Mean-field glassy phase of the random-field Ising model

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(Received 20 July 2001; revised manuscript received 15 April 2002; published 1 July 2002)

The emergence of glassy behavior of the random-field Ising model (RFIM) is investigated using an extended mean-field theory approach. Using this formulation, systematic corrections to the standard Bragg-Williams theory can be incorporated, leading to the appearance of a glassy phase, in agreement with the results of the self-consistent screening theory of Mezard and Young. Our approach makes it also possible to obtain information about the low-temperature behavior of this glassy phase. We present results showing that within our mean-field framework, the hysteresis and avalanche behavior of the RFIM is characterized by power-law distributions, in close analogy with recent results obtained for mean-field spin-glass models.

DOI: 10.1103/PhysRevB.66.014413 PACS number(s): 64.70.Pf, 75.60.Ej, 75.50.Lk

I. INTRODUCTION

The random-field Ising model (RFIM) (Ref. 1) is one of the simplest models used to describe the frustration introduced by disorder in interacting many-body systems. Despite its simplicity, its behavior proved to be the source of much controversy, primarily due to the lack of reliable theoretical methods that one can use for such systems. Still, efforts to elucidate the basic properties of the RFIM continue to attract considerable attention, primarily because of its direct relevance to a number of important physical problems. These include not only the behavior of diluted magnets in external magnetic fields,[2] but also several aspects of electronic transport in disordered insulators[3] and systems near the metal-insulator transition.[4,5] In addition, the nonequilibrium behavior of the RFIM has been used to model the physics of hysteresis and avalanche behavior and the origin of self-organized criticality.[6] Finally, nondisordered models with infinitesimal random fields have been studied[7] in order to investigate self-generated glassy behavior[8] observed in systems such as supercooled liquids,[9] or even underdoped cuprates.[10]

The simplest effect of turning on a weak random field is the resulting depression of the critical temperature for uniform ordering, while for sufficiently strong randomness the ordered phase completely disappears. This behavior is apparent even in the simplest Bragg-Williams (BW) mean-field theory (MFT), but understanding the disorder-induced modification of the relevant critical behavior proved much more difficult. Very early on, perturbative renormalization-group (RG) results of Parisi and Sourlas[11] suggested the existence of a ''dimensional reduction'' by which the random problem belongs to the same universality class as a clean one in two dimensions less.

Unfortunately, the beautiful result of PS was found to be in conflict not only with the heuristic argument of Imry and Ma,[12] but also with the exact results of Imbrie,[13] and thus deemed incorrect. More recently, the origin of these discrepancies was traced[14] to the implicit assumption of Parisi and Sourlas that outside the ordered phase, a single thermodynamically stable state exists. This assumption may not be warranted if another phase transition, presumably of glassy character, would precede any uniform ordering, which would lead to the breakdown of dimensional reduction. Of course, such a glassy phase is not found in the naive BWMFT, so that more sophisticated theoretical schemes have to be used in order to identify the corresponding instability of the high-temperature paramagnetic phase. Such a theory was formulated by Mezard and Young,[14] who utilized the self-consistent screening (SCS) approach of Bray,[15] and identified the glassy phase by carrying out a replica-symmetry-breaking stability analysis. Similar results were obtained by numerically solving the mean-field equations for a fixed realization of disorder by Lancaster *et al.*,[16] confirming the existence of the glassy phase. Finally, Mezard and Monasson,[17] and De Dominicis *et al.*[18] presented arguments that the glass phase should persist even at weak disorder, and everywhere precede the uniform ordering, in agreement with the breakdown of the dimensional reduction.

While these approaches provided important information on the RFIM, several aspects remained unsatisfactory. The SCS scheme, while being able to identify the glass phase, proved of considerable complexity, making it difficult to obtain simple analytical results. In addition, the physical content of this formulation does not appear very transparent, making it difficult to establish what crucial ingredients are necessary for a theory in order to be able to identify and describe such a glass phase. Finally, the low-temperature properties of the glass phase also seem very difficult to establish using this approach, which presents a severe limitation in applying the RFIM to the problem of the ''Coulomb glass''[5] and the related theory of Efros and Shklovskii.[3]

The main goal of the present paper is to identify the simplest possible approach that is capable of providing a description of the glassy phase. We will show that to do this, two requirements have to be satisfied: (i) the theory has to identify the correct order parameters, and (ii) it has to incorporate spatial fluctuations beyond the naive BW theory. Both of these are automatically satisfied by the SCS theory, but we will show how the same goals can be achieved in a simpler

and more transparent fashion, by examining systematic corrections to the BW theory. To do this, we follow an approach used by Plefka[19] to derive the Thouless-Anderson-Palmer (TAP) equations for spin glasses.[20] A similar formulation was subsequently used by Georges *et al.*,[21] to obtain systematic corrections to the MFT of the Sherrington-Kirkpatrick (SK) model. This approach fixes the desired order parameters by introducing appropriate source fields, and then evaluates the corresponding Gibbs free energy by an expansion in powers of the interaction J. A brief description of some of our results in the electronic context has already been presented in Ref. 5, but the present paper provides many more details and a number of other interesting results.

The rest of the paper is organized as follows. In Sec. II, we begin our discussion by examining the RFIM on a Bethe lattice, where a particularly simple derivation of the BW theory and its leading corrections can be obtained, resulting in the emergence of the glass phase. A more general strategy based on the Plefka approach is presented in Sec. III, showing that these leading corrections take the same form on arbitrary lattices. The low-temperature structure of the glass phase is discussed in Sec. IV, where the hysteresis and avalanche behavior in our extended MFT is examined, showing the emergence of self-organized criticality similar to that recently discovered by Pazmandi *et al.*[22] for the SK model. The role of higher-order corrections to our theory and its relation to the $1/z$ expansion approach is discussed in Sec. V, where we show that the leading nontrivial corrections to the BWMFT represent dominant contributions in the joint limit of large coordination *and* large random fields. In this section, we also comment on the relevance of our approach to the controversial question of self-generated glassy behavior in systems without disorder, which we discuss by examining the limit of weak random fields. Our conclusions are summarized in Sec. VI, where we also outline some interesting directions for future work.

II. BETHE LATTICE

The simplest theoretical formulation of the random-field Ising model (RFIM) is obtained by considering the large coordination limit, where the standard Bragg-Williams mean-field theory (BWMFT) becomes exact. However, one has to go beyond this limit in order to obtain nontrivial results, and we address this question by examining systematic corrections to the BWMFT. The large coordination limit and the leading corrections are particularly easily formulated in the special case of the Bethe lattice, where a simple recursive procedure can be used. This approach also gives some insight in the mechanism for the emergence of the glass phase, so we begin our discussion by concentrating on the Bethe lattice in the limit of large coordination.

The Hamiltonian of the random-field Ising model is given by

$$H_{int} = -\sum_{\langle ij \rangle} J_{ij} S_i S_j - \sum_i h_i S_i . \tag{1}$$

Here, $S_i = \pm 1$, and $J_{ij} = J/z$ are uniform ferromagnetic interactions between nearest-neighbor sites, rescaled with the coordination number z in order to obtain a finite result in the $z \to \infty$ limit. The random magnetic fields h_i are assumed to be Gaussian distributed, with zero mean and a variance $\langle h_i^2 \rangle = H_{RF}^2$.

Using standard replica methods,[20] we can formally average over disorder, and the resulting partition function takes the form ($\alpha = 1, \ldots, n;\ n \to 0$)

$$Z^n = \mathrm{Tr} \exp\left[\frac{\beta J}{z} \sum_{\alpha} \sum_{\langle ij \rangle} S_i^\alpha S_j^\alpha + \frac{1}{2} (\beta H_{RF})^2 \sum_i \left(\sum_\alpha S_i^\alpha \right)^2 \right]. \tag{2}$$

We proceed by taking advantage of the tree-like structure of the Bethe lattice, by formally summing over all the degrees of freedom in one branch. The resulting functional $\Phi(S_0^\alpha)$ is a function only of the variable S_0^α at the branch origin, and can be easily seen to obey the following self-consistent equation

$$\Phi(S_0^\alpha) = \mathrm{Tr}_{S_1^\alpha} \left\{ \exp\left[\frac{\beta J}{z} \sum_\alpha S_0^\alpha S_1^\alpha + \frac{1}{2} (\beta H_{RF})^2 \left(\sum_\alpha S_1^\alpha \right)^2 \right] \Phi^{z-1}(S_1^\alpha) \right\}. \tag{3}$$

A. Bragg-Williams theory

In order to examine the large z limit, it is convenient to define a single-site effective action by

$$L[S^\alpha] \equiv -\frac{1}{2} (\beta H_{RF})^2 \left(\sum_\alpha S_1^\alpha \right)^2 - \ln[\Phi^{z-1}(S^\alpha)]. \tag{4}$$

In the $z \to \infty$ limit, this expression simplifies, since the interaction has been scaled by $1/z$, and the functional $\Phi(S_0^\alpha)$ can be obtained by expanding the self-consistency condition, Eq. (3), in powers of the interaction J. To leading order, only the terms *linear* in J survive, and we find

$$L^{(1)}[S^\alpha] = -\beta J \sum_\alpha S^\alpha m^\alpha - (\beta H_{RF})^2 \sum_{\alpha < \beta} S^\alpha S^\beta. \tag{5}$$

Here, the index (1) indicates that only terms linear in J are retained. As we expect for a Bragg-Williams theory, this local effective action corresponds to a single spin which, in addition to the local random field, also experiences the presence of a ''molecular'' field Jm^α due to interaction with the neighbors.

