Thermal conductivity of single-crystalline MgB$_2$

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The ab-plane thermal conductivity κ of single-crystalline hexagonal MgB$_2$ has been measured as a function of magnetic field H with orientations both parallel and perpendicular to the c axis and at temperatures between 0.5 and 300 K. In the mixed state, $\kappa(H)$ measured at constant temperatures reveals features that are not typical for common type-II superconductors. The observed behavior may be associated with the field-induced reduction of two superconducting energy gaps, significantly different in magnitude. A nonlinear temperature dependence of the electronic thermal conductivity is observed in the field-induced normal state at low temperatures. This behavior is at variance with the law of Wiedemann and Franz, and suggests an unexpected instability of the electronic subsystem in the normal state at $T \approx 1$ K.

I. INTRODUCTION

The discovery of superconductivity in MgB$_2$ below an unexpectedly high critical temperature T_c of the order of 40 K (Ref. 1) initiated intensive studies of its physical properties. Numerous results indicate that the superconducting state of MgB$_2$ is conventional in the sense that the electron pairing is mediated by the electron-phonon interaction. In most reports the superconducting energy gap is claimed to be nodeless, compatible with s-wave pairing. However, various types of experiments,[2-13] mainly using powder or polycrystalline samples and often surface sensitive, have given evidence for two gaps of different magnitude in the quasiparticle excitation spectrum of this superconductor. Calculations of the Fermi surface of this material[14] reveal three-dimensional sheets (π bands) and two-dimensional (2D) tubes (σ bands), and it seems quite possible that the gap values for these different parts of the Fermi surface differ substantially. It has been argued[15] that the holelike quasiparticles on the 2D parts of the Fermi surface experience the larger superconducting gap with a maximum value close to $1.76k_BT_c$, as predicted by the original weak-coupling BCS theory.[16] The second and smaller gap is associated with the 3D sheets of the Fermi surface. This intriguing situation and other possible anomalous features of this seemingly simple metallic compound ought to be checked experimentally on single-crystalline material.

Below we present the results of measurements of the thermal conductivity κ and the electrical resistivity ρ parallel to the basal ab plane of the hexagonal crystal lattice of MgB$_2$ as a function of temperature T between 0.5 and 300 K, and varying magnetic fields H between 0 and 50 kOe, oriented both parallel and perpendicular to the c axis. In the mixed state, the observation of a rapid increase of the electronic thermal conductivity with increasing H at constant $T \ll T_c$ is consistent with a field-induced reduction of the smaller of the two superconducting energy gaps. Another important observation is that of an unusual nonlinear temperature dependence of the electronic thermal conductivity at $T \ll T_c$ in the field-induced normal state, in obvious disagreement with the law of Wiedemann and Franz.

II. SAMPLE AND EXPERIMENT

The investigated single crystal with dimensions of 0.5 $\times 0.17 \times 0.035$ mm^3 was grown with a high-pressure cubic anvil technique, similar to the one presented in Ref. 17, but with a slightly different thermal treatment, described in Ref. 18. The high quality of similar crystals from the same batch was confirmed by single-crystal x-ray diffraction and energy dispersive x-ray (EDX) analysis. A standard uniaxial heat flow method was used for the $\kappa(T,H)$ measurements. Temperatures between 0.5 and 2.4 K were achieved in a noncommercial ^{3}He cryostat and the regime between 2 and 300 K was covered by using a ^{4}He flow cryostat. The thermometers for monitoring the temperature difference in the $\kappa(T,H)$ measurements were Chromel-Au+0.07% Fe thermocouples with thin wires (25 μm diameter) for $T \geq 2$ K and a pair of ruthenium oxide thermometers below 2.4 K. In the region of temperature overlap, the results of the measurements of $\kappa(T)$ gave identical values for both types of thermometers. Since the thermopower of the Au+Fe alloy is strongly field dependent at low temperatures, special efforts were made to ensure a reliable calibration of the thermocouples in magnetic fields. Possible errors due to the thermal conductivity of connecting wires are estimated to be below 1% of the total measured thermal conductivity. Additional measurements of the electrical resistivity $\rho(T)$ in the ab plane in magnetic fields oriented along the hexagonal c axis were made as well. The electrical resistivity was measured by employing a four-contact configuration with the same contacts used for measuring the voltage and the temperature difference, respectively.

