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Quasiparticle thermal conductivities in a type-II superconductor at high magnetic fields

Saša Dukan* and T. Paul Powell
Department of Physics and Astronomy, Goucher College, Baltimore, Maryland 21204

Zlatko Tešanović
Department of Physics and Astronomy, Johns Hopkins University, Baltimore, Maryland 21218
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We present a calculation of the quasiparticle contribution to the longitudinal thermal conductivities $\kappa_{xx}(H,T)$ (perpendicular to the external field) and $\kappa_{zz}(H,T)$ (parallel to the external field) as well as the transverse (Hall) thermal conductivity $\kappa_{xy}(H,T)$ of an extreme type-II superconductor in a high magnetic field $(H_{c1} \ll H < H_{c2})$ and at low temperatures. In the limit of frequency and temperature approaching zero $(\Omega \to 0, T \to 0)$, both longitudinal and transverse conductivities upon entering the superconducting state undergo a reduction from their respective normal state values by the factor $(\Gamma/\Delta)^2$, which measures the size of the region at the Fermi surface containing gapless quasiparticle excitations. We use our theory to numerically compute the longitudinal transport coefficient in borocarbide and A-15 superconductors. The agreement with recent experimental data on $LuNi_2B_2C$ is very good.

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I. INTRODUCTION

The *low-temperature, high-field* region in the H-T phase diagram of an extreme type-II superconductor is the regime where the Landau-level quantization of the electronic energies *within* the superconducting state is well defined, i.e., the cyclotron energy $\hbar\omega_c > \Delta, T, \Gamma$ where $\Delta \equiv \Delta(T,H)$ is the BCS gap, T is the temperature, and $\Gamma \equiv \Gamma(\omega)$ is the scattering rate due to disorder. This regime should be contrasted with the more familiar opposite limit of low magnetic fields and high temperatures where electrons occupy a huge number of Landau levels and where the temperature and/or impurity scattering broaden these levels and reduce the significance of Landau quantization. With the Landau level structure fully accounted for, one discovers a qualitatively new nature of quasiparticle excitations at high fields: for fields H below but near H_{c2} the quasiparticle spectrum is *gapless* at a discrete set of points on the Fermi surface. These gapless excitations reflect a coherent quasiparticle propagation over many unit cells of a closely packed vortex lattice with fully overlapping vortex cores.[1–3] The presence of such low-lying excitations makes an *s*-wave, conventional, superconductor in a high magnetic field somewhat similar to an anisotropic, unconventional, superconductor with nodes in the gap. In the low-temperature, high-field regime, however, the nodes in the gap reflect the *center-off-mass* motion of the Cooper pairs in the magnetic field, in contrast to *d*-wave superconducting cuprates where such nodes are due to the *relative orbital* motion. This gapless behavior in bulk systems is found to persist to surprisingly low magnetic fields $H^* \sim (0.2–0.5)H_{c2}$. Below H^* gaps start to open up in the quasiparticle spectrum, and the system eventually reaches the low-field regime of localized states in the cores of isolated, well-separated vortices.[3] At the present time the strongest evidence of the quantization of quasiparticle orbits *within* the superconducting state comes from observations of the de Haas–van Alphen (dHvA) oscillations in various superconducting materials.[4] The persistence of the dHvA signal deep within the mixed state, with the frequency of oscillations still maintaining the normal-state value, can be attributed to the presence of a small portion of the Fermi surface containing gapless quasiparticle excitations, surrounded by regions where the gap is large.[5,6]

Another useful probe of low-energy excitations in superconductors is a measurement of their thermal transport. The simultaneous measurements of the field-dependent longitudinal $\kappa_{xx}(H,T)$ and transverse $\kappa_{xy}(H,T)$ thermal conductivities are now experimentally feasible, and can yield information on both quasiparticle dynamics and the pairing mechanism. The dependence of transport coefficients on magnetic field is currently a hotly debated issue in the scientific community in light of the experimental observation of field-independent plateaus in the longitudinal thermal conductivity of high temperature superconductors at low fields $(H_{c1} < H \ll H_{c2})$.[7] The field-independent κ_{xx} is attributed to the $d_{x^2-y^2}$ pairing mechanism at low fields and to the nodal structure of the resulting quasiparticle excitations.[8,9] The presence of propagating gapless quasiparticles in the superconducting state at low temperatures and high magnetic fields should also lead to transport properties qualitatively different from those found in *s*-wave superconductors at low fields, where the number of thermally activated quasiparticles is exponentially small and the only contribution to the thermal conduction is found along the field direction and originates from the bound states within vortex cores. Recently, the thermal conductivity of the borocarbide superconductor $LuNi_2B_2C$ was measured down to $T = 70$ mK by Boaknin *et al.*[10] in a magnetic field perpendicular to the heat current from $H = 0$ to above $H_{c2} = 7$ T. In the limit of $T \to 0$, a considerable thermal transport was observed in the mixed state of the superconductor $(H_{c1} < H \leq H_{c2})$, indicating the presence of delocalized low-energy excitations at the Fermi surface. On the other hand, no thermal transport was observed at zero field, a result consistent with the *s*-wave superconducting gap without nodes at the Fermi surface.

