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Current-driven plasma instabilities in parallel quantum-wire systems

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A theory of current-driven plasma instability in a double-quantum-wire system is presented. It is shown that, if the wires carry steady currents in opposite directions, the quasi-one-dimensional (quasi-1D) plasma waves propagating along the wires become unstable when the electron drift velocity falls between the phase velocities of the acoustic and optical plasma modes of the Coulomb-coupled quantum wires. Such an instability occurs at experimentally achievable drift velocities because of the softening of the plasma waves in 1D wires. The condition for plasma instability in a lateral double-quantum-wire superlattice is also determined.

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Spurred by experimental achievements in quasi-one-dimensional (quasi-1D) semiconductor structures, the study of collective plasma excitations in 1D electron systems has been vigorously pursued.[1–5] The existence of 1D plasmons in quantum wires was well established experimentally by means of infrared spectroscopy[1] and resonant inelastic light scattering.[2] Their theoretical description in the random-phase approximation (RPA) (Refs. 3 and 4) proved to be in quantitative agreement with the experimental data. In addition to intrinsic physical interest, 1D quantum wire plasmon phenomenology may be of practical importance in regard to possible device applications based on the use of radiative plasmon decay in tunable solid-state sources of far-infrared electromagnetic radiation.[6]

The key element for experimental realization of a plasmon-based source of electromagnetic radiation is the generation of a plasma instability that can be produced by a strong steady current, with energy transferred from the current to the growing plasma wave. Such current-driven plasma instabilities are well known in gaseous plasmas,[7] occuring when the drift velocity of the carriers exceeds a threshold value. The same effect takes place in bulk solid-state plasmas, but the required threshold drift velocities are very large and cannot be achieved experimentally because of low electron mobilities in bulk semiconductors.

In low-dimensional electron semiconductor systems, conditions for the onset of current-driven plasma instabilities are much more favorable. The threshold value of the drift velocity is of the order of the phase velocity of the plasma waves.[6,8,9] Softening of the plasma modes in low-dimensional systems in comparison to those of bulk systems[10] leads to a substantial reduction of the plasmon phase velocity. Given the relatively high electron mobilities in these systems and the capability of varying electron concentration over a very wide range, low-dimensional semiconductor structures offer a very attractive vehicle for the experimental realization of tunable devices based on current-driven plasma instabilities.

Various aspects of the current-driven plasma instability phenomenon, including determination of the energy transfer involved, have been considered theoretically for a number of low-dimensional semiconductor structures such as planar quantum wells and superlattices, single quantum wires as well as other systems.[6,8] Although the estimated threshold drift velocities were found to be close to and somewhat above realistic experimental possibilities, the primary obstacle to achieving a *sufficiently* low threshold drift velocity remains the higher plasmon phase velocities at the low wave vectors q used experimentally. This is a consequence of low-dimensional plasmon dispersion laws [$\omega^2 \propto q$ (2D) (Ref. 10) and $\omega^2 \propto q^2 \ln q$ (1D) (Ref. 4) at small q], with phase velocity $v_p = \omega/q$ diverging in the long wavelength limit. To avoid these difficulties, plasmon modes with lower frequencies are required.

Further softening of low-dimensional plasma excitations occurs when two parallel layers of 2D electron gas or two parallel wires of 1D electron gas are brought close to each other.[4,11] These systems have been realized experimentally in double quantum wells of 2D electron gas[12] and, very recently, in double quantum wire structures.[13,14] Coulomb interaction between degenerate plasmon modes in the individual wells or wires splits the modes into high-frequency optical and low-frequency acoustic branches. Such an acoustic branch is a very promising candidate to accommodate plasma instability. Current-driven plasma instability in a double-layer system of 2D electron gas was considered in Ref. 9: in hydrodynamic approximation the threshold of instability was found to depend on separation of the layers, and under appropriate conditions (low electron concentration, small separation between the layers) it may approach experimentally achievable values.

In this paper, we consider current-driven plasma instability in a double-quantum-wire system, in which two parallel quantum wires are coupled by Coulomb forces, but electron tunneling between them is taken to be negligible. We assume that the length of the wires is comparable with the elastic mean-free path for electrons, so the conditions of quasiballistic transport are met in both wires. Within the framework of this model, we examine current-driven plasma instability when the individual wires carry different steady currents.