We emphasize that this procedure *automatically* defines the order parameters entering the local effective action. In the $z \to \infty$ limit only the magnetization m^α appears, which from Eq. (3) satisfies the following self-consistency condition:

$$m^\alpha = \langle S^\alpha \rangle_{L[S]}. \tag{6}$$

Since the interaction term does not mix the replicas, they trivially decouple, and the self-consistency condition for the magnetization $m^\alpha = m$ takes the form

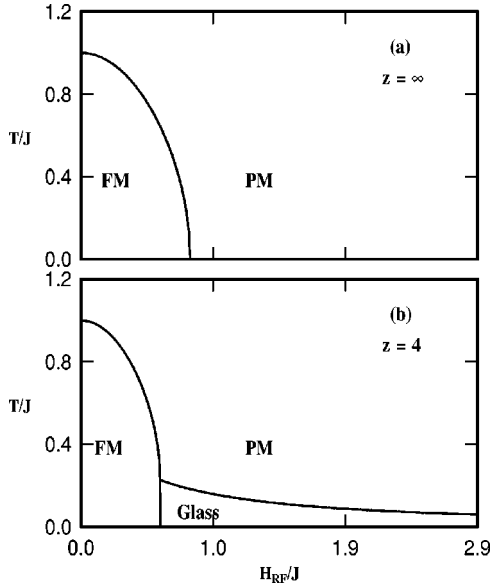

FIG. 1. Phase diagram for coordination numbers $z=\infty$ (a) and $z=4$ (b). Ferromagnetic (FM), paramagnetic (PM), and glass phases are found. Note that the glass phase does not exist for $z=\infty$.

$$m=\int Dx \tanh(\beta H_{RF}x+\beta Jm), \tag{7}$$

where $Dx=dx\exp(-x^2/2)/\sqrt{2\pi}$. This equation is a straightforward generalization of the well-known Bragg-Williams condition to include the effect of random fields. The critical temperature where the magnetization vanishes is easily computed, and is found to vanish at a critical strength $H_{RF}^c/J=\sqrt{2/\pi}$ of the random fields, as shown in Fig. 1(a). Outside this ferromagnetic phase the local effective action of Eq. (5) reduces to that of noninteracting spins in the presence random fields, and no further phase transition can be found.

The reason for this limitation of the BWMFT is very simple, and can be appreciated by considering the (thermally averaged) Weiss field $h_i^W=\langle\Sigma_j J_{ij}S_j\rangle_T$ produced by the neighboring spins on a given site. In a uniform system, h_i^W is the same on every site, but in presence of randomness, it may display appreciable spatial fluctuations. This is especially important outside the any uniformly ordered phase, where the spatial average $\overline{h}=\langle h_i^W\rangle_{site}$ vanishes, but the local value h_i^W of the Weiss field may remain finite, reflecting the *local* breaking of the up-down symmetry. This behavior is encountered in spin glasses, where $\overline{h_i^W}=0$, but $\overline{(h_i^W)^2}$ becomes finite below a temperature corresponding to the glassy freezing. In the simple BWMFT, only the first moment $\overline{h_i^W}\sim m$ is retained, and thus its inability to describe any glassy ordering.

B. Leading corrections and glassy freezing

In order to search for the existence of nontrivial behavior outside the uniformly ordered phase, we have to go beyond the BWMFT, i.e., the $z=\infty$ limit. The leading correction is obtained by iterating the self-consistency condition Eq. (3) to second order in the interaction J, and in the expression for the local effective action a new term, quadratic in J, appears:

$$L=L^{(1)}+L^{(2)},$$

$$L^{(2)}=-\frac{1}{z}(\beta J)^2\sum_{\alpha<\beta}S^\alpha S^\beta[q^{\alpha\beta}-(m^\alpha)^2]. \tag{8}$$

The order parameter m^α is still given by Eq. (6) [although the average is now computed with respect to the extended action of Eq. (8)], but the new order parameter $q^{\alpha\beta}$ appears, which is self-consistently determined by

$$q^{\alpha\beta}=\langle S^\alpha S^\beta\rangle_{L[S]}. \tag{9}$$

This quantity is nothing but the familiar Edwards-Anderson order parameter.[20] Note that in our Bethe lattice procedure, it appears automatically, as a result of an expansion in powers of the interaction J to the lowest nontrivial order beyond the BWMFT. Its presence reflects the fact that in finite-coordinated lattices, local Weiss fields h_i are random numbers with a finite dispersion. More precisely, they are described by the distribution of local fields $P(h_i)$, the explicit form of which is determined by the replica matrix $q^{\alpha\beta}$.

As in standard spin-glass theory,[20] the solution of these equations assumes the simplest form in the high-temperature phase, where replica symmetry is valid, such that $m^\alpha=m$ and $q^{\alpha\beta}=q$. In this case, our self-consistency conditions take the form

$$m=\int Dx\tanh\times\left[\left((\beta H_{RF})^2+\frac{(\beta J)^2}{z}(q-m^2)\right)^{1/2}x+\beta Jm\right],$$

$$q=\int Dx\tanh^2\times\left[\left((\beta H_{RF})^2+\frac{(\beta J)^2}{z}(q-m^2)\right)^{1/2}x+\beta Jm\right]. \tag{10}$$

This is sufficient to determine the ferromagnetic (FM) phase boundary which, as before, is determined by setting $m=0$. By numerically solving these equations, we find that the FM phase is only slightly reduced due to the fluctuation corrections, as shown in Fig. 1(b).

Identifying the glassy freezing is more difficult. For standard spin-glass models, the glassy freezing coincides with the breaking of the up-down symmetry, so that the glass transition temperature can be identified even within replica symmetric theory, as the point where the Edwards-Anderson order parameter $q=\overline{\langle S_i\rangle^2}$ assumes a nonzero value. In our case, the random magnetic field plays a role of a source conjugate to the order parameter, locally breaking the up-down symmetry. The situation is similar as in spin-glass models in a uniform external field,[20] where the replica symmetric order parameter q remains nonzero for any temperature, and thus cannot be used to identify glassy freezing. Instead, we follow Mezard and Young,[14] and look for an

instability to replica symmetry breaking (RSB) within the paramagnetic phase. To do this, we can set $m=0$, and note that the remaining equation for $q^{\alpha\beta}$ is in fact *identical* to that describing the Sherrington-Kirkpatrick model[20] in presence of random magnetic fields. This model is also described with the Hamiltonian of Eq. (1), but this time with J_{ij}'s being Gaussian random variables with zero mean and variance $\langle J_{ij}^2\rangle \equiv J^2/zN$, where J is the interaction of the original lattice model.

The advantage of mapping our equations to those of an infinite range model will be also used in Secs. IV and V, where we examine the low-temperature structure of the glass phase. In addition, the calculation for obtaining the replica symmetry-breaking (RSB) boundary can be carried out using standard methods,[20,23] following the approach of de Almeida and Thouless[23] (AT). A more general strategy for performing the RSB stability analysis, which is applicable for arbitrary lattices, and to higher-order fluctuation corrections will be presented and discussed in Sec. V. Here we just quote the result valid to the leading nontrivial order, which takes the form

$$1=\frac{(\beta J)^2}{z}\int Dx\,\cosh^{-4}\left[\left((\beta H_{RF})^2+\frac{(\beta J)^2 q}{z}\right)^{1/2}x\right]. \tag{11}$$

As expected, in the large coordination ($z\to\infty$) limit, the glass transition temperature vanishes, and our results reduce to standard BWMFT predictions. For finite z, the SG phase emerges, in agreement with the results of Mezard and Young.[14] The above equation can easily be solved numerically, and the results are shown in Fig. 1(b). In contrast to the predictions of the SCS approach, our SG phase emerges only for sufficiently strong random fields. This is a simple result of the fact that in simple mean-field treatments such as ours, different phases do not directly affect each other, since there is no ''precursor'' of the ordering in the disordered phase.

C. Limit of large random fields

It is interesting to note that the situation is especially simple in the limit of large random fields. In the extreme case $H_{RF}\to\infty$, the interactions can be ignored and all the spins tend to align with their local random fields. In this case there is only one thermodynamic state, so that the emergence of a multitude of metastable states associated with the RSB instability is clearly suppressed. We conclude that the glass transition temperature must be depressed to zero as $H_{RF}\to\infty$, just as in the case of spin glasses in uniform external fields discussed by de Almeida and Thouless. However, the asymptotic form of $T_G(H_{RF})$ proves to be different in our case, which may be significant for several experimental systems.