The purpose of this work is to examine the contribution of low-energy quasiparticles to the thermal transport of conventional, i.e., extreme type-II superconductors in the regime of high magnetic fields and low temperatures. The paper is organized as follows: In Sec. II we develop the Kubo formalism for the transport coefficients within the Landau level pairing mechanism, while in Sec. III we incorporate disorder into a Green's function description of a three-dimensional superconductor in a high magnetic field. We use the formalism of Secs. II and III in Sec. IV to examine both longitudinal $\kappa_{xx}(H,T)$ and $\kappa_{zz}(H,T)$ as well as transverse $\kappa_{xy}(H,T)$ conductivities in the regime of the freqency $\Omega\to 0$ and the temperature $T\to 0$. Finally, in Sec. V we report on numerical calculations of the thermal transport in the borocarbide $LuNi_2B_2C$ and the A-15 superconductor V_3Si, and compare our theoretical plots with the available experimental data.

II. KUBO FORMALISM IN THE LANDAU LEVEL PAIRING SCHEME

Thermal conductivities can be calculated within the framework of the Kubo formalism as a linear response of a system to an external perturbation,[11]

$$\frac{\kappa_{ij}(\Omega,T)}{T} = -\frac{1}{T^2}\,\frac{\mathrm{Im}\Pi_{ij}^{ret}(\Omega)}{\Omega} \tag{1}$$

where $\Pi_{ij}^{ret}(\Omega)=\Pi_{ij}(i\Omega\to\Omega+i\delta)$, and

$$\Pi_{ij}(i\Omega)=\int d^3x_1 d^3x_2 \Pi_{ij}(1,2;i\Omega),$$

$$\Pi_{ij}(1,2;i\Omega) = -\int_0^\beta d\tau e^{i\Omega\tau}\langle T_\tau j_i(x_1,\tau) j_j(x_2,0)\rangle \tag{2}$$

is the spatially averaged, finite temperature thermal current-current correlation function tensor, and $\beta\equiv 1/k_BT$. In order to derive the heat current operators $\mathbf{j}(1)$ and $\mathbf{j}(2)$ at the space-time point $1=(\mathbf{x}_1,\tau)$ and $2=(\mathbf{x}_2,0)$ we follow the standard s-wave derivation in zero field,[12] and generalize it to our case of a nonuniform gap at high fields. A similar approach was recently utilized in Ref. 9 for a d-wave superconductor in zero field. The heat current carried by the quasiparticles can be computed within the standard variational procedure as

$$\mathbf{j} = \frac{\partial \mathcal{L}}{\partial(\nabla\psi)}\dot{\psi} + \dot{\psi}^\dagger \frac{\partial\mathcal{L}}{\partial(\nabla\psi^\dagger)} \tag{3}$$

from the Lagrangian density

$$\mathcal{L} = -\frac{1}{2m}\nabla\psi_\alpha^\dagger\cdot\nabla\psi_\alpha + \frac{e}{2mci}(\psi_\alpha^\dagger\nabla\psi_\alpha - \nabla\psi_\alpha^\dagger\psi_\alpha)\cdot\mathbf{A} + \frac{e^2}{2mc^2}|\mathbf{A}|^2\psi_\alpha^\dagger\psi_\alpha - \frac{1}{2i}(\psi_\alpha^\dagger\dot{\psi}_\alpha - \dot{\psi}_\alpha^\dagger\psi_\alpha) - \frac{1}{2}g\psi_\alpha^\dagger\psi_{-\alpha}^\dagger\psi_{-\alpha}\psi_\alpha, \tag{4}$$

where all the energies are measured with respect to the chemical potential and $\dot{\psi}\equiv\partial\psi/\partial t$. Nambu's two-component field operators $\psi_\alpha\equiv\psi_\alpha(\mathbf{r})$ are written in a compact notation for the sake of brevity. In extreme type-II superconductors, as soon as the magnetic field satisfies $H\gg H_{c1}(T)$ the vector potential $\mathbf{A}\equiv\mathbf{A}(\mathbf{r})$ can be safely assumed to be entirely due to the external field $\mathbf{H}=\nabla\times\mathbf{A}$. This holds over most of the $H-T$ phase diagram. We have used a simple BCS-model point interaction $V(\mathbf{r}_1-\mathbf{r}_2)=-g\,\delta(\mathbf{r}_1-\mathbf{r}_2)$ in expression (4). To lowest order in the concentration of impurities, the electron-impurity interaction can be omitted in computing the heat current. The effect of disorder will be included later in the Green's functions for a superconductor. Variational procedure (3) yields

$$\mathbf{j}(1) = -\frac{1}{2m}\left[\frac{\partial}{\partial\tau_1}\left(\nabla' - \frac{e\mathbf{A}'}{ci}\right) + \frac{\partial}{\partial\tau_1'}\left(\nabla + \frac{e\mathbf{A}}{ci}\right)\right]\psi_\alpha^\dagger(1')\psi_\alpha(1)\big|_{1'=1} \tag{5}$$

for the heat current operator. With this definition it is straightforward to calculate the correlation tensor $\Pi_{ij}(1,2;i\Omega)$ within the usual Hartree-Fock approximation (i.e., the bare bubble approximation) defined by

$$\langle T_\tau\psi_i(1)\psi_k(2)\psi_l^\dagger(2')\psi_j^\dagger(1')\rangle \to G_{il}(1,2')G_{kj}(2,1'), \tag{6}$$

where G_{kl} is the Nambu's matrix Green's function. Inserting Eq. (5) into Eq. (2), and with the help of Eq. (6), the current-current correlator becomes

