Theory of the quantum Hall Smectic Phase. I. Low-energy properties of the quantum Hall smectic fixed point

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We develop an effective low-energy theory of the quantum Hall (QH) smectic or stripe phase of a two-dimensional electron gas in a large magnetic field in terms of its Goldstone modes and of the charge fluctuations on each stripe. This liquid-crystal phase corresponds to a fixed point that is explicitly demonstrated to be stable against quantum fluctuations at long wavelengths. This fixed-point theory also allows an unambiguous reconstruction of the electron operator. We find that quantum fluctuations are so severe that the electron Green function decays faster than any power law, although slower than exponentially, and that consequently there is a deep pseudo-gap in the quasiparticle spectrum. We discuss, but do not resolve, the stability of the quantum Hall smectic to crystallization. Finally, the role of Coulomb interactions and the low-temperature thermodynamics of the QH smectic state are analyzed.

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Recent experiments on extremely high mobility two-dimensional electron gases (2DEG's) in large magnetic fields by Lilly and co-workers[1] and by Du and co-workers,[2] have revealed an unusually large and strongly temperature-dependent anisotropy in the transport properties. These, and subsequent experiments,[3] have made it clear that this anisotropy is an intrinsic property of a new, anisotropic metallic phase of the 2DEG. The anisotropic metal, rather than the fractional quantum Hall effect, apparently dominates the physics of all partially filled Landau levels (LL's) with LL index $N \geqslant 2$.

Motivated by these experiments and theoretical work on Hartree-Fock states with stripe order,[4–7] and exploiting an analogy with the stripe related phases of other strongly correlated electron systems,[8] two of us proposed[9] that the ground states of quantum Hall systems with partially filled Landau levels with $N \geqslant 2$ are predominantly electronic liquid crystalline. These phases, with broken rotation and sometimes translation symmetry, are intermediate between the isotropic quantum fluid and the quasiclassical electronic crystal. In particular, we argued for the existence of quantum Hall smectic, nematic, and hexatic phases.

The simplest liquid-crystalline phase to visualize is the smectic or stripe ordered phase, which breaks translation symmetry in one direction, but is nonetheless a fluid in the sense that there is no gap to current carrying states and there is a noninteger number of electrons per magnetic unit cell. As mentioned above, early Hartree-Fock calculations suggested that the ground state of the 2DEG in large magnetic fields is a unidirectional charge density wave (CDW).[4] These results are manifestly incorrect in the lowest Landau level, where the fractional quantum Hall effect occurs, and the ground state is, to good approximation, the Laughlin state.[10] But, for a high enough Landau level index N, a stripe phase can be shown to be a reasonable ground state of the 2DEG for filling factors close to half-filling of the partially filled Landau level.[5–7] Here, the stripes are the result of a competition between *effective attractive short-range forces* and the familiar (long range) Coulomb interactions. We will see below that these mean field pictures of the stripe state are a good starting point for a description of a quantum Hall smectic. However, at Hartree-Fock level, the smectic phase can easily be seen[9,11,12] to be unstable to crystallization, that is to the occurrence of a charge-density wave along the stripe direction which breaks translational symmetry and produces an insulating state which can be thought of as an anisotropic Wigner crystal.

In the present paper, we will develop a theory of the unpinned quantum Hall smectic phase, and investigate its low energy properties. Based on our earlier work,[9,13] it is our belief that it is the quantum Hall nematic state, not the smectic, that is the most experimentally important phase. However, these are new phases of matter, and their precise characterization is important in its own right, and for possible relevance to future experiments. Moreover, a mean-field theory of the quantum Hall nematic, to serve as a starting point for a similar analysis, has not yet been developed although important progress in this direction has been made.[14] Serious progress has also been made towards understanding the nematic Fermi fluid in zero external magnetic field.[15]

The central result of this paper is an effective low-energy theory for the QH smectic which can be expressed in terms of two canonically conjugate sets of degrees of freedom: the displacement fields of the stripes, i.e., the Goldstone mode of the spontaneously broken translational symmetry, and the chiral edge modes of each stripe. In a separate publication[16] two of us we will give a microscopic derivation of this effective low-energy theory.

The quantum smectic is also interesting from a broader conceptual point of view, in that the Goldstone modes are so soft that coupling to them destroys all coherent single-electron motion, even at temperature $T=0$. Indeed, we find that the electron Green function $G(x,y,t)$ is highly anisotropic. It falls off more slowly than an exponential, but more

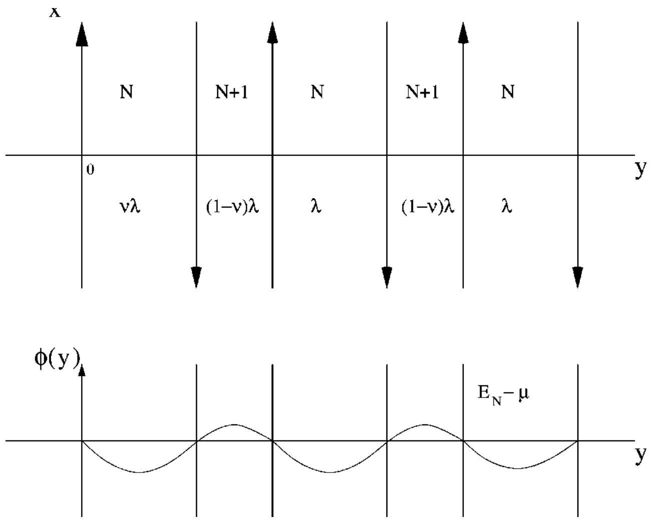

FIG. 1. Schematic representation of the stripe solution. Top panel: the chiral edge states of each stripe; here N and $N+1$ label the number of filled Landau levels in each region, λ is the period of the stripe (wavelength), and ν is the effective filling factor of the partially filled Landau level. Bottom panel: effective potential $\phi(y)$ for the stripe solution; y is the coordinate perpendicular to the stripe, μ is the chemical potential, and E_N is the energy of Landau level N.

rapidly than any power law as a function of time t or distance along a stripe x; it falls at least exponentially as a function of displacement perpendicular to the stripes y. For instance, for zero spatial separation $G(0,0,t) \sim \exp[-A\ln^2(t/t_0)]$, which implies a strong pseudogap in the local density of states. Despite this, for a system with short range interactions, the specific heat at low T, due to the soft Goldstone modes, is $C_V \sim T\ln(T_0/T)$—there are even more low energy modes than in a Fermi liquid. For the case of Coulomb interactions we find instead a T linear law.

A smectic state (quantum or classical) is a state with spontaneously broken continuous symmetries.[17,18] In the zeroth order description, given by the Hartree-Fock theory, the sample is spontaneously divided into strips with filling factors that alternate between two successive integers N and $N+1$, as shown in Fig. 1. At this level, the high density regions are separated from the low density regions by (straight) edge states with alternating chirality. As the filling factor of the partially filled level changes the width and period of these strips changes, which implies that this state is compressible. As a consequence, the Hall conductivity varies linearly as the filling factor varies across the partially filled Landau level.

It is tempting to think of the smectic as an array of chiral, one-dimensional wires (internal edge states), interacting by some (possibly complicated) effective interaction induced by the fact that the edges are self-consistently generated, and hence fluctuating. This approach was advocated in some of our recent papers.[9,19] It was also advocated in an insightful paper by MacDonald and Fisher[12] who proposed an effective theory for a system of coupled chiral Luttinger liquids compatible with rotational invariance, as required for a true smectic state. In fact, the theory of MacDonald and Fisher is essentially equivalent to the theory for the quantum hall smectic that we introduce and discuss in this paper.

However, we find that the picture of the quantum Hall smectic as an array of coupled Luttinger liquids in fact can obscure the underlying physics of this state. This picture suggests that some sort of quasi-one-dimensional low-energy theory can explain the physics of this state and in particular that it can be used to analyze its stability. Thus, on the basis of this analysis, it was argued in Refs. 9 and 12 that even beyond Hartree-Fock theory the smectic phase would always unstable to crystallization. While this approach is reasonable in the case in which the internal edges are pinned by an external, periodic potential, we will see, below, that this stability analysis based on a $(1+1)$-dimensional scaling may not be safe in the absence of explicit symmetry breaking. The physical reason is that the Goldstone modes of the smectic are so soft that they dominate the low-energy physics, and the resulting fixed point is a strongly anisotropic $(2+1)$-dimensional system.

In fact, recently, Fertig and co-workers,[11,20,21] have carried out numerical time-dependent Hartree-Fock calculations, and showed that their numerical results can be described qualitatively by a quantum smectic with essentially the same properties that we find here. In particular, they find that the unpinned quantum Hall smectic state can be stable. In addition, Côté and Fertig have analyzed their time-dependent Hartree-Fock results in terms of an effective elastic theory similar to ours. Recently, Fogler and Vinokur studied the classical hydrodynamics at finite temperature of the Hall smectic.[22] These conflicting results, and our earlier arguments[8] of a first-order transition for the smectic-crystal transition in the absence of pinning and of a second-order transition[19] in the presence of pinning, show that this subject is still open. We will not resolve the issue of the stability of the QH smectic against crystallization in this paper, as it turns out to be very subtle, but we will discuss the present status of this problem.

While this paper was being completed we became aware of the recent work by Lopatnikova and co-workers[23] who have derived an effective low-energy theory for the quantum Hall smectic which is essentially equivalent to the one we present in this paper. Although, for the most part, the results of the present paper, as well as those of our microscopic theory which will be discussed elsewhere,[16] are in agreement with the results of Ref. 23, there are differences in the analysis of the consequences of the effective low-energy theory.

This paper is organized as follows. In Sec. I, we present a simple, phenomenological derivation of the quantum Hall smectic fixed point Hamiltonian. In Sec. II we use this effective field theory to compute the correlation functions of the quantum Hall smectic, including the electron propagator. Since the fixed-point Hamiltonian is formally scale invariant, one generally expects these correlation functions to be power laws, with exponents determined by the scaling dimension of the fields; this expectation is met, in large measure, but there are various logarithms that appear in certain limits which violate scaling. This is the origin of the subtleties in the stability analysis. In Sec. III we repeat the previous calculations in the presence of unscreened Coulomb interactions. In

Sec. IV we discuss the low-temperature thermodynamic properties of the QH smectic and in particular we compute the specific heat. In Sec. V we discuss the issue of the stability of the quantum Hall smectic towards crystallization. Finally, in Sec. VI we present our conclusions.