III. RESULTS

A. Electrical resistivity

The electrical resistivity $\rho(T)$ in zero magnetic field is presented in Fig. 1. The narrow ($\Delta T_c = 0.15$ K) superconducting transition occurs at $T_c = 38.1$ K. As is shown in the lower inset of Fig. 1, in the temperature region between T_c and 130 K, $\rho(T)$ may well be approximated by $\rho = \rho_0 + AT^3$, where ρ_0 and A are constants. At temperatures below 50 K, $\rho \approx \rho_0 = 2.0$ $\mu\Omega$ cm. A cubic $\rho(T)$ dependence has
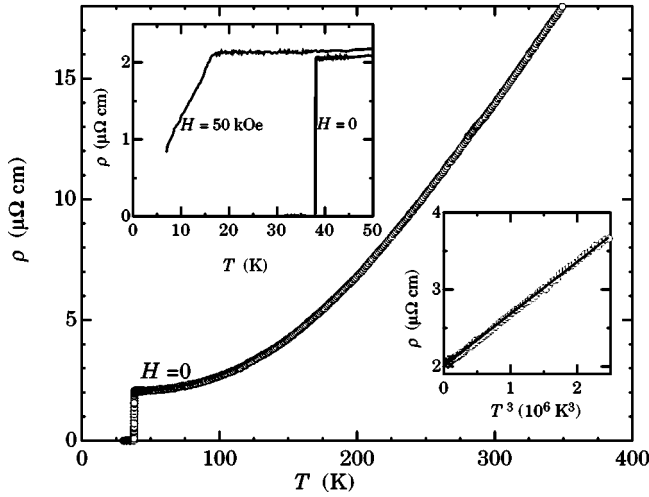
FIG. 1. In-plane electrical resistivity $\rho(T)$ of hexagonal MgB_2. The upper inset emphasizes the low-temperature part for $H=0$ and 50 kOe oriented along the c axis. The lower inset demonstrates the cubic temperature dependence of $\rho(T)$ for $T \lesssim 130$ K.

often been observed in multiband transition metals and is associated with interband electron-phonon scattering.[19,20] The upper inset of Fig. 1 emphasizes $\rho(T)$ close to the superconducting transitions in fields of $H=0$ and $H=50$ kOe, respectively. For our sample, the bulk upper critical field for the field direction parallel to the c axis, H_{c2}^c, is about 30 kOe at zero temperature and decreases with increasing temperature,[21] as shown in the inset of Fig. 3. Therefore the data for $H=50$ kOe were obtained with the bulk of the sample in the normal state. The abrupt slope change in $\rho(T,H=50$ kOe$)$ at 17 K is most likely related to the onset of spurious superconductivity in the surface region of the sample.[21,22]
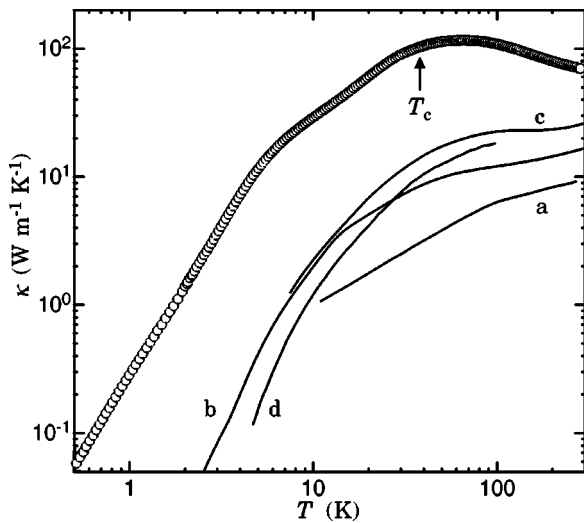

FIG. 2. Thermal conductivity vs temperature in the ab plane of MgB_2 in zero magnetic field. The solid lines represent $\kappa(T)$ measured on polycrystalline samples [data from (a) Ref. 25, (b) Ref. 26, (c) Ref. 23, and (d) Ref. 24].
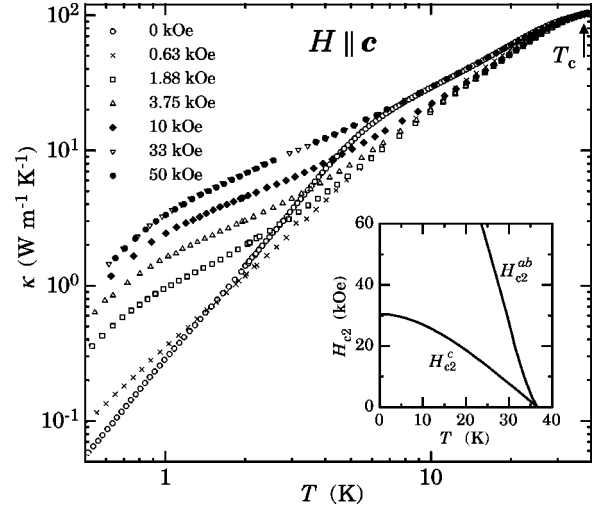

FIG. 3. Thermal conductivity vs temperature in the ab plane of MgB_2 for several values of magnetic fields parallel to the c axis. The arrow indicates the zero-field critical temperature T_c. The temperature dependences of the upper critical fields H_{c2}^c and H_{c2}^{ab} as established in Ref. 21 are shown in the inset.

B. Thermal conductivity

The thermal conductivity data $\kappa(T)$ in zero magnetic field are presented in Fig. 2. The $\kappa(T)$ values are about an order of magnitude higher than previously reported for polycrystalline samples.[23–26] Also the overall temperature dependence of κ is quite different from those earlier data. Instead of a monotonous increase with temperature we note a distinct maximum of $\kappa(T)$ at $T \sim 65$ K. The cause of these differences is obviously the strong influence of intergrain boundaries on the heat transport in polycrystals, which masks the intrinsic mechanisms of quasiparticle scattering. As may be seen in Fig. 2, no anomaly in $\kappa(T)$ provides evidence for the superconducting transition at T_c. A distinct change of slope in $\kappa(T)$ at approximately 6 K is observed, however.