$$\begin{aligned}\Pi_{ij}(1,2;\Omega) &= \frac{1}{4m^2\beta}\sum_\mu\left[-i(\Omega+\mu)\left(\nabla_{1'} - \frac{e\mathbf{A}(1')}{ci}\right) + i\mu\left(\nabla_1 + \frac{e\mathbf{A}(1)}{ci}\right)\right] \\ &\times\left[i(\Omega+\mu)\left(\nabla_2 + \frac{e\mathbf{A}(2)}{ci}\right) - i\mu\left(\nabla_{2'} - \frac{e\mathbf{A}(2')}{ci}\right)\right] \\ &\times \mathrm{Tr}\,[\tau_3 G(1,2',\Omega+\mu)\tau_3 G(2,1',\mu)]_{1\to 1',2\to 2'},\end{aligned} \tag{7}$$

where τ_3 is a Pauli matrix, $\mu=2\pi m/\beta$ are bosonic Matsubara frequencies, and $1\equiv\mathbf{x}_1$.

It was shown in Ref. 1 that the mean-field Hamiltonian corresponding to the Lagrangian density [Eq. (4)] can be diagonalized in terms of the basis functions of the magnetic sublattice representation (MSR), characterized by the quasi momentum $\mathbf{q}$ perpendicular to the direction of the magnetic field. The eigenfunctions of this representation in the Landau gauge $\mathbf{A}=H(-y,0,0)$ and belonging to the mth Landau level, are

$$\phi_{k_z,\mathbf{q},m}(\mathbf{r})=\frac{1}{\sqrt{2^n n!\sqrt{\pi}l}}\sqrt{\frac{b_y}{L_xL_yL_z}}\exp(ik_z\zeta)\times\sum_k\exp\left(i\frac{\pi b_x}{2a}k^2-ikq_yb_y\right)\times\exp\left[i\left(q_x+\frac{\pi k}{a}\right)x-\frac{1}{2}\left(\frac{y}{l}+q_xl+\frac{\pi k}{a}l\right)^2\right]\times H_m\left[\frac{y}{l}+\left(q_x+\frac{\pi k}{a}\right)l\right], \tag{8}$$

where ζ is the spatial coordinate and k_z is the momentum along the field direction, $\mathbf{A}=(a,0)$ and $\mathbf{b}=(b_x,b_y)$ are the unit vectors of the triangular vortex lattice, $l=\sqrt{\hbar c/eH}$ is the magnetic length, and $L_xL_yL_z$ is the volume of the system. $H_m(x)$ is the Hermite polynomial of order m. Quasimomenta $\mathbf{q}$ are restricted to the first magnetic Brillouin zone MBZ spanned by vectors $\mathbf{Q_1}=(b_y/l^2,-b_x/l^2)$ and $\mathbf{Q_2}=(0,2a/l^2)$.

Normal and anomalous Green's functions for a clean superconductor in this representation can be constructed as

$$G_{11}(1,2;\omega)\equiv\mathcal{G}(1,2;\omega)=\sum_{n,k_z,\mathbf{q}}\phi_{n,k_z,\mathbf{q}}(1)\phi^*_{n,k_z,\mathbf{q}}(2)G_n(k_z,\mathbf{q};\omega),$$

$$G_{21}(1,2;\omega)\equiv\mathcal{F}^\dagger(1,2;\omega)=\sum_{n,k_z,\mathbf{q}}\phi^*_{n,-k_z,-\mathbf{q}}(1)\phi^*_{n,k_z,\mathbf{q}}(2)F^*_n(k_z,\mathbf{q};\omega), \tag{9}$$

where $\omega=(2m+1)\pi/\beta$ are the electron Matsubara frequencies. Similar expressions can be written for the remaining two Nambu matrix elements. In writing Eq. (9) we have taken into account only diagonal (in Landau level index n) contributions to the Green's functions. This is a good approximation in high magnetic fields where $\Delta/\hbar\omega_c\ll 1$ and the number of occupied Landau levels n_c is not too large, which is the case for the extreme type-II systems under consideration. In this situation we are justified in using the diagonal approximation,[1,3] in which the BCS pairs are formed by electrons belonging to mutually degenerate Landau levels located at the Fermi surface while the contribution from Landau levels separated by $\hbar\omega_c$ or more is included in the renormalization of the effective coupling constant $[g\to\tilde{g}(H,T)]$.[13] As long as the magnetic field is larger than some critical field $H^*(T)$, the off-diagonal pairing does not change the qualitative behavior of the superconductor in a magnetic field. The critical field H^* at $T\sim 0$ can be estimated from the dHvA experiments to be $\sim 0.5H_{c2}$ for A-15 and $\sim 0.2H_{c2}$ for borocarbide superconductors.[4]

When the Nambu matrix [Eq. (9)] is inserted into Eq. (7) and the space average in Eq. (2) is performed, the longitudinal $\Pi_{xx}(i\Omega)=\Pi_{yy}(i\Omega)$ and transverse (Hall) $\Pi_{xy}(i\Omega)=-\Pi_{yx}(i\Omega)$ current-current correlation functions become

$$\Pi_{ij}(i\Omega)=\frac{1}{4m^2l^2\beta}\sum_\omega\sum_{n,k_z,\mathbf{q}}(\Omega+2\omega)^2\times\frac{n+1}{2}\mathrm{Tr}\,[\tau_3G_n(k_z,\mathbf{q},i\Omega+i\omega)\tau_3\times G_{n+1}(k_z,\mathbf{q},i\omega)\pm\tau_3G_{n+1}(k_z,\mathbf{q},i\Omega+i\omega)\times\tau_3G_n(k_z,\mathbf{q},i\omega)] \tag{10}$$