I. EFFECTIVE FIELD THEORY

Consider a stripe crystal in a large magnetic field. In the classical ground state, there is a charge density wave with fixed wavelength running along each stripe and the stripes (labeled by an index j) are spaced by distance λ and are straight and parallel; without loss of generality they can be taken to run in the $\hat{x}$ direction. Smooth deformations of this state can be described in terms of the local displacement of the charge-density wave (CDW) along the stripe direction $\phi_j(x)$ and the transverse displacement of the stripe from its classic ground state position $u_j(x)$. In terms of these variables, the action describing the dynamics of this system is

$$S=\lambda\sum_j \int dt dx\{\mathcal{L}_{\rm sm}+\mathcal{L}_{\rm lock}+\mathcal{L}_{\rm irr}\}, \tag{1.1}$$

$$\mathcal{L}_{\rm sm}=\frac{eB}{\lambda}u_j\partial_t\phi_j-\frac{\kappa_\parallel}{2}(\partial_x\phi_j)^2 -\frac{\kappa_\perp}{2}\left(\frac{u_j-u_{j+1}}{\lambda}\right)^2-\frac{Q}{2}(\partial_x^2u_j)^2, \tag{1.2}$$

$$\mathcal{L}_{\rm lock}=V\cos[\alpha(\phi_j-\phi_{j+1})+\beta\partial_x u], \tag{1.3}$$

$$\mathcal{L}_{\rm irr}=\frac{1}{2}[M_1(\dot{u}_j)^2+M_2(\dot{\phi}_j)^2]+\gamma(\partial_x\phi_j)(\partial_x u_j)^2+\cdots,$$

where we have used that the current on the jth stripe is $\partial_t\phi_j$. The first term, $\vec{j}\cdot\vec{A}$ in the gauge $\vec{A}=By\hat{x}$, gives rise to the Lorentz force law. κ_a are the various elastic constants and Q is the bending stiffness of a stripe.

The term which tends to lock the relative phases of the CDW's on neighboring stripes $\mathcal{L}_{\rm lock}$ is the principal term which distinguishes the crystal from the smectic—it vanishes in a smectic phase. α^{-1} is the wavelength of the CDW in appropriate units and $\beta=\lambda\alpha$ is necessary to insure that the system is rotationally invariant.[24] This term has been omitted in the published literature on this problem, but its necessity can be seen readily. Consider a rotated version of the classical stripe crystal ground state: $u_j=x\sin(\theta)+j\lambda[1-\cos(\theta)]/\cos(\theta)$ and $\phi_j=-j\lambda\sin(\theta)$. Since this is also a classical ground state, it must be that $S=0$. The effective action we have considered, with the stated value of β, is invariant under this transformation for θ sufficiently small that we can ignore the nonlinear terms, i.e., $[1-\cos(\theta)]/\cos(\theta)\approx 0$. To insure rotational invariance under large rotations, we would have to include additional nonlinear terms in the fields, as is well known[18] in the classical liquid-crystal literature; however, these terms do not affect any results of the present paper. Côté and Fertig[20] have explicitly shown, in the Hartree-Fock approximation, that V is very small, and indeed, we will ignore it for now, and then assess its perturbative relevance, below.

The final term $\mathcal{L}_{\rm irr}$ consists of all remaining, higher order terms which, as we will see, are irrelevant at the quantum Hall smectic fixed point. We have explicitly exhibited three of the most interesting of these—inertial terms, which can be neglected at low energies and large magnetic fields, and the leading geometric coupling between the stripe geometry and the density wave order along the stripe, which plays a key role[8,19] in determining the stability of the smectic phase in the absence of a magnetic field.

If we take the continuum limit of this action, then the leading order terms in $\mathcal{L}_{\rm sm}$ have the interpretation as the fixed point action for the quantum Hall smectic. This limit is obtained by replacing $\lambda\Sigma_j\to\int dy$, $u_j(x)\to u(x,y)$, $\phi_j(x)\to\phi(x,y)$, and $\mathcal{L}_{\rm sm}\to\mathcal{L}^*_{\rm sm}$:

$$\mathcal{L}^*_{\rm sm}=\frac{eB}{\lambda}u\partial_t\phi-\frac{\kappa_\parallel}{2}(\partial_x\phi)^2-\frac{\kappa_\perp}{2}(\partial_y u)^2-\frac{Q}{2}(\partial_x^2 u)^2. \tag{1.4}$$

In a separate publication[16] we give a detailed microscopic derivation of this effective action.

This action is a scale invariant fixed point action with respect to the anisotropic transformation

$$x\to rx,\quad y\to r^2y,\quad t\to r^3t,$$
$$u\to r^{-1}u,\quad \theta\to\theta,\quad \phi\to r^{-2}\phi, \tag{1.5}$$

where θ is the dual field defined below, in Eq. (1.14). Thus, the effective space-time dimension is 6. All the operators included in the fixed point Lagrangian $\mathcal{L}^*_{\rm sm}$ are marginal since they have scaling dimension 6.

If we then perform a standard scaling analysis, all operators with dimension larger than 6 are irrelevant while operators with dimension smaller than 6 are relevant. It would then be straightforward to assess the perturbative relevance of $S_{\rm lock}$; in the continuum limit, we can approximate $1-\cos[\alpha(\phi_j-\phi_{j+1})+\beta\partial_x u]=(\lambda^2\alpha^2/2)(\partial_y\phi+\partial_x u)^2+\cdots$, from which we deduce that this term is the energy associated with a shear deformation. By power counting, at the smectic fixed point this operator is a combination of operators with dimensions 4, 6, and 8, respectively. The operator $(\partial_x u)^2$ is *relevant*, according to this scaling analysis, suggesting that the smectic is unstable to formation of an unpinned crystalline state. The higher order terms can be easily seen to have higher dimension. Similar analysis leads to the conclusion that the explicit terms in $\mathcal{L}_{\rm irr}$ have dimensions 8 (M_1), 10 (M_2), and 7 (γ), respectively.

It is easy to see that external periodic potentials (e.g., lattice pinning) are relevant: an operator of the form $\cos(2\pi u/\lambda)$ has scaling dimension 2 and is strongly relevant, while operators of the form $\cos(\beta\phi)$ (where β is a constant) have dimension 4 and are also relevant. Finally, an explicit term of the form $(\partial_x u)^2$ (as opposed to the one generated by expanding $\mathcal{L}_{\rm lock}$) has scaling dimension 4 and is also relevant. Such terms break rotational invariance explicitly and can be generated for instance by an in-plane magnetic field (or by any other perturbation that generates an anisotropic effective mass for the electrons).

The Lagrangian as written is even under $u_j \rightarrow -u_j$ and $B \rightarrow -B$ (related to particle-hole symmetry); there is another operator[25] that should appear in $\mathcal{L}_{\rm sm}$ that we have not included, namely, $(u_{j+1}-u_j)\partial_x \phi \rightarrow \lambda(\partial_y u)(\partial_x \phi)$. However, its effect is equivalent to a renormalization of the elastic constants. Although by power counting it is marginal, we treat it as a redundant operator and will ignore it.

Another issue, raised originally by MacDonald and Fisher, is whether there are two distinct sets of low-energy degrees of freedom or not. Clearly, since the dynamical term in the action linearly couples u and ϕ, they are mixed, so there is only one low energy mode, not two. Formally, this means that we can integrate out one set of degrees of freedom, leaving an action with a quadratic kinetic energy in terms of the other. For example, if we Fourier transform the effective action, and then integrate out the shape modes, we are left with

$$S^*_{\rm sm} = \frac{1}{2} \sum_{\vec{k},\omega} \frac{\kappa_\parallel k_x^2}{\epsilon^2(\vec{k})} [\omega^2 - \epsilon^2(\vec{k})] |\phi_{\vec{k},\omega}|^2, \tag{1.6}$$

where

$$\epsilon^2(\vec{k}) = \kappa_\parallel \left(\frac{\lambda}{eB} \right)^2 k_x^2 [Q k_x^4 + \kappa_\perp k_y^2] \tag{1.7}$$

which is the dispersion relation of the Goldstone modes. Notice that it has a line of values of $\vec{k} = (0, k_y)$ with zero energy.

However, the price we have to pay for integrating out the Goldstone modes u, is a singular dependence of the action on ω and $\vec{k}$, reflecting the presence of highly nonlocal interactions. It is a matter of convenience, then, whether we treat the problem in terms of a nonlocal action with a minimal number of degrees of freedom, or a local action with twice as many degrees of freedom; we feel that the latter representation makes the basic physics clearer. In any case, the effective action we obtain here is equivalent to that of MacDonald and Fisher.[12] In summary, we conclude that the displacement field u, representing the fluctuations of the shape of the stripe, and the Luttinger filed ϕ, representing the charge fluctuations on each stripe, are canonically conjugate variables and thus are not independent degrees of freedom, in agreement with the arguments of MacDonald and Fisher.[12] This discussion corrects some of the arguments given earlier by two of us in Ref. 9.

Another way to look at the effective action is as a phase-space path integral—in this case, ϕ_j and

$$\Pi_j \equiv -eBu_j \tag{1.8}$$

are interpreted as a field and its conjugate momentum, $[\phi_j(x,t), \Pi_k(x',t)] = i\delta_{jk}\delta(x-x')$. (This reflects the well known Landau level physics, in which the different components of the position operator become canonically conjugate variables, and so fail to commute.) We can thus express the same physics in terms of a (discretized) Hamiltonian density operator $\mathcal{H} = \Sigma_j \mathcal{H}_j$, where

$$\mathcal{H}_j = \frac{\lambda}{2(eB)^2} \left[Q(\partial_x^2 \hat{\Pi}_j)^2 + \frac{\kappa_\perp}{\lambda^2} (\hat{\Pi}_j - \hat{\Pi}_{j+1})^2 \right] + \frac{\lambda \kappa_\parallel}{2} (\partial_x \hat{\phi}_j)^2. \tag{1.9}$$

This formulation is convenient for studying the effect of single particle (fermionic) excitations in the smectic state.

