Free flux flow resistivity in a strongly overdoped high-T_c cuprate: The purely viscous motion of the vortices in a semiclassical d-wave superconductor

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(Received 13 March 2002; published 17 July 2002)

We report the free flux flow (FFF) resistivity associated with a purely viscous motion of the vortices in a moderately clean d-wave superconductor Bi:2201 in the strongly overdoped regime ($T_c = 16$ K) for a wide range of the magnetic field in the vortex state. The FFF resistivity is obtained by measuring the microwave surface impedance at different microwave frequencies. It is found that the FFF resistivity is remarkably different from that of conventional s-wave superconductors. At low fields ($H < 0.2H_{c2}$) the FFF resistivity increases linearly with H with a coefficient which is far larger than that found in conventional s-wave superconductors. At higher fields, the FFF resistivity increases in proportion to $\sqrt{H}$ up to H_{c2}. Based on these results, the energy dissipation mechanism associated with the viscous vortex motion in ''semiclassical'' d-wave superconductors with gap nodes is discussed. Two possible scenarios are put forth for these field dependences: the enhancement of the quasiparticle relaxation rate and the reduction of the number of the quasiparticles participating the energy dissipation in d-wave vortex state.

DOI: 10.1103/PhysRevB.66.014527 PACS number(s): 74.25.Fy, 74.25.Nf, 74.60.Ec

I. INTRODUCTION

When a vortex line in a type-II superconductor moves in the superfluid, the frictional force is determined by the damping viscosity, which in turn depends on the energy dissipation processes of quasiparticles. The problem of the energy dissipation associated with the viscous motion of the vortices has continued to hold the attention of researchers for years. To gain an understanding of the energy dissipation, the experimental determination of the free flux flow (FFF) resistivity is particularly important. Hereafter the term FFF will refer to a purely viscous motion of the vortices, which is realized when the pinning effect on the vortices is negligible. The FFF resistivity is known to be one of the most fundamental quantities in the superconducting state. In fully gapped s-wave superconductors, the flux flow state has been extensively studied and by now a rather good understanding of the energy dissipation processes has been achieved.[1–7] In s-wave superconductors, the quasiparticles trapped inside the vortex core play a key role in the dissipation processes. Moreover, it has been shown that there is a fundamental difference in the quasiparticle energy relaxation processes among dirty ($\xi > l$), moderately clean ($\xi < l < \xi\varepsilon_F/\Delta$), and superclean ($l > \xi\varepsilon_F\Delta$), s-wave superconductors, where ξ is the coherence length, l is the mean free path, ε_F is the Fermi energy, and Δ is the superconducting energy gap.

A renewed interest in the problem concerning the quasiparticle dissipation is owed to recent developments in the investigation of unconventional superconductors. The latter are characterized by superconducting gap structures which have nodes along certain crystal directions. In the last two decades unconventional superconductivity has been found in several heavy fermion, organic, and oxide materials. From the viewpoint of the physical properties of the vortex state, perhaps the most relevant effects of the nodes are the existence of gapless quasiparticles extending outside the vortex core.[8–10] In fact recent studies of heat capacity,[11] thermal conductivity,[12] and NMR relaxation rate[13] provide a strong evidence that these quantities are governed by delocalized quasiparticles. However, despite these extensive studies of the vortex state of unconventional superconductors, the microscopic mechanisms of the energy dissipation associated with viscous vortex motion is still far from being completely understood, exposing explicitly our incomplete knowledge of the vortex dynamics in type-II superconductors. Thus it is particularly important to clarify whether the arguments of the energy dissipation are sensitive to the symmetry of the pairing state.[14]

Recently, the flux flow resistivities in the f-wave superconductor UPt_3 and d-wave high-T_c cuprates, both with line nodes, were demonstrated to be quite unusual. However, these materials may not be suitable for the study of the typical behavior of the flux flow resistivity in unconventional superconductors. The T vs H phase diagram of UPt_3, which still is controversial, is considered to consist of various phases with different superconducting gap functions, which complicates considerably the interpretation of the FFF resistivity.[15,16] The flux flow resistivities of $YBa_2Cu_3O_{7-\delta}$

and $Bi_2Sr_2CaCu_2O_{8+\delta}$ in the underdoped and optimally doped regimes have been measured by several groups but here again there are several difficulties in interpreting them.[17–22] For instance, the measurements could not cover a wide field range in the vortex state due to the extremely large upper critical field H_{c2}. Moreover, very recent scanning tunneling microscope (STM) measurements have demonstrated that the vortex core structure of these high-T_c cuprates is very different from that expected in the semiclassical *d*-wave superconductor,[24,25] possibly due to the extremely short coherence lengths and the strong antiferromagnetic fluctuation effect within the core.

The situation therefore calls for the need for a textbook example of the FFF resistivity of unconventional superconductors with nodes, in which the semiclassical description of the vortex core discussed in the literature, e.g., Refs. 8–10 and 26, applies. Especially the FFF resistivity in the "semiclassical" superconductors in the moderately clean regime is strongly desired, because almost all unconventional superconductors fall within this regime. It should be noted that the determination of the FFF resistivity is not only important for understanding the electronic structure in the vortex state but is also relevant for analyzing the collective motion of the vortices, such as flux creep phenomena. This is easily understood if one recalls that the motion of the vortices in the vortex liquid and solid phase in high-T_c cuprates has been analyzed by assuming the Bardeen-Stephen relation for an individual vortex, as discussed in Sec. V. Then, if the FFF resistivity strongly deviates from the Bardeen-Stephen relation, the interpretation of the collective motion of the vortices should be modified.