In Fig. 3 we display $\kappa(T)$ in the range between 0.5 and 40 K, measured at selected fixed magnetic fields, oriented parallel to the c axis. The practically overlapping curves for $H=33$ and 50 kOe indicate that for this field orientation and these H values, the normal state has been reached in the whole covered temperature range. The initial decrease of κ with increasing field is relatively large at higher temperatures, but is progressively reduced with decreasing temperatures. It finally turns into an increase of $\kappa(H)$ at constant temperatures below approximately 1 K. This behavior of the low-temperature thermal conductivity in the mixed state $0 < H < H_{c2}$ is better illustrated in Fig. 4 where we present the $\kappa(H)$ curves, measured at constant temperatures below 8 K for field directions both parallel and perpendicular to the c axis. The typical features of these curves are the rapid initial decrease of κ with increasing field, narrow minima in $\kappa(H)$ at field values that are low with respect to H_{c2}, and a subsequent S-shape-type increase of κ with further increasing field. The latter feature is particularly pronounced for the $\kappa(H)$ curves at the lowest temperatures. The low-field increase is practically independent of the field direction as may clearly be seen in Fig. 4, where data for $H\|c$ (open symbols)
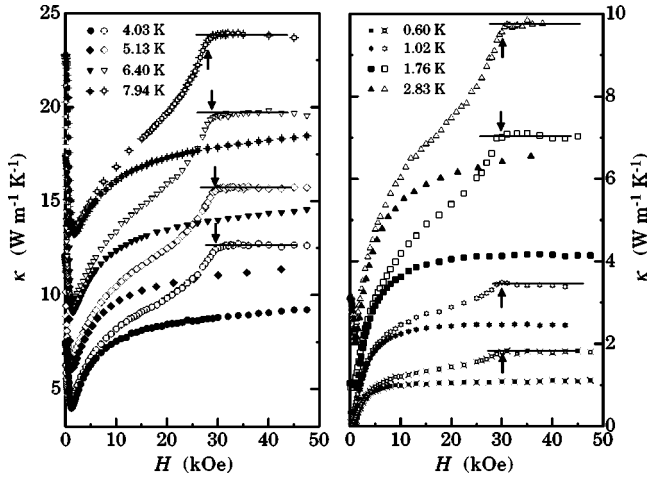
FIG. 4. Thermal conductivity in the basal plane of MgB$_2$ vs H at several fixed temperatures. The arrows denote the upper critical field H_{c2} for $H \parallel c$. The solid and open symbols correspond to the field direction perpendicular and parallel to the c axis, respectively.

and $H \perp c$ (solid symbols) are displayed for comparison. The increasing slope at higher fields and $H \parallel c$ is certainly caused by approaching the normal state at H_{c2}. This trend is not observed for $H \perp c$, for which H_{c2} is estimated to be about 130 kOe at low temperatures, obviously far beyond our experimental possibilities.

IV. DISCUSSION

The thermal conduction of a superconductor is usually provided by electronic quasiparticles (κ_e) and phonons (κ_{ph}), such that

$$\kappa = \kappa_e + \kappa_{ph}. \qquad (1)$$

Upon decreasing the temperature to below T_c in zero magnetic field, the reduction of the number of unpaired electrons leads to a decrease of κ_e and an increasing κ_{ph}. The overall behavior of $\kappa(T)$ in the superconducting state depends on the relative magnitudes of κ_e and κ_{ph} and also on the strength of the electron-phonon interaction. Applying external magnetic fields induces vortices in the sample. The quasiparticles associated with the vortices not only provide additional contributions to phonon scattering and hence a reduction of κ_{ph}, but also enhance κ_e. The competition of these two processes leads to $\kappa(H)$ curves as shown in Fig. 4.

In what follows, we first discuss the low-temperature part of $\kappa(T)$ in the field-induced normal state, i.e., for $H \geq H_{c2}$. Subsequently we discuss $\kappa(T)$ in the superconducting state for $H = 0$ and finally turn to the data of $\kappa(H)$, measured at constant temperatures in the mixed state, i.e., $H < H_{c2}$.

A. Thermal conductivity in the normal state

One of the main problems encountered in the analysis of the thermal conductivity of a conductor is that a separate identification of the two terms in Eq. (1) at arbitrary temperature is not straightforward. In the normal state, a convenient and often used way to estimate κ_e is to employ the

Wiedemann-Franz law (WFL), relating the electrical resistivity and the electronic contribution to the thermal conductivity via

$$\kappa_e(T) = L_0 T / \rho(T), \qquad (2)$$

where $L_0 = 2.45 \times 10^{-8}$ W Ω K^{-2} is the Lorenz number. The validity of this law requires an elastic scattering of electrons and it is well established that Eq. (2) is applicable if the scattering of electrons by defects dominates. This is usually true at low temperatures, where $\rho(T) \approx \rho_0$. In our case, the data in Fig. 1 suggest that Eq. (2) is applicable at temperatures below about 50 K. Inserting the appropriate numbers into Eq. (2) suggests that, at temperatures $T_c \leq T \leq 50$ K, $\kappa_e(T)$ provides about half of the total thermal conductivity. As we demonstrate below, the applicability of the WFL for MgB$_2$ at low temperatures is questionable, however.