Standard methods[26] of bosonization for the 1DEG permit us to reconstruct the electron operators for the jth stripe from the collective bosonic fields. The operators that create an electron on the jth stripe are related to the left and right moving fields, $\psi_{\pm,j}(x,t)$ in the usual manner

$$\Psi_j(x,t) = e^{ik_F^j x} \psi_{+,j}(x,t) + e^{-ik_F^j x} \psi_{-,j}(x,t). \tag{1.10}$$

The right and left moving Fermi fields have the bosonized form, in terms of the field ϕ_j on each stripe

$$\psi_{\pm,j}(x,t) = \frac{\mathcal{U}_j}{\sqrt{2\pi a}} e^{i\sqrt{\pi}[\theta_j(x,t) \pm \phi_j(x,t)]}, \tag{1.11}$$

where $a = l$ is the short distance cutoff. Here the dual field θ_j is defined by

$$\theta_j(x,t) = \int_{-\infty}^{x} dx' \, \Pi_j(x',t) \tag{1.12}$$

and the operators U_j are the Klein factors

$$U_j = \prod_{i<j} e^{i\sqrt{\pi}\int_{-\infty}^{\infty} dx \partial_x \phi_j(x,t)} \tag{1.13}$$

which satisfy the relations $U_j^\dagger \theta_k(x) U_j = \theta_k(x) + i\sqrt{\pi}\delta_{jk}$ and $[U_i, U_j] = [U_i, \phi_j] = 0$.

It is simple to check that $\Psi_j(x,t)$ constructed in this way satisfies canonical anticommutation relations for a fermionic field, and adds a charge e to the system. Because the bosonized Hamiltonian is quadratic, it is also straightforward (although a bit complicated) to compute the electron Green function; this is done in Sec. II. Remarkably, we find that the fluctuations are so severe that the fermion Green function falls off as a function of imaginary time as $G(t) \sim \exp[-A \ln^2(t)]$, which is faster than any power law, although slower than exponential. As exponential fall-off implies a gap in the spectrum, this behavior implies a strong pseudogap with a characteristic energy scale. This is *not* the behavior one observes in a set of coupled one-dimensional Luttinger liquids, i.e., in a pinned smectic. This is discussed further in Sec. II.

It is useful to find an effective Lagrangian for the dual field θ. This can be done straightforwardly by noting that the dual field θ and the displacement field u are related by

$$\partial_x \theta = -\frac{eB}{\lambda} u. \tag{1.14}$$

Thus, upon integrating out the ϕ fields in Eq. (1.4), we find that the dynamics of the dual field θ is governed by the local effective Lagrangian

$$\mathcal{L}[\theta]=\frac{1}{2\kappa_{\|}}(\partial_t\theta)^2-\frac{\kappa_{\perp}\lambda^2}{2e^2B^2}(\partial_x\partial_y\theta)^2-\frac{Q\lambda^2}{2e^2B^2}(\partial_x^3\theta)^2. \tag{1.15}$$

Since θ is dimensionless, all three terms in this effective Lagrangian correctly have scaling dimension 6. In Fourier space the Lagrangian takes the form

$$S[\theta]=\int\frac{d^3k}{(2\pi)^3}\frac{1}{2\kappa_{\|}}[\omega^2-\epsilon^2(\vec{k})]|\theta(\vec{k},\omega)|^2. \tag{1.16}$$

This result shows that it is the dual field θ that can most naturally be related with the collective modes of the quantum Hall smectic phase. We conclude this section with a few observations concerning the properties of the smectic fixed point.

Symmetries: There is a peculiar ''semilocal'' sliding symmetry of the effective field theory. Because the Hamiltonian has no terms which depend on the variation of ϕ in the y direction, $\mathcal{L}_{\rm sm}$ is invariant under the transformation

$$\phi(x,y,t)\rightarrow\phi(x,y,t)+f(y), \tag{1.17}$$

where $f(y)$ is an arbitrary function of the (continuum) stripe index y. This symmetry corresponds to the independent translations or shifts along each stripe.[27] Superficially this symmetry is similar to a local gauge symmetry. However, since it is not truly local, as the allowed transformations depend only on the y coordinate, this symmetry can be completely broken by a suitable choice of boundary conditions, e.g., open boundary conditions (in contrast to a genuine gauge invariance which requires gauge fixing). There is an analogous sliding symmetry for the θ fields. Nevertheless, this semilocal sliding symmetry has profound consequences on the behavior of both the ϕ and θ correlation functions which are infrared divergent for $y\neq 0$. We shall also see that there are a number of striking additional features of the correlation functions which stem from this symmetry. (The locking term $\mathcal{L}_{\rm lock}$, if relevant, breaks this symmetry.) Although the smectic state spontaneously breaks translational symmetry in the y direction, the underlying symmetry of space is reflected in the invariance of S under

$$u(x,y,t)\rightarrow u(x,y,t)+a. \tag{1.18}$$

Similarly, the underlying rotational symmetry is reflected in the invariance of S under

$$u\rightarrow u+\theta x,$$
$$\phi\rightarrow\phi-\theta y. \tag{1.19}$$

Finally, absent the redundant term $(\partial_x\phi)(\partial_y u)$, the action is invariant under the particle-hole transformation, $u\rightarrow -u$ and $B\rightarrow -B$.

Coulomb interactions: In the present discussion, we have assumed all interactions are short-ranged. (See Sec. II for the effects of Coulomb interactions.)

Breakdown of the scaling assumption: Finally we note that the scaling analysis implied by Eq. (1.5) assumes that the correlation functions of the fields θ, ϕ and u obey simple albeit anisotropic power laws. We will show in the next section that this is (almost) true for the correlation functions of the displacement fields u, but that the Luttinger field ϕ and the dual field θ develop logarithmic singularities along the stripe direction. In fact the correlation functions are not only highly anisotropic but have also a complex singularity structure, determined by both rotational invariance and by the sliding symmetry. In Sec. V we will argue what this structure, and in particular the existence of a divergent number of low-energy degrees of freedom, raises questions concerning the validity of naive scaling, and calls for a more careful renormalization group analysis than we have attempted in this paper.

II. CORRELATION FUNCTIONS OF THE QUANTUM HALL SMECTIC

In this section, we use the effective field theory to compute the correlation functions of the displacement field u (which is the Goldstone boson of the smectic) and of the Luttinger fields ϕ and θ, which directly represent the charge fluctuations on each stripe. We will see that the correlation functions can be written in a scaling form which explicitly displays the scaling laws dictated by the fixed point.

We define the imaginary time correlators

$$C_u(x,y,t)\equiv\langle u(x,y,t)u(0,0,0)\rangle \tag{2.1}$$

(and analogously for ϕ and θ), and we will denote the subtracted correlators by

$$\tilde{C}_\phi(x,y,t)\equiv-\frac{1}{2}\langle[\phi(x,y,t)-\phi(0,0,0)]^2\rangle. \tag{2.2}$$

We will find below that due to the smectic symmetry the unsubtracted propagator of operators that are not invariant under the shift symmetry of Eq. (1.17) are infrared divergent in the limit $x\rightarrow 0$ and $t\rightarrow 0$ with y fixed. While this fact is true in any theory with gapless excitations, these infrared divergences are particularly important due to the feature of the dispersion relation $\epsilon(\vec{k})$ which vanishes at $k_x=0$ *for all* k_y, Thus, the propagators of the Luttinger field ϕ and of the dual field θ need to be subtracted. In contrast, the propagator of the Goldstone modes u needs no subtraction in the thermodynamic limit since the u fields are invariant under shifts. Among other things, this implies that the quantum fluctuations of u are bounded, and so do not necessarily destroy the smectic order. However, the semi-local symmetry of the smectic results in expressions for C_ϕ and C_θ which diverge in the thermodynamic limit; only the subtracted version of these correlators, which are invariant under this symmetry, are well defined. Notice, however, that the subtraction of Eq. (2.2) only removes the uniform shift and not the semilocal shifts. Thus, even the subtracted propagators for ϕ and θ will become infrared divergent in the limit $x\rightarrow 0$, $t\rightarrow 0$, with y fixed. We will deal explicitly with this issue below.

A. The dual field correlation function

We will begin by calculating the subtracted propagator of the dual field θ in imaginary time

$$\tilde{C}_{\theta}(x,y,t)=\int \frac{d^2k}{(2\pi)^2}\frac{\kappa_{\parallel}}{2\epsilon(\vec{k})}[e^{-|t|\epsilon(\vec{k})+i\vec{k}\cdot\vec{x}}-1]. \quad (2.3)$$

In principle this integral must be done with finite ultraviolet cutoffs in both k_x and k_y, which we can take to be Λ, the large momentum cutoff of the edge modes, and $1/\lambda$, where λ is the wavelength of the stripe state, typically a number of the order of a few magnetic lengths. Also, for a finite system of linear size $L_x=L_y=L$, $1/L$ will be the infrared low momentum cutoff. However, it turns out that this subtracted propagator is finite in the thermodynamic limit $L\to\infty$ except in the regime $x\to 0$ and $t\to 0$ with y fixed. This infrared divergence is a consequence of the local shift invariance (or sliding symmetry) of the quantum Hall smectic phase. In addition, the subtracted propagator has a finite limit as $\Lambda\to\infty$ and $\lambda\to 0$, for all space-time points (x,y,t) except on the axes (which will be discussed below). Thus, for generic values of (x,y,t) the subtracted propagator is faithfully described by the scaling form

$$\tilde{C}_{\theta}(x,y,t)=\frac{1}{2\pi^2}\sqrt{\frac{\kappa_{\parallel}}{\kappa_{\perp}}}\frac{eB}{\lambda}F_{\theta}(\xi,\eta), \quad (2.4)$$

where $F_{\theta}(\xi,\eta)$ is the scaling function

$$F_{\theta}(\xi,\eta)=\frac{1}{2}\Re\int_0^{\infty}\frac{du}{u}\int_{-\infty}^{\infty}\frac{dz}{\sqrt{1+z^2}}\times[e^{-\eta u^3\sqrt{1+z^2}+i\xi u+iu^2 z}-1]. \quad (2.5)$$

Here ξ and η are the scaling variables

$$\xi=\left(\frac{\kappa_{\perp}}{Q}\right)^{1/4}\frac{|x|}{\sqrt{|y|}},$$

$$\eta=\frac{\lambda}{eB}\sqrt{\kappa_{\parallel}\kappa_{\perp}}\left(\frac{\kappa_{\perp}}{Q}\right)^{1/4}\frac{|t|}{|y|^{3/2}}. \quad (2.6)$$

Notice that the correlation function $\tilde{C}_{\theta}(x,y,t)$ depends on its arguments only through the scaling variables ξ and η of Eq. (2.6), which are scale invariant according to the rules of Sec. I, Eq. (1.5). This feature follows from the scaling properties of the dual field θ which, according to Eq. (1.5), is invariant under scale transformations.