We stress here that high-T_c cuprates in the strongly overdoped regime are particularly suitable for the above purpose because of the following reasons. (i) Most importantly, it appears that the semiclassical description of the electronic structure of the vortex core is adequate in strongly overdoped materials.[8–10,26] This is because many experiments have revealed that in the overdoped regime the electron correlation and antiferromagnetic fluctuation effects, which might change the vortex core structure dramatically as observed in STM measurements, are much weaker than those in optimally doped and underdoped materials. In fact most of the physical properties in the overdoped materials are well explained within the framework of Fermi liquid theory. (ii) Low H_{c2} enables us to measure the FFF resistivity for a wide range of the vortex state. (iii) The large coherence lengths and small anisotropy ratio reduce the superconducting fluctuation effect which make interpretation of flux flow resistivity complicated. In fact, as we discuss in Sec. IV, the resistive transition of the overdoped materials in a magnetic field is much sharper than that of optimally doped and underdoped materials. (iv) The flux flow Hall angle which complicates analysis of the flux flow state is very small.[18,27]

The purpose of this work is to present and discuss our experimental results on the FFF resistivity ρ_f of moderately clean *d*-wave superconductors. The experiments were carried out using strongly overdoped Bi:2201. This system is an excellent choice for studying the FFF resistivity. It has a comparatively simple crystal structure (no chain, single CuO_2 layer) and hence the band structure is simple. H_{c2} is within laboratory reach over a very broad range of temperatures. A major cause of difficulty in obtaining the FFF resistivity in high-T_c cuprates was the strong pinning effect. To overcome this difficulty, we have measured the microwave surface impedance at different frequencies. High-frequency methods are suitable for this purpose because they probe the vortex response at very low currents when the vortices undergo reversible oscillations and they are less sensitive to flux creep.[23,28] We show that the FFF resistivity of the "semiclassical" *d*-wave superconductor is very different from that of conventional *s*-wave superconductors. On the basis of the results, we discuss the dissipation mechanism associated with viscous motion of the vortices in unconventional superconductors.

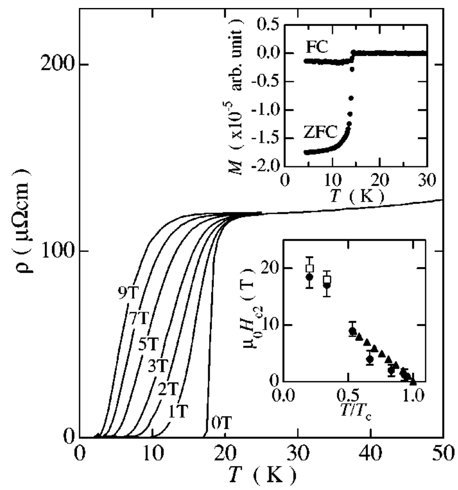

FIG. 1. The resistive transition in magnetic field of overdoped Bi:2201 in the same batch with $T_c = 18$ K. Inset (upper): the magnetization at 5 Oe of the same sample used for the microwave measurements under the conditions of zero field cooling (ZFC) and field cooling (FC). Inset (lower): T dependence of H_{c2} determined by three different methods. The solid triangles denote H_{c2} defined by the dc resistive transition in the main panel, using the criterion $\rho = 0.5\rho_n$. The solid circles denote H_{c2} defined by the magnetic field at which ρ_1 becomes frequency independent. The open squares denote H_{c2} defined by the field at which R_s reaches to a normal-state value. H_{c2} is estimated to be ~ 20 T below 5 K.

II. EXPERIMENT

High-quality single crystals of Bi:2201 ($Bi_{1.74}Pb_{0.38}Sr_{1.88}Cu_{1.00}O_y$) in the overdoped regime with transition temperature $T_c = 16$ K were grown by the floating zone method.[29] The sample size used for the microwave measurement was ~ 0.8 mm$\times$0.7 mm$\times$0.04 mm. The upper inset of Fig. 1 depicts the magnetization at the superconducting transition for the same sample used for the microwave measurements. The normal-state resistivity in the ab plane, ρ_n, depends on T as $\rho_n \propto T^\beta$ with $\beta \sim 2$, the typical Fermi liquid behavior which can be seen in the overdoped high-T_c cuprates. The resistive transition of the sample in the same batch with $T_c = 18$ K is also shown in Fig. 1. Both resistive transition in zero field and magnetization measurements show a sharp superconducting transition.

