Upconversion and migration in erbium-doped silica waveguides in the continuous-wave excitation switch-off regime

S. Sergeyev,* D. Khoptyar, and B. Jaskorzynska

Royal Institute of Technology, Department of Microelectronics and Information Technology, Laboratory of Optics, Photonics and Quantumelectronics, Electrum 229, SE-164 40 Kista, Sweden

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Based on an approach for averaging solution of rate equations over possible ensemble configurations, we derive integral equation for population inversion taking into account nonlinear homogeneous upconversion and migration in Er^{3+}-doped silica waveguides for the case of 1480-nm CW excitation switch-off (CWES) regime. Because of pump for CWES is off, only migration tends to refill the holes in excitation distribution caused by upconversion. Thus the upconversion is weaker than that for the CW operation, except for $t=0$ and $t\rightarrow\infty$, upconversion.

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I. INTRODUCTION

The recent interest in Er-doped planar amplifiers is motivated by their potential use for integrated active devices in optical communication systems at the 1.5-μm transmission window. In order to obtain sufficiently large gain in a short planar device two orders of magnitude higher Er concentrations (10^{20}–10^{21} cm^{-3}) are required in comparison with a typical fiber amplifier. However, at high concentrations, parasitic interactions between Er ions are strongly enhanced.[1–5] The resulting upconversion processes reduce achievable gain and have to be properly taken into account while modeling the amplifier performance. It has been observed that the dependence of the upconversion rate (W) on the population inversion (n) is approximately linear for small population inversions ($n\rightarrow 0$) whereas it becomes nonlinear when the population inversion increases.[1–5]

Our statistical model derived in Ref. 5 well describes the nonlinear dependence of the upconversion rate on the population inversion without introducing the concept of Er^{3+} clusters,[1,3] and does not require time-consuming Monte Carlo simulations.[4] Although the short-pulse excitation and continuous-wave (CW) operation considered in Ref. 5 are generic for the theories of migration and quenching, the simplest experiments for determination of the upconversion rate in Er^{3+}-doped fibers have been done for continuous-wave excitation switch-off (CWES) regime.[2] So, the model considered in Ref. 5 should be generalized to the case of CWES.

In this paper, by means of approach considered in Ref. 5, we derive integral equation for the population inversion taking into account the upconversion and migration of excitation in Er^{3+}-doped waveguides for the case of CWES. We also compare the resulting upconversion characteristics with those obtained for CW operation[5] and the experimental results of Ref. 2.

II. THEORETICAL MODEL

We start from the rate equation derived in Ref. 5 for the case of a short-pulse (δ-pulse) excitation,

$$\frac{dn(x_1,x_2,t)}{dt}=-n(x_1,x_2,t)\left(1+\frac{k^2n(t)^2}{4x_1^2}\right)-[n(x_1,x_2,t)-n(t)]\frac{k^2r}{8x_2^2}, \qquad (1)$$

where

$$n(t)=\langle n(x_1,x_2,t)\rangle_{x_1,x_2}=\int_0^\infty\int_0^\infty n(x_1,x_2,t)f(x_1)f(x_2)dx_1dx_2.$$

Following Ref. 5, we also introduce initial conditions that correspond to the continuous-wave excitation at 1480 nm,

$$n(x_1,x_2,0)=\frac{\alpha}{1+\beta\alpha}\frac{1}{1+\frac{k^2n_0^2}{4x_1^2(1+\beta\alpha)}+\frac{k^2r}{8x_2^2(1+\beta\alpha)}}+\frac{\frac{k^2rn_0}{8x_2^2(1+\beta\alpha)}}{1+\frac{k^2n_0^2}{4x_1^2(1+\beta\alpha)}+\frac{k^2r}{8x_2^2(1+\beta\alpha)}}. \qquad (2)$$