where the $+$ sign corresponds to $\Pi_{xx}(i\Omega)$, the $-$ sign corresponds to $i\Pi_{xy}(i\Omega)$, and $\omega=(2m+1)\pi/\beta$ are electronic Matsubara frequencies. On the other hand, the longitudinal (parallel to the external magnetic field) current-current correlation function $\Pi_{zz}(i\Omega)$ becomes

$$\Pi_{zz}(i\Omega)=\frac{1}{4m^2\beta}\sum_\omega\sum_{n,k_z,\mathbf{q}}k_z^2(\Omega+2\omega)^2\times\mathrm{Tr}[\tau_3G_n(k_z,\mathbf{q},i\Omega+i\omega)\tau_3G_n(k_z,\mathbf{q},i\omega)]. \tag{11}$$

In order to perform the summation over the Matsubara frequencies ω, we introduce a spectral representation for the Nambu matrix $G_n(k_z,\mathbf{q},\omega)$ as

$$G_n(k_z,\mathbf{q},\omega)=\int_{-\infty}^{\infty}d\omega_1\frac{A_n(k_z,\mathbf{q},\omega_1)}{i\omega-\omega_1}, \tag{12}$$

where the spectral function matrix $A_n(k_z,\mathbf{q},\omega)$ is defined as

$$A_n(k_z,\mathbf{q},\omega)=-\frac{1}{\pi}\mathrm{Im}G_n^{ret}(k_z,\mathbf{q},\omega). \tag{13}$$

When the spectral representation of the Green's functions [Eq. (12)] is used back in Eqs. (10) and (11), respectively, we obtain

$$\Pi_{ij}(i\Omega)=\frac{1}{4m^2l^2\beta}\sum_{n,k_z,\mathbf{q}}\int d\omega_1\int d\omega_2\frac{n+1}{2}\times\mathrm{Tr}[\tau_3A_n(k_z,\mathbf{q},\omega_1)\tau_3A_{n+1}(k_z,\mathbf{q},\omega_2)\pm\tau_3A_{n+1}(k_z,\mathbf{q},\omega_1)\tau_3A_n(k_z,\mathbf{q},\omega_2)]\times S \tag{14}$$

and

$$\Pi_{zz}(i\Omega)=\frac{1}{4m^2\beta}\sum_{n,k_z,\mathbf{q}}k_z^2\int d\omega_1\int d\omega_2\times\mathrm{Tr}[\tau_3A_n(k_z,\mathbf{q},\omega_1)\tau_3A_n(k_z,\mathbf{q},\omega_2)]\times S, \tag{15}$$

where S contains Matsubara sums, i.e.,

$$S=\frac{1}{\beta}\sum_{\omega}(\Omega+2\omega)^2\frac{1}{(i\Omega+i\omega-\omega_1)(i\omega-\omega_2)}. \quad (16)$$

The sum can be evaluated in the standard way by picking up the contributions from each of the poles of the summand.[11] After the analytic continuation $i\Omega\rightarrow\Omega+i\delta$, we obtain the retarded function S_{ret},

$$S_{ret}=\frac{(2\omega_2+\Omega)^2 n_F(\omega_2)-(2\omega_1-\Omega)^2 n_F(\omega_1)}{\omega_2-\omega_1+\Omega+i\delta}, \quad (17)$$

where $n_F(\omega)$ is the Fermi function.

In order to obtain the imaginary part of $\Pi_{ij}(\Omega)$ we need to find an imaginary part of S_{ret} when calculating the longitudinal conductivity $\kappa_{xx}(\Omega,T)$ and $\kappa_{zz}(\Omega,T)$. On the other hand, since S_{ret} enters the expression for $i\Pi_{xy}(\Omega)$ in Eq. (14), we need to find the real part of $-S_{ret}$. Using the identity

$$\frac{1}{x+i\delta}=P\frac{1}{x}-i\pi\delta(x), \quad (18)$$

and taking the imaginary part of Eq. (16), we find that the diagonal conductivities become

$$\begin{aligned}\frac{\kappa_{xx}}{T}=\frac{\kappa_{yy}}{T}&=\frac{\pi}{4m^2l^2}\sum_n\sum_{k_z,\mathbf{q}}\\&\times\int_{-\infty}^{+\infty}d\omega\frac{(2\omega+\Omega)^2}{T^2}\frac{n_F(\omega)-n_F(\omega+\Omega)}{\Omega}\\&\times\frac{n+1}{2}\mathrm{Tr}\ [\tau_3A_n(k_z,\mathbf{q},\omega+\Omega)\tau_3A_{n+1}(k_z,\mathbf{q},\omega)\\&+\tau_3A_{n+1}(k_z,\mathbf{q},\omega+\Omega)\tau_3\mathbf{A_n}(k_z,\mathbf{q},\omega)] \quad (19)\end{aligned}$$

and

$$\begin{aligned}\frac{\kappa_{zz}}{T}&=\frac{\pi}{4m^2}\sum_{n,k_z,\mathbf{q}}k_z^2\int_{-\infty}^{+\infty}d\omega\frac{(2\omega+\Omega)^2}{T^2}\frac{n_F(\omega)-n_F(\omega+\Omega)}{\Omega}\\&\times\mathrm{Tr}\ [\tau_3A_n(k_z,\mathbf{q},\omega+\Omega)\tau_3A_n(k_z,\mathbf{q},\omega)]. \quad (20)\end{aligned}$$