The microwave surface impedance $Z_s = R_s + iX_s$, where R_s and X_s are the surface resistance and surface reactance, respectively, was measured by the standard cavity perturbation technique using cylindrical cavity resonators made by oxygen-free copper operated in TE_{011} mode. The resonance frequencies of these cavities were approximately 15 GHz, 30 GHz, and 60 GHz. The sample was placed in an antinodes of the oscillatory magnetic field H_{ac}, such that H_{ac} lies parallel to the c axis of the sample. The external dc magnetic field was applied perpendicular to the ab plane. In this configuration, the two-dimensional pancake vortices respond to an oscillatory driving current induced by H_{ac} within the ab planes. The cavities at 15 GHz and 30 GHz were operated at 1.7 K and sample temperatures were controlled by hot finger techniques using a sapphire rod. The sample temperature in the cavity at 60 GHz was controlled by changing the temperature of the cavity. The Q values of each cavity are 6.2×10^4 for 15 GHz, 2.3×10^4 for 30 GHz at 4.2 K, and 2×10^4 at 4.2 K and 1.5×10^4 at 20 K for 60 GHz. According to the cavity perturbation theory, R_s and X_s can be obtained by

$$R_s = G\left(\frac{1}{2Q_s} - \frac{1}{2Q_0}\right) = G\Delta\left(\frac{1}{2Q}\right) \tag{1}$$

and

$$X_s = G\left(-\frac{f_s - f_0}{f_0}\right) + C = G\left(-\frac{\Delta f}{f_0}\right) + C, \tag{2}$$

where Q_s and f_s are the Q factor and the resonance frequency of the cavity in the presence of a sample, and Q_0 and f_0 are those without a sample. G is a geometrical factor and C is a metallic shift constant.

In Figs. 2(a) and 2(b), the T dependences of R_s and X_s for Bi:2201 at 15 GHz are shown. The measurements in a magnetic field have been performed in the field cooling condition. We first discuss R_s and X_s in zero field. In zero field, both R_s and X_s decrease rapidly with decreasing T below the transition. Let us quickly recall the behavior of Z_s in the superconductors. In the normal state, the microwave response is dissipative and $R_s = X_s = \mu_0 \omega \delta$, where μ_0 is the vacuum permeability, $\omega/2\pi$ is the microwave frequency, and $\delta_n = \sqrt{2\rho_n/\mu_0\omega}$ is the normal-state skin depth. In Bi:2201, l is estimated to be ~ 200 Å, which is much shorter than δ_n at the onset in our frequency range. We therefore can determine the absolute value of R_s and X_s from the comparison with ρ_n assuming $R_s = X_s$ (Hagen-Rubens relation). In the Meissner phase, the microwave response is purely reactive and $R_s \simeq 0$ and $X_s = \mu\omega\lambda_{ab}$, where λ_{ab} is the London penetration depth in the ab plane. Using $\rho_n = 130$ $\mu\Omega$ cm for Bi:2201 at the onset, we obtained $\lambda_{ab} = 1500$ Å at $T = 0$. This value is slightly smaller than the penetration depth in $YBa_2Cu_3O_{7-\delta}$ and $Bi_2Sr_2CaCu_2O_{8+\delta}$. In the inset of Fig. 2(b), $\Delta\lambda = \lambda(0) - \lambda(T)$ at low temperatures is plotted as a function of T^2. Here $\Delta\lambda$ is proportional to T^2. The relation $\Delta\lambda \propto T^2$ has been observed in many high-T_c cuprates and discussed in terms of the superfluid density in d-wave superconductors with the impurity state.[30]

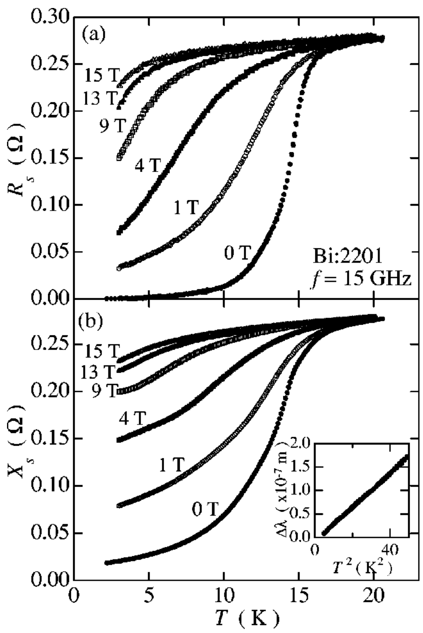

FIG. 2. T dependence of the surface resistance R_s (a) and surface reactance X_s (b) at 15 GHz in a magnetic field. Both the microwave magnetic field $\mathbf{H}_{ac}$ and dc magnetic field $\mathbf{B}$ are applied parallel to the c axis ($\mathbf{H}_{ac} \| \mathbf{B} \| c$). In this configuration, the energy dissipation is caused by oscillation of the two-dimensional pancake vortices. The measurements have been done under the field cooling condition. The absolute values of R_s and X_s were determined by the normal-state dc resistivity. Inset: $\Delta\lambda = \lambda(0) - \lambda(T)$ at low temperatures is plotted as a function of T^2.

III. SURFACE IMPEDANCE IN THE VORTEX STATE

We now focus on the surface impedance in the vortex state. Figure 3 shows the H dependence of R_s and X_s of Bi:2201 at 15 GHz. In these measurements R_s and X_s are obtained by sweeping H. The hysteresis due to the effect of the trapped field in the crystal is very small. Moreover, both R_s and X_s obtained by sweeping H well coincide with those obtained under the field cooling conditions shown in Fig. 2. These results indicate that neither inhomogeneous field distribution inside the crystal nor magnetostriction[42] caused by sweeping H seriously influences the analysis of Z_s.