Our calculations show that, for a given wave vector q, plasmon instability arises when the drift velocity of the carriers $v_{\rm dr}$ lies in the interval $v_{p-}(q)<v_{\rm dr}<v_{p+}(q)$, where $v_{p\pm}(q)$ are the phase velocities of the optical $(+)$ and acoustic $(-)$ plasmon modes of the coupled quantum wires. The instability problem is also examined for a lateral double-quantum-wire superlattice consisting of two sublattices, shifted with respect to each other in the transverse direction and carrying different steady currents. We analyze dependence of the instability region on quantum wire characteristics and geometry of the system.

To obtain the collective modes in a double-quantum-wire system with steady currents, we employ the standard RPA formulation of longitudinal plasmon dispersion.[15] Within this approach, the dispersion equation for plasmons in a two-component confined electron plasma was previously derived for isolated 2D electron layers[16] and 1D quantum wires[4,17] with the inclusion of two subbands, as well as for two spatially separated Coulomb-coupled parallel 2D electron layers[11] and parallel 1D quantum wires.[4] The generalization of these results for a double-quantum-wire system with steady currents is rather straightforward. The presence of currents only changes the electron distribution functions of the wires. For a system of two identical parallel wires with only their lowest lateral energy subbands, E_1, involved (higher lateral energy subbands are taken to be energetically inaccessible) and no interwire tunneling, the dispersion equation for coupled intrasubband plasmons has the familiar form[4]

$$\left(1-\frac{2e^2}{\varepsilon_b}v_{11}\Pi_1\right)\left(1-\frac{2e^2}{\varepsilon_b}v_{11}\Pi_2\right)-\left(\frac{2e^2}{\varepsilon_b}\right)^2 v_{12}^2\Pi_1\Pi_2=0, \tag{1}$$

where

$$\Pi_i(q_x,\omega)=\int\frac{dk_x}{2\pi}\frac{f_i(E_{1k_x-q_x})-f_i(E_{1k_x})}{\hbar\omega+E_{1k_x-q_x}-E_{1k_x}}, \tag{2}$$

and

$$v_{11}(q_x)=\int dy\,dy'\,\phi_1^2(y)\phi_1^2(y')K_0(q_x|y-y'|), \tag{3a}$$

$$v_{12}(q_x)=\int dy\,dy'\,\phi_1^2(y)\phi_1^2(y'-d)K_0(q_x|y-y'|). \tag{3b}$$

In these equations the x axis is taken in the wire direction. Both wires are positioned in the plane $z=0$ and are separated by distance d. The wires are assumed to have finite width in the y direction and zero thickness in the z direction. The electron energy eigenvalues are $E_{1k_x}=E_1+\hbar^2k_x^2/2m^*$ and the corresponding unperturbed energy eigenfunctions of the ith wire ($i=1,2$) are $\psi_{ik_x}=(1/\sqrt{L_x})e^{ik_xx}\phi_1(y-d_i)\chi(z)$ with $\chi^2(z)=\delta(z)$ $(d_1=0,d_2=d)$; $f_i(E)$ is the electron distribution function in the ith wire, ε_b is the background dielectric constant, and $K_0(x)$ is the zeroth-order modified Bessel function of the second kind that arises from the x Fourier transform of the Coulomb potential.

Addressing a weakly nonequilibrium situation with steady currents, we denote the drift velocity in the ith wire by $v_{\rm dr}^{(i)}$ and assume the unperturbed electron distribution in the ith wire to be a drifted (equilibrium) Fermi function, $f_i=f_0(k_x-m^*v_{\rm dr}^{(i)}/\hbar)$. This approximation is justified in the quasiballistic regime when electrons in the wires move in an environment nearly free of scattering, with the drift velocity determined by an external circuit.[18] The polarizability $\Pi_i(q_x,\omega)$ at zero temperature is evaluated as

$$\Pi_i(q_x,\omega)=\frac{m^*}{\pi\hbar^2q_x}\ln\frac{(\omega-q_xv_{\rm dr}^{(i)})^2-(E_{q_x}-q_xv_F)^2}{(\omega-q_xv_{\rm dr}^{(i)})^2-(E_{q_x}+q_xv_F)^2}, \tag{4}$$