In the limit of large random fields, our results simplify considerably, since the RSB boundary resides at very low temperatures. The leading-order behavior can be obtained by replacing the $q(T)$ in Eq. (11) by its zero-temperature limit $q(0)=1$, and we find

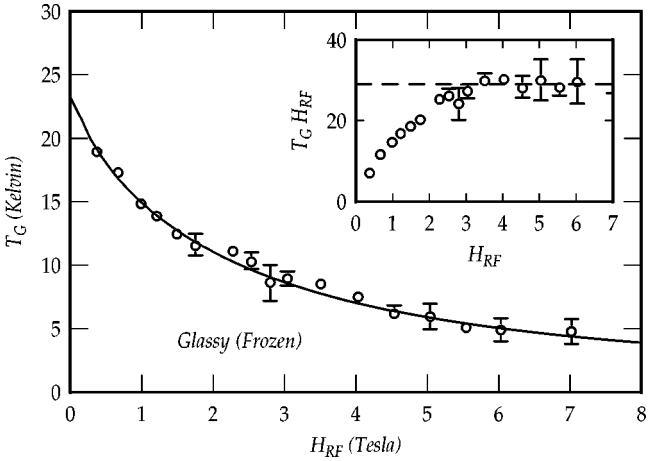

FIG. 2. Glass transition temperature as a function of the random-field strength. Open circles are the experimental data for $Fe_{0.31}Zn_{0.69}F_{0.2}$ (from Ref. 2), and the line is the prediction of Eq. (11). The inset shows how $T_G\sim 1/H_{RF}$ at large fields.

$$T_G\approx\frac{4J^2}{3zH_{RF}}\sim 1/H_{RF}\,. \tag{12}$$

It is interesting that our glass transition temperature thus decreases very *slowly* with the random field strength, in contrast to the form of the de Almeida–Thouless line[23,20] (infinite ranged spin glasses in a uniform field) where $T_G\sim\exp(-H^2/2J^2)$. This fact could be particularly significant for strongly disordered electronic systems,[4,5] where it would suggest the possibility to observe the glassy behavior of electrons at finite temperatures. The physical reason for this behavior in our case is not obvious, but we will see that it reflects some very subtle features of the low-temperature glass phase, which will be discussed in Sec. IV.

From the physical point of view, an instability to RSB, such as we find, is known[20,23] to describe the emergence of an extensive number of metastable states, and the associated slowing down in the relaxational dynamics of the spins. Experimentally, this results in the onset of the history dependence of cooling, and the related bifurcation of field-cooled (FC) and zero-field-cooled (ZFC) spin susceptibilities. Such experiments have been carried out on diluted antiferromagnets in uniform external fields, which have long been believed to be realizations of the RFIM. Here, the effective strength of the random field can be varied by modifying the magnitude of the external magnetic field. In one such experiment,[2] the field dependence of the ''irreversibility line'' has been determined, defined as the temperature where the FC and ZFC susceptibilities start to differ. Interestingly, these experiments show a rather slow decrease of this glass transition temperature, in agreement with our predictions. We have digitized the data from Ref. 2, and compared them to our predictions, as shown in Fig. 2, where an apparent confirmation of our $T_G\sim 1/H_{RF}$ law can be seen (see inset). While this agreement of our theory and experiment is encouraging, it should not be taken too seriously, given the uncertainties of the precise correspondence of the experi-

mental system and the RFIM that we consider. More experiments on other related systems would be welcome to test our predictions in more detail.

III. GENERAL LATTICES: LEGENDRE TRANSFORM APPROACH

So far, we have seen how leading corrections to the BW theory can be obtained on the Bethe lattice, resulting in the emergence of the glass phase. This example was useful, because it automatically introduces the correct order parameters, thus allowing the emergence of the glass phase. However, we would like to formulate a more general approach, in order to demonstrate the generality of our conclusions, and also to be able to systematically examine higher-order fluctuation corrections. In the case of the RFIM, the desired order parameters cannot be introduced as for standard spin-glass models, where one decouples the disorder-averaged interaction term using a Hubbard-Stratonovich transformation. In absence of random interactions, the ''bare'' disorder-averaged Hamiltonian has only terms linear in the interaction J, but higher order terms can be generated by fluctuations. In this case, one has to introduce the order parameters ''by hand,'' and then expand the free energy to the lowest nontrivial order in the interaction J, in order to obtain glassy ordering. To do this, we follow a strategy introduced by Plefka[19] and Georges *et al.*,[21] and use a Legendre transform approach, introducing external source fields $\xi_i^{\alpha\beta}$ in order to fix the Edwards-Anderson (EA) order parameters $q_i^{\alpha\beta}$. Here and in the rest of the paper, we are mostly interested in the emergence of the glassy phase of the RFIM, so we concentrate on the nonmagnetic ($m=0$) solutions. The disorder-averaged Helmholtz free energy takes the form

$$-\beta F=\lim_{n\to 0}\frac{\partial}{\partial n}[-\beta F_n], \tag{13}$$

where

$$-\beta F_n=\ln \mathrm{Tr}\exp\left[\frac{\beta J}{z}\sum_{\alpha}\sum_{\langle ij\rangle}S_i^{\alpha}S_j^{\alpha}+\frac{(\beta H_{RF})^2}{2}\sum_i\left(\sum_{\alpha}S_i^{\alpha}\right)^2+\sum_i\sum_{\alpha<\beta}\xi_i^{\alpha\beta}S_i^{\alpha}S_i^{\beta}\right]. \tag{14}$$

The EA order parameters are given by

$$q_i^{\alpha\beta}=\frac{\partial(-\beta F)}{\partial\xi_i^{\alpha\beta}}=\langle S_i^{\alpha}S_i^{\beta}\rangle, \tag{15}$$

and the corresponding Gibbs free energy is

$$-\beta G=\lim_{n\to 0}\frac{\partial}{\partial n}[-\beta G_n];$$

$$-\beta G_n=-\beta F_n-\sum_i\sum_{\alpha<\beta}\xi_i^{\alpha\beta}q_i^{\alpha\beta}. \tag{16}$$

A. Extended mean-field theory

To obtain an extended mean-field theory, we can expand $-\beta G$ to a desired order in powers of the reduced interaction $\varepsilon\equiv\beta J$, while fixing the value of the independent variables $q_i^{\alpha\beta}$. In doing this, one has to keep in mind that source fields $\xi_i^{\alpha\beta}$ are by Eq. (15) implicit functions of the order parameters $q_i^{\alpha\beta}$. The *form* of these functions is also a function of βJ, so in order to be consistent, one has to expand the fields $\xi^{\alpha\beta}$ to a given order in βJ as well, while fixing $q_i^{\alpha\beta}$'s. Defining $-\beta G_n/N\equiv g$, we can write

$$g(\varepsilon)=g_o+g_{int}(\varepsilon), \tag{17}$$

$$g_{int}(\varepsilon)=\sum_{k=1}^{\infty}\frac{\varepsilon^k}{k!}g_k. \tag{18}$$

In this expression, the coefficients g_k are functions of $q^{\alpha\beta}$ evaluated at $\varepsilon=0$. Explicitly, we find

$$g_o[q]=f_o[\xi]-\frac{1}{N}\sum_i\sum_{\alpha<\beta}\xi_i^{\alpha\beta}q_i^{\alpha\beta}, \tag{19}$$

where

$$f_o[\xi]=\frac{1}{N}\ln \mathrm{Tr}\exp\{-L_o\}, \tag{20}$$

and the reference effective action is defined by

$$L_o=-\sum_i\sum_{\alpha<\beta}[\xi_i^{\alpha\beta}+(\beta H_{RF})^2]S_i^{\alpha}S_i^{\beta}. \tag{21}$$

The interaction terms take the form

$$g_1=\frac{1}{N}\langle\phi\rangle_o \tag{22}$$

$$g_2=\frac{1}{N}\langle\phi(\phi-\langle\phi\rangle_o+\chi-\langle\chi\rangle_o)\rangle_o, \tag{23}$$

$$\vdots$$

where we have defined

$$\phi=\frac{1}{z}\sum_{\alpha}\sum_{\langle ij\rangle}S_i^{\alpha}S_j^{\alpha}, \tag{24}$$

$$\chi=\sum_i\sum_{\alpha<\beta}S_i^{\alpha}S_i^{\beta}\frac{\partial}{\partial\varepsilon}\xi_i^{\alpha\beta}. \tag{25}$$

In these expressions, the averages are taken with respect to the reference effective action L_o, which is a function of the external fields $\xi_i^{\alpha\beta}$. Note that since the coefficients g_k are evaluated at $\varepsilon=0$, after the cumulants are evaluated, the external fields should be set equal to $\xi_i^{\alpha\beta}(\varepsilon=0)\equiv\xi_o^{\alpha\beta}$, which are implicit functions of the independent variables $q^{\alpha\beta}$, as determined by the following constraint condition:

$$q^{\alpha\beta}=\frac{\partial}{\partial\xi_i^{\alpha\beta}}f_o[\xi]|_{\xi_i^{\alpha\beta}=\xi_o^{\alpha\beta}}=\langle S_i^\alpha S_i^\beta\rangle_o\,. \tag{26}$$

At this stage, we have taken the order parameter $q^{\alpha\beta}$ to be uniform, since the system is translationally invariant after averaging over disorder.