Before discussing the validity of WFL at low temperatures, we note the complications that arise from unusual features in the $\rho(T)$ data shown in Fig. 1, most likely caused by the influence of superconductivity in minor regions of the sample with enhanced T_c and H_{c2}, different from the corresponding bulk values. The onset of this superconductivity is clearly seen in the upper inset of Fig. 1 by inspecting $\rho(T)$ measured in a field of 50 kOe, substantially exceeding $H_{c2}(0)$, the maximum bulk upper critical field of the same sample.[21] Superconducting traces in a minute fraction of the sample may cause a considerable reduction of the total measured electrical resistivity but leave the thermal conductivity virtually unchanged. In this case a failure of the WFL would not be surprising. However, because the electrical resistivity of the sample above T_c is practically temperature independent, we can, with a great deal of certainty, expect that the intrinsic electrical resistivity of the *bulk* remains constant also at lower temperatures. The constant residual resistivity ρ_0 is caused by defects which actually set the maximum mean free path for electrons, independent of temperature. Indeed, results by Xu *et al.*[27] of $\rho(T)$ measurements on single-crystalline MgB$_2$ at magnetic fields which presumably exceed the upper critical field of the minor phase demonstrate that in, e.g., $H = 90$ kOe the electrical resistivity remains constant with decreasing temperature down to at least 2 K.

In Fig. 5, we plot as a solid line, the normal-state electronic thermal conductivity κ_e^{WFL} below 8 K, calculated using Eq. (2) with the experimental value $\rho_0 = 2.1$ $\mu\Omega$ cm for $H = 33$ kOe. The measured total thermal conductivity, shown by open circles, is considerably higher than κ_e^{WFL} across the entire covered temperature range. For the estimate of the upper limit of the phonon contribution we assume that the minima of $\kappa(H)$ shown in Fig. 4 are caused by the competition of a decreasing κ_{ph} and an increasing κ_e. With this interpretation it is clear that the values of $\kappa_{\min}(H)$ represent at most the maximum value of the lattice contribution κ_{ph}. The smooth interpolation between these minimum values of $\kappa(H)$ for different temperatures, denoted as $\kappa_{ph}^{\max}$, is shown as the dashed line in Fig. 5. The difference between the measured thermal conductivity at $H = 33$ kOe and the upper limit of the phonon contribution, $\kappa_e^{\min} = \kappa - \kappa_{ph}^{\max}$, obviously repre-
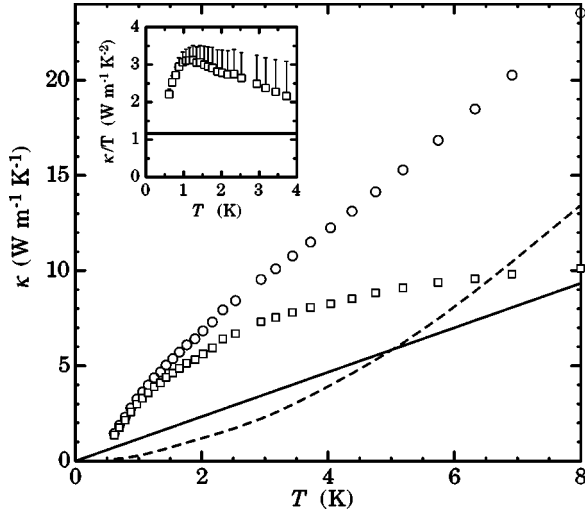
FIG. 5. Normal-state thermal conductivity measured in $H = 33$ kOe between 0.6 and 8 K (open circles). The upper limit of the phonon thermal conductivity κ_{ph}^{max} (dashed line) and the lower limit of the electronic contribution κ_e^{min} (open squares) are calculated as described in the text. The solid line represents the electronic contribution κ_e^{WFL} calculated using the Wiedemann-Franz law in Eq. (2). In the inset, we plot κ_e^{min} (open squares) and L_0/ρ_0 (solid line), respectively. The error bars in the inset mark the maximum uncertainty of κ_e/T.

senting the lower limit of the electronic contribution, is shown in Fig. 5 by open squares. It may be seen that at least below 8 K, the electronic contribution is considerably larger than predicted by the WFL. With increasing temperature, κ_e^{min} approaches the WFL prediction, as is demonstrated in the inset of Fig. 5. Nevertheless, it is impossible to identify the temperature limit where the validity of the WFL is recovered if such a limit exists at all. Considering our procedure it is clear that the true electronic contribution exceeds our estimate, particularly towards the upper end of the considered temperature range. However, at very low temperatures where $\kappa(H=0) \ll \kappa_n$ (e.g., at 0.60 K the zero-field thermal conductivity is less than 6% of the normal-state thermal conductivity), κ_e^{min} must be very close to the true κ_e. Thus the observed deviation from the WFL and in particular its temperature dependence, revealing a peaklike structure of κ_e/T vs T, shown in the inset of Fig. 5, is a reliable result of our investigation.