It is direct consequence of scaling, and of the arguments presented above, that $F_{\theta}(\xi,\eta)$ is a smooth differentiable function for all values of the scaling variables ξ and η, except close to $(\xi,\eta)=(0,0)$ (i.e., $x\to 0$ and $t\to 0$ with y fixed) where the subtracted propagator becomes infrared divergent, and for $\xi\to\infty$ (i.e., $y\to 0$ with x fixed) or $\eta\to\infty$ (i.e., $y\to 0$ with t fixed) where the scaling function develops branch cut singularities. These branch cuts will show up in the form of logarithmic dependences on x and t for $y\to 0$. The scaling function has also an infrared divergence as a function of y as $x\to 0$ and $t\to 0$. This infrared divergence is a manifestation of the shift invariance, Eq. (1.17), of the quantum smectic.

It would be natural then to further subtract this (divergent) contribution from the scaling function which would now be infrared finite everywhere. However, in order to keep things simple, we will refrain from doing this but we will keep in mind the existence of these divergent contributions. Thus, we see that for generic values of ξ and η the correlation function is a finite scale-invariant function only of ξ and η. Notice that fixing both ξ and η generally corresponds to a curve in space and time.

We will now discuss the three special regimes.

(a) $x=0$, t fixed and $y\to 0$: The autocorrelation function of the dual field θ has a strong logarithmic infrared singularity as $(x,y,t)\to(0,0,0)$,

$$C_{\theta}(0,0,0)=\frac{3}{4\pi^2}\frac{eB}{\lambda}\sqrt{\frac{\kappa_{\parallel}}{\kappa_{\perp}}}\ln^2\left(\frac{L}{\lambda}\right). \quad (2.7)$$

This regime corresponds to setting $\xi=0$ and taking $\eta\to\infty$, where the scaling function F_{θ} takes the limiting form

$$F_{\theta}(0,\eta)\to\ln^2\eta. \quad (2.8)$$

Hence, the subtracted correlation function $\tilde{C}_{\theta}(x,y,t)$ is infrared finite and has the leading long time behavior

$$\tilde{C}_{\theta}(0,0,t)=\frac{1}{2\pi^2}\frac{eB}{\lambda}\sqrt{\frac{\kappa_{\parallel}}{\kappa_{\perp}}}\ln^2\left(\frac{|t|}{t_0}\right)\cdots, \quad (2.9)$$

where

$$t_0=\left(\frac{\lambda}{\sqrt{\kappa_{\perp}}}\right)^{3/2}\frac{Q^{1/4}}{\sqrt{\kappa_{\parallel}}}\frac{eB}{\lambda}. \quad (2.10)$$

Notice that $t_0\to 0$ as the UV cutoff in y vanishes, $\lambda\to 0$ (naturally, this is done only inside the cutoff factor which carries the 3/2 power, and not in the dimensional factor eB/λ). Thus a finite but small y acts as a short distance cutoff for the time correlation function.

(b) $t=0$, x fixed and $y\to 0$: In this regime, $\eta=0$ and $\xi\to\infty$, where the scaling function behaves as

$$F_{\theta}(\xi,0)\to\ln^2\xi \quad (2.11)$$

as $\xi\to\infty$. Hence, the equal-time subtracted correlation function "on the same stripe" (i.e., as $y\to 0$), has the long distance behavior

$$\tilde{G}_{\theta}(x,0,0)=\frac{1}{2\pi^2}\frac{eB}{\lambda}\sqrt{\frac{\kappa_{\parallel}}{\kappa_{\perp}}}\ln^2\left(\frac{|x|}{x_0}\right), \quad (2.12)$$

where

$$x_0=\left(\frac{\lambda^2 Q}{4\kappa_{\perp}}\right)^{1/4}. \quad (2.13)$$

Notice that here too a small but finite y acts as a short distance cutoff for the x correlator.

(c) $x=t=0$ and y fixed: In this regime both $\xi\to 0$ and $\eta\to 0$ and the scaling function develops an infrared divergence. We find that the equal-time *subtracted* correlation function on different stripes has the behavior

$$\tilde{C}_\theta(0,y,0)=\frac{1}{8\pi^2}\frac{eB}{\lambda}\sqrt{\frac{\kappa_\parallel}{\kappa_\perp}}\ln^2\left(\frac{\pi^2}{2}\sqrt{\frac{Q}{\kappa_\perp}}\frac{|y|}{L^2}\right)+\cdots, \tag{2.14}$$

where once again L is the linear size of the system. This correlation function is also infrared divergent as a consequence of the local sliding symmetry of the θ field.

B. The displacement field correlation function

We will analyze the propagator of the displacement field along the same lines used above for the θ correlator. The propagator of the displacement field u at space-time separation $x=(x,y,t)$, where t is *imaginary time*, is given by

$$C_u(x,y,t)=\int\frac{d^2k}{(2\pi)^2}\kappa_\parallel\left(\frac{\lambda}{eB}\right)^2\frac{k_x^2}{2\epsilon(\vec{k})}e^{i\vec{k}\cdot\vec{r}-|t|\epsilon(\vec{k})}. \tag{2.15}$$

The same line of argument used for the θ correlator now implies that the propagator of the displacement field u for finite (x,y,t) has the scaling form in the limit of $L\to\infty$, and, after removing all the ultraviolet cutoffs, we get

$$C_u(x,y,t)=\frac{1}{2\pi^2}\frac{\lambda}{eB}\sqrt{\frac{\kappa_\parallel}{Q}}\frac{1}{|y|}F_u(\xi,\eta), \tag{2.16}$$

where $F_u(\xi,\eta)$ is the scaling function

$$F_u(\xi,\eta)=\frac{1}{2}\Re\int_0^\infty du\, u\int_{-\infty}^{\infty}\frac{dz}{\sqrt{1+z^2}}\times e^{-\eta u^3\sqrt{1+z^2}+i\xi u+iu^2z} \tag{2.17}$$

which converges provided ξ and η, defined in Eq. (2.6), are finite. Hence, consistent with the predictions of the scaling laws of Eq. (1.5), the full propagator is the product of the universal scaling function $F_u(\xi,\eta)$ and the power law factor of $1/|y|$, i.e., u scales as $1/r$ and its propagator as $1/r^2$ (where r is the scale factor).

(a) *The auto correlation function*: In the regime $x\to 0$ and $y\to 0$ with t fixed, i.e., $\xi=0$ and $\eta\to\infty$, we find that the propagator has the asymptotic behavior

$$C_u(0,0,t)=-\frac{A_t}{|t|^{2/3}}, \tag{2.18}$$

where

$$A_t=\frac{1}{6\pi^2}\left(\frac{\Gamma(1/3)}{2^{2/3}}\right)^2\left(\frac{\lambda}{eBQ^4}\right)^{1/3}. \tag{2.19}$$

This behavior is consistent with the scaling laws.

(b) *The equal-time correlation function*: The equal-time correlation function $C_u(x,0,0)$ for the displacement field has the asymptotic behavior

$$C_u(x,0,0)=-\frac{A_x}{|x|^2}\ln\left(\frac{x_0}{|x|}\right) \tag{2.20}$$

which is also consistent with scaling. The constant A_x is given by

$$A_x=\frac{1}{2\pi^2}\sqrt{\frac{\kappa}{\kappa_\perp}}\frac{\lambda}{eB}. \tag{2.21}$$

This result can also be obtained by differentiating twice the scaling function for the θ field with respect to x, at $\eta=0$.

(c) *Equal-time correlation on different stripes*: Unlike the θ field correlation function, discussed in Sec. II A and the Luttinger field correlator to be discussed below, the equal-time correlation function for the displacement field u on different stripes, $C_u(0,y,0)$ is infrared finite and has the expected scaling form

$$C_u(0,y,0)=-\frac{A_y}{|y|}, \tag{2.22}$$

where

$$A_y=\frac{1}{8\pi}\frac{\lambda}{eB}\sqrt{\frac{\kappa_\parallel}{Q}}. \tag{2.23}$$

The infrared finiteness of the u correlator is required by the smectic symmetry since the u fields are the Goldstone bosons of the symmetries broken spontaneously by the quantum Hall smectic state.