Similarly, taking the real part of Eq. (16) with the help of Eq. (18) yields, for the off-diagonal conductivity,

$$\begin{aligned}\frac{\kappa_{xy}}{T}=-\frac{\kappa_{yx}}{T}&=\frac{1}{4m^2l^2}\sum_n\sum_{k_z,\mathbf{q}}\int_{-\infty}^{+\infty}d\omega\\&\times\frac{(2\omega+\Omega)^2}{T^2}\frac{n_F(\omega)-n_F(\omega+\Omega)}{\Omega}\\&\times\frac{n+1}{2}\mathrm{Tr}\ [\tau_3B_{n+1}(k_z,\mathbf{q},\omega+\Omega)\tau_3A_n(k_z,\mathbf{q},\omega)\\&-\tau_3B_n(k_z,\mathbf{q},\omega+\Omega)\tau_3A_{n+1}(k_z,\mathbf{q},\omega)], \quad (21)\end{aligned}$$

where the function $B_n(k_z,\mathbf{q},\omega+\Omega)$ is defined as

$$B_n(k_z,\mathbf{q},\omega+\Omega)=\int_{-\infty}^{+\infty}d\omega_1\frac{A_n(k_z,\mathbf{q},\omega_1)}{\omega-\omega_1+\Omega}. \quad (22)$$

III. GREEN'S FUNCTIONS IN THE PRESENCE OF DISORDER

Before further discussing expressions (19), (20), and (21), we should go back to the question of spectral functions or, alternatively, Green's functions for the superconductor in a magnetic field. The Green's functions for the clean superconductor can be easily found following Ref. 14, with their "Fourier transforms" in the quasimomentum space expressed in the Nambu formalism as

$$\begin{aligned}G_n(k_z,\mathbf{q},i\omega)&=\frac{1}{(i\omega)^2-E_n(k_z,\mathbf{q})}\\&\times\begin{pmatrix}i\omega+\epsilon_n(k_z) & -\Delta_{nn}(\mathbf{q})\\ -\Delta^*_{nn}(\mathbf{q}) & i\omega-\epsilon_n(k_z)\end{pmatrix} \quad (23)\end{aligned}$$

where

$$E_{n,p}(k_z,\mathbf{q})=p\hbar\omega_c\pm\sqrt{\epsilon_n^2(k_z)+|\Delta_{n+p,n-p}(\mathbf{q})|^2},$$

$$\epsilon_n(k_z)=\frac{\hbar^2k_z^2}{2m}+\hbar\omega_c(n+1/2)-\mu \quad (24)$$

is the quasiparticle excitation spectrum of the superconductor in a high magnetic field near points $k_z=\pm k_{Fn}=\sqrt{2m[\mu-\hbar\omega_c(n+1/2)]/\hbar^2}$ calculated within the diagonal approximation,[1,3] where $\Delta/\hbar\omega_c\ll1$. For the quasiparticles near the Fermi surface ($k_z\sim k_{Fn}$) it suffices to consider only the $E_{n,p=0}$ bands. The gap $\Delta_{nn}(\mathbf{q})$, which in the MSR representation can be written as

$$\begin{aligned}\Delta_{nm}(\mathbf{q})=\frac{\Delta}{\sqrt{2}}\frac{(-1)^m}{2^{n+m}\sqrt{n!m!}}\sum_k\exp\Bigg(i\pi\frac{b_x}{a}k^2+2ikq_yb_y\\-(q_x+\pi k/a)^2l^2\Bigg)H_{n+m}[\sqrt{2}(q_x+\pi k/a)l],\end{aligned} \quad (25)$$

turns to zero on the Fermi surface at the set of points in the MBZ with a strong linear dispersion in q. The excitations from other bands, $p\neq0$ in Eq. (24), are gapped by at least a cyclotron energy, and their contribution to the quasiparticle transport can be neglected at low temperatures [$T\ll\Delta(T,H)\ll\hbar\omega_c$]. Once the off-diagonal pairing in Eq. (9) is included, the excitation spectrum cannot be written in the simple form of Eq. (24), and a closed analytic expression for the superconducting Green's function cannot be found. Nevertheless, when these off-diagonal terms are treated pertubatively as in Ref. 3, the qualitative behavior of the quasiparticle excitations, characterized by the nodes in the MBZ, remains the same. This statement is correct in all orders of perturbation theory, and therefore is exact as long as the pertubative expansion itself is well defined, i.e., as long as $H>H^*(T)$. Once the magnetic field is lowered below H^*, gaps start to open up at the Fermi surface, signaling the crossover to the low-field regime of quasiparticle states localized in the cores of widely separated vortices.[15]

In a dirty but homogenous superconductor, with a coherence length ξ much longer than the effective distance ξ_{imp} over which the impurity potential changes ($\xi/\xi_{imp} \gg 1$), the superconducting order parameter is not affected by the impurities and still forms a perfect vortex lattice. For such a system, the bare Green's function in Eq. (9) is dressed via scattering through the diagonal (normal) self-energy $\Sigma^N(i\omega)$ and off-diagonal (anomalous) self-energy $\Sigma^A_{nn}(\mathbf{q},i\omega)$.[14] A dressed Green's function is obtained by replacing ω with $\tilde{\omega}$ and $\Delta_{nn}(\mathbf{q})$ with $\tilde{\Delta}_{nn}(\mathbf{q})$ in Eq. (9), where

$$i\tilde{\omega} \equiv i\omega - \Sigma^N(i\omega),$$

$$\tilde{\Delta}_{nn}(\mathbf{q}) \equiv \Delta_{nn}(\mathbf{q}) + \Sigma^A_{nn}(\mathbf{q},i\omega). \tag{26}$$