The equation of state is obtained from the saddle-point condition, giving

$$0=\frac{\partial g[q]}{\partial q^{\alpha\beta}}=\sum_{\gamma\delta}\,[\tilde{\partial}_{\gamma\delta}f_o-q_{\gamma\delta}]\partial_{\alpha\beta}\xi_o^{\gamma\delta}-\xi_o^{\alpha\beta}+\partial_{\alpha\beta}\,g_{int}\,, \tag{27}$$

where we have used the notation $\partial_{\alpha\beta}\equiv\partial/\partial q^{\alpha\beta}$, and $\tilde{\partial}_{\alpha\beta}\equiv\partial/\partial\xi^{\alpha\beta}$. Using the constraint condition Eq. (26), we can eliminate $\xi_o^{\alpha\beta}$ and write

$$q^{\alpha\beta}=\tilde{\partial}_{\alpha\beta}f_o[\partial\,g_{int}]. \tag{28}$$

The described expansion can be carried out to any desired order in ε, generating fluctuation corrections to the Gibbs potential. It is worth noting that, since this expansion is a series in powers of the interaction J_{ij}, it generates corrections due to *short-ranged* fluctuations, in contrast to the usual loop expansion that is dominated by the long-wavelength fluctuations. Accordingly, we expect that the prediction for the critical behavior of the order parameter and response functions to maintain the mean-field character if the expansion is truncated at any finite order.

B. Leading fluctuation corrections: the J^2 theory

The leading fluctuation corrections to the BW theory are obtained by retaining terms up to second order in ε, which is the lowest order to which we can identify a glass phase. Using the up-down symmetry of the Hamiltonian, we find (see Appendix A) that $\langle\phi\rangle_o=0$ and $\chi(\varepsilon=0)=0$, so that the above expressions simplify giving

$$g_1=0, \tag{29}$$

$$g_2=\frac{1}{N}\langle\phi^2\rangle_o=\frac{1}{2z}\left[n+2\sum_{\alpha<\beta}\,q^{\alpha\beta}q^{\alpha\beta}\right]. \tag{30}$$

This expression is valid to any order in ε, but using in the leading theory [terms to $O(\varepsilon^2)$], we find

$$\partial_{\alpha\beta}g_{int}=\frac{\varepsilon^2}{z}q^{\alpha\beta}, \tag{31}$$

giving

$$q^{\alpha\beta}=\tilde{\partial}_{\alpha\beta}f_o\left[\frac{1}{z}(\beta J)^2q\right]. \tag{32}$$

This equation of state is precisely the same as the $m=0$ limit of the expression [Eq. (9)] that we have obtained on the Bethe lattice. We have thus demonstrated that to leading nontrivial order beyond the BW theory, we find the same glassy phase for arbitrary lattices. To this order, which corresponds to introducing the Onsager reaction field correction, the results are independent of the lattice geometry, as also found in other problems.[20,21]

IV. $T=0$ GLASS PHASE: SELF-ORGANIZED CRITICALITY

The nature of the glass phase is in general much more complicated than the usual ordered phases, and is characterized by a large density of low-energy excitations and low-lying metastable states. Within the class of mean-field models for spin glasses, the structure of the ordered phase has been investigated most extensively in the vicinity of the glass transition, where the Parisi theory is analytically tractable. Much less is known about the low-temperature behavior, although some of the most interesting phenomena are most pronounced there, including the hysteresis and avalanche behavior.

A. Hysteresis and avalanches behavior in presence of random fields

To investigate the glass phase, we concentrate on the $T=0$ behavior of our model. To do this, we limit our attention to the J^2 theory described above, where the RFIM maps to the SK spin-glass model in presence of additional random magnetic fields. This mapping allows us to use different methods to investigate the $T=0$ behavior, which would be difficult to address using the Parisi theory. In particular, we investigate the hysteresis and avalanche behavior within our extended mean-field theory of the RFIM, by following recent work of Pazmandi, Zarand, and Zimanyi (PZZ).[22] In their original calculation, PZZ have examined the standard SK model, and the only modification that we introduce is the additional presence of random magnetic fields. Our procedure is as follows. One first introduces a large external magnetic field (in addition to fixed random fields), so that all the N spins are aligned with it. The external field is then slowly reduced and the stability of the spin configuration is examined with respect to any single spin flips. As soon as the system becomes unstable, the spin configuration is allowed to relax to a local energy minimum, causing an ''avalanche.'' The procedure is then repeated, resulting in the system following a hysteresis loop. If the external field is swept to large negative values (such that all the spins align in the negative direction), and then the procedure reversed, then the state of the system follows a ''major'' (external) hysteresis loop. If the field is instead reversed before the major loop is completed, then the system embarks on one of the minor hysteresis loops. This procedure is illustrated is Fig. 3 where a typical ''hysteresis spiral'' is presented.

B. Distribution of local fields

To characterize spin-glass state, we examine the probability distribution $P(H_i)$ of local magnetic fields $H_i=h_i+\Sigma_j J_{ij}S_j$ acting on a given spin S_i. In the high-temperature phase, $P(H_i)$ has a simple Gaussian distribution. To see this, we recall that above the glass transition, the replica symmetry remains valid, so that the presence of the interaction term

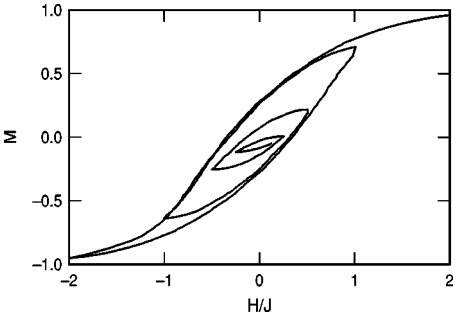

FIG. 3. A typical hysteresis spiral.

in the local effective action [see Eqs. (9) and (10); we use $m=0$ here] simply renormalizes the effective distribution of random fields, such that $H_{RF}^{eff}=\sqrt{(H_{RF})^2+J^2q/z}$. The distribution $P(H_i)$ therefore remains a simple Gaussian of width given by H_{RF}^{eff}. From Parisi theory,[20] we expect this distribution to acquire a nontrivial form upon replica symmetry breaking, but we would like obtain its specific form in the spin-glass state.

To calculate this quantity on the hysteresis loop, we follow a simulation procedure identical to that used for the SK model by Pazmandi, Zarand, and Zimanyi (PZZ).[22] In this procedure, one considers a finite-size system of N spins, with a given realization of random interactions J_{ij} and random fields h_i, and let the system explore the metastable states sampled on the hysteresys loop. The values of the local fields H_i are then computed for every spin, and the procedure is repeated for many realizations of disorder in order to generate a large ensemble from which the desired distribution can be computed. To implement this procedure, we have carried large-scale simulations using systems with up to $N=3200$ spins, and obtaining ensembles of $M=500\,000$ data points from which the distribution histograms were obtained. The resulting distributions for several values of the random-field strength are shown in Fig. 4. We find that the distribution is characterized by the emergence of a universal pseudogap of the form

$$P(H_i)\approx CH_i^{\alpha},(H_i\ll J), \tag{33}$$

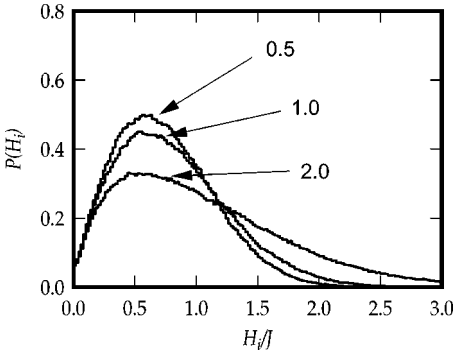

FIG. 4. Distribution of local fields for $N=3200$, as a function of random-field strength, for $H_{RF}\sqrt{z}/J=0.5$, 1.0, 2.0 (as indicated by arrows). Note the universal form of the pseudogap. The finite value of $P(0)\sim 1/\sqrt{N}$ is a finite-size effect (Ref. 22).

where $\alpha=1$ and $C=z/J^2$ is independent of the random-field strength. This universality is consistent with the findings of Pazmandi, Zarand, and Zimanyi,[22] who have shown that it reflects the self-organized criticality which characterizes the $T=0$ behavior of the mean-field glassy systems. Our results confirm that the conclusions of Pazmandi, Zarand, and Zimanyi remain valid in presence of random fields, thus representing a very robust property of glassy phases, at least within the confines of the considered mean-field descriptions.

By following these procedures, we have carefully verified that all the findings that Pazmandi, Zarand, and Zimanyi have established[22] for the SK model also hold in the presence of random fields, and thus also apply to the RFIM within the present mean-field formulation. In particular, we have confirmed that the avalanche sizes are distributed on all scales, and are characterized by a power-law distribution, characteristic of *self-organized criticality*. It is most remarkable that this critical nature is not confined to the ground state, but persists for all the metastable states within the hysteresis loop. It is particularly interesting, as we have explicitly verified by simulation, that not all local minima of the energy have this property. Instead, the critical states form a *subset* of metastable states that can be reached by the described hysteresis procedure. In this way, the ground state seems not to have any special features, but rather to share the same properties with an extensive number of critical metastable states. This notion offers a natural origin for the criticality that is found, since all the states along the hysteresis loop can be considered to be *on the brink of an avalanche*, and are therefore inherently unstable to weak perturbation, making the criticality possible. In fact, it is precisely the requirement for marginal stability of the hysteresis states that was used by Pazmandi, Zarand, and Zimanyi[22] to derive the universal form of the local-field distribution. This argument examines the modification of the local fields H_i upon flipping a given set of n_{flip} other spins. This is given by

$$\delta H_i=H_i'-H_i+2\sum_{jflipped}J_{ij}S_j\,. \tag{34}$$

One then computes the probability that the local field is reversed, so that instabilities are created, triggering avalanches. Note, however, that the variations δH_i is *independent* of the value of the (external) random fields h_i present in our case. As a result, the rest of the argument goes as in Ref. 22 giving the above marginality condition.