C. The Luttinger field correlation function

We now turn to the (subtracted) propagator of the Luttinger field ϕ. In imaginary time and for finite (x,y,t), the subtracted propagator of the Luttinger field is given by

$$\tilde{C}_\phi(x,y,t)=\int\frac{d^2k}{(2\pi)^2}\frac{\epsilon(\vec{k})}{2\kappa_\parallel k_x^2}[e^{i\vec{k}\cdot\vec{r}-|t|\epsilon(\vec{k})}-1]. \tag{2.24}$$

Once again, we will write this propagator in scaling form

$$\tilde{C}_\phi(x,y,t)=\frac{1}{2\pi^2}\sqrt{\frac{\kappa_\perp}{\kappa_\parallel}}\frac{\lambda}{eB}\frac{1}{y^2}F_\phi(\xi,\eta) \tag{2.25}$$

which is consistent with scaling. The scaling function $F_\phi(\xi,\eta)$ is

$$F_\phi(\xi,\eta)=\frac{1}{2}\Re\int_0^\infty du\, u^3\int_{-\infty}^{\infty}dz\,\sqrt{1+z^2}\times[e^{-\eta u^3\sqrt{1+z^2}+i\xi u+iu^2z}-1]. \tag{2.26}$$

This scaling function is well defined for all values of ξ and η except near (0,0) (namely, for $x\to 0$ and $t\to 0$ with y fixed) where it develops infrared singularities, and for $\xi\to\infty$ ($y\to 0$ with x fixed) or $\eta\to\infty$ ($y\to 0$ with t fixed) where it too develops branch cut singularities. We will consider now the following three regimes.

(a) $x=0$ $y\to 0$ with t fixed. The autocorrelation function of the Luttinger field ϕ has a logarithmic infrared divergence as $t\to 0$,

$$C_\phi(0,0,0)=\frac{\Gamma(4/3)}{4\pi^2}\,\frac{\lambda}{eB}\left(\frac{\sqrt{\kappa_\perp}}{\lambda^3\kappa_\parallel}\right)^{1/2}\ln\left(\frac{L}{x_0}\right), \qquad (2.27)$$

where L is the linear size of the system. In contrast, the subtracted autocorrelation function is infrared finite and has the asymptotic behavior for $|t|\gg|y|^{3/2}$,

$$\tilde{C}_\phi(0,0,t)=-\frac{\Gamma(4/3)}{3\pi^2}\,\frac{\lambda}{eB}\left(\frac{\sqrt{\kappa_\perp}}{\lambda}\right)^{3/2}\frac{1}{\sqrt{\kappa_\parallel\kappa_\perp}}\ln\left|\frac{t}{t_0}\right| \qquad (2.28)$$

which seemingly violates scaling. However we will see below that this results also follows from the scaling function.

(b) $t=0$ and $y\to 0$ with x fixed. By direct evaluation of the scaling function $F_\phi(\xi,0)$ we find that as $\xi\to\infty$ it behaves as

$$F_\phi(\xi,0)\to \mathrm{ci}(\xi)-\ln\xi, \qquad (2.29)$$

where $\mathrm{ci}(\xi)$ is the cosine integral. Thus, for large ξ the scaling function has a logarithmic term similar to the one discussed above. Hence, the equal-time subtracted correlation function of the Luttinger field ϕ has the $|x|\gg\sqrt{y}$ behavior

$$\tilde{C}_\phi(x,0,0)=-\frac{6}{\pi^2}\,\frac{1}{\lambda^2}\sqrt{\frac{\kappa_\perp}{\kappa_\parallel}}\frac{\lambda}{eB}\ln\left|\frac{x}{x_0}\right| - \frac{3}{2\pi^2}\,\frac{Q}{\sqrt{\kappa_\perp\kappa_\parallel}}\left(\frac{\lambda}{eB}\right)\frac{1}{x^4}+\cdots. \qquad (2.30)$$

Hence we see that, for arbitrary finite values of the scaling variables $\xi\sim|x|/\sqrt{|y|}$ and $\eta\sim|t|/|y|^{3/2}$, the subtracted propagator of the Luttinger field obeys strictly the scaling laws, and it develops a logarithmic singularity as either the t or the x axes are approached.

(c) $x=0$ and $t=0$ with y fixed. The equal-time correlation function for ϕ on different stripes is also strongly infrared divergent as $L\to\infty$. In particular, since it can be seen that in this case $\lim_{y\to 0}C_\phi(y)=0$, there is no need to subtract the correlation function. Thus the correlation function is

$$C_\phi(0,y,0)=\frac{1}{2\pi^2}\left(\frac{\lambda}{eB}\right)\frac{1}{\sqrt{\kappa_\perp\kappa_\parallel}}\left(\frac{\lambda}{|y|}\right)^2\ln\left[\frac{\lambda L}{x_0|y|}\right] \qquad (2.31)$$

which diverges in the thermodynamic limit $L\to\infty$. The physical origin of this infrared divergence is, once again, the local shift symmetry of the Luttinger field on each stripe.

D. The electron propagator in the smectic phase

In this subsection we will use the fixed point theory to reconstruct the electron operator. We are interested in finding out several things. To begin with, we would like to know if the quantum smectic state does support sharp excitations with the quantum numbers of the electron. At the Hartree-Fock level there clearly are electronlike excitations in the spectrum. However, the connection between this problem and Luttinger models suggests that it may be possible to find a behavior akin to an array of quasi-one-dimensional systems in which the electron fractionalizes into a set of suitably defined solitons. In fact, if the displacement fields were to be gapped out, say by lattice commensurability effects, this is indeed what it may well happen as discussed in Refs. 19 and 28. Thus, instead of a Luttinger-like power law behavior, we will find that the autocorrelation function of the electron, and the associated spectral function, vanishes as a function of either $|x|$ or $|t|$ faster than any power but more slowly than an exponential. This "pseudogap" behavior is indicative of a pronounced suppression of final states for electron tunneling in the quantum Hall smectic phase. In the remainder of this section we will derive this result.

We are interested in computing the Green function of the electron operators as defined in Sec. I. In this section we will use the results of Sec. II to compute the fermion Green function directly in the continuum limit along the y direction. Thus, from now on, we will replace the discrete stripe label j by the continuum coordinate y. Due to the smectic symmetry,[19,27,29] Eq. (1.17), of the effective Lagrangian of Eq. (1.4), the (continuum) fermion propagator

$$G(x,y,t;x',y',t')=\langle T\Psi(x,y,t)\Psi^\dagger(x',y',t')\rangle \qquad (2.32)$$

vanishes identically for $y\neq y'$. Thus it is sufficient to compute the propagator on a single stripe, i.e., as $|y'-y|\to 0$. Below we will compute the propagator for the right movers. The propagator for the left movers follows trivially from it.

Using the bosonization formulas, the propagator for the right moving fermions at fixed y (i.e., on the same stripe) $\psi_+(x,y,t)$ is

$$\langle\psi_+(x,y,t)^\dagger\psi_+(0,y,0)\rangle=\frac{1}{2\pi a}e^{-\pi\Phi_+(x,t)}, \qquad (2.33)$$

where, for imaginary time $t>0$ and $x>0$, we find

$$\Phi_+(x,t)=\lim_{y\to 0}[\tilde{C}_\theta(x,y,t)+\tilde{C}_\phi(x,y,t)], \qquad (2.34)$$

where we have dropped the contributions from the cross correlations between θ and ϕ. The only role of these terms is to insure the correct anticommutation relations on the fermion operator, i.e., that the fermion propagator is an odd, antiperiodic in time, function of the coordinates. It is straightforward to check that these conditions are met.

Elsewhere in this section we showed that the correlator of the dual field $\tilde{C}_\theta$ is always more singular than the propagator of the Luttinger field $\tilde{C}_\phi$. Using these results we find that, at equal (imaginary) times and at $y=0$, the function $\Phi_+(x,0,t)$ has the has the asymptotic behavior for large $|x|$

$$\Phi_+(x,0,0)=\frac{1}{2\pi^2}\,\frac{eB}{\lambda}\sqrt{\frac{\kappa_\parallel}{\kappa_\perp}}\ln^2\left(\frac{|x|}{x_0}\right)+\cdots. \qquad (2.35)$$

We can easily see that this asymptotic behavior is due to the equal-time correlation function of the dual fields $\tilde{C}_\theta(x)$ of Eq. (2.12). Consequently, the equal-time fermion correlation function behaves as

$$G_+(x,0,0) \propto \mathrm{sgn}(x)\, e^{-(1/2\pi)(eB/\lambda)\sqrt{\kappa_\parallel/\kappa_\perp}\ln^2(|x|/x_0)}, \tag{2.36}$$

where the sgn(x) factor shows that it is a correlation function of a *fermion*. This correlation function exhibits the same analytic behavior as the structure factor in DNA-lipid complexes.[29,30] Notice that it decays *faster* than any power but *more slowly* that an exponential decay.

Conversely, the fermion autocorrelation function (in imaginary time) is, for $t>0$, and on the same stripe ($y=0$) is given by

$$G_+(0,0,t) \propto e^{-\pi\Phi_+(0,0,t)} \tag{2.37}$$

whose long time behavior, at zero temperature and as finite temperature, is once again dominated by $\tilde{C}_\theta(t)$, the autocorrelation function of the dual field. Hence, we find that the fermion autocorrelation function has the long time behavior

$$G_+(0,0,t) \propto e^{-(1/2\pi(eB/\lambda)\sqrt{\kappa_\parallel/\kappa_\perp}}\ln^2(|t|/t_0) \tag{2.38}$$

with the same t_0 defined above in this section.

The fermion Green function on different stripes can be found by similar means. Using these methods, the equal-time Green function on different stripes, $y\neq 0$, is found to vanish:

$$G_+(0,y,0) \propto e^{-\pi\Phi_+(0,y,0)} \to 0 \tag{2.39}$$

in the thermodynamic limit $L\to\infty$ due to the infrared divergence in both $\tilde{C}_\theta(0,y,0)$ and in $\tilde{C}_\phi(0,y,0)$, which are both a consequence of the sliding symmetry of the quantum Hall smectic state.

The behavior of the electron auto-correlation function of Eq. (2.38) clearly shows the strong suppression of electron states at low energies. It implies a dramatic suppression of the *tunneling density of states* for electrons into the quantum Hall smectic. Notice that this is a much more dramatic effect than the power law behavior of one-dimensional Luttinger liquids. It is due to the strong fluctuations of the shape of the stripes, a hallmark of a quantum smectic.[8] It easy to see that in the presence of a periodic pinning potential along the direction perpendicular to the stripes, this behavior is superseded by a conventional Luttinger picture. This is expected since the resulting state is equivalent to the smectic metal of Ref. 19. Similarly, it is easy to see that at conventional Luttinger behavior is recovered at short times once irrelevant operators which lead to a linear dispersion relation are included.