In the vortex state, Z_s is governed by the vortex dynamics. We may roughly estimate R_s in the limit of large and negligible rf field penetration as follows. In the flux flow state when the pinning frequency $\omega_p/2\pi$ is negligible compared to the microwave frequency ($\omega_p \ll \omega$), two characteristic length scales—namely, λ_{ab} and the flux flow skin depth $\delta_f \sim \sqrt{2\rho_f/\mu_0\omega}$—appear in accordance with the microwave field penetration. At low fields, λ_{ab} greatly exceeds δ_f ($\lambda_{ab} \gg \delta_f$). In this regime, R_s and X_s are given as $R_s \sim \rho_f/\lambda_{ab}$ and $X_s \sim \mu_0\omega\lambda_{ab}$. On the other hand, at high fields where δ_f greatly exceeds λ_{ab} ($\delta_f \gg \lambda_{ab}$), the viscous loss becomes dominant and the response is similar to the normal

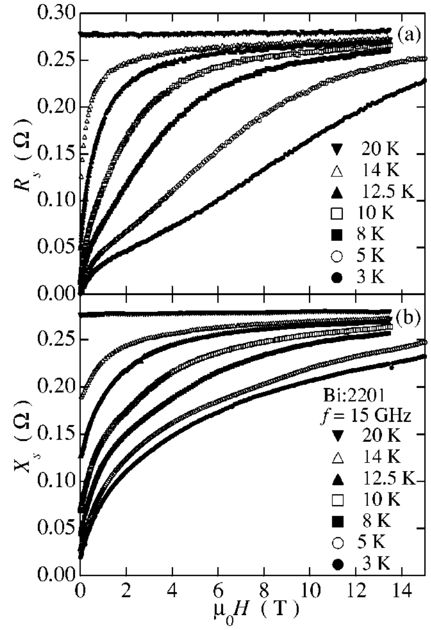

FIG. 3. Field dependence of the surface resistance R_s (a) and surface reactance X_s (b) at 15 GHz measured by sweeping H.

state ($R_s \simeq X_s$) except that δ_n is replaced by δ_f. In the presence of pinning centers of the vortices, R_s is reduced as discussed below.

We here analyze the field dependence of Z_s in accordance with the theory of Coffey and Clem.[28] The equation of vortex motion for the vortex line velocity $\mathbf{u}$,

$$\eta \mathbf{u} + \kappa_p \mathbf{x} = \Phi_0 J \times \hat{\mathbf{z}}, \tag{3}$$

where η and κ_p are the viscous drag constant and pinning parameter, respectively, and $\hat{\mathbf{z}}$ the unit vector parallel to $\mathbf{B}$ (we take $\mathbf{J} \| \mathbf{x}$). According to Coffey and Clem, the field dependence of Z_s in the Meissner and vortex phases is expressed as

$$Z_s = i\mu_0 \omega \lambda_{ab} \left[\frac{1-(i/2)\delta_v^2/\lambda_{ab}^2}{1+2i\lambda_{ab}^2/\delta_{nf}^2} \right]^{1/2}, \tag{4}$$

where $\delta_v^2 = \delta_f^2 (1 - i\omega_p/\omega)^{-1}$ with $\omega_p/2\pi = \kappa_p/2\pi\eta$ being the pinning frequency. Writing Z_s in terms of the complex resistivity $\rho = \rho_1 + i\rho_2$ as $Z_s = \sqrt{i\omega\mu_0(\rho_1 + i\rho_2)}$, we have

$$\rho_1 = \mu_0 \omega \frac{\lambda_{ab}^2 s}{1+s^2} + \rho_f \frac{1}{1+s^2} \frac{1+sp}{1+p^2} \tag{5}$$

and

$$\rho_2 = \mu_0 \omega \frac{\lambda_{ab}^2}{1+s^2} + \rho_f \frac{1}{1+s^2} \frac{p-s}{1+p^2}, \tag{6}$$

where $s = 2\lambda_{ab}^2/\delta_{nf}^2$ and $p = \omega_p/\omega$. In Eqs. (5) and (6), the first terms on the right-hand side are ρ_1 and ρ_2 at zero field, and second terms represent the field dependence. In what follows we discuss the microwave response, focusing on ρ_1 obtained from R_s and X_s. Figure 4 shows the field dependence of $\Delta\rho_1(H) = \rho_1(H) - \rho_1(0)$ at three different microwave frequencies. The field dependence of $\Delta\rho_1$ is frequency dependent; ρ_1 increases with increasing frequency. Since ρ_1 is reduced by the vortex pinning effect, as seen in Eq. (5), this result indicates that the pinning effect of the vortices is not negligible for analysis of the flux flow resistivity in our microwave frequency range. Therefore, it is necessary to determine the pinning frequency for an accurate determination of the FFF resistivity.