From the historical perspective, evidence of marginal stability of the spin-glass phase in mean-field models has long been appreciated based on Parisi and the Thouless-Anderson-Palmer (TAP) theory.[20] In addition, stability arguments requiring $\alpha\geqslant 1$ have been presented by Palmer and Pond,[24] based on early ideas of TAP.[20] However, it was not clear why this bound has to be satisfied, or that even the prefactor C assumes a universal value. In this sense, the hysteresis and avalanche study of Ref. 22 has provided an important conceptual advance, linking this universality with the self-organized criticality of the mean-field models.

C. Self-organized criticality and the AT line

We have seen that the glass transition temperature in our random-field case decreases very slowly, as $T_G \sim 1/H_{RF}$ at large random fields. This is very different than in the case of the familiar AT line[23] of the SK model in a uniform field, where the RSB temperature decreases *exponentially* with the uniform field strength. In the following we present a simple heuristic argument that explains the physical origin of both behaviors based on the self-organized criticality of the glassy ground states in these mean-field models.

What emerges from the analysis of Ref. 22 for the SK model, and which also applies in our extended mean-field formulation for the RFIM, is the phenomenon that at $T=0$ the distribution of local fields assumes a universal form at $H_i \to 0$, as given by Eq. (33). We have established this property for all the states within the hysteresis loop following the methods of Ref. 22. However, it is very likely that a similar universal distribution of local fields also characterizes the exact ground state of the system. In fact, this possibility has been proposed a long time ago both by Thouless, Anderson, and Palmer[20] and by Palmer and Pond,[24] consistent with the notion of the marginal stability of the spin-glass state for infinite-range models. Similarly to those of Ref. 22, the arguments of Palmer and Pond can also be extended to include the addition of random magnetic fields, resulting in a stability bound that remains universal, i.e., independent of the random-field strength. To establish more firmly this universal form of the pseudogap for the ground state, one would have to carry out more elaborate numerical simulations. Efficient optimization methods needed for such computations are available,[25] but this requires extensive efforts which are beyond the scope of this paper.

For our purposes, we will assume that the pseudogap retains a universal form for all $T=0$ critical states, including the hysteresis states as well as the ground state. If this is true, we can now make a simple estimate of the ''condensation energy'' gained by glassy ordering, which should scale with the gap size. Having in mind that the replica symmetric (high-temperature) distribution $P_{RS}(H_i)$ is a Gaussian of width given by H_{RF}^{eff}, and the fact that the pseudogap has a universal, linear form, we conclude that the gap size should scale as

$$E_{gap} \sim P_{RS}(0) \sim 1/H_{RF}, \tag{35}$$

at $H_{RF} \to \infty$. Here, we have used the fact that $q(T=0)=1$, so to leading order $H_{RF}^{eff} \approx H_{RF}$ for large random fields. Using this result, we can immediately estimate the glass transition temperature, which should also scale with this gap size, since E_{gap} is the only energy scale characterizing the ground state. We thus conclude that $T_g \sim 1/H_{RF}$, in agreement with our analytical calculations based on the RSB analysis. It is worth emphasizing here that the simple relationship between the gap energy and the random-field strength directly follows from the universality of the pseudogap form. Has the gap had another functional form (e.g., a randomness-dependent exponent α), this relationship would be modified, resulting in a different dependence of $T_g(H_{RF})$. In this way, the agreement between our analytical RSB results and the presented heuristic arguments provides additional evidence for the self-organized critical nature of the ground state in these models.

Finally, it is interesting to examine how the presented heuristic argument applies to the usual SK model in a uniform field. In this case, the only modification is introduced by the fact that in the high-temperature phase, the uniform field simply shifts the entire Gaussian distribution of local fields. As a result, $P_{RS}(H_i=0,H) \sim \exp\{-H^2/2J^2\}$, and we find that the AT temperature decreases exponentially with uniform field, in agreement with the analytical results[23] of de Almeida and Thouless.

D. Relevance for the Coulomb glass problem

The nontrivial nature of mean-field glass models may be particularly significant for the Coulomb glass problem, relevant for disordered insulators. The Hamiltonian for the Coulomb glass is given by

$$H = \sum_{ij} \frac{e^2}{r_{ij}} (n_i - \langle n \rangle)(n_j - \langle n_j \rangle) - \sum_i \varepsilon_i n_i . \tag{36}$$

Here, $n_i = 0,1$ are the occupation numbers, r_{ij} is the intersite distance, and ε_i are the random site energies. The transformation $S_i = 2n_i - 1$ immediately maps this Hamiltonian to an *antiferromagnetic* RFIM with long-range interactions. Note that the presence of long-range antiferromagnetic interactions leads to considerably stronger frustration than that in the ferromagnetic RFIM considered in this paper, which is found only for sufficiently strong randomness. These differences may be crucial in low dimensional systems, possibly leading to a low-temperature glassy phase for the Coulomb system even if a similar behavior is suppressed for the ferromagnetic RFIM case. However, on the mean-field level, even the standard RFIM displays such glassy ordering, the character of which may be closely related to that of the Coulomb glass.

The best established property of the Coulomb glass model is the existence of a ''Coulomb gap,'' as predicted by Efros and Shklovskii (ES).[3] According to them, the electronic system would be unstable, unless the single-particle density of states (which corresponds to our local-field distribution) has a pseudogap of the form

$$\rho(\varepsilon) = C(d) \varepsilon^{d-1}, \tag{37}$$

where $C(d)$ is a universal constant in d dimensions. More precisely, Efros and Shklovskii have examined the stability of the system with respect to one-electron excitations, showing that the form of Eq. (37) represents an *upper bound* for the density of states. If this bound would be saturated, then the pseudogap would assume a universal form, but no convincing arguments have been presented why this should happen. However, large-scale numerical studies[26] have obtained results similar to the ES predictions, failing to produce any evidence of the hard gap. Still, the reason for saturating the ES bound has remained a mystery.

Another aspect of the Coulomb glass that has not been properly clarified is the presence or absence of a finite-temperature glass transition,[27] and the nature of the glass

phase. In this respect, the main difficulty was the absence of an appropriate order parameter that would allow identifying the transition. Since the random site energies in this model play a role of random fields, the situation is identical as in the usual RFIM, and the usual EA order parameter cannot be used.[28] As we have seen, the transition at best can have a character of an AT line, which should be most easily identified in changes of the dynamics as the temperature is lowered. Some numerical evidence that such a dynamical transition may exist has been obtained by using the ''damage-spreading'' algorithms,[29] giving hints of ergodicity breaking below a certain temperature. In addition, several experiments[30] have reported history-dependent transport and other glassy features in disordered insulators.

Despite the successes of the ES theory, several basic questions remain unanswered. Most importantly, it is not clear whether the emergence of the universal ES gap is related to the possible low-temperature glassy state of the model. In this respect, the scenario that we have presented for the RFIM provides an interesting possibility. It is conceivable that, as in our mean-field theory, a glassy phase exists below a well-defined transition temperature, which corresponds to the emergence of a large number of metastable states. If this phase were characterized by self-organized criticality, then the associated marginal stability would naturally explain the saturation of the ES bound, and the resulting universality of the Coulomb gap. An an interesting way to address these questions would consist of examining the $T=0$ hysteresis properties of this model, following the approaches of Pazmandi, Zarand, and Zimanyi, but this direction will be pursued elsewhere.

V. HIGHER-ORDER CORRECTIONS AND THE 1/*z* EXPANSION

So far, we have examined the leading-order corrections to the BW theory, producing the glass phase. On general grounds, one does not expect that higher-order fluctuations corrections of a finite order in J would produce qualitative modifications in this mean-field context. Nevertheless, we shall examine the next-to-leading terms, in order to assert the convergence properties of such an expansion.

A. J^4 theory

In Sec. III, we have already calculated the terms up the to order J^2 in the expansion of the Gibbs free energy. Terms up to order J^4 can be computed using the same procedures, where we use the average up-down symmetry of the problem outside the ferromagnetic phase. In contrast to the J^2 theory, these additional terms depend on the specific form of the lattice. To be specific, we concentrate on the hypercubic lattice in d dimensions (so that $z=2d$). After lengthy algebra, the resulting contributions up to order J^4 take the form

$$g_3=0, \tag{38}$$

$$g_4=\frac{1}{N}[\langle\phi^4\rangle_o^c+3\langle\phi^2\chi'\rangle_o^c]. \tag{39}$$

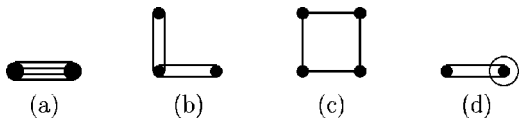

FIG. 5. Diagrams contributing to order J^4. The diagrams (a), (b), and (c) correspond to the $\langle\phi^4\rangle_o^c$ term, and (d) to the $\langle\phi^2\chi'\rangle_o^c$ term. Here, the ϕ bonds are represented by a full line, and the local χ' term is represented by an open circle.