III. THE COULOMB FIXED POINT

In the previous sections we have discussed in detail the properties of the quantum Hall smectic phase for a 2DEG with short-range interactions. Here we will consider the effects of long range Coulomb interactions. Using a time-dependent Hartree-Fock approach Côté and Fertig[20] have investigated the quantum Hall smectic state in a 2DEG with Coulomb interactions, and found that the dispersion relation of the collective modes is modified. In this section we will investigate how these changes modify the picture of the fixed point developed above.

Rather than reworking the mean field theory we have used for the short range case, we will work directly with the effective Lagrangian of Eq. (1.4) and modify it to reflect the effects of Coulomb interactions. There is a simple and direct way to account for their effects at the level of the effective theory. First of all, the term $\frac{1}{2}\kappa_\parallel(\partial_x\phi)^2$ represents short-range density-density interactions (recall that in bosonization the forward scattering part of the electron local density is given by $\partial_x\phi$). Thus long-range interactions lead to a nonlocal term in the *action*. Let $\tilde{V}(\vec{k})$ be the Fourier transform of the two-dimensional Coulomb interaction, i.e., $\tilde{V}(\vec{k})\propto 1/|\vec{k}|$. In momentum space this amounts to modify $\kappa_\parallel$ by a momentum-dependent factor

$$\kappa_\parallel \to \kappa_\parallel(\vec{k}) = \frac{\overline{\kappa_\parallel}}{|\vec{k}|}, \tag{3.1}$$

where $\overline{\kappa}$ is an effective coupling constant. Similarly, long-range Coulomb interactions will also modify the elastic modulus $\kappa_\perp$ along the direction perpendicular to the stripes. Hence, $\kappa_\perp$ will also have to be changed in a similar fashion,

$$\kappa_\perp \to \kappa_\perp(\vec{k}) = \frac{\overline{\kappa_\perp}}{|\vec{k}|}. \tag{3.2}$$

Thus the effective low-energy theory of the quantum Hall smectic with Coulomb interactions is obtained by replacing $\kappa_\parallel \to \kappa_\parallel(\vec{k})$ and $\kappa_\perp\to\kappa_\perp(\vec{k})$ at the level of the effective action, in Fourier space. The new dispersion relation is

$$\epsilon(\vec{k}) = \frac{\lambda}{eB}\sqrt{\frac{\overline{\kappa_\parallel}}{|\vec{k}|}}|k_x| \sqrt{Qk_x^4 + \frac{\overline{\kappa_\perp}}{|\vec{k}|}k_y^2}. \tag{3.3}$$

Thus, the dispersion relation is nonlocal. However, we will show below that for the analysis of the infrared behavior of the displacement fields correlators it is sufficient to use an approximate, simpler, version of the dispersion relation obtained by setting $|\vec{k}|\approx|k_x|$. In this limit, the dispersion relation becomes

$$\epsilon(\vec{k}) \equiv \frac{\lambda}{eB}\sqrt{\overline{\kappa_\parallel}}\sqrt{Q|k_x|^5 + \overline{\kappa_\perp}k_y^2}. \tag{3.4}$$

In particular, at $k_y=0$, we find $\epsilon(k_x,0)\propto|k_x|^{5/2}$, a result first obtained by Côté and Fertig.[20]

The dispersion relation of Eq. (3.4) shows that the dimensional counting for the case of Coulomb interactions is $\omega^2 \sim k_y^2 \sim k_x^5$, which suggests new scaling transformations, with scale factor r, of the form

$$x\to rx, \quad y\to r^{5/2}y, \quad t\to r^{5/2}t,$$
$$u\to r^{-1}u, \quad \phi\to r^{-5/2}\phi, \quad \theta\to\theta. \tag{3.5}$$

Hence, in the case of Coulomb interactions the effective dimension is $D=1+5/2+5/2=6$. Note that the same caveats that were raised about the short range scaling laws also apply here.

However, unlike the short range case, for $k_x=0$ this approximate dispersion relation now vanishes only at $k_y=0$, rather than on the entire line $k_x=0$ as it is the case for the exact dispersion relation as a consequence of the sliding symmetry. However, this is not a problem for the calculation of the correlator of the displacement field since this field is invariant under the sliding symmetry. For the same reason this approximation cannot be used to calculate the corraltors of the Luttinger and the dual fields since these are not invariant under the sliding symmetry.

We have computed explicitly the behavior of the low energy and long distance behavior of the correlation functions. We found the following results.

A. The dual field propagator

We computed the subtracted correlator of the dual field θ for the Coulomb case. For x, y and t close to the axes we find the following.

(1) For $t=y=0$, the (subtracted) correlation function has the behavior

$$\tilde{C}_{\theta}(x,0,0)=\frac{5}{8\pi^2}\frac{eB}{\lambda}\sqrt{\frac{\overline{\kappa_\parallel}}{\overline{\kappa_\perp}}}\ln^2[(4\overline{\kappa_\perp}/Q\lambda^2)^{1/5}|x|]. \tag{3.6}$$

(2) In the regime $x=0$, $y\to 0$ and t finite, we get

$$\tilde{C}_{\theta}(0,0,t)=-\frac{2}{5\pi^2}\frac{eB}{\lambda}\sqrt{\frac{\overline{\kappa_\parallel}}{\overline{\kappa_\perp}}}\ln^2\left(t\frac{\lambda}{eB}\sqrt{\overline{\kappa_\parallel}\,\overline{\kappa_\perp}}\Lambda\right), \tag{3.7}$$

where Λ is the UV cutoffs, which we took to be the same along the x and y directions for simplicity. Here we have kept only the $\ln^2 t$ terms. Thus $\tilde{C}_{\theta}(t)$ and $\tilde{C}_{\theta}(x)$ have a similar infrared behavior.

(3) In contrast, for $x=t=0$ we find instead the infrared divergent behavior

$$\tilde{C}_{\theta}(0,y,0)=\frac{1}{16\pi^2}\frac{eB}{\lambda}\sqrt{\frac{\overline{\kappa_\parallel}}{\overline{\kappa_\perp}}}\ln^2\left[\left(\frac{\pi}{L}\right)^{5/2}\sqrt{\frac{Q}{\overline{\kappa_\perp}}}|y|\right] \tag{3.8}$$

similar to what we found for the case of short-range interactions.

B. The propagator of the displacement field

We verified that for the calculation of the infrared behavior of the correlation functions of the displacement field it is correct to replace the full dispersion by the approximate one.

We find that the propagator of the displacement field u takes the scaling form

$$\tilde{C}_u(x,y,t)=\frac{1}{2\pi^2}\frac{\lambda}{eB}\sqrt{\frac{\overline{\kappa_\parallel}}{\overline{\kappa_\perp}}}\left(\frac{\overline{\kappa_\perp}}{Q}\right)^{2/5}\frac{1}{|y|^{4/5}}F_u^c(\xi,\eta), \tag{3.9}$$

where $F_u^c(\xi,\eta)$ is the scaling function

$$F_u^c(\xi,\eta)=\frac{1}{2}\mathrm{Re}\int_0^\infty du\,u\int_{-\infty}^{\infty}\frac{dz}{\sqrt{1+z^2}}\times e^{-\eta u^{5/2}\sqrt{1+z^2}+i\xi u+iu^{5/2}z}. \tag{3.10}$$

In limiting cases we find the asymptotic behaviors

$$C_u(x,0,0)=-A_x\frac{\ln(\mu|x|)}{|x|^2},$$

$$C_u(0,y,0)=\frac{A_y}{|y|^{5/2}},$$

$$C_u(0,0,t)=\frac{A_t}{|t|^{5/2}} \tag{3.11}$$

which, up to logarithms, clearly obey the scaling laws of Eq. (3.5). Here we have set

$$A_x=\frac{5}{8\pi^2}\frac{\lambda}{eB}\sqrt{\frac{\overline{\kappa_\parallel}}{\overline{\kappa_\perp}}},$$

$$\mu=(4\overline{\kappa_\perp}/Q)^{1/5},$$

$$A_y=\frac{(\sqrt{5}-1)}{20\pi^2}\Gamma(2/5)\Gamma(1/10)\Gamma(4/5)\frac{\overline{\kappa_\parallel}(\lambda/eB)^2}{(\overline{\kappa_\perp}Q^4)^{2/10}},$$

$$A_t=\frac{(\Gamma(2/5))^2}{5\pi^2}\left(\frac{\lambda}{eB}\right)\left(\frac{\overline{\kappa_\parallel}}{\overline{\kappa_\perp}}\right)^{1/10}\left(\frac{eB}{2\sqrt{2}\lambda Q}\right)^{4/5}. \tag{3.12}$$

Hence, even in the presence of Coulomb interactions the propagator of the displacement field u obeys scaling. Notice that here too this propagator is free of the infrared singularities that we find in both the θ and in the ϕ correlators. Once again this feature is dictated by the smectic symmetry.

C. The propagator for the Luttinger field

The subtracted propagator of the Luttinger field does not have a simple scaling form in the Coulomb case. We will only consider the limiting regimes.

(1) For $t=0$ and $y=0$, the subtracted correlation function becomes

$$\tilde{C}_{\phi}(x,0,0)=-\frac{1}{8\pi^2}\frac{1}{\lambda^2}\sqrt{\frac{\overline{\kappa_\perp}}{\overline{\kappa_\parallel}}}\frac{\lambda}{eB}\ln\left(\frac{|x|}{x_0}\right)+\cdots, \tag{3.13}$$

where $x_0=(Q/\overline{\kappa_\perp})^{1/5}\lambda^{2/5}$. This behavior is similar to what we found in the short range case.