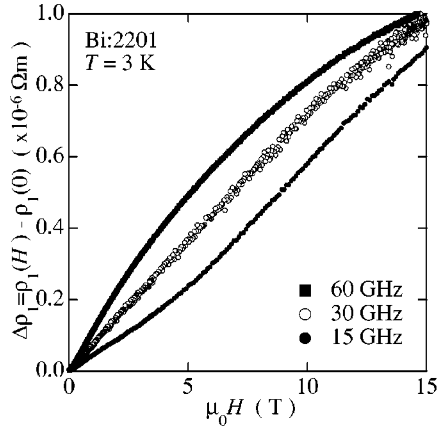

FIG. 4. The field dependence of $\Delta\rho_1(H) = \rho_1(H) - \rho_1(0)$ obtained from R_s and X_s at three different microwave frequencies. ρ_1 increases with increasing microwave frequency.

In Fig. 5, $\Delta\rho_1$ at $T=3$ K is plotted as a function of the microwave frequency. The solid lines show the results of the fitting by $\Delta\rho_1(H,\omega) = \rho_f \omega^2/(\omega^2 + \omega_p^2)$. It should be noted that since $s \ll 1$ except the vicinity of H_{c2}, the H dependence of s little influences the present analysis. Nevertheless, we restrict our analysis at $H \lesssim 10$ T to avoid the influence of the H dependence of s. The fitting parameters are ω_p and ρ_f. The ambiguity for determining ω_p and ρ_f is small. The H dependence of the pinning frequency obtained by the fitting is depicted in Fig. 6. At low field, $\omega_p/2\pi$ is approximately 22 GHz at $T=3$ K and 17 GHz at 5 K. These values are much

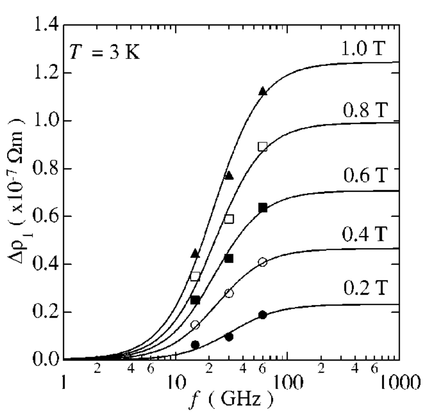

FIG. 5. Frequency dependence of $\Delta\rho_1(H)$ at $T=3.0$ K. [Solid triangles (1.0 T), open squares (0.8 T), solid squares (0.6 T), open circles (0.4 T), solid circles (0.2 T)]. The solid lines are the results of the fitting by $\Delta\rho_1(B,\omega) = \rho_f \omega^2/(\omega^2 + \omega_p^2)$. For details, see the text.

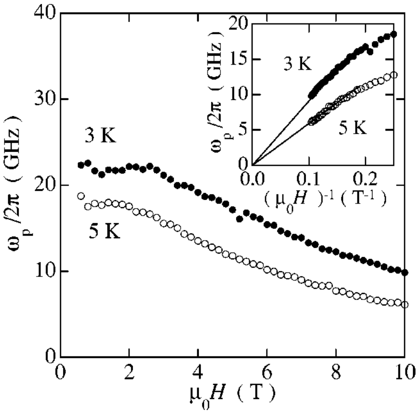

FIG. 6. The field dependence of the pinning frequency $\omega_p/2\pi$ at $T=3$ K and 5 K obtained by the fitting shown in Fig. 4. Inset: same data plotted as a function of $1/H$. $\omega_p/2\pi$ decays in proportion to $1/H$. The solid lines show the relation $\omega_p/2\pi \propto 1/H$.

larger than the pinning frequency in $Bi_2Sr_2CaCu_2O_{8+\delta}$ but much smaller than $\omega_p/2\pi$ in $YBa_2Cu_3O_7$.[19,22,23] At low field, ω_p decreases gradually, while at $\gtrsim 1.5$ T ω_p decreases approximately as $\omega_p \propto H^{-1}$, as shown in the inset of Fig. 6.

IV. FREE FLUX FLOW RESISTIVITY OF Bi:2201

Before discussing the FFF resistivity, it will prove useful to first comment on H_{c2} of Bi:2201. It is well known that the resistive transitions of high-T_c cuprates are significantly broadened in magnetic field due to the strong thermal fluctuation effect and the vortex dynamics. Although in overdoped Bi:2201 such a broadening effect is relatively small, it still becomes an obstacle in determining H_{c2}.[31] In the lower inset of Fig. 1, we plot H_{c2} determined by three different methods. The solid triangles represent H_{c2} defined by the dc resistive transition in Fig. 1, using a criterion $\rho = \frac{1}{2}\rho_n$. The solid circles are H_{c2} defined by the magnetic field at which ρ_1 becomes frequency independent. The open squares represent H_{c2} defined by the field at which R_s reaches a normal-state value. The values of H_{c2} obtained from the three different methods do not differ significantly. A striking divergence in H_{c2} as the temperature approached zero was reported in the overdoped Tl:2201 in the transport measurements,[32] while such a divergent behavior was not observed in the specific heat and Raman scattering measurements.[33] The divergent behavior of H_{c2} was discussed in terms of several proposed models, such as Josephson-coupled small grains with T_c higher than the bulk.[34,35] However, in the present Bi:2201 such anomalies are not observed in H_{c2} at least above 2 K. At present we do not know the reason for this difference. From these measurements, H_{c2} is estimated to be approximately 20 T below 5 K.