where $E_{q_x}=\hbar q_x^2/2m^*$ and $v_F=\hbar k_F/m^*$ is the electron Fermi velocity. In the semiclassical limit $(q_x\ll k_F)$ we have,

$$\Pi_i(q_x,\omega)=\frac{n}{m^*}\frac{q_x^2}{(\omega-q_xv_{\rm dr}^{(i)})^2-q_x^2v_F^2}, \tag{5}$$

where $n=2k_F/\pi$ is the linear electron density in the wires. Substituting Eq. (5) into Eq. (1) we obtain the following dispersion equation:

$$\begin{aligned}&[(\omega-q_xv_{\rm dr}^{(1)})^2-q_x^2v_F^2][(\omega-q_xv_{\rm dr}^{(2)})^2-q_x^2v_F^2]\\&\quad-v_{11}u_0^2q_x^2[(\omega-q_xv_{\rm dr}^{(1)})^2+(\omega-q_xv_{\rm dr}^{(2)})^2-2q_x^2v_F^2]\\&\quad+(v_{11}^2-v_{12}^2)u_0^4q_x^4=0,\end{aligned} \tag{6}$$

where $u_0^2=2ne^2/m^*\varepsilon_b$. With the new variable $x=\omega-q_x(v_{\rm dr}^{(1)}+v_{\rm dr}^{(2)})/2$, Eq. (6) reduces to a biquadratic equation with the solution

$$x_\pm^2=q_x^2\overline{v}_{\rm dr}^2+q_x^2(v_F^2+v_{11}u_0^2)\pm q_x^2\sqrt{4\overline{v}_{\rm dr}^2(v_F^2+v_{11}u_0^2)+v_{12}^2u_0^4}, \tag{7}$$

where $\overline{v}_{\rm dr}=|v_{\rm dr}^{(1)}-v_{\rm dr}^{(2)}|/2$. If $v_{\rm dr}^{(1)}=v_{\rm dr}^{(2)}=0$, Eq. (7) yields known double-wire dispersion relations[4] for optical (ω_+) and acoustic (ω_-) plasmon branches as

$$\omega_\pm^2=q_x^2[v_F^2+(v_{11}\pm v_{12})u_0^2]. \tag{8}$$

It should be pointed out that these plasma modes are not Landau damped, in consequence of Eq. (8). Instability arises when $x_\pm^2<0$. Using Eq. (7) one can easily show that this inequality is satisfied for x_- when

$$\sqrt{v_F^2+(v_{11}-v_{12})u_0^2}<\overline{v}_{\rm dr}<\sqrt{v_F^2+(v_{11}+v_{12})u_0^2}. \tag{9}$$

Equation (9) determines the region of current-driven instability for double-wire plasma waves. When both wires carry the same current in opposite directions, i.e., $v_{\rm dr}^{(1)}=-v_{\rm dr}^{(2)}\equiv v_{\rm dr}$ we have

$$v_{p-}(q_x)<v_{\rm dr}<v_{p+}(q_x), \tag{10}$$

where $v_{p\pm}=\omega_\pm(q_x)/q_x$ are the phase velocities of the optical and acoustic plasmon branches given by Eq. (8).

These results for the double-quantum-wire system can be easily generalized to the case of a lateral double-quantum-wire superlattice (in x-y plane) consisting of two sublattices, each of period a, shifted with respect to each other by distance d in the transverse y direction. As above, the wires are along the x axis. We take the sublattices to carry different steady currents and neglect tunneling. The energy eigenfunctions for electrons in the lowest lateral subband in the wires can be constructed in the form of Bloch combinations for each sublattice as

$$\psi_{i\mathbf{k}}(\vec{\mathbf{r}})=\frac{e^{ik_x x}}{\sqrt{L_x}}\frac{1}{\sqrt{N}}\sum_{j=-N/2}^{N/2}e^{ik_y ja}\phi_1(y-d_i-ja)\chi(z). \qquad (11)$$

Here $i=1,2$ is the sublattice index, k_y is y component of the electron wave vector $\mathbf{k}, -\pi/a<k_y<\pi/a$; $N=L_y/a$ is the number of quantum wires in each sublattice with periodic boundary conditions. All other notations are the same as above.