The violation of the Wiedemann-Franz law at low temperatures is very unusual because the validity of this law is expected to hold for the Fermi-liquid ground state of common metals. A similar set of data of Hill and coworkers for $(Pr,Ce)_2CuO_4$ was interpreted as evidence for a breakdown of the Fermi-liquid theory for this oxide material.[28] The non-Fermi-liquid behavior of cuprates was speculated to be the consequence of possible spin-charge separation, a scenario that is considered in relation with high-T_c superconductivity.[29] The same arguments are hardly relevant for the case of MgB2 where a spin-charge separation is not expected. Another explanation for the results of Ref. 28 was recently offered in Ref. 30. It was argued that a peak-type structure of κ_e/T plotted versus T might be the consequence

of some kind of transition leading to a gap formation in the electronic excitation spectrum of the normal state. This scenario would lead to a peak of κ_e/T at T_0, close to the transition temperature, an exponential decrease of κ_e/T below T_0, and a gradual approach to the Wiedemann-Franz value of L_0/ρ_0 at $T > T_0$. Such a peak at $T_0 \sim 1$ K indeed follows from our analysis of the MgB2 data (see the inset of Fig. 5). Unfortunately our data set does not extend to low enough temperatures in order to confirm the exponential temperature dependence of κ_e well below T_0.

B. Thermal conductivity in the superconducting state ($H=0$)

A rather unexpected feature in the temperature dependence of κ in zero magnetic field is the absence of even the slightest manifestation of the transition at $T_c = 38.1$ K, as has already been mentioned in previous reports on $\kappa(T)$ for polycrystalline materials.[23–26] This observation is quite unusual for superconductors with non-negligible phonon heat transport, because the opening of the superconducting gap rapidly reduces the rate of phonon scattering on electrons and should lead to a fast increase of κ_{ph} below T_c. The assumptions that either the phonon-electron scattering is much weaker than phonon-defect scattering, or that κ_{ph} is negligibly small in the vicinity of T_c, which, in principle, might explain the absence of a $\kappa(T)$ feature at T_c, are all incompatible with the observation that applying a relatively weak external magnetic field of 0.63 kOe, introducing some additional quasiparticles in the cores of vortices, considerably reduces the thermal conductivity at intermediate temperatures (see Fig. 3). The possibility that the enhancement of κ_{ph} below T_c is exactly compensated by a reduction of κ_e is not considered because, for a BCS superconductor, the slope change in $\kappa_{ph}(T)$ at T_c is much more pronounced than that in $\kappa_e(T)$.[31] The latter statement is not true for special cases of extremely clean samples of strong-coupling superconductors, such as Pb and Hg,[32,33] where the scattering of electrons by defects is negligibly small. Our sample of MgB2 with $\rho(300)/\rho(0) \approx 6.8$ cannot really be regarded as fulfilling the extreme clean-limit criteria. It is possible, however, to account for both the absence of a feature in $\kappa(T)$ at T_c and the obvious slope change in $\kappa(T)$ centered around 6 K by postulating that the superconducting energy gap $\Delta(T)$ for quasiparticles which strongly interact with low-frequency phonons is considerably smaller than the values given by BCS theory.

In the simplest approximation, the phonon thermal conductivity can be calculated as

$$\kappa_{ph} = (v^2/3) \int C(\omega)\tau(\omega)d\omega, \quad (3)$$

where v is the mean sound velocity, and $C(\omega)$ and $\tau(\omega)$ are the specific heat and the average relaxation time of a phonon mode with frequency ω, respectively. The total phonon relaxation rate may be calculated by assuming that the simultaneous influence of all independent phonon scattering mechanisms τ_i can be accumulated in the form

$$\tau^{-1} = \sum_i \tau_i^{-1}. \qquad (4)$$

The phonon-electron relaxation time $\tau_{p\text{-}e}$ changes most rapidly upon the opening of the superconducting gap. In Ref. 31, the phonon-electron scattering time in the superconducting state takes the form

$$\tau_{p\text{-}e}^s(\omega) = g(\omega/T, \Delta/T)\,\tau_{p\text{-}e}^n(\omega), \qquad (5)$$