In Fig. 7(a), we plot ρ_f/ρ_n as a function of H/H_{c2} at 3 K, assuming $H_{c2}=19$ T. If we assume $H_{c2}=17$ T at $T=5$ K, both ρ_f almost exactly coincide with ρ_f at 3 K, as shown in Figs. 7(a) and 7(b). The field dependence of ρ_f is convex. We found that the there are two characteristic regimes in the H dependence of ρ_f. In the low-field region ($H/H_{c2}<0.2$), ρ_f increases linearly with H as

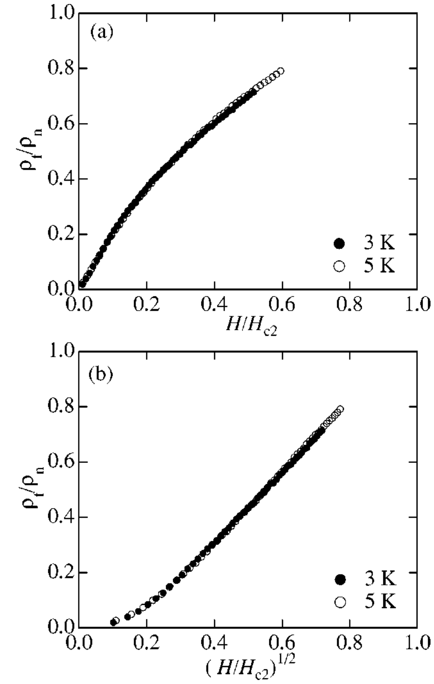

FIG. 7. (a) The flux flow resistivity at $T=3$ K and 5 K as a function of H/H_{c2}. We assumed $H_{c2}=19$ T at 3 K and 17 T at 5 K. The flux flow resistivity is normalized by the normal-state value. (b) Same data plotted as a function of $\sqrt{H/H_{c2}}$.

$$\rho_f = \alpha \frac{H}{H_{c2}} \rho_n, \tag{7}$$

with $\alpha \simeq 2$. A deviation from H-linear dependence is clearly observed at higher field. In Fig. 7(b), ρ_f/ρ_n is plotted as a function of $\sqrt{H/H_{c2}}$. We found that ρ_f increases as

$$\rho_f \propto \sqrt{\frac{H}{H_{c2}}} \tag{8}$$

at $H/H_{c2} \gtrsim 0.2$. Since the linear extrapolation of ρ_f/ρ_n in Fig. 7(b) points to $\rho_f/\rho_n=1$ at $H/H_{c2}=1$, it is natural to expect that the relation of Eq. (8) continues all the way up to H_{c2}.

V. DISCUSSION

A. Flux flow in *s*-wave superconductors

In order to contrast the present results with the FFF resistivity of isotropic *s*-wave superconductors, we first briefly review the flux flow state in *s*-wave superconductors.

For isotropic *s*-wave pairing in the dirty regime, the Bardeen-Stephen model appears to be quite successful in describing the energy dissipation.[1–3] Bardeen-Stephen theory models the vortex core as a cylinder whose radius is the coherence length. It is assumed that the core is a normal metallic state inside of which the energy dissipation is dominated by the impurity scattering, similar to the ordinary resistive process. This is a good approximation for dirty superconductors with $l<\xi$. It follows from this model that the FFF resistivity in dirty *s*-wave superconductors is proportional to the normal-state resistivity and is to the number of the vortices,

$$\rho_f = \rho_n H/H_{c2}. \tag{9}$$

The validity of this Bardeen-Stephen relation has been confirmed in most dirty *s*-wave superconductors almost throughout the whole Abrikosov phase: $H_{c1}<H<H_{c2}$.[3]

However, the description of the vortex core as a normal metal is limited to dirty *s*-wave superconductors. In moderately clean and superclean *s*-wave superconductors with $l>\xi$, the quasiparticle response to an electromagnetic field is radically different from that of normal electrons, since the model of a normal metallic core breaks down.[6,7] The difference lies in the fact that the quasiparticles in the core are subject to Andreev reflections by the pair potential and form the bound states of Caroli, de Gennes, and Matricon[36,37] before getting scattered by impurities. The largest energy difference between the bound states is roughly estimated as $\hbar\Omega_0 \sim \Delta^2/\varepsilon_F$, where Ω_0 is the angular velocity. The electric conduction in the vortex state is governed by the scattering time between the Andreev bound states in the presence of impurities. The effects of these quasiparticles on the vortex dynamics have been considered in a number of papers. For moderately clean *s*-wave superconductors, the FFF resistivity has been calculated as[2,7]

$$\rho_f \sim \rho_n \frac{1}{\ln\left(\dfrac{\Delta}{k_B T}\right)} \frac{H}{H_{c2}}. \tag{10}$$

The logarithmic factor results from the shrinkage of the vortex core at low temperature and logarithmic energy dependence of the impurity scattering rate of the Andreef bound state (Kramer-Pesch effect).[38] Thus, in spite of the fundamental difference of the character of the quasiparticles within the vortex core, the FFF resistivity in the moderately clean *s*-wave superconductors increases in proportion to *H*, which is similar to that in the dirty superconductors. In fact, the FFF resistivity of several moderately clean *s*-wave superconductors discovered recently was found to be proportional to *H*, though the logarithmic correction at very low temperature has never been reported so far.[39]