Using the wave functions of Eq. (11) as a basis set, the dispersion equation for plasma modes in this system can be derived in a manner completely similar to that of the double-quantum-wire system.[4,5,19] The result is still given by Eq. (1), with only a superlattice modification of the Coulomb matrix elements.[4] These matrix elements of Eq. (1) are to be replaced as follows:

$$v_{11}\to\overline{v}_{11}(q_x,q_y)=\int dy dy'\,\phi_1^2(y)\phi_1^2(y')F(q_x,q_y,y-y'), \qquad (12a)$$

$$v_{12}\to|\overline{v}_{12}(q_x,q_y)|=\left|\int dy dy'\,\phi_1^2(y)\phi_1^2(y'-d)\times F(q_x,q_y,y-y')\right|, \qquad (12b)$$

where

$$F(q_x,q_y,y-y')=\frac{\pi}{a}\sum_{l=-\infty}^{\infty}\frac{e^{-i(q_y+\frac{2\pi l}{a})(y-y')}}{\sqrt{q_x^2+\left(q_y+\frac{2\pi l}{a}\right)^2}}. \qquad (13)$$

Employing Poisson's summation formula in Eq. (13), Eq. (12) take the form

$$\overline{v}_{11}(q_x,q_y)=v_{11}(q_x)+2\sum_{m=1}^{\infty}\int dy dy'\,\phi_1^2(y)\phi_1^2(y')\times K_0(q_x|y-y'+ma|)\cos mq_y a, \qquad (14a)$$

$$\overline{v}_{12}(q_x,q_y)=v_{12}(q_x)+\sum_{m=1}^{\infty}\int dy dy'\,\phi_1^2(y)\phi_1^2(y'-d)\times[e^{imq_y a}K_0(q_x|y-y'+ma|)+e^{-imq_y a}\times K_0(q_x|y-y'-ma|)]. \qquad (14b)$$

[It is readily verified from Eqs. (14) that $\overline{v}_{11}\to v_{11}$ and $\overline{v}_{12}\to v_{12}$ when $a\to\infty$.] The region of instability is determined by an inequality analogous to Eq. (9),

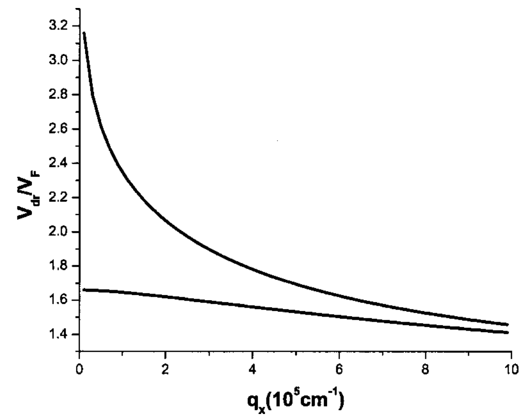

FIG. 1. Drift velocity boundaries of the instability region for a double-quantum-wire system as a function of q_x at $d=300$ Å Here $n=8.5\times10^5$ cm^{-1}, $m^*=0.067m_e$, and $\varepsilon_b=13$.

$$\sqrt{v_F^2+(\overline{v}_{11}-|\overline{v}_{12}|)u_0^2}<\overline{v}_{\rm dr}<\sqrt{v_F^2+(\overline{v}_{11}+|\overline{v}_{12}|)u_0^2}. \qquad (15)$$

Equations (9), (10), and (15) defining the regions of current-driven plasma instability for a double-quantum-wire system and a double-quantum-wire superlattice are the main results of this paper. For simplicity, we consider the situation when both wires (sublattices) carry the same current in opposite directions. In this case the instability condition for a double-quantum-wire system is given by Eq. (10), i.e., instability occurs when $v_{\rm dr}$ lies between the phase velocities of the acoustic and optical plasmons for a given q_x. A similar instability condition was derived for a planar double 2D layer system in Ref. 9. Since the phase velocity of a 1D plasmon—in particular, the 1D acoustic plasmon which determines the threshold drift velocity for instability at given q_x—is much less than its 2D counterpart, the condition for the onset of instability is more favorable in a double-quantum-wire system than in a planar double 2D layer system.

