangle_{ph}$ and $I_{n,n'}\equiv\langle\phi_{q_{n'}}|e^{i\vec{Q}\cdot\vec{r}}|\phi_{q_n}\rangle=\int d^2r\,\phi^*_{n'}(\vec{r})e^{i\vec{Q}\cdot\vec{r}}\phi_n(\vec{r})$. The phase (or longitudinal) correlation function has nonzero values only for $l'+n=l+n'$. It depends only on the relative variables $\nu=n'-n=l'-l$ and $\mu=l-n=l'-n'$, and can be written in the form $C_4(l',n,l,n')=e^{i\overline{\chi}\mu\nu}\tilde{C}_4(\mu,\nu)$. The transverse correlation function $\langle I_{n,n'}I^*_{l,l'}\rangle$ is given by the expression

$$\langle I_{n,n'}I^*_{l,l'}\rangle=\delta_{n',n+\nu}\delta_{l',l+\nu}\langle e^{iQ_y(q_l-q_n)/2}\times\delta^{(L_x)}(q_{n+\nu}-q_n-Q_x)\,\delta^{(L_x)}(q_{l+\nu}-q_l-Q_x)\rangle, \tag{19}$$

where $\delta^{(L_x)}(q)\equiv\int_{-L_x/2}^{L_x/2}dx\,e^{ixq}$ and $L_x=\sqrt{N}a_x$ is the length of the 2D superconductor along the $x$ axis. Thus the structure factor can be reduced to the more compact form

$$S(\vec{Q})=\frac{\pi|c_0|^4}{2N}e^{-Q^2/4}\sum_{n,\mu,\nu}e^{i\overline{\chi}\mu\nu}\tilde{C}_4(\mu,\nu)\times e^{i(\pi\mu/a_x)Q_y}\Gamma_{n,\mu,\nu}(\vec{Q}), \tag{20}$$

where

$$\Gamma_{n,\mu,\nu}(\vec{Q})=\int_{-L_x/2}^{L_x/2}dx\,dx'\times\exp\left[i\left(\frac{2\pi\nu}{a_x}-Q_x\right)(x+x')\right]\overline{\Gamma}_{n,\mu,\nu}(x,x';Q_y) \tag{21}$$

and

$$\overline{\Gamma}_{n,\mu,\nu}(x,x';Q_y)=\left\langle\exp i\left[x(\xi_{n+\nu}-\xi_n)+x'(\xi_{n+\mu+\nu}-\xi_{n+\mu})+\frac{Q_y}{2}(\xi_{n+\mu}-\xi_n)\right]\right\rangle.$$

The average over the transverse fluctuations in $\overline{\Gamma}_{n,\mu,\nu}(x,x';Q_y)$ can be readily calculated in the harmonic approximation (15). It is seen to be independent of $n$, so that the sum over $n$ in Eq. (20) produces a factor $\sqrt{N}$. At temperatures, $T_m\ll T\ll T_{al}$, the reduced phase correlation function appearing in Eq. (20) has the limiting liquid-like behavior [obtained at $\tau\ll 1$ (Ref. 6)] $\tilde{C}_4(\mu,\nu)\to\delta_{\nu,0}+\delta_{\mu,0}$

$-\delta_{\mu,0}\delta_{\nu,0}$. Evidently, the contribution to the structure factor from the third term can be neglected in the thermodynamic limit $N\to\infty$. For the first term ($\nu=0$) we find $\overline{\Gamma}_{\mu,\nu=0}(x,x';Q_y)=e^{-|\mu|Q_y^2/8\tau_{al}}$, which is independent of both $x$ and $x'$, so that $\Gamma_{\mu,\nu=0}(\vec{Q})=L_x^2\delta_{Q_x,0}e^{-|\mu|Q_y^2/8\tau_{al}}$. The remaining sum over $\mu$ in Eq. (20) leads to

$$S(Q_x=0,Q_y) = \frac{\pi a_x^2|c_0|^4}{2}\sqrt{N}e^{-Q_y^2/4}S_{1D}(Q_y), \tag{22}$$

where

$$S_{1D}(Q_y)=\frac{\sinh\dfrac{Q_y^2}{8\tau_{al}}}{\cosh\dfrac{Q_y^2}{8\tau_{al}}-\cos\dfrac{\pi Q_y}{a_x}}$$