where $\tau_{p\text{-}e}^n(\omega)$ is the normal-state relaxation time. The function $g(\omega/T, \Delta/T)$ is quite complicated, but its main feature is a steplike increase of the phonon relaxation rate at the phonon frequency $\omega = 2\Delta/\hbar$, a consequence of the fact that a phonon with an energy less than 2Δ cannot brake a Cooper pair and interacts only with quasiparticles that have already been excited above the gap.[34] In the "dominant phonon" approximation[35] it is assumed that, at temperature T, the main contribution to the heat transport in the lattice is due to phonons with frequencies close to ω_{dom}, where $\hbar\omega_{\text{dom}} \approx 3.8k_B T$. For weak-coupling BCS superconductors, where $\Delta(0) = 1.76k_B T_c$ and $\Delta(T)$ is a standard function tabulated in Ref. 36, $\hbar\omega_{\text{dom}}(T) = 2\Delta(T)$ is always fulfilled at the temperature $0.73T_c$, i.e., not far below T_c. This is the reason why $\kappa_{\text{ph}}(T)$ increases rapidly below T_c, leading to a maximum of the measured κ, typically close to $T_c/2$.[34] For MgB$_2$, instead of a peak near $T_c/2$, $\kappa(T)$ exhibits a distinct feature at about $T_c/6 \approx 6$ K, which can also be regarded as a peak-type structure on top of the background which decreases monotonously with decreasing T. This suggests that the relevant superconducting energy gap is equal to $\hbar\omega_{\text{dom}}$ at much lower temperatures than $0.73T_c$. Hence, for the quasiparticles which scatter phonons most effectively, the energy gap $\Delta(T)$ is about 3 times smaller than the values given by the original BCS theory.

In principle, more information could have been extracted from our data by direct comparison with existing theories for the thermal conductivity in multiband superconductors.[34,37–39] However, since the WFL seems to be invalid in the normal state of MgB$_2$ at low temperatures, any attempt to analyze $\kappa(T)$ quantitatively in the superconducting state is hampered by the difficulties in separating κ_e and κ_{ph} in a reliable manner.

C. Thermal conductivity in the mixed state

Before discussing the features of κ in magnetic fields, it is important to note that the zero-field values of κ at temperatures $T \ll T_c$ are almost entirely due to the phonon contribution. The electronic thermal conductivity in the superconducting state $\kappa_{e,s}(T)$ can be estimated using the theory of Bardeen, Rickayzen, and Tewordt.[31] In their model,

$$\kappa_{e,s} = \kappa_{e,n} f(y), \qquad (6)$$

where

$$f(y) = \frac{2F_1(-y) + 2y\ln(1+e^{-y}) + \dfrac{y^2}{1+e^y}}{2F_1(0)} \quad (T < T_c)$$

$$f(y) = 1 \quad (T \gtrsim T_c), \qquad (7)$$

as well as $F_n(-y) = \int_0^\infty z^n (1+e^{z+y})^{-1} dz$, $y = \Delta(T)/k_B T$. Using for $\kappa_{e,n}$ the values of $\kappa_{e,n}^{\min}$ shown in Fig. 5 and taking into account the lowest previously claimed value of $\Delta(0) = 1.7$ meV,[10] it may be shown that $\kappa_{e,s}$ is negligibly small below 4 K and, therefore, $\kappa(H=0) \approx \kappa_{\text{ph}}$.

As may clearly be seen in Fig. 4, the $\kappa(H)$ curves reveal very similar features at all temperatures below 8 K. The most intriguing aspect of these curves is the very rapid increase of κ at relatively weak fields, after the initial decrease of κ_{ph}. This increase is undoubtedly due to a field-induced enhancement of the number of electronic quasiparticles and thus to an increase of κ_e. For common type-II superconductors in their mixed state, the features of $\kappa_e(H)$ are expected to depend on the ratio between the electron mean free path l and the coherence length ξ_0. A rough estimate of l can be obtained from the residual resistivity ρ_0 employing the Drude relation $\rho_0 = 3/[N_0 l v_F e^2]$, where N_0 is the electronic density of states at the Fermi level and $v_F/3$ is the average in-plane component of the Fermi velocity. Using the values $N_0 = 0.7$ states/(eV unit cell) (Ref. 14) and $v_F = 4.9 \times 10^7$ cm/sec (Ref. 4), we obtain $l \approx 80$ nm. Since this value is considerably larger than the in-plane coherence length $\xi_{ab,0} = 11.8$ nm (Ref. 21), our sample is in the moderately clean limit. In this limit, κ_e is expected to be small at all fields below H_{c2}, except close to H_{c2}, where it grows according to

$$\kappa_e = \kappa_{e,n}[1 - C_T(H_{c2} - H)^{1/2}], \qquad (8)$$

where $\kappa_{e,n} \propto T$ is the normal-state electronic thermal conductivity above H_{c2} and C_T is a temperature-dependent coefficient.[40] As may be seen in Fig. 4, for MgB$_2$, the largest positive slope $d\kappa/dH$ is observed well below H_{c2} for both orientations of the magnetic field. Although the form of Eq. (8) implies that $\kappa(H)$ should scale with the value of H_{c2}, the thermal conductivity depends only weakly on the field direction up to approximately 6 kOe. This is amazing because the anisotropy of the upper critical field $H_{c2}^{ab}/H_{c2}^c \approx 4.2$ at low temperatures.[21]

For $H \perp c$, after a steep initial increase, $\kappa(H)$ reaches a region where it exhibits only a relatively weak H dependence. The same tendency is also observed for $H \| c$ but it is partly masked by yet another increase of $\kappa(H)$ close to H_{c2}. For $H \perp c$ and $T \lesssim 8$ K, $H_{c2}^{ab} \sim 130$ kOe (Ref. 21) and is not accessible in our experimental setup. Therefore the region of weak H dependence extends to the highest fields reached in this study.