Here $k=\sqrt{\pi}c_{\mathrm{Er}}/c_{\mathrm{up}}$, c_{Er} is the concentration of Er^{3+} ions, $c_{\mathrm{up}}=[(4\pi/3)R_{\mathrm{up}}^3]^{-1}$ is the critical concentration for the upconversion, $r=(R_m/R_{\mathrm{up}})^6$, R_{up} and R_m are the critical distances for upconversion and migration,[6,7] $\alpha=\sigma_aP_a\tau\Gamma_a/Ah\nu_a$, $\beta=(\sigma_a+\sigma_e)/\sigma_a$, σ_a and σ_e are the absorption and emission cross sections, τ is the first excited state lifetime, P_a is a pump power at frequency ν_a, Γ_a is the overlap factor between light-field mode and the erbium distribution, A is the effective area of the erbium distribution, x_i are random variables related to the migration and upconversion probabilities and described by the distribution functions[5]

$$f(x_i)=\frac{2}{\sqrt{\pi}}\exp(-x_i^2) \quad (i=1,2), \qquad (3)$$

$n_0=\langle n(x_1,x_2,0)\rangle_{x_1,x_2}$ is found from Eq. (2)

$$n_0=\frac{\alpha(n_0+\sqrt{r/2})}{1+\beta\alpha}\frac{F\left(\frac{k(n_0+\sqrt{r/2})}{2\sqrt{1+\beta\alpha}}\right)}{\left(n_0+\sqrt{r/2}F\left(\frac{k(n_0+\sqrt{r/2})}{2\sqrt{1+\beta\alpha}}\right)\right)} \tag{4}$$

with $F(x)=1-\sqrt{\pi}x\exp(x^2)\mathrm{erfc}(x)$. Thus, Eq. (1) with initial condition (2) describes the evolution of the population inversion for the cw pump switched off at $t=0$.

We find the solution of Eq. (1) and average it over variables x_1, x_2 with the distribution functions (3), which results in the following integral equation for $n(t)$:

$$n(t)=n_0\zeta(t)\exp(-t),$$

$$\zeta(t)=F_0(t)+\int_0^t\zeta(\tau)F_1(t,\tau)\frac{\partial F_2(t-\tau)}{\partial\tau}d\tau, \tag{5}$$

where

$$F_0(t)=\frac{\alpha}{(1+\beta\alpha)n_0}\int_0^\infty\exp(-y)G_1(y)G_2(y)dy-\int_0^\infty\exp(-y)G_1(y)\frac{\partial G_2(y)}{\partial y}dy,$$

$$G_1(y)=\exp\left(-kn_0\left\{\int_0^t\zeta^2(\tau)\exp(-2\tau)d\tau+\frac{y}{1+\beta\alpha}\right\}^{1/2}\right),$$

$$G_2(y)=\exp\left(-k\sqrt{r/2}\left\{t+\frac{y}{1+\beta\alpha}\right\}^{1/2}\right),$$

$$F_1(t,\tau)=\exp\left(-n_0k\sqrt{\int_\tau^t\zeta^2(\tau)\exp(-2\tau)d\tau}\right),$$

$$F_2(x)=\exp\left(-\frac{k\sqrt{r}}{\sqrt{2}}\sqrt{x}\right). \tag{6}$$

It follows from Eqs. (5) and (6) that for $t\to 0$

$$\zeta(t)=1-\left(\frac{\alpha(1-\beta n_0)}{n_0}-1\right)t+O(t^{3/2}),$$

$$W_{\mathrm{up}}=-\frac{1}{n(t)}\frac{dn(t)}{dt}-1=-\frac{1}{\zeta}\frac{\partial\zeta}{\partial t}=\frac{\alpha(1-\beta\cdot n_0)}{n_0}-1. \tag{7}$$

As expected, the upconversion asymptote (7) is equal to the CW upconversion rate obtained in Ref. 5.
To find the asymptote for $t\to\infty$, we rewrite Eq. (5) in the following form:

$$\zeta(t)=F_0(t)+\int_0^t\zeta(\tau)c(t,\tau)\frac{\partial G(t,\tau)}{\partial\tau}d\tau, \tag{8}$$

where

$$G(t,\tau)=\exp\left(-n(