Here, $\chi'=\partial\chi/\partial\varepsilon$, and the brackets $\langle\cdots\rangle_o^c$ indicates that only connected diagrams, which give nonvanishing contributions, should be retained. This refers to diagrams obtained by representing the quantity ϕ [see Eq. (24)] by a bond, since it stems from the interaction terms connecting two nearest neighbors on a lattice. While each power of ϕ involves a sum over all possible embeddings of such a bond, only diagrams consisting of a close loop of such bonds produce nonvanishing results, due to the average up-down symmetry of the problem. Four different classes of such diagrams are shown in Fig. 5, corresponding to the J^4 contributions that we consider. The evaluation of these terms is straigthforward. Here, we only emphasize the following properties of these diagrams, that are valid even to higher order in J, as follows. (i) Diagrams (a), (b), and (d) contain vertices with four emerging bonds, that in the replica calculation give rise to expressions which include moments of the form $r_{\alpha\beta\gamma\delta}=\langle S_i^\alpha S_i^\beta S_i^\gamma S_i^\delta\rangle_o$, with $\alpha\neq\beta\neq\gamma\neq\delta$. Such moments cannot be simply expressed in terms of the order parameter $q^{\alpha\beta}$. In contrast, the ''loop'' diagram (c) is expressed as a power series involving only powers of $q^{\alpha\beta}$. (ii) The lattice embedding factors, which specify the z dependence, do not depend on the specific replica decorations that have to be carried out on each diagram, but are determined only by the topology of the diagram. (iii) We find that diagrams (b) and (d) result in identical expressions, except for the lattice embedding prefactors, which are different. As a result, cancellations occur, such that the sum of (b) and (d) produces a contribution to the free energy which is of order J^4/z^3, as is the contribution of (a). (iv) The leading contribution, of order J^4/z^2, follows only from the loop diagram (c). (v) The diagrams (a), (b), and (d) consist of self-retracting paths, and as such are identical for both a hypercubic and the Bethe lattice with the same z. The ''open loop'' diagram (c) is specific to the hypercubic, but absent for the Bethe lattice.

In examining the J^4 (and higher-order) corrections, we are primarily interested in identifying a limit where these terms are ''small,'' so that only the leading J^2 contributions may be retained. On a Bethe lattice, all the contributions are of order $1/z^3$, and thus can be ignored in the limit of large coordination. It is easy to see that the same conclusion applies to arbitrary lattices with purely random interactions such that $\langle J_{ij}\rangle=0$, as found in spin-glass models,[21] since in this case only self-retracting paths survive. In contrast, for the RFIM on general lattices, where the interactions are uniform, the open loop diagrams (c) survive, and provide the leading contribution, which is of the same order as in the J^2 term. We expect that similar conclusions apply to higher-order contributions as well. This property, that the open loop diagrams provide a leading contribution in large dimensions, is well

known,[31] and has been extensively used to study models of strong electronic correlation in large dimensions.[32] To get the leading contributions for large z, one would therefore have to sum up such open loop contributions to all orders in J. In absence of randomness such re-summations have been carried out,[31,32] but at least for the electronic models, the results were not qualitatively different from those obtained on the Bethe lattice, where only the second-order terms may be retained.

In our case such re-summations are more difficult to carry out due to the presence of randomness (replica indices), and will not be attempted here. Instead, we will show that if one examines the *joint* limit of large coordination and strong random fields, even the J^4 terms represent subleading contributions, and the simple J^2 theory suffices. To show this, we only consider the leading loop contributions [Fig. 5(c)], which takes the form

$$g_4=\frac{3}{z^2}\Bigg[6\sum_{\alpha\neq\beta}(q^{\alpha\beta})^2+4\sum_{\alpha\neq\beta\neq\gamma}q^{\alpha\beta}q^{\beta\gamma}q^{\gamma\alpha}+\sum_{\alpha\neq\beta}(q^{\alpha\beta})^4 +2\sum_{\alpha\neq\beta\neq\gamma}(q^{\alpha\beta})^2(q^{\beta\gamma})^2+\sum_{\alpha\neq\beta\neq\gamma\neq\delta}q^{\alpha\beta}q^{\beta\gamma}q^{\gamma\delta}q^{\delta\alpha}\Bigg]. \tag{40}$$

B. 1/z corrections to the equation of state: RS solution

A general expression for the equation of state, valid to arbitrary order, is given by Eq. (28). To compute the relevant J^4 corrections in the replica symmetric case, we calculate the variation of g_{int}, which to this order reads

$$\partial_{\alpha\beta}\, g_{int}|_{RS}=\frac{\varepsilon^2}{z}q+3\left(\frac{\varepsilon^2}{z}\right)^2 q(1-q)^2, \tag{41}$$

where we have taken the $n\to 0$ limit.

We are especially interested in examining the form of the solution in the large H_{RF} limit, where the RSB transition occurs at low temperatures. We examine the relative magnitude of the terms appearing in g_{int}. At first sight, the J^4 terms seem to be dominant in the low-T limit, since it is of order $\varepsilon^4=(\beta J)^4$. However, note that this term is also proportional to $\delta q^2=(1-q)^2$, which is small at low temperatures, since $q\to 1$ at $T\to 0$. To see this, it suffices (to leading order) to compute $q(T)$ at $J=0$, giving

$$q^{J=0}=1-\frac{2}{\sqrt{\pi}}\frac{1}{\beta H_{RF}}\cdots. \tag{42}$$

This gives $\delta q\sim(\beta H_{RF})^{-1}$, so that $(\beta J)^4 q(1-q)^2 \sim(\beta J)^2(H_{RF}/J)^{-2}$. The J^4 term is down by a factor $(H_{RF}/J)^{-2}$, thus contributing only to *subleading* order in the limit of large random fields.

C. RSB stability analysis: A general approach

Within the J^2 theory (Secs. II and III), we have mapped the RFIM to an equivalent infinite-range model, making it possible to carry out the RSB analysis similarly as for the SK model. In the following, we present a general approach to the RSB stability analysis, which can be used even if higher-order terms are retained.

To identify the RSB instability we follow de Almeida and Thouless,[23] and examine the variations of the Gibbs free energy with respect to the deviations from replica symmetry $q^{\alpha\beta}=q+\delta q^{\alpha\beta}$. From the definition of the Gibbs free energy, Eq. (19), the corresponding stability matrix is

$$\partial_{\alpha\beta,\gamma\delta}\, g=-\partial_{\alpha\beta}\xi^{\gamma\delta}+\partial_{\alpha\beta,\gamma\delta}\, g_{int}. \tag{43}$$

Here, we have used the notation $\partial_{\alpha\beta,\gamma\delta}\equiv\partial^2/\partial q^{\alpha\beta}\partial q^{\gamma\delta}$. To eliminate the quantity $\partial_{\alpha\beta}\xi^{\gamma\delta}$, we take a variation of the constraint condition Eq. (26), and obtain

$$0=\delta_{\alpha\beta,\gamma\delta}-\sum_{\mu\nu}\tilde{\partial}_{\gamma\delta,\mu\nu}\, f_o[\xi]\partial_{\alpha\beta}\xi^{\mu\nu}, \tag{44}$$

where we have used $\tilde{\partial}_{\alpha\beta,\gamma\delta}\equiv\partial^2/\partial\xi^{\alpha\beta}\partial\xi^{\gamma\delta}$. To write these expressions in a more compact form, we introduce a matrix notation $(\hat{g}'')_{\alpha\beta,\gamma\delta}\equiv\partial_{\alpha\beta,\gamma\delta}\, g$; $(\hat{g}''_{int})_{\alpha\beta,\gamma\delta}\equiv\partial_{\alpha\beta,\gamma\delta}\, g_{int}$; $(\hat{\xi}')_{\alpha\beta,\gamma\delta}\equiv\partial_{\alpha\beta}\,\xi^{\gamma\delta}$; $(\hat{f}''_{int})_{\alpha\beta,\gamma\delta}\equiv\tilde{\partial}_{\alpha\beta,\gamma\delta}\, f_o[\xi]$, giving

$$\hat{g}''=-\hat{\xi}'+\hat{g}''_{int}, \tag{45}$$

$$0=\hat{I}-\hat{f}''\cdot\hat{\xi}'. \tag{46}$$

In this form, the matrix $\hat{\xi}'$ can be eliminated and we get

$$\hat{g}''=-(\hat{f}'')^{-1}+\hat{g}''_{int}. \tag{47}$$

To simplify the calculation further, we note that in the high-temperature phase all the eigenvalues of $\hat{g}''$ are positive, but the RSB instability is identified[23] when at least one of its eigenvalues vanishes, such that the *determinant* (at fixed n) vanishes. Using this property, an equivalent approach to the RSB stability analysis can be formulated by examining the stability of an auxiliary matrix,