The field dependence of κ_e, although very different from what one would expect from a conventional superconductor, can qualitatively be explained in terms of a two-band model with two energy gaps of different magnitude associated with each band. Nakai and co-authors[41] analyzed such a model where one band with strong pairing (L band) is responsible for superconductivity, and superconductivity in the second band (S band) is induced by Cooper pair tunneling. Consequently, the two bands are characterized by a smaller gap Δ_S and a larger gap Δ_L, and normal-state electronic densities of

states at the Fermi level $N_{0,S}$ and $N_{0,L}$, respectively. The analysis presented in Ref. 41 shows that the quasiparticle states in the vortices are highly confined in the L band but only loosely bound in the S band. Therefore the quasiparticle states of the vortices in the S band start to overlap already in weak fields and the resulting density of states equals that of the normal-state $N_{0,S}$ at $H \ll H_{c2}$. The situation can be visualized as a vortex lattice involving the L-band states, coexisting with the normal state in the S band where the energy gap is suppressed. This model explains very well the behavior of the electronic specific heat in a magnetic field.[2,42] The field-induced suppression of the smaller gap is claimed to be consistent with the results of point-contact measurements[3,9] and recent scanning tunneling spectroscopy experiments.[43] In terms of the two-gap model, the saturation of the thermal conductivity much below H_{c2} may be regarded as the result of the closing of the energy gap in the S band. The heat transport via quasiparticles of the band associated with the larger gap is significant only in the vicinity of H_{c2}, and, eventually, above H_{c2} the full normal-state electronic heat transport is restored. The lack of a substantial dependence of κ_e on the field orientation for $H \ll H_{c2}$ is an obvious consequence of the weakly anisotropic 3D nature of the π bands. From this we conclude that the smaller gap must open in the π band. The rapid increase of the number of quasiparticles in the π bands also naturally explains the very fast drop of $\kappa_{ph}(H)$ in small fields, because the corresponding excited quasiparticles are the dominant scattering centers for phonons at low temperatures.

A more quantitative analysis of $\kappa_e(H)$ can be made for the lowest temperatures where, as may be seen from Fig. 3, the phonon contribution is relatively small in comparison with the field-induced electronic contribution. At very low temperatures, the phonon scattering by electronic quasiparticles is less effective[35]; therefore, $\kappa_{ph}(H>0)$ should not much deviate from $\kappa_{ph}(H=0)$. Indeed, at temperatures of 0.60 and 1.02 K, there is no initial decrease of $\kappa(H)$ in small fields. Assuming that the phonon contribution is essentially H independent and that the smaller gap is completely suppressed in fields exceeding 20 kOe, we may establish the individual contributions to κ_e of the quasiparticles associated with either the σ or π band. This is illustrated in Fig. 6. A possible reduction of κ_{ph} with increasing H could slightly change this ratio in favor of $\kappa_{e,\pi}$, but only by a few percent. The ratio $\kappa_{e,\pi}/\kappa_{e,\sigma}$ is 0.57/0.43, as estimated from the $\kappa(H)$ data at $T=0.60$ K. This ratio is remarkably close to the ratio of the densities of electronic states in the two bands, $N_{0,\pi}/N_{0,\sigma}$ of 0.58/0.42, as calculated by Liu *et al.*[15] and 0.55/0.45 by Belashchenko *et al.*[44] Similar ratios of 0.55/0.45 and 0.50/0.50 have been extracted from tunneling spectroscopy measurements[43] and from specific heat experiments,[2] respectively. From this comparison, we reach the important conclusion that the electron mean free paths on different sheets of the Fermi surface are close to being equal. Indeed, the electronic thermal conductivity may be calculated from $\kappa_e = C_e v_F l/3$, where C_e is the electronic specific heat. Since $C_{e,i} \propto N_{0,i}$ and the ab components of the Fermi velocity $v_{F,i}$ ($i=\pi,\sigma$) are similar for different sheets of the Fermi surface in MgB_2,[45] the equality $\kappa_{e,\pi}/\kappa_{e,\sigma}$
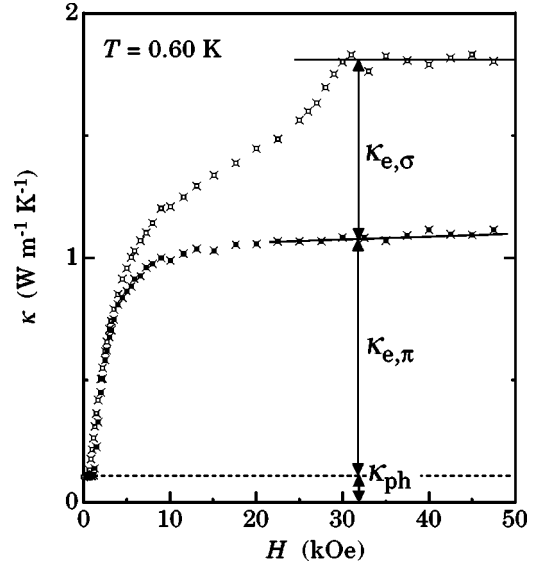

FIG. 6. Separation of the individual contributions of the σ- and π-band quasiparticles, and the phonons to the normal-state thermal conductivity of MgB_2 at $T=0.60$ K.