In order to calculate the spectral functions in Eq. (19) the analytical continuation should be performed so that $G_{ret}(k_z,\mathbf{q},\omega) = G(k_z,\mathbf{q},i\omega \rightarrow \omega + i\delta)$, where $\Sigma^{N,A}_{ret}(\omega) = \Sigma^{N,A}(i\omega \rightarrow \omega + i\delta)$, with the impurity scattering rate in the superconducting state defined as $\Gamma(\omega) = -\mathrm{Im}\Sigma^N_{ret}(\omega)$. It was shown by us in Ref. 14 that the anomalous self-energy does not qualitatively change the form of the gap function $\Delta_{nn}(\mathbf{q})$ at low energies, and therefore $\Sigma^A_{nn}(\mathbf{q},\omega)$ will be neglected in further calculations. At the same time, the real part of the normal self-energy $\Sigma^N(\omega)$ can be either neglected or absorbed into $\epsilon_n(k_z)$.

IV. THERMAL CONDUCTIVITIES IN THE $T \rightarrow 0$ LIMIT

We are interested in calculating thermal conductivities in Eqs. (19) and (21) in the limit of $\Omega \rightarrow 0$ and small T such that $T \ll \Gamma(\omega)$. In the limit of $\Omega \rightarrow 0$ the difference of Fermi functions in Eq. (19) becomes

$$\frac{n_F(\omega+\Omega) - n_F(\omega)}{\Omega} \rightarrow \frac{\partial n_F}{\partial \omega}. \tag{27}$$

This function is sharply peaked around $\omega = 0$ at very low temperatures, so that we are justified in expanding the integrand in Eq. (19) and (21) around $\omega = 0$ up to second order in ω, and setting the scattering rate to a constant $\Gamma = \Gamma(\omega = 0)$. In the high-field superconductors the largest contribution to the thermal conductivity comes from the quasiparticle excitations at the Fermi surface with momenta $\mathbf{q}$ such that $\Delta(\mathbf{q}) \leq \max(T,\Gamma)$, while the excitations gapped by large $\Delta(\mathbf{q})$ give exponentially small contributions. Therefore, in order to simplify the integration over the MBZ and summation over the Landau level index in Eqs. (19) and (21), we linearize the excitation spectrum [Eq. (24)] around nodes at the Fermi surface.[5] This enables us to obtain approximate but analytic expresions for thermal transport coefficients which capture the qualitative behavior near H_{c2}. Keeping this in mind and with the help of the Sommerfeld expansion,[16] the longitudinal conductivity $\kappa_{xx}(H,T) = \kappa_{yy}(H,T)$ and $\kappa_{zz}(H,T)$ at low temperatures become

$$\frac{\kappa_{xx}(H,T)}{\kappa^N_{xx}(H,T)} = \left(\frac{4}{\pi} - 1\right)\left(\frac{\Gamma}{\Delta}\right)^2 + \frac{7\pi^2}{5}\left(1 - \frac{3}{\pi}\right)\left(\frac{k_B T}{\Delta}\right)^2 \tag{28}$$

and

$$\frac{\kappa_{zz}(H,T)}{\kappa^N_{xx}(H=0,T)} = \left(\frac{\Gamma}{\Delta}\right)^2 + \frac{7\pi^2}{15}\left(\frac{k_B T}{\Delta}\right)^2. \tag{29}$$

$\kappa^N_{xx}(H,T)$ is the thermal conductivity of the normal metal in a magnetic field,[17]

$$\kappa^N_{xx}(H,T) = \frac{\pi^2}{3}\frac{n_e}{2m^*\Gamma}\frac{4\Gamma^2}{(\hbar\omega_c)^2 + 4\Gamma^2}T, \tag{30}$$

where $n_e = (1/2\pi l^2)\Sigma_n k_{Fn}$ is electronic density in the system. On the other hand, the transverse conductivity $\kappa_{xy}(H,T) = -\kappa_{yx}(H,T)$ becomes

$$\frac{\kappa_{xy}(H,T)}{\kappa^N_{xy}(H,T)} = \left(\frac{4}{\pi} - 1\right)\left(\frac{\Gamma}{\Delta}\right)^2 + \frac{7\pi^2}{5}\left(1 - \frac{3}{\pi}\right)\left(\frac{k_B T}{\Delta}\right)^2, \tag{31}$$

where $\kappa^N_{xy}(H,T)$ is the off-diagonal thermal conductivity of the normal metal in a magnetic field:[17]

$$\kappa^N_{xy}(H,T) = \frac{\pi^2}{3}\frac{n_e}{m^*\omega_c}\frac{(\hbar\omega_c)^2}{(\hbar\omega_c)^2 + 4\Gamma^2}T. \tag{32}$$