$$\hat{g}''_1=\hat{f}''\cdot\hat{g}''. \tag{48}$$

This is true, since

$$\det(\hat{g}''_1)=\det(\hat{f}'')\det(\hat{g}''), \tag{49}$$

and it can be verified by explicit calculation that the matrix $\hat{f}''$ remains nonsingular in the temperature range of interest. In other words, to identify the RSB instability, we need to compute the eigenvalues of the auxiliary stability matrix

$$\hat{g}''_1=\hat{f}''\cdot\hat{g}''_{int}-\hat{I}. \tag{50}$$

We now present a general strategy for computing the eigenvalues of this matrix. Our first observation is that any replica matrix of the form considered has at most three different matrix elements, as discussed by de Almeida and Thouless.[23] In addition,[23] for any such matrix there can be found three different (degenerate) eigenvectors. Most remarkably, the *form* of these eigenvectors *does not* depend on the value of the corresponding matrix elements. Thus any replica matrix has the same eigenvectors, and only the cor-

responding eigenvalues depend on the value of the matrix elements. Since a product of two replica matrices is again a replica matrix, we conclude that the relevant eigenvectors in our case are identical to those computed by de Almeida and Thouless, and we only need to evaluate the relevant eigenvalues. Only one of the eigenvalues vanishes at the RSB transition, call it λ_3, and the corresponding eigenvector $\vec{x}_3$. The eigenvalue λ_3 in our case can be computed by acting on $\vec{x}_3$ with the matrix $\hat{g}_1''$, and we find

$$\lambda_3 = \lambda_3^f \cdot \lambda_3^{int} - 1, \tag{51}$$

where λ_3^f and λ_3^{int} are the respective eigenvalues of the matrices $\hat{f}''$ and $\hat{g}_{int}''$. We emphasize that this strategy is not specific to our J^4 theory, but is valid to arbitrary order in the expansion.

To obtain the desired J^4 corrections, we need to compute the corresponding corrections to the matrices $\hat{f}''$ and $\hat{g}_{int}''$, as done in Appendix B. For general H_{RF}, we find that the J^4 corrections are of the *same* order as the J^2 ones, leading to a glass transition temperature $T_G \sim 1/\sqrt{z}$. Clearly, all the higher-order terms coming from the ''loop'' diagrams are also of the same order, and would have to be included as well, in order to collect all the leading contributions in a $1/z$ expansion, similarly as in other problems in large dimensions.[32]

However, as shown in Appendix B the expressions simplify in the limit of large random fields, where all the J^4 contributions are down by a factor $(J/H_{RF})^2$, and to leading order expressions are obtained by simply retaining only the J^2 terms. We conclude that the J^2 theory, which provides the leading nontrivial order in our extended mean-field theory, represents an asymptotically exact formulation in the *joint* limit of large coordination and large random fields. These conclusions have been obtained by examining the example of a simple hypercubic lattice with nearest-neighbor interactions, in the limit of large coordination. Since the general structure of the diagrammatics and the relevant z dependence is qualitatively the same for general lattices, we expect these results to hold for any model with short-range interactions.

The situation is more complicated for models with long-range interactions, such as the Coulomb glass. In such cases, the diagrammatic expansion cannot be truncated to any finite order in the interaction, in order to avoid well-known divergences associated with the screening processes.[33] The simplest consistent treatment has to sum up all the ''chain'' diagrams, leading to the Debye-Huckel approximation,[33] which is also known as the random-phase approximation in the electronic context.[34] This class of diagrams is, in fact, equivalent to the class of loop diagrams in our expansion of the free energy, which provides the leading contributions in the limit of large coordination. Thus, to address the behavior of the Coulomb glass model, one should extend our calculation to sum up all the loop diagrams, which can straightforwardly be done even within our formulation. The resulting theory should be capable of addressing the interplay of screening effects and glassy freezing, a topic of great relevance for disordered electronic systems.

D. Weak random fields and self-generated glass

Another instance where terms not contained in the simplest J^2 theory may be important is the limit of weak randomness. On general grounds, one there expects the system at low temperature to assume some uniform order. In many cases the corresponding transition has a first-order character, so that upon rapid cooling the system may remain trapped in a meta-stable state and undergo glassy freezing. This process is believed to be even more likely in the presence of competing uniform interactions, which typically can depress the uniform ordering down to very low temperatures. A model for this behavior has been proposed a long time ago,[9] based on earlier work[35] that emphasized the relation between mode-coupling theories[36] of supercooled liquids and a special class of infinite-range spin-glass models displaying a first-order glass transition scenario. The possibility of glassy freezing in the absence of randomness has recently attracted renewed attention, and several studies[8,7] have concentrated on uniformly frustrated infinite-range models where these processes can be studied in detail. However, a more general approach would be even more useful, where one could examine the competition between uniform and glassy orderings in a controlled scheme, and which could be applied to models with realistic interactions and lattice geometries.

In principle, these questions can be addressed within our approach by examining the fate of the glassy phase in the limit of weak random fields, an idea that was introduced a long time ago.[9] In the simplest J^2 theory, and outside the ferromagnetic phase, our model maps to the SK model in the presence of random fields, leading to the glass transition line given in Fig. 1(b). As the random fields are reduced, the glassy phase is enhanced, but for sufficiently weak randomness, glassy freezing is pre-empted by uniform ferromagnetic ordering. On the other hand, if we restrict our attention to the nonmagnetic $(m=0)$ solution, then the glass transition line can be extended to $H_{RF}=0$, leading to $T_G(H_{RF}=0)=J/\sqrt{z}$. In this way, our formulation may be considered the simplest approach that can lead to glassy behavior in the absence of randomness. However, the prediction of this lowest-order approximation cannot be considered as reliable for weak random fields, since corrections to any order in J make contributions of the comparable magnitude, even in the limit of large coordination. In fact, using expressions that we have obtained within the J^2 theory, it is not difficult to compute the resulting correction to the glass transition temperature, which remains of order $1/\sqrt{z}$, but with an *increased* prefactor. The details will not be elaborated, since stopping at any finite order in J is clearly not sufficient. The enhancement of the glass phase in the limit of weak random fields due to these fluctuation corrections may indicate the possibility that, once all the leading corrections are retained, the glass transition would *precede* any uniform ordering as suggested by the SCS theory of Mezard and Young.[14] If this is correct, it would indicate that the convergence of the $1/z$ expansion is not uniform as a function of the random-field strength, since $T_G(H_{RF}\to\infty)\sim 1/\sqrt{z}$, but $T_G(H_{RF}\to 0)\sim O(1)$. In that case, the correct mean-field theory should not be formulated by performing a z-dependent rescaling of the interactions and

then letting $z \to \infty$. Instead, the formulation should retain all the leading $1/z$ corrections, in finite dimensions. It is interesting to note that recent work of Lopatin and Ioffe[7] is closely related to the approach that we propose, since it singles out all the leading $1/d$ corrections for a specific uniformly frustrated model, in the limit of large dimensions. In this particular model, the interactions are sufficiently frustrated, precluding any uniform ordering, and resulting in a particularly simple large coordination limit. In more general cases, competition between uniform and glassy ordering and the possibilities of having either first- or second-order transitions should be considered and may be determined by the details of the interactions or the presence of disorder.

VI. CONCLUSIONS

In this paper, we have presented a systematic approach that can incorporate short-range fluctuation corrections to the standard Bragg-Williams theory of the random-field Ising model. We have shown that if the correct order parameters are introduced, corrections to even the lowest nontrivial order immediately result in the appearance of a glassy phase for sufficiently strong randomness. This low-order treatment is shown to be sufficient for large randomness, where it provides the leading corrections in the limit of large coordination. The structure of the resulting glassy phase is very similar to that found in familiar infinite-range spin-glass models, and is characterized by universal behavior emerging from the self-organized criticality of the ground state.

The major puzzle that remains to be resolved is the extent to which the application of these mean-field ideas is relevant to the low-temperature behavior of low-dimensional systems with short-range interactions. An alternative approach, based on droplet arguments[37] presents a very different scenario, particularly in situations where external fields, either uniform or random, explicitly breaks the symmetry of the Hamiltonian. In this instance, droplet arguments would preclude the existence of any finite-temperature glass transition, in contrast to the mean-field predictions. In addition, recent numerical results[38] on $d=3$ RFIM have also been used to argue against the existence of a finite-temperature transition in a field. In this context, it is worth noting that self-organized criticality is *not* found in recent studies[6] of hysteresis and avalanche behavior of RFIM with short-range interactions[6] in low dimensions. For these models, although hysteresis behavior is present, the distribution of avalanche sizes is bounded, and criticality is found only by fine tuning the parameters of the system to a particular point of the phase diagram. Similar results have been obtained in studies that have examined the sensitivity of the $d=1,2$ RFIM to small random perturbations of the quenched disorder.[39]

In our opinion, more general emergence of self-organized criticality similar to that found in mean-field glassy models most likely requires the existence of longer-range spin-spin interactions and/or high spatial dimensions. On the other hand, experiments[40] measuring the Barkhausen noise on several ''hard magnets'' have indicated power-law distribution of avalanche sizes and avalanche times, consistent with self-organized criticality. Such behavior may be a result of the fact that in such systems the dominant interactions have a dipolar and thus longer-range character, bringing the behavior of these materials closer to the predictions of mean-field glassy models. These features may also be of particular importance in applications of the RFIM to disordered electronic systems and the related physics of the Coulomb glass behavior.

In this work, have also outlined how our theory could be extended to examine models with either longer-range interactions or the limit of weak random fields, which is of particular importance to the long-puzzling question of the self-generated glassiness in uniform systems. Our theory is closely related to other recent approaches[20,14,8,7] that address the emergence of glassy phases on a mean-field level. These theories taken as a whole appear to shed light on a number of experimentally relevant systems, and present a fairly complete and consistent picture of glassy behavior.

ACKNOWLEDGMENTS

We thank E. Dagotto, A. Moreo, T. R. Kirkpatrick, G. Kotliar, A. E. Ruckenstein, G. Zimanyi, and G. Zarand for useful discussions. This work was partially supported by NSF Grant No. DMR-9974311, the National High Magnetic Field Laboratory, and the Alfred P. Sloan Foundation (V.D.).

APPENDIX A: CANCELLATIONS DUE TO UP-DOWN SYMMETRY

In investigating the glassy phase of the RFIM outside the FM phase, and in zero uniform field, we can make use of the fact that after averaging, the system respects up-down symmetry. As a result, a number of terms vanish, so that the expressions simplify. In the following, we discuss these cancellations in some detail.

1. Moments of spins on different sites

The term $\langle \phi \rangle_o$ contains expectation values of a product of spins on different sites. Since the averages are computed at $\varepsilon = 0$, such terms very generally factor out, so that we can write

$$\langle S_{i_1}^{\alpha_1} \cdots S_{i_k}^{\alpha_k} \rangle_o = \langle S_{i_1}^{\alpha_1} \rangle_o \cdots \langle S_{i_k}^{\alpha_k} \rangle_o = m^k, \tag{A1}$$

where m is the magnetization. Outside the FM phase, and at $H=0$, the magnetization and thus all such moments vanish.

2. Evaluation of $\langle \phi \rangle_o$ and $\langle \chi \rangle_o$

An immediate consequence of the above factorization property is the fact that $\langle \phi \rangle_o = 0$, since this expression contains products of two spins on different sites. The expression for χ contains a derivative of the source field of the form $(\partial/\partial\varepsilon)\xi_i^{\alpha\beta}$. To compute this derivative, we use the fact that due to the definition of the Legendre transform, we can write

$$\xi_i^{\alpha\beta} = \frac{\partial}{\partial q_i^{\alpha\beta}} (\beta G). \tag{A2}$$

To calculate $\langle\chi\rangle_o$, we need to compute the derivative of $\xi_i^{\alpha\beta}(\varepsilon)$, evaluated at $\varepsilon=0$, and we get

$$\left.\frac{\partial\xi_i^{\alpha\beta}}{\partial\varepsilon}\right|_{\varepsilon=0}=\frac{\partial}{\partial q_i^{\alpha\beta}}\frac{\partial}{\partial\varepsilon}(\beta G)\bigg|_{\varepsilon=0}=-\frac{\langle\phi\rangle_o}{\partial q_i^{\alpha\beta}}=0. \quad \text{(A3)}$$

We therefore conclude that $\langle\chi\rangle_o=0$ as well.

3. Terms with odd powers of ϕ

In evaluating higher order terms in the ε expansion, terms of the form $\langle\phi^p\psi(\chi',\chi'')\rangle_o$ appear, where $\psi(\chi',\chi'')$ is an arbitrary polynomial function of χ' and χ'', and p is an odd number. To evaluate such terms, we note that as before, spin moments on different sites factor out, but we still have to compute nontrivial spin moments of the form $\langle S_i^{\alpha_1}\cdots S_i^{\alpha_r}\rangle_o$, with r having the same parity as p, i.e., r is odd. Here, we have used the fact that quantities $\chi'=(\partial/\partial\varepsilon)\chi$ and $\chi''=(\partial^2/\partial\varepsilon^2)\chi$ are *quadratic in local variables*, i.e., contain products of the form $S_i^\alpha S_i^\beta$. In addition, the considered moments will have an odd number of spins only if the considered lattices have no odd-membered rings, such as found for example on a triangular lattice. If any of the replica indices coincide, then $(S_i^\alpha)^2=1$, and an *even* number of spins drop out, but the remaining expression still has the form

$$M_s=\langle S_i^{\alpha_1}\cdots S_i^{\alpha_s}\rangle_o, \quad \alpha_1<\alpha_2<\cdots\alpha_s. \quad \text{(A4)}$$

Using well-known properties of replicas,[20] it is readily seen that

$$M_s=\overline{\langle S_i\rangle^s}=\overline{m^s}=0. \quad \text{(A5)}$$

Therefore the expression of the considered form vanish as well for lattices with no odd-membered rings.

APPENDIX B: VARIATIONS OF THE GIBBS FREE ENERGY

A general procedure needed to obtain the equation of state and the RSB stability criterion involves computing the variations of the Gibbs free energy with respect to the order parameter $q^{\alpha\beta}$. In the following we outline how these variations can be computed by concentrating only on the leading-order contributions from the loop diagram.

1. Calculation of $\partial_{\alpha\beta}g_{int}$

The calculation of $\partial_{\alpha\beta}g_{int}$ to order J^2 is already computed in Sec. III A. Here, we compute the J^4 correction. Defining the matrix $\hat{A}=\hat{I}+\hat{Q}$, we can write $g_4=(3/z^2)\mathrm{Tr}[\hat{A}^4]$, and find

$$\partial_{\alpha\beta}\,g_4=\sum_{\gamma\delta}\frac{\partial g_4}{\partial A_{\gamma\delta}}\frac{\partial A_{\gamma\delta}}{\partial q^{\alpha\beta}}=\frac{24}{z^2}(\hat{A}^3)_{\alpha\beta}. \quad \text{(B1)}$$

In the RS limit $q^{\alpha\beta}=q$ and taking $n\to0$ this reduces to the expression of Eq. (41).

2. Calculation of $\hat{g}''_{int}$

From Eq. (31), the J^2 contribution reads

$$\partial^2_{\alpha\beta,\gamma\delta}\,g_2=\frac{2}{z}\delta_{\alpha\beta,\gamma\delta}. \quad \text{(B2)}$$

The J^4 contribution can be calculated using the same procedure as for the first variation, and we find

$$\partial^2_{\alpha\beta,\gamma\delta}\,g_4=\frac{48}{z^2}\left[\delta_{\alpha\gamma}\sum_\mu A_{\beta\mu}A_{\mu\delta}+A_{\alpha\gamma}A_{\beta\delta}+\delta_{\beta\delta}\sum_\mu A_{\alpha\mu}A_{\mu\gamma}\right]. \quad \text{(B3)}$$

In the replica symmetric theory, the resulting matrix elements of $\hat{g}''_{int}$ are given by

$$\tilde{P}=(\hat{g}''_{int})_{\alpha\beta,\alpha\beta}=\frac{\varepsilon^2}{z}+\frac{1}{2}\left(\frac{\varepsilon^2}{z}\right)^2[2(1-q^2)+1],$$

$$\tilde{Q}=(\hat{g}''_{int})_{\alpha\beta,\alpha\delta}=\frac{1}{2}\left(\frac{\varepsilon^2}{z}\right)^2[2q(1-q)+q],$$

$$\tilde{R}=(\hat{g}''_{int})_{\alpha\beta,\gamma\delta}=\frac{1}{2}\left(\frac{\varepsilon^2}{z}\right)^2q^2. \quad \text{(B4)}$$

Given these matrix elements, one can immediately evaluate the relevant eigenvalue[23] of the matrix $\hat{g}''_{int}$, which takes the form

$$\lambda_3^{int}=\tilde{P}-2\tilde{Q}+\tilde{R}=\frac{\varepsilon^2}{z}+\frac{3}{2}\left(\frac{\varepsilon^2}{z}\right)^2(1-q)^2. \quad \text{(B5)}$$

In this expression, we note that similarly as in the computation of the J^2 correction to the RS equation of state, the J^4 correction is proportional to $\delta q=1-q$, and is therefore down by a factor $(J/H_{RF})^2$ compared to the leading term. We again conclude that in the limit of large random fields, to leading order it suffices to retain the J^2 contribution.

3. Calculation of $\hat{f}''$

The functional $f_o[\xi]$ is the free energy of free spins in presence of fields $\xi^{\alpha\beta}$, and we are interested in computing its second variation at the saddle point where $\xi^{\alpha\beta}=\partial_{\alpha\beta}g_{int}$. Therefore this evaluation is almost identical as for the SK model.[20] In addition, since this quantity is evaluated in the RS theory, $\partial_{\alpha\beta}g_{int}$ assumes the form that we have already discussed when we examined the RS equation of state. We conclude that for this quantity as well, to leading order in the limit of large random fields, the argument of $\hat{f}''$ can be replaced by its J^2 approximation, and we straightforwardly obtain Eq. (11).

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