To illustrate our results, we consider a double-quantum-wire system and rewrite Eq. (10) in the following form:

$$\sqrt{1+(v_{11}-v_{12})r_s}<v_{\rm dr}/v_F<\sqrt{1+(v_{11}+v_{12})r_s}, \qquad (16)$$

where $r_s=4m^*e^2/\pi\hbar^2\varepsilon_b k_F$ is a dimensionless electron gas density parameter. The lower and upper boundaries of the instability region in Eq. (16) are shown as functions of q_x in Fig. 1. In this numerical calculation the Coulomb matrix elements of Eqs. (3) were evaluated assuming harmonic-oscillator wave functions due to the confinement in the y direction. The wire width b, defined as full width at half maximum of the ground-state harmonic-oscillator wave function, was taken to be 150 Å. Other parameters were taken as $n=8.5\times10^5$ cm^{-1}, $d=300$ Å, $m^*=0.067m_e$, $\varepsilon_b=13$. For these parameters, $r_s\approx0.93, E_F\approx10$ meV, and the separation between the lowest lateral subbands is about 15 meV. Our results show that the boundaries of the instability region shift towards larger $v_{\rm dr}$ and the region of instability expands as q_x decreases. In the long wavelength limit ($q_x\to0$) the upper boundary diverges, going to infinity, whereas the threshold lower boundary approaches a finite limit, $v_{\rm dr}^{\rm th}$.

The value of $v_{\rm dr}^{\rm th}$ can be found from Eqs. (3) and (16). Employing the asymptotic behavior of the Bessel function, $K_0(x)\approx -\ln(x/2)-\gamma$ as $x\to 0$, where $\gamma=0.577\cdots$ is Euler constant, we obtain

$$v_{\rm dr}^{\rm th}=v_F\sqrt{1+r_s\left[\ln\left(\frac{1.67d}{b}\right)+\frac{1}{2}(\gamma+\ln 2)\right]}. \qquad (17)$$

For the parameters given above, $v_{\rm dr}^{\rm th}\approx 1.65 v_F$. If $v_{\rm dr}>v_{\rm dr}^{\rm th}$, plasmons with arbitrarily small q_x are unstable. The width of the instability region for a given q_x also depends on the separation d between the wires. When d increases the Coulomb coupling between the wires weakens [see Eq. (3b)], the frequencies of acoustic and optical plasma modes in Eq. (8) approach the same limiting value, and the width of the instability region determined by Eqs. (9) or (10) decreases.

A similar analysis is applicable to the lateral double-quantum-wire superlattice. In this case the boundaries of the instability region have an additional dependence on the wave vector q_y [Eqs. (14 and (15)]. The width of the interval of $v_{\rm dr}$—in which plasmons having a given q_x are unstable—takes its maximum value at $q_y=0$ and decreases as $q_y\to\pm\frac{\pi}{a}$. In particular, for $a=2d$ (equidistant wires), we obtain from Eq. (14b),

$$|\overline{v}_{12}(q_x,q_y)|=2\left|\sum_{m=1}^{\infty}\int dy\,dy'\,\phi_1^2(y)\,\phi_1^2(y')\times K_0[q_x|y-y'+(2m-1)d|]\times\cos(2m-1)q_y d\right|. \qquad (18)$$

When $q_y\to\pm\pi/2d$, we have $|\overline{v}_{12}|\to 0$ and the width of the instability interval reduces to zero.

In summary, we have presented a theory of current-driven plasma instability in a double-quantum-wire system, and also in the lateral double-quantum-wire superlattice. We showed that if the wires carry steady currents, the quasi-1D plasma waves propagating along the wires become unstable when the electron drift velocity falls within clearly determined limits. Double-quantum-wire plasmon instability has been shown to occur at a lower drift velocity than that of planar double 2D layer plasmon instability, because of the softening of the plasma waves in 1D as compared to 2D systems. Furthermore, the effect of a superlattice on instability is manifested in a modulation of the instability boundaries dependent on the component of the plasmon wave vector along the superlattice axis.

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