Relations (28), (29), and (31), obtained when $\Gamma/\Delta \ll 1$ within the ''linearized spectrum aproximation,'' tell us that there is still a considerable thermal transport in the mixed state of the superconductor. This is in stark contrast to the exponential suppresion of transport characteristic of an s-wave superconductor in zero field. Furthermore, relations (28), (29), and (31) indicate that when passing from the normal state to the superconducting state, both longitudinal and transverse transport coefficients κ/T are reduced from their respective normal-state values by the factor $\sim(\Gamma/\Delta)^2$ [the term linear in $(T/\Delta)^2$ is negligible at low tempartures even for very clean superconductors]. The factor $\sim(\Gamma/\Delta)^2$ measures the fraction of the Fermi surface $\mathcal{G}$ containing gapless quasiparticle excitations at $T=0$. The size of $\mathcal{G}$ is determined by both the total number of nodes in the excitation spectrum [Eq. (24)] and the areas in different branches where the BCS gap Δ is very small but not necessarily zero. This result, obtained here for the thermal coefficients, is consistent with the behavior of some other superconducting observables that measure the presence of low-energy excitations at the Fermi surface. One such experimentally confirmed behavior is the reduction of the de Haas–van Alphen (dHvA) oscillation's amplitude in both A-15 and borocarbide superconductors when the sample becomes superconducting.[4] The drop in the overall amplitude in passing from the normal to the superconducting state reflects the presence of a small portion of the Fermi surface $\sim\mathcal{G}$ containing coherent gapless excitations while the rest is gapped by large Δ.[5]

V. COMPARISON WITH EXPERIMENT

Recently, the longitudinal thermal conductivity of the borocarbide superconductor $LuNi_2B_2C$ was measured down to $T=70$ mK by Boaknin *et al.*[10] in a magnetic field from

$H=0$ to above $H_{c2}=7$ T. In the limit of $T\rightarrow 0$, a considerable thermal transport is observed in the mixed state of the superconductor ($H_{c1}<H\leqslant H_{c2}$), indicating the presence of delocalized low-energy excitations at the Fermi surface. The authors argue that this observation is strong evidence of a highly anisotropic gap function in $LuNi_2B_2C$, possibly with nodes. On the other hand, no sizable thermal conductivity was observed in zero field, the result expected for a superconducting gap without nodes.

The result of Boaknin *et al.* is consistent with the observation of dHvA oscillations down to fields $H^*\sim H_{c2}/5$ in YNi_2B_2C, a close cousin of $LuNi_2B_2C$, as well as in V_3Si where the oscillations persist down to fields $H^*\sim H_{c2}/2$.[4] It was shown by us that the drop in the dHvA amplitude observed in these experiments can be atributed to the quantization of quasiparticle orbits within the superconducting state, which results in the formation of nodes in the gap.[5] This quantum regime behavior in fields $H^*<H<H_{c2}$ is due to the center-off-mass motion of the Cooper pairs, in contrast to the *d*-wave or the anisotropic *s*-wave, where nodes in the gap are due to the relative orbital motion. Therefore, it makes sense to compare the theory developed in this paper with the experimental data in Ref. 10. A quick check tells us that expression (28), where $\Gamma(H)/\Delta(H)<1$, does not hold through the entire range of fields used in the experiment. Therefore, we *numerically* compute the longitudinal thermal conductivity directly from Eq. (19), without using any additional aproximations, for both the borocarbide superconductor $LuNi_2B_2C$ as well as for the A-15 superconductor V_3Si.

In the limit of $\Omega\rightarrow 0$ and $T\rightarrow 0$ expression (19) yields

$$\frac{\kappa}{T}=\frac{\pi}{12m}\frac{\Gamma^2}{\hbar\omega_c}\sum_{n}^{n_c}(n+1)\times\sum_{k_z,\mathbf{q}}\frac{1}{E^2_{n,p=0}(k_z,\mathbf{q})+\Gamma^2}\,\frac{1}{E^2_{n+1,p=0}(k_z,\mathbf{q})+\Gamma^2}, \tag{33}$$

where the number of Landau levels involved in superconducting pairing $n_c=E_F/\hbar\omega_c$ varies as a function of magnetic field. In the borocarbide superconductor $LuNi_2B_2C$ n_c can be estimated as $n_c\sim 33$ at H_{c2} and $n_c\sim 1147$ at a field of $H=0.2$ T (these numbers were obtained using an effective mass of $0.35m_e$ and a Fermi velocity of $v_F=2.76\times 10^7$cm/s, as reported in Ref. 18). On the other hand, the number of occupied Landau levels in the A-15 superconductor V_3Si is much larger: $n_c\sim 241$ at $H_{c2}=18.5$ T and $n_c\sim 4470$ at $H=1$ T (we used an effective mass of $1.7m_e$ and a Fermi velocity of $v_F=2.8\times 10^7$cm/s from Ref. 19 in this estimate). The scattering rate $\Gamma=\Gamma(\omega=0)$ in Eq. (33) is, in general, modified relative to the normal state scattering rate Γ_0 when the system becomes superconducting. Indeed, the self-consistent calculation of Γ in Ref. 14 gives $\Gamma(H)=\sqrt{\Gamma_0\Delta(H)/2}$. We assume $\Delta(H)=\Delta\sqrt{1-H/H_{c2}}$ which is a good approximation for the range of fields used in the experiment.