is the well-known static structure factor for a fluctuating 1D lattice,[23] which has a series of thermally broadened, Lorentzian-like Bragg peaks centered at $Q_y=G_{y,m}\equiv 2\pi m/b_y$, $m=\pm1,\pm2,\ldots$. The width of the $m$th diffraction peak in the low-temperature region $T\ll T_{al}$ is $\gamma_m\approx m^2(a_x^3/2\pi)(T/T_{al})$. Averaging over all equivalent directions, the structure factor will restore the original hexagonal symmetry, preserving, however, the different dependence on the radial, $\delta Q\simeq\delta Q_y$, and azimuthal, $\delta\theta\simeq\delta Q_x/Q_y$, components of the wave vector. Thus, the radial width is predicted to increase linearly with $T$, reflecting the underlying chain structure.

It should be noted here that at low temperatures quantum fluctuations of the 1D lattice under study could drastically smear this diffraction structure.[24] A simple analysis of the vortex dynamics in transversely fluctuating chains shows, however, that the inertial mass of a fluctuating chain is macroscopically large (i.e., $\sim N$), implying that the effect of such transverse quantum fluctuations can be neglected in the thermodynamic limit.

The structure factor for $Q_x\neq0$ is determined by the terms with $\mu=0$. In this case the Gaussian correlation function $\overline{\Gamma}_{\mu=0,\nu}(x,x';Q_y)=e^{-2\pi^2|\nu|(x+x')^2/a_x^2\tau_{al}}$ leads to the size-independent structure factor

$$S(Q_x\neq0,Q_y)=\pi a_x|c_0|^4e^{-\vec{Q}^2/4} \times\sum_{\nu\neq0}\frac{\pi^{1/2}}{\gamma_{x,\nu}}\exp\left[-\frac{1}{2}\left(\frac{G_{x,\nu}-Q_x}{\gamma_{x,\nu}}\right)^2\right], \tag{23}$$

where $G_{x,\nu}=2\pi\nu/a_x$ are points of reciprocal lattice in the $x$ direction with $\gamma_{x,\nu}=\sqrt{|\nu|/\tau_{al}}\propto T^{1/2}$.

Thus we conclude that at $T\sim T_{al}$, which is well above the melting point $T_m$, the quasi-long-range positional order is totally destroyed by transverse fluctuations of chains. Furthermore, even at much lower temperatures the recovery of translational order is significantly slowed down by the 1D nature of the fluctuations. The preserved nematic order along the $x$ direction is reflected in the $\sqrt{N}$ prefactor appearing in the structure factor for $Q_x=0$ [Eq. (22)], regardless of the temperature.

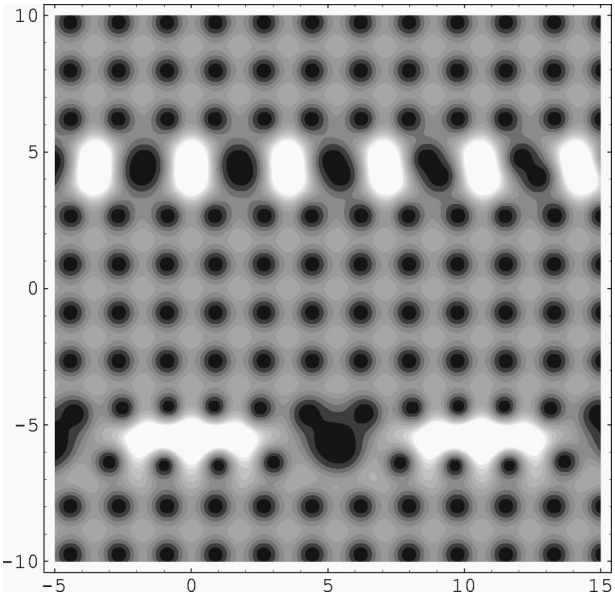


FIG. 6. Superfluid density contour of a square Abrikosov vortex lattice in which two Landau orbitals are injected at $q_{-3}^{(1)}=(2\pi/a_x)\times(-3+0.51)$ and $q_3^{(1)}=(2\pi/a_x)(3+0.17)$. Note that the length of the created domains increases with the decreasing distance of the nearest regular chain to the injected orbital.

## V. ISOTROPIC VORTEX FLUID

In the high-temperature region, where $k_BT$ is of the order of the SC condensation energy $\varepsilon_0$, the nematic liquid crystalline order is gradually destroyed by a complicated mixture of amplitude and phase fluctuations. The characteristic energy scale of the amplitude fluctuations can be estimated by considering the effective free energy functional, Eq. (9), after neglecting the weak interchain couplings. The corresponding variance at $k_BT\ll\varepsilon_0$ is

$$\frac{\langle|c_n|^2\rangle-|c_0|^2}{|c_0|^2}\simeq\left(\frac{k_BT}{\pi\varepsilon_0}\right)^{1/2}e^{-\varepsilon_0/k_BT}, \tag{24}$$

showing that these fluctuations become significant only for $k_BT\sim\varepsilon_0\simeq\pi\alpha^2/2\beta\sim k_BT_c$. Furthermore, amplitude fluctuations of the quasicontinuous set of coefficients $c_q$ which vanish at low temperatures can become important in this high-temperature regime. As will be shown below this type of fluctuations leads to destruction of the chain structure. Besides, phase fluctuations associated with shear motions within Bragg families of large Miller indices are also operative at these high temperatures.

Let us consider in some detail the second type of amplitude fluctuations. As a first step we consider the relatively simple case when a single Landau orbital is injected into an ''interstitial site,'' e.g., between the $n=3$ and $n=4$ regular chains shown in Fig. 6. In the superfluid density contour this injection appears as additional vortices injected between the

corresponding regular chains of vortices. However, since the total number of vortices should be conserved, the local accumulation of vortices is compensated by the depletion of vortices in nearby regions, thus leading to a breakdown of the chain structure around the injected Landau orbital and to the formation of a quasiperiodic domain structure along the chain direction. The size of a domain increases as the injected orbital approaches the regular one (see Fig. 6). Evidently the effect is quite local so that a global destruction of the chain structure requires, at least, such $\sqrt{N}$ injected chains.

To estimate the energy cost of this type of fluctuations let us imagine that $\sqrt{N}$ orbitals, all with equal amplitude, are injected into the regular system in such a way that a single additional guiding center appears between any two regular ones along the $y$ axis. The corresponding order parameter $\Delta(x,y)$ can be thus written as a linear combination, $\Delta(x,y)=\Delta_0(x,y)+\Delta_1(x,y)$, of two condensate wave functions:

$$\Delta_j(x,y)=\sum_{n=-\sqrt{N}/2+1}^{\sqrt{N}/2}\exp[iq_n^{(j)}x-(y+q_n^{(j)}/2)^2],$$

with $j=0,1$, where $q_n^{(0)}=2\pi n/a_x$ and $q_n^{(1)}=q_n^{(0)}+\xi_n$. Note that for the sake of simplicity we selected the vortex lattice described by $\Delta_0(x,y)$ to be a square lattice. Also note that these wave functions are orthogonal for any $|\xi_n|<\pi/a_x$.

As reflected by Eq. (6), in the thermodynamic limit $N\to\infty$, when the fluctuations of the global superfluid density $\tilde{\Delta}^2$ vanish, the SC free energy is determined solely by the generalized Abrikosov parameter $\beta_a(\Delta)$. Thus, in the present estimate, we may evaluate the contributions to the free energy from the selected configurations $\Delta_j(x,y)$ by considering the corresponding values of $\beta_a(\Delta_j)$. Using normalized wave functions, i.e., setting $\overline{|\Delta_j|^2}=1$, and taking advantage of the orthogonality condition and the related properties $\overline{|\Delta_j|^2\Delta_0\Delta_1^*}=\overline{\Delta_0^2\Delta_1^{*2}}=0$, the calculation of the Abrikosov parameter yields

$$\beta_a=\frac{1}{4}[\beta_A+\beta_a(\Delta_1)]+\overline{|\Delta_0|^2|\Delta_1|^2}, \tag{25}$$

where $\beta_A=\beta_a(\Delta_0)$. Now, it is easy to show that $\overline{|\Delta_0|^2|\Delta_1|^2}\approx 1$, so that since $\beta_a(\Delta_1)\geqslant\beta_A\approx 1$, one finds that $\beta_a\gtrsim 1.5$. This is a remarkable result, since it implies that the energy cost of the fluctuations under study,

$$F_{GL}-F_{MF}=\pi N\left(\frac{\alpha^2}{2\beta}\right)\left(\frac{1}{\beta_A}-\frac{1}{\beta_a}\right)\gtrsim\frac{1}{3}|F_{MF}|,$$

so that their influence becomes significant only in the high-temperature region near $T_c$.

This type of fluctuations may become important, however, at temperatures well below $T_c$ if the relative amplitude $\varepsilon$ of the injected chains is selected to be much smaller than that of the regular ones. Indeed, writing down the total order parameter as

$$\Delta(\vec{r})=\Delta_0(\vec{r})+\varepsilon\Delta_1(\vec{r}), \tag{26}$$

where $\varepsilon\ll 1$, and calculating the correction to $\beta_a$ up to second order in $\varepsilon$, we obtain

$$\beta_a(\Delta)=\beta_a(\Delta_0)+2(2-\beta_a(\Delta_0))\varepsilon^2\approx\beta_A+2\varepsilon^2. \tag{27}$$

Note that in Eq. (25) the only term which depends on the location of the injected orbital with respect to the regular ones is the overlap integral $\overline{|\Delta_0|^2|\Delta_1|^2}\approx 1$, so that the energy cost of an injected orbital is nearly independent of its location with respect to the regular ones. Thus the distribution of injected orbitals should be completely random.

In investigating the influence of these randomly injected orbitals on the structure factor of the vortex lattice, the weak effect associated with the small amplitude $\varepsilon$ is expected to dramatically weaken further due the random nature of the injections. Similar to diffraction patterns obtained from crystals with randomly distributed point defects,[25] the structure factor in our case is dominated by the sharp peaks associated with the coherent scattering by the regular vortex chains. The incoherent scattering by the randomly injected chains introduces a weak, nearly uniform background into the structure factor.

This can be seen by considering the Fourier component $I(\vec{Q})=\int d^2r|\Delta_0(\vec{r})+\varepsilon\Delta_1(\vec{r})|^2e^{(\vec{Q}\cdot\vec{r})}$ in Eq. (17), which leads to the structure factor

$$S(\vec{Q})=S_0(\vec{Q})+\Delta S(\vec{Q})+o(\varepsilon^4), \tag{28}$$

where $S_0(\vec{Q})$ is the structure factor derived for the regular chains. The leading correction is determined by the cross term

$$\Delta S(\vec{Q})\propto\frac{\varepsilon^2}{N}\int d^2r_1d^2r_2\exp[i(\vec{Q}\cdot\vec{\rho})]\times\Delta_0(\vec{r}_1)\Delta_0^*(\vec{r}_2)\langle\Delta_1(\vec{r}_2)\Delta_1^*(\vec{r}_1)\rangle+\text{c.c.}, \tag{29}$$

where $\vec{\rho}=\vec{r}_1-\vec{r}_2$ and the average is over the distribution of the injected orbitals. Note that the linear terms in $\varepsilon$ vanish after this averaging. Substituting in Eq. (29) the correlation function for a random distribution of injected orbitals,[26] $\langle\Delta_1(\vec{r}_2)\Delta_1^*(\vec{r}_1)\rangle\sim|c_0|^2e^{i2\rho_xR_y}e^{-\vec{\rho}^2/2}$, where $R_y=(y_1+y_2)/2$, and using for simplicity the form of $\Delta_0(x,y)$ corresponding to a square lattice, one finds

$$\Delta S(Q_x,Q_y)\sim\varepsilon^2\exp\left(-\frac{1}{4}Q^2\right).$$

Thus, in contrast to the macroscopically intense (i.e., with intensity $\sim N^\sigma$, $0<\sigma\leqslant 1$), sharp peaks of $S_0(\vec{Q})$, the correction $\Delta S(Q_x,Q_y)$ due to the injected orbitals is a very smooth function of $\vec{Q}$, with a very small magnitude (i.e., of the order of $\varepsilon^2$).

We therefore conclude that the destructive effect of the injected orbitals on the long-range nematic order becomes significant only at $T\sim T_c\gg T_m$. In this temperature range other types of fluctuations are also important. For example,

phase fluctuations corresponding to shear motions along Bragg chains with large Miller indices have also a characteristic energy of the order of $\varepsilon_0$ and so play a significant role in transforming the Bragg liquid into an isotropic fluid.

Finally, one should note that the rotational symmetry of the GL free energy functional for infinite uniform superconductors implies that the vortex lattice is actually disordered even at low temperatures due to the presence of the superconductor boundaries and of random pinning interactions in the bulk. The remarkable rigidity of the chain structure found in this paper thus suggests that local pinning potentials and boundary conditions should be noticeable not only locally but also in the whole sample.

## VI. CONCLUSION

In this article we provide a detailed analysis of the Landau orbital description of the vortex state in extremely type-II, 2D superconductors at high perpendicular magnetic fields. The apparent advantage of this representation in the low-temperature region near the vortex lattice melting point, where shear fluctuations along the principal axis of the vortex lattice are dominant, is due to the small energy scale $\lambda^2\varepsilon_0$, $\lambda \approx 0.066$, of these fluctuations, as compared with the SC condensation energy $\varepsilon_0$. Similar shear fluctuations along any other direction, determined by a Bragg family of chains in the Abrikosov vortex lattice, cost significantly more energy, and therefore the first stage of the vortex lattice melting process is discontinuous. The corresponding melting temperature $T_m$ is a small $(\sim\lambda^2)$ fraction of $T_c$. The vortex state around this melting point is characterized by short-range superconducting order, quasi-long-range positional order,[6,9] and long-range orientational order along a principal Bragg chain direction. At the melting point the orientational symmetry is not completely broken; it just changes discontinuously to a different symmetry group. This vortex phase is everywhere locally incompressible.

In extending the discussion to the vortex liquid state well above the melting temperature $T_m$ the Bragg chain structure still plays a significant role. As the temperature increases above $T_m$ a special type of amplitude fluctuations, which can be described as classical transverse vibrations of vortex chains around their equilibrium periodic positions (while vortices are accumulated and depleted alternately in individual chains), leads to the destruction of the quasi-long-range 2D positional order, but without breaking the long-range orientational order of the chains. Due to the 1D nature of these vibrations, the structure factor suffers thermal broadening of the diffraction peaks, which remains effective at temperatures far below the mean field $T_c$.

All other types of fluctuations, which destroy this ''nematic'' orientational order, have characteristic energies of the order of the SC condensation energy $\varepsilon_0$, so that their influence become significant only near $T_c$.

Experimental confirmation of this picture may relay on the small-angle neutron scattering (SANS) technique,[27] in which the magnetic structure factor of the superconductor can be measured directly. Taking into account the crucial role played by defects and the sample boundaries in pinning clusters of vortex chains along random directions in the superconductor, one may use the approximation expression, Eq. (28), for the structure factor only after averaging over this random distribution of directions. This average modifies the coherent scattering term $S_0(\vec{Q})$ significantly, yielding temperature-independent broadening of the diffraction peaks and eliminating the intrinsic anisotropy which characterizes $S_0(\vec{Q})$ in an infinite, ideal superconductor. The transverse vibrations, discussed in Sec. IV, further broaden the diffraction peaks. Since this broadening is temperature dependent, it can, in principal, be extracted from the experimental linewidth; in the temperatures range $T_m \ll T \ll T_c$, we predict that the width of the peaks increases linearly with the temperature, in a similar manner to that observed in single-crystal niobium by Lynn *et al.*[28]

Finally, on the basis of this picture it seems plausible to model the vortex liquid state in the broad region between the melting point and the mean-field superconducting transition as a gas of weakly interacting vortex chains. Such a description has been recently shown,[29,30] to account very well for the SC-induced damping of the dHvA oscillations observed in the vortex liquid state of the quasi-2D organic charge transfer salt $\kappa$-$(\mathrm{BEDT\text{-}TTF})_2\mathrm{Cu}(\mathrm{NCS})_2$.[7,8,31]

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[1] A. Schilling, R.A. Fisher, N.E. Phillips, U. Welp, D. Dasgupta, W.K. Kwok, and G.W. Crabtree, Nature (London) **382**, 791 (1996).

[2] E. Zeldov, D. Majer, M. Konczykowski, V.B. Geshkenbein, V.M. Vinokur, and H. Shtrikman, Nature (London) **375**, 373 (1995).

[3] S. Friemel, C. Pasquier, Y. Loirat, D. Jerome, Physica C **259**, 181 (1996).

[4] J. Wosnitza, J. Low Temp. Phys. **117**, 1701 (1999).

[5] E. Brezin, D.R. Nelson, and A. Thiaville, Phys. Rev. B **31**, 7124 (1985).

[6] V.N. Zhuravlev and T. Maniv, Phys. Rev. B **60**, 4277 (1999).

[7] T. Sasaki, W. Biberacher, K. Neumaier, W. Hehn, K. Andres, T. Fukase, Phys. Rev. B **57**, 10 889 (1998).

[8] P.J. van der Wel, J. Caufield, S.M. Hayden, J. Singelton, M. Springford, P. Messon, W. Hays, M. Kurmoo, and P. Day, Synth. Met. **70**, 831 (1995).

[9] For a recent review see T. Maniv, V. Zhuravlev, I.D. Vagner, and P. Wyder, Rev. Mod. Phys. **73**, 867 (2001).

[10] S. Doniach and B.A. Huberman, Phys. Rev. Lett. **42**, 1169 (1979).

[11] Daniel S. Fisher, Phys. Rev. B **22**, 1190 (1980).

[12] J.M. Kosterlitz and D.J. Thouless, J. Phys. C **6**, 1181 (1973); B.I. Halperin and D. Nelson, Phys. Rev. Lett. **41**, 121 (1978); A.P. Young, Phys. Rev. B **19**, 1855 (1979).
[13] Zlatko Tesanovic and A.V. Andreev, Phys. Rev. B **49**, 4064 (1994).
[14] Yusuke Kato and Naoto Nagaosa, Phys. Rev. B **48**, 7383 (1993).
[15] R. Sasik and D. Stroud, Phys. Rev. B **49**, 16 074 (1994).
[16] J. Hu and A.H. MacDonald, Phys. Rev. B **56**, 2788 (1997).
[17] M. Franz and S. Teitel, Phys. Rev. Lett. **73**, 480 (1994).
[18] A. Tanaka and X. Hu, Physica C **357-360**, 438 (2001).
[19] T. Maniv, A.I. Rom, I. Vagner, and P. Wyder, Phys. Rev. B **46**, 8360 (1992).
[20] F.D. Haldane and E.H. Rezayi, Phys. Rev. B **31**, 2529 (1985).
[21] H.E. Stanley, *Introduction to Phase Transitions and Critical Phenomena* (Clarendon, Oxford, 1971).
[22] R. Sasik, D. Stroud, and Z. Tesanovic, Phys. Rev. B **51**, 3042 (1995).
[23] V.J. Emery and J.D. Axe, Phys. Rev. Lett. **40**, 1507 (1978).
[24] R. Spal, C.E. Chen, T. Egami, P.J. Nigrey, and A.J. Heeger, Phys. Rev. B **21**, 3110 (1980).
[25] W. Moritz, in *Determination of Surface Structure by LEED* edited by P.M. Marcus and F. Jona (Plenum, New York, 1984).
[26] M.J. Stephen, Phys. Rev. B **45**, 5481 (1992).
[27] X.S. Ling, S.R. Park, B.A. McClain, S.M. Choi, D.C. Dender, and J.W. Lynn, Phys. Rev. Lett. **86**, 712 (2001).
[28] J.W. Lynn, N. Rosov, T.E. Grigereit, H. Zhang, and T.W. Clinton, Phys. Rev. Lett. **72**, 3413 (1994).
[29] V.N. Zhuravlev, T. Maniv, I. Vagner, and P. Wyder, cond-mat/9812040 (unpublished).
[30] V.N. Zhuravlev, T. Maniv, I. Vagner, and P. Wyder, Physica B **315**, 47 (2002).
[31] H. Ito, S.M. Hayden, P.J. Meeson, M. Springford, and G. Saiyo, J. Supercond. **12(3)**, 525 (1999).