$\approx N_{0,\pi}/N_{0,\sigma}$ is tantamount to saying that, at low temperatures, the ratio of the electron mean free paths in different bands l_π/l_σ is close to unity. This observation is essential in view of the current discussion of the possibly different impurity scattering rates in different bands of electronic states of MgB_2.[46–48]

Although the absence of any particular feature at T_c in the zero-field $\kappa(T)$ data, discussed in Sec. IV B, gives only qualitative support for the existence of parts of the Fermi surface with a gap much smaller than predicted by the standard BCS theory, the magnetic-field-induced variation of the low-temperature thermal conductivity may be regarded as strong evidence in favor of the multigap scenario.

At the same time we believe that the model of Haas and Maki[49] for explaining the thermodynamic and optical properties of MgB_2 is not appropriate. They proposed the **k** dependence of a single energy gap to adopt the form of a prolate ellipsoid. Our claim is based on the comparison of our data set with similar results for materials with strongly anisotropic gap functions. In Fig. 7, we redraw a figure from Ref. 50, which compares the field-induced variation of κ_e at temperatures well below T_c for different conventional and unconventional superconductors, amended by our data for MgB_2 at $T=0.60$ K. The Nb data[51] reveal the typical salient features of a clean, almost isotropic s-wave superconductor, and confirm the very weak energy transport by quasiparticles far below H_{c2}, in agreement with Eq. (8). A considerably faster, almost linear in H, increase of κ_e is observed for a superconductor with nodes in $\Delta(\mathbf{k})$, here exemplified by UPt_3.[52] A similar H variation of κ_e has been observed for $LuNi_2B_2C$, which led the authors of Ref. 50 to claim an anisotropy of the energy gap $\Delta_{max}/\Delta_{min} > 10$. The increase of $\kappa_e(H)$ in MgB_2 is much faster than in any of these materials. For the field direction $H \perp c$, more than half of the normal-state thermal conductivity is restored already at $H = 0.05 H_{c2}$. This means that, with increasing H, instead of the
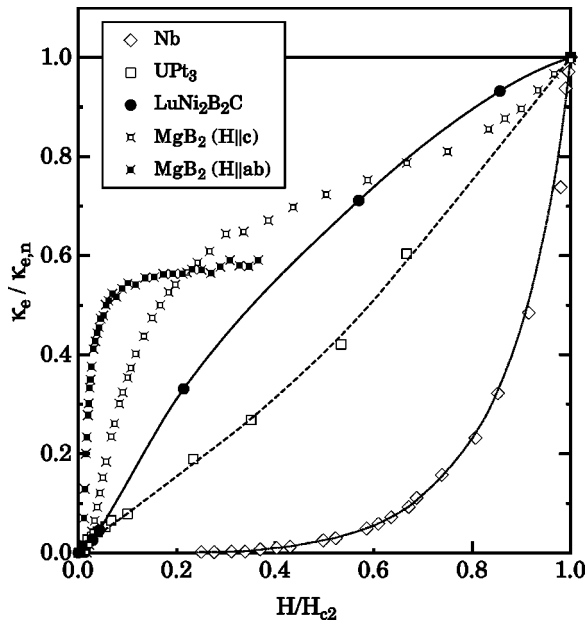
FIG. 7. The electronic thermal conductivity normalized to its normal state value vs H/H_{c2}. The data for MgB$_2$ are from this work; the results for Nb, UPt$_3$, and LuNi$_2$B$_2$C are from Refs. 51, 52, and 50, respectively.

gradual increase of the number of quasiparticles contributing to the heat transport, which is characteristic of single-gap anisotropic superconductors, we are dealing with the suppression of an energy gap on a significant portion of the total

Fermi surface by relatively weak magnetic fields. Thus the features of $\kappa_e(H)$ of MgB$_2$ displayed in Fig. 7 may be regarded as a natural consequence of the existence of two different gaps.

V. SUMMARY

Our $\kappa(T,H)$ data provide evidence for a rapid field-induced enhancement of quasiparticles in the superconducting state of MgB$_2$ well below H_{c2}, consistent with an efficient field-induced closing of the smaller energy gap, thus provoking a fast growth of the electronic thermal conductivity. At higher fields, the growth of $\kappa_e(H)$ tends to saturate until, in the vicinity of H_{c2}, the contribution to κ from the electrons of the band associated with the larger energy gap grows rapidly, merging into the practically field-independent thermal conductivity in the normal state above H_{c2}.

At low temperatures, the electronic thermal conductivity of the field-induced normal state is nonlinear in T and deviates considerably from the prediction of the Wiedemann-Franz law. This deviation peaks at about 1 K, suggesting the existence of some transition provoking a gap formation in the electronic excitation spectrum close to this temperature.

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