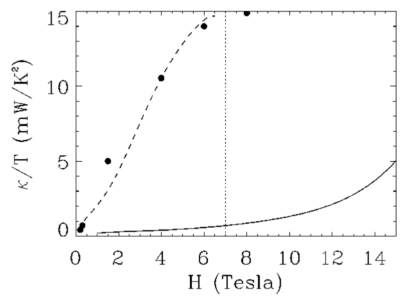

FIG. 1. Magnetic field dependence of the quasiparticle longitudinal thermal conductivity computed from Eq. (33) for $LuNi_2B_2C$ (dashed line) and V_3Si (full line). Full circles represent experimental data of Boaknin *et al.* (Ref. 10) The vertical dotted line indicates the normal-superconducting transition at $H_{c2}=7$ T for $LuNi_2B_2C$. The upper critical field for V_3Si at $H_{c2}=18.5$ T is not shown. For $LuNi_2B_2C$ we have used experimentally determined values for $\Delta=4.4$ meV and $\Gamma_0=0.5\Delta$ from Ref. 10 as well as the effective mass $m^*=0.35m_e$ from Ref. 18. For V_3Si, $\Delta=2.6$ meV, $\Gamma_0=0.61$ meV, and $m^*=1.7m_e$ were taken from Ref. 19.

The dashed line in Fig. 1 shows the magnetic field dependence of the quasiparticle thermal conductivity κ/T for the borocarbide superconductor $LuNi_2B_2C$ in the limit of $T\rightarrow 0$ obtained by a numerical evaluation of Eq. (33), where values for the BCS gap Δ and normal state inverse scattering rate Γ_0 are taken from Ref. 10. The full circles in Fig. 1 are the experimental data of Boaknin *et al.*[10] for the same superconductor. The full line in Fig. 1 shows the theoretical plot obtained by numerical evaluation of Eq. (33) for the A-15 superconductor V_3Si, where values for Δ and Γ_0 are taken from Ref. 19. There is a significant difference in the behavior of κ/T in these two superconducting systems, characterized by much smaller thermal transport in V_3Si, when compared to the transport in $LuNi_2B_2C$ in the same range of magnetic fields. This observation indicates that the number of gapless or near-gapless excitations at the Fermi surface in the V_3Si is very small at $H\ll H_{c2}$. In order to understand this difference, one has to note that in magnetic fields $H<H^*=0.5H_{c2}$ the number of occupied Landau levels in the V_3Si system is huge ($\leqslant 4500$), and that V_3Si is away from the regime of coherent gapless excitations of high fields. On the other hand, the number of occupied Landau levels in $LuNi_2B_2C$ is much smaller ($\leqslant 1000$), and it seems that there are still many gapless excitations left at low fields. It is suprising though that significant thermal transport exists down to $H\sim 0.015H_{c2}$, a field much smaller than the critical field $H^*=0.2H_{c2}$ for this system, where most of the quasiparticle spectrum should be gapped. Note, however, that a possible source for such a significant transport at low fields might be the highly anisotropic *s*-wave gap function in $LuNi_2B_2C$, as suggested in Ref. 10. If there is such anisotropy along one or more directions on the Fermi surface, the range of validity of our theory may be extended to fields lower than the simple estimate for H^*.[20] In this regard, we alert the reader that at the lowest fields in Fig. 1 ($H\sim 0.04H_{c2}$) our theory is stretched to its very limits and its quantitative accuracy diminishes.

VI. CONCLUSIONS

In this paper we develop expressions for the longitudinal and transverse quasiparticle thermal conductivities for an extreme type-II superconductor in a magnetic field. We utilize the Landau level formalism of superconducting pairing in a magnetic field to obtain, within the Kubo mechanism of linear response to an external perturbation, thermal currents perpendicular and parallel to the external magnetic field. From there, current-current correlation functions are introduced within the Matsubara finite temperature mechanism in order to derive closed expressions for thermal conductivities $\kappa_{ij}(\Omega,T)$. We examine the transport coefficients κ_{ij}/T in the limits of $\Omega\rightarrow 0$ and $T\rightarrow 0$, and find that there is considerable thermal transport in the mixed state of a superconductor with an *s*-wave symmetry due to the creation of gapless exitations in the magnetic field. This is in contrast to the zero-field thermal transport, which is exponentially small for an *s*-wave superconductor with no nodes in the gap. Furthermore, when passing from the normal state to the superconducting state the thermal coefficients κ_{ij}/T become reduced with respect to their normal state values by a factor $\sim(\Gamma/\Delta)^2$ which measures the fraction of the Fermi surface that contains coherent gapless or near gapless excitations in a magnetic field. In this respect, thermal conductivities behave similarly to the dHvA oscillations, in which the amplitude is also reduced at the superconducting transition. Finally, we numerically compute the longitudinal thermal conductivity for two realistic superconducting systems; the borocarbide $LuNi_2B_2C$ and the A-15 superconductor V_3Si. The thermal transport in $LuNi_2B_2C$ is much larger in magnitude than the thermal transport of V_3Si at the same field. This result indicates that the borocarbide $LuNi_2B_2C$ might still be in the regime of delocalized quasiparticle states even at fields much lower than the critical field $H^*\sim 0.2H_{c2}$ (estimated from the dHvA experiments[4,20]). The agreement of our theoretical plot with the experimental data for $LuNi_2B_2C$ taken by Boaknin *et al.*[10] over a wide range of fields used in the experiment is suprisingly good.

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*Electronic address: dukan@monk.goucher.edu

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$$x^3=\frac{2\Delta^4}{\pi E_F(\hbar\omega_c 2)^3}(1-x)^2,$$

and yields $H^*\sim(0.2\text{–}0.5)H_{c2}$ for the systems in question (see Ref. 4). This estimate depends on the value of the *s*-wave gap function and can be much smaller than $(0.2\text{–}0.5)H_{c2}$ if the value of minimum gap in a strongly anisotropic *s*-wave case is significantly different from the accepted BCS value.