B. Flux flow in *d*-wave superconductors

We are now in position to discuss the FFF resistivity of semiclassical *d*-wave superconductors. It is obvious from Figs. 7(a) and 7(b) that *the field dependence of ρ_f expressed as Eqs. (7) and (8) is markedly different from that of conventional s-wave superconductors expressed as Eqs. (9) and (10).*

We first discuss the low-field behavior of the FFF resistivity in Bi:2201. The linear dependence of ρ_f on the magnetic field means that the energy dissipation per vortex does not depend on the magnetic field or the intervortex spacing. We can interpret this fact naturally if the energy dissipation is assumed to occur mainly near each vortex even in the superconductors with gap nodes. In fact, this assumption is justified by a numerical result on the ac response of the *d*-wave vortex.[6,43] Comparing Eq. (7) with Eq. (10), the coefficient of the *H*-linear term in *d*-wave superconductors is found to be nearly as twice as that in *s*-wave superconductors. This behavior is similar to UPt_3 with line nodes, in which ρ_f at low field is larger than that found in conventional *s*-wave superconductors.[15] It should be noted that a similar result was reported in very recent measurements of the high-purity borocarbide superconductor YNi_2B_2C with a very anisotropic superconducting gap, presumably anisotropic *s*-wave symmetry.[39,40] These results led us to conclude that a large initial slope is a common feature in the FFF resistivity of the superconductors with nodes. In what follows, we discuss possible origins for the enhancement of the FFF resistivity at low fields on the basis of the theoretical results available at the present stage.

According to Kopnin and Volovik, the vortex transport in semiclassical *d*-wave superconductors is governed by the dynamics of quasiparticles which form Andreev bound states around a vortex, much like in *s*-wave superconductors.[41] The excitation spectrum of those quasiparticles is given by

$$E(L,\theta) = -\Omega(\theta)L \tag{11}$$

in terms of the angle θ in momentum space and *L*, the angular momentum. In this expression, $\Omega(\theta)$ denotes the angular velocity, which depends on the direction θ. Roughly speaking, $\Omega(\theta)$ is proportional to the square of the energy gap, $\Delta(\theta)$ [$\propto\cos(2\theta)$ for $d_{x^2-y^2}$ states]. This branch corresponds to the Caroli-de Gennes-Matricon mode in the isotropic *s*-wave superconductors (in *s*-wave symmetry, Ω_0 is θ independent).[36,37] The quasiparticles with θ away from the nodes in *d*-wave vortex are well localized near vortex cores and they are similar, in nature, with those in an *s*-wave vortex. As the angle θ approaches a nodal direction, however, the quasiparticles become more extended and farther away from the vortex cores. In this way the character of quasiparticles in the *d*-wave vortex is very different from that of quasiparticles in the *s*-wave vortex.

According to the theory by Kopnin and Volovik based on the relaxation time approximation, the FFF resistivity is given by

$$\rho_f = \frac{B}{\langle\Omega(\theta)\rangle \tau_{\mathrm{v}} n_{\mathrm{e}} |e| c}, \tag{12}$$

where $\langle\cdots\rangle$ denotes the average over the Fermi surface, τ_{v} is the relaxation time of quasiparticles, and n_e is the carrier density in the vortex state.[26] In the theory of the relaxation time approximation, the transport coefficients are given in the form of the parallel circuit; the conductivity is expressed as a sum of the contribution from each part of the Fermi surface. Then magnitude of resistivity in vortex state expressed by Eq. (12) is governed by the largest value of $\Omega(\theta)$ on the Fermi surface. This fact is physically interpreted in the following way. The quasiparticles with smaller $\Omega(\theta)$ come from the vicinity of nodes. They are only weakly excited by vortex motion, because such quasiparticles are extended in regions far away from vortex cores. On the other hand, quasiparticles with larger $\Omega(\theta)$ are localized near vortex cores. Therefore it is likely that such quasiparticles are excited substantially by vortex motion and an appreciable deviation of the distribution function from the equilibrium state may occur. Thus, when the gap has nodes, portions of the Fermi surface near the nodal directions do not contribute to $\langle\Omega(\theta)\rangle$. This is in marked contrast to the isotropic *s*-wave superconductors, in which every part of the Fermi surface can contribute to $\langle\Omega(\theta)\rangle$. The reduction of the number of quasiparticles available for the energy dissipation in the superconductors with nodes gives rise to the enhanced flux flow resistivity. This scenario has been adopted in Ref. 15 to discuss the flux flow resistivity of UPt_3. Although this argument explains the low-field ($H<0.2H_{c2}$) behavior expressed as Eq. (7), it gives no account for the $\sqrt{H}$-dependence of ρ_f expressed as Eq. (7) observed in the almost whole regime at higher field ($0.2H_{c2}\lesssim H<H_{c2}$).