(2) For $x=y=0$, we find that the subtracted correlator has the behavior

$$\tilde{C}_{\phi}(0,0,t)=-\frac{\lambda}{eB}\sqrt{\frac{\overline{\kappa_\perp}}{\kappa_\parallel}}\frac{1}{4\pi^2}\Lambda^2\ln\left(t\frac{\lambda}{eB}\sqrt{\overline{\kappa_\parallel\kappa_\perp}}\Lambda\right)+\cdots, \tag{3.14}$$

where $\Lambda=\Lambda-x=\Lambda_y$ is the UV cutoff. Thus, the subtracted autocorrelation function has a $\ln t$ behavior.

(3) For $\eta=0$ and $\xi\to 0$, we now find

$$\tilde{C}_{\phi}(0,y,0)=-\frac{5}{8\pi^2}\frac{\lambda}{eB}\sqrt{\frac{\overline{\kappa_\perp}}{\overline{\kappa_\parallel}}}\frac{1}{y^2}\ln[(Q\lambda^2/\overline{\kappa_\perp})^{1/5}L]. \tag{3.15}$$

Hence, here this propagator is infrared divergent too.

(4) The strong multiplicative infrared divergences that we found in $C_{\phi}(t)$ imply that, for Coulomb interactions too, the fermion propagator vanishes for $y\neq 0$. However, just as in the case of short-range interactions, at $y=0$ the fermion propagator is dominated by the contribution of the correlator of the dual field θ, which here too behaves as $\ln^2 x$ or $\ln^2 t$. Hence, also in the case of Coulomb interactions, the fermion autocorrelation function has the same ''pseudogap'' behavior found for the case of short-range interactions, i.e., exp $[-A\ln^2(t/t_0)]$ (where A is a constant).

IV. THERMODYNAMIC PROPERTIES OF THE QUANTUM HALL SMECTIC

The quantum Hall smectic phase has remarkable thermodynamic properties. These can be deduced directly from the effective low-energy theory.

It is an elementary exercise in statistical mechanics to show that the internal energy density U is

$$U=\int\frac{d^2k}{4\pi^2}\frac{\epsilon(\vec{k})}{e^{\epsilon(\vec{k})/T}-1}. \tag{4.1}$$

We will apply this formula for both cases, short-range and Coulomb interactions. For short-range interactions we find that the internal energy density at low temperatures has the behavior

$$U(T)=\frac{1}{9}\frac{\lambda}{eB}\sqrt{\frac{\kappa_\parallel}{\kappa_\perp}}T^2\ln\left(\frac{T_0}{T}\right)+\cdots, \tag{4.2}$$

where $T_0=1/t_0$. Hence, the low-temperature specific heat $c(T)$ of the quantum Hall smectic phase obeys the law

$$c(T)\propto T\ln\frac{T_0}{T}+O(T,T^3\ln T). \tag{4.3}$$

Hence, for short-range interactions, the low-temperature specific heat of the 2DEG in the quantum Hall smectic phase is larger than the specific heat of either Fermi or Luttinger liquid phases (both being linear).

Coulomb interactions modify the above laws only slightly. In particular we find that the internal energy density at low temperatures now obeys a pure T^2 law (without the logarithm), and that the specific heat has a T linear behavior $c(T)\sim T$.

V. IS THE QUANTUM HALL SMECTIC PHASE STABLE?

We have shown that, although the Goldstone modes of the smectic are very soft, the quantum zero-point fluctuations of the long-wavelength modes do not destroy the stripe order. This is not a trivial issue—at finite temperature, the harmonic analysis would lead to a linearly diverging mean-square stripe displacement $\langle u^2\rangle\sim TL$. Of course, what this really means is that there is no smectic phase at finite temperatures.

It is a much more subtle issue whether the smectic phase is unstable to the formation of a stripe crystal, i.e., whether there is inevitably translation symmetry breaking along the stripe direction. There are two sorts of analysis that have been applied to answer this question, although we feel there are possible flaws with both.

In Sec. I we defined a scale invariant smectic fixed point Lagrangian. Treating this Lagrangian as one would in studying critical phenomena, one can assess the relevance of various physically allowed perturbations using standard dimensional analysis. By this analysis, there is one operator, which originates from the shear piece of the locking term in the stripe crystal Lagrangian, which is apparently relevant. One might conclude from this that the smectic is unstable to crystallization. A bizarre aspect of this analysis, which causes us to have reservations concerning its validity, is that the different pieces of the locking term, whose relative strength is ultimately determined by rotational symmetry, have different scaling dimensions. This analysis was generalized for the case of Coulomb interactions in Sec. III; the stability analysis yields the same answer. The basic assumption behind the scaling analysis is that the correlation functions have a simple analytic structure reflecting the scaling laws, although x, y, and t enter with different effective exponents, in just the same way that space and time can enter differently in quantum critical phenomena.

However, in Secs. II and III we found that while the Goldstone boson field u follows the scaling laws (up to a multiplicative logarithmic correction), the Luttinger field ϕ and the dual field θ have a much more complex singularity structure. In particular the correlation functions of the fields ϕ and θ for space-time points on the same stripe (that is, for equal y coordinates) have the leading equal-time behavior $\ln(|x|/x_0)$ and $\ln^2(|x|/x_0)$, respectively (and an analogous behavior as a function of time). Along other directions the ϕ and θ are finite only when properly subtracted. Otherwise they exhibit infrared divergences, as a consequence of the sliding symmetry. The consequent breakdown of scaling is ''weak''—it only involves logarithms. However, the RG analysis, although based on examining lowest order perturbation theory in the additional couplings, is ultimately nonperturbative; it is based on the assumption that the correlation functions scale. Without a careful analysis of the singularity structures that occur in higher order perturbation theory, it is not possible to determine whether the breakdown of scaling is significant or not. Indeed, the length and time scales x_0 and t_0 discussed in Sec. II appear in various correlation functions—for instance, the fermion propagator has a ''pseudogap'' behavior $\exp[-\text{const } \ln^2(|t|/t_0)]$, instead of the familiar power law of Luttinger liquids. These properties seemingly imply that, unlike arrays of Luttinger liquids, the QH smectic is not a critical state. Instead it behaves much more similar to a stable state of matter, characterized by the finite length and time scales.

The second approach is based on the observation that the on-stripe pure logarithmic behavior of the ϕ correlator is reminiscent of that encountered in weakly coupled Luttinger liquids. This singularity has been used to suggest that the locking perturbations may ultimately drive the QH smectic state to a crystalline state.[9,12,23] Moreover, if the structure of lowest order perturbation theory in the $\mathcal{L}_{\text{lock}}$ is examined, it is found to be dominated by large values of k_y, so the characteristic two-dimensional dispersion of the smectic Goldstone modes does not significantly affect the results.

When the stripes are pinned (i.e., if the transverse Goldstone behavior is suppressed), we have no doubt of the validity of this approach. In such a state the Goldstone bosons are gapped and can be integrated out. Their net effect is to induce and renormalize the interstripe and intrastripe forward scattering interactions. Hence, the pinned smectic reduces to an array of Luttinger liquids. On a technical level, this approach actually treats the system as a distinct Luttinger liquid for each value of k_y. Consequently, instead of a Luttinger parameter and Fermi velocity, there is a ''Luttinger function'' $K(k_y)$ and a velocity function $v_F(k_y)$. The phase diagram of such systems was considered recently by us,[19] by Vishwanath and Carpentier,[28] and by Sondhi and Yang.[31] It was found that as the parameters of the Luttinger function are changed, there is a complex phase diagram which includes both crystalline and smectic metallic phases.

However, there is reason to worry about the validity of this approach for the unpinned smectic. Specifically, this analysis is completely insensitive to the small k_y behavior of the Luttinger and velocity functions. But the Goldstone behavior of the smectic implies a vanishing Fermi velocity as $k_y \to 0$. This pathology has no consequences in lowest order perturbation theory, which is why it does not affect the scaling analysis that is based on it. However, it is responsible for the the form of the low temperature specific heat and the non-Luttinger-liquid (pseudogap) form of the fermion propagator in the quantum smectic state. It seems too good to be true that this salient physics should have no affect on the stability of the state. Again, it is only by an analysis of the structure of higher order perturbation theory (which we do not attempt) that it can be determined whether the pathologies associated with small k_y are truly unimportant for this purpose, or whether they become more important in higher order terms.

Leaving this issue aside, we can approach the stability question from the viewpoint of coupled Luttinger liquids. MacDonald and Fisher[12] showed that rotational invariance constrains the Luttinger functions to reach simple limiting values as a function of k_y. They further argued that if these functions are monotonically increasing with k_y, and if charge-conjugation symmetry is respected, then the CDW lock-in interactions are always relevant and the QH smectic is ultimately unstable to crystallization. This is in apparent contradiction with the results obtained by Fertig and co-workers[11,20,21] (using the same method of analysis) in which numerical solutions of the time-dependent Hartree-Fock equations were used to compute the values of $K(k_y)$. It is important to remember that, in this case, the scaling analysis performed in Sec. I cannot be used to discard a variety of additional operators from the fixed-point Hamiltonian. In particular, there is an infinite number of marginal operators, representing forward scattering density-density interactions, which affect the scaling properties.[19] The analysis of MacDonald and Fisher[12] could in principle be reconciled with these results if the assumption of monotonicity of the Luttinger function on the transverse momentum is dropped. It is apparent that these neglected terms could, in principal, do this.

VI. CONCLUSIONS AND OPEN PROBLEMS

In this paper we presented an effective theory for the low energy degrees of freedom of the quantum smectic or stripe phase of the 2DEG in a large magnetic field. In the quantum Hall smectic phase the 2DEG spontaneously breaks both translational (in the direction perpendicular to the stripes) and rotational invariance. We showed that the form of the effective theory is dictated by very general principles: the native symmetries of the smectic state and the Lorentz force law. We determined the spectrum of collective modes which exhaust the low-energy degrees of freedom. For a 2DEG with short range (screened) interaction, these Goldstone modes have a dispersion which is exceedingly soft, vanishing as $\epsilon \sim k_x^3$ at $k_y = 0$ and with a whole line of zero-energy states at $k_x = 0$ and $k_y \neq 0$. These Goldstone bosons should be detectable in Raman scattering experiments in 2DEG in heterostructures. A tangible reflection of how soft these Goldstone modes are is seen from the temperature dependence of the specific heat $c_V \sim T|\ln(T/T_0)|$ at low T. This is larger than that of a Fermi liquid. In contrast, we found that for long-range Coulomb interactions the Goldstone bosons behave as $\epsilon \sim k_x^{5/2}$ at $k_y = 0$. Nevertheless, the specific heat obeys the familiar T linear law.

We also used the fixed point theory to compute the electron propagator. We found that for short-range interactions there should be a pseudogap in the single particle density of states, as deduced from the tunneling conductance at low temperatures. For Coulomb interactions we found a similar behavior.

The question of the stability of the stripe state has been a matter of discussion and controversy for some time.[8,9,12] We have shown here that although it is true that by a suitable parametrization of the degrees of freedom it is possible to write the effective theory in terms of what looks similar to an array of coupled Luttinger liquids, this is not a true quasi-one-dimensional system. Instead, it is a strongly anisotropic, fully two-dimensional system whose the symmetries force an anisotropic form of scaling described in Sec. I.

Nevertheless, clearly the quantum Hall smectic has very unusual scaling properties. For instance, although the correlation functions obey scaling, as shown in great detail in Secs. II and III, the scaling functions are not analytic everywhere in the plane defined by the scaling variables ξ and η, and develop singularities in the extreme regimes $\xi\to\infty$ or $\eta\to\infty$. However, these are very soft, logarithmic, integrable singularities on a set of measure zero of the space-time coordinates. Thus, when the effects of any typical perturbation is considered, for instance, the coupling of the CDW order parameters on different stripes, it may well be that these additional singularities may not affect the scaling analysis as they do not lead to true singular behavior of a true two-dimensional system. In any case a more careful renormalization group analysis is required to reach a definitive conclusion. However, if either translation invariance is broken explicitly (say by an external periodic potential in the direction perpendicular to the stripes) the behavior will indeed cross over to a quasi-one-dimensional regime quite similar to what was discussed in Ref. 19. In other words, a pinning potential changes the properties of this state substantially and in fact it enhances the effects of fluctuations whose main effect is to make it more unstable to crystallization. It has also been suggested recently[31] that the pinned stripe state may also be unstable to a quantum Hall state.

An open and very interesting question is the possible existence of a quantum nematic state of the 2DEG in large magnetic fields at zero temperature. This is an important question both conceptually and experimentally as it appears to be consistent with the experimental data.[13] Recently some of us[15] developed a theory of a quantum nematic Fermi fluid at zero external magnetic field in the proximity of an isotropic Fermi liquid phase. It will be particularly interesting to construct a theory of the quantum melting of the smectic by a dislocation unbinding mechanism.[32]

It is also intriguing to explore the relation of the quantum Hall smectic with other phases of the quantum Hall system. There is some evidence from exact diagonalization studies in small systems[33,34] of a direct transition to a paired quantum Hall state.[35,36] We would expect such a transition, if it occurs, to be first order. More exciting is the possibility of a relation between the quantum Hall smectic and the other well known compressible state of a half-filled Landau level.

Finally it is interesting to note that the imaginary time form of the effective action for the dual field θ, Eq. (1.15), bears a close formal resemblance to the free energy for the sliding columnar phase of DNA-lipid complexes.[29,30,37,38] In momentum space the free energy of the sliding columnar (SC) phase is given by[38]

$$F_{\rm SC}=\int \frac{d^3q}{(2\pi)^3}[Bq_z^2+Kq_x^4+K_yq_x^2q_y^2]|u_z(\vec{q})|^2, \quad (6.1)$$

where $\hat{y}$ is the direction normal to the layers, $\hat{z}$ is the direction normal to the DNA strands, u_z is the displacement field of the DNA strands parallel to the lipid layers, and B, K, and K_y are effective elastic constants. Although the effective elastic theory are quite similar, the coordinates x, y, and z do not scale as the coordinates x, y, and t of the quantum smectic. While in the case of the quantum hall smectic the effective dimension is $D=6$, in the sliding columnar phase it is $D=5$. A direct consequence of these scaling properties is that the nonlinear corrections to the elastic strain tensor introduce additional interactions which are marginally relevant in the sliding columnar phase,[37] but are irrelevant in the quantum Hall smectic phase. Remarkably, although the details of the models are different, some of the correlation functions in the quantum Hall smectic phase that we discussed in Secs. II and III have the same behavior as certain correlation functions in the sliding columnar phases.

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[1] M.P. Lilly, K.B. Cooper, J.P. Eisenstein, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. **82**, 394 (1999).

[2] R.R. Du, D.C. Tsui, H.L. Stormer, L.N. Pfeiffer, K.W. Baldwin KW, and K.W. West, Solid State Commun. **109**, 389 (1999).

[3] W. Pan, J.S. Xia, V. Shvarts, E.D. Adams, R.R. Du, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, K.W. Baldwin, and K.W. West, Physica A **6**, 14 (2000); M.P. Lilly, K.B. Cooper, J.P. Eisenstein, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. **83**, 824 (1999); W. Pan, R.R. Du, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, K.W. Baldwin, and K.W. West, *ibid.* **83**, 820 (1999); K.B. Cooper, M.P. Lilly, J.P. Eisenstein, L.N. Pfeiffer, and K.W. West, Phys. Rev. B **60**, R11 285 (1999).

[4] H. Fukuyama, P. Platzman, and P.W. Anderson, Phys. Rev. B **19**, 5211 (1979).

[5] A.A. Koulakov, M.M. Fogler, and B.I. Shklovskii, Phys. Rev. Lett. **76**, 499 (1996); M.M. Fogler, A.A. Koulakov, and B.I. Shklovskii, Phys. Rev. B **54**, 1853 (1996); M.M. Fogler and A.A. Koulakov, *ibid.* **55**, 9326 (1997).

[6] R. Moessner and T.J. Chalker, Phys. Rev. B **54**, 5006 (1996).

[7] T. Stanescu, I. Martin, and P. Phillips, Phys. Rev. Lett. **84**, 1288 (2000).

[8] S.A. Kivelson, E. Fradkin, and V.J. Emery, Nature (London) **393**, 550 (1998).

[9] Eduardo Fradkin and Steven A. Kivelson, Phys. Rev. B **59**, 8065 (1999).

[10] R.B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).
[11] H. Fertig, Phys. Rev. Lett. **82**, 3693 (1999).
[12] A.H. MacDonald and Matthew P.A. Fisher, Phys. Rev. B **61**, 5724 (2000).
[13] Eduardo Fradkin, Steven A. Kivelson, Efstratios Manousakis, and Kwangsik Nho, Phys. Rev. Lett. **84**, 1982 (2000).
[14] C. Wexler and A. Dorsey, Phys. Rev. B **64**, 115312 (2001).
[15] Vadim Oganesyan, Steven Kivelson, and Eduardo Fradkin, Phys. Rev. B **64**, 195109 (2001).
[16] Daniel Barci and Eduardo Fradkin, following paper, Phys. Rev. B **65**, 245320 (2002).
[17] P.G. de Gennes and J. Prost, *The Physics of Liquid Crystals* (Oxford University Press, New York, 1998).
[18] P.M. Chaikin and T.C. Lubensky, *Principles of Condensed Matter Physics* (Cambridge University Press, Cambridge, UK).
[19] V.J. Emery, E. Fradkin, S.A. Kivelson, and T.C. Lubensky, Phys. Rev. Lett. **85**, 2160 (2000).
[20] R. Côté and H.A. Fertig, Phys. Rev. B **62**, 1993 (2000).
[21] Hangmo Yi, H.A. Fertig, and R. Côté, Phys. Rev. Lett. **85**, 4156 (2000).
[22] M. Fogler and V. Vinokur, Phys. Rev. Lett. **84**, 5828 (2000).
[23] Anna Lopatnikova, Steven H. Simon, Bertrand I. Halperin, and Xiao-Gang Wen, cond-mat/0105079 (unpublished).
[24] We are grateful to Bert Halperin for bringing to our attention the linear combination $\partial_y \phi + \partial_x u$, representing shear strain, the only combination allowed in a rotationally invariant system.
[25] We thank Alan Dorsey (private communication) for raising the question of the role of this operator to us.
[26] See for instance V.J. Emery in *Highly Conducting One-Dimensional Solids*, edited by J. Devreese *et al.* (Plenum, New York, 1979); *Bosonization*, edited by M. Stone, (World Scientific, Singapore, 1994).
[27] C.S. O'Hern, T.C. Lubensky, and J. Toner, Phys. Rev. Lett. **83**, 2745 (1999).
[28] Ashvin Vishwanath and David Carpentier, Phys. Rev. Lett. **86**, 676 (2001).
[29] L. Golubović and M. Golubović, Phys. Rev. Lett. **80**, 4341 (1998); **81**, 5704(E) (1998).
[30] C.S. O'Hern and T.C. Lubensky, Phys. Rev. Lett. **80**, 4345 (1998).
[31] S.L. Sondhi and Kun Yang, Phys. Rev. B **63**, 054430 (2001).
[32] L. Radzihovsky and A. Dorsey, cond-mat/0110083 (unpublished).
[33] Exact diagonalization of 2DEG's with small numbers of electrons suggest this possibility; see E.H. Rezayi, F.D.M. Haldane, and Kun Yang, Phys. Rev. Lett. **85**, 5396 (2000).
[34] E.H. Rezayi, F.D.M. Haldane and Kun Yang, Phys. Rev. Lett. **83**, 1219 (1999).
[35] N. Read and G. Moore, Nucl. Phys. B **360**, 362 (1991).
[36] M. Greiter, X.G. Wen, and F. Wilczek, Phys. Rev. B **46**, 9586 (1992).
[37] C.S. O'Hern and T.C. Lubensky, Phys. Rev. E **58**, 5948 (1998).
[38] L. Golubović, T.C. Lubensky, and C.S. O'Hern, Phys. Rev. E **62**, 1069 (2000).