There is, however, another scenario. In the following part, we show that the reduction of $\tau_{\rm v}$ in *d*-wave vortex states explains consistently both Eqs. (7) and (8). Here we regard the impurity scattering as the main process of relaxation in the cuprates. Within the Born approximation, $\tau_{\rm v}$ is inversely proportional to the density of states (DOS) of quasiparticles available as the outgoing states in the scattering process of localized quasiparticles. On the other hand, the low-energy DOS of quasiparticles in *d*-wave vortex states is known to be larger than that in *s*-wave vortex states theoretically.[8,26,44] In Ref. 26, Kopnin and Volovik calculated the density of states $N_{\rm v}(E)$ per each *d*-wave vortex for energy E to obtain

$$N_{\rm v}(E)\sim N_0\xi^2(\Delta/E)\sim N_0\xi r(E), \tag{13}$$

where N_0 denotes the DOS on the Fermi surface in the normal state and $r(E)=\hbar v_F/E$ with v_F the Fermi velocity. The singularity at $E=0$ is removed by a cutoff length. According to Ref. 26, for energy E satisfying $r(E)>R_{\rm B}$ with intervortex distance $R_{\rm B}\sim\xi\sqrt{H_{c2}/B}$, $r(E)$ should be replaced by $R_{\rm B}$ for *pure* superconductors without impurity scattering. For *impure and clean* superconductors, instead, we speculate that $r(E)$ should be replaced by $R_{\rm B}$ or the mean free path $l_{\rm v}$ ($=v_F\tau_{\rm v}$), whichever is smaller. We then expect that

$$N_{\rm v}(0)/(N_0\xi^2)\sim\begin{cases} l_{\rm v}/\xi, & l_{\rm v}<R_{\rm B},\\ \sqrt{H_{c2}/B}, & R_{\rm B}<l_{\rm v},\end{cases} \tag{14}$$

for $E=0$. The quasiparticle DOS per each isotropic *s*-wave vortex is given by $N_0\xi_0^2$.[36] Therefore, the left-hand side in Eq. (14) gives the ratio of the DOS in *d*-wave vortex states to that in the isotropic *s*-wave vortex state. From this fact and Eq. (14), we expect that

$$\tau_v(d\text{-wave})/\tau_v(s\text{-wave})\sim\begin{cases} \xi/l_{\rm v}, & l_{\rm v}<R_{\rm B},\\ \sqrt{B/H_{c2}}, & R_{\rm B}<l_{\rm v}.\end{cases} \tag{15}$$

This reduction of the relaxation time in *d*-wave vortex also yields an enhancement of the flux flow resistivity ρ_f. If we assume here that this reduction of $\tau_{\rm v}$ *alone* leads to the enhancement of ρ_f, i.e.,

$$\rho_f(d\text{-wave})/\rho_f(s\text{-wave})\sim\tau_v(s\text{-wave})/\tau_v(d\text{-wave}) \tag{16}$$

and $\rho_f(s\text{-wave})\sim\rho_n(B/H_{c2})$, we obtain

$$\rho_f(d\text{-wave})/\rho_n\sim\begin{cases} (l_{\rm v}/\xi)(B/H_{c2}), & l_{\rm v}<R_{\rm B},\\ \sqrt{B/H_{c2}}, & R_{\rm B}<l_{\rm v}.\end{cases} \tag{17}$$

We then see the upshot of the hyposesis (17). The expression (17) is consistent with the experimental results on ρ_f both in low fields, Eq. (7), and in high fields Eq. (8). From the relation $l_{\rm v}\sim R_{\rm B}$ at the crossover field 2–3 T from Eq. (7) to Eq. (8), we obtain $l_{\rm v}=280$–340 Å. From this value of $l_{\rm v}$ and $\xi\sim42$ Å (estimated from $H_{c2}=20$ T), we obtain $l_{\rm v}/\xi=6.6$–8. This value is somewhat larger than $\alpha\sim2$. With consideration of the crudeness of our estimation, however, we should say that these two values are of the same order.

At the present state of the study, we do not know whether the dominant source for quasiparticle energy dissipation comes from the reduction of the number of the quasiparticles or the enhancement of the carrier scattering rate. A detailed numerical calculation for the energy dissipation especially when each vortex overlaps with its neighborhood would be necessary.

VI. SUMMARY

The microwave surface impedance measurements in the vortex state of overdoped Bi:2201 demonstrate that the free flux flow resistivity in moderately clean *d*-wave superconductors with gap nodes is remarkably different from that in conventional fully gapped *s*-wave superconductors. At low fields, the free flux flow resistivity increases linearly with H with a coefficient which is far larger than that found in conventional *s*-wave superconductors. At higher fields, the flux flow resistivity increases in proportion to $\sqrt{H}$ up to H_{c2}. Two possible scenarios are put forth for these field dependences: the enhancement of the quasiparticle relaxation rate and the reduction of the number of the quasiparticles participating the energy dissipation in the *d*-wave vortex state. The present results indicate that the physical mechanism of energy dissipation associated with the purely viscous motion of the vortices is sensitive to the symmetry of the pairing state.

ACKNOWLEDGMENTS

We thank T. Hanaguri, T. Kita, A. Maeda, and A. Tanaka for helpful discussions. This work was supported by a Grant-in-Aid for Scientific Research on Priority Area ‘‘Novel Quantum Phenomena in Transition Metal Oxides’’ from the Ministry of Education, Culture, Sports, Science, and Technology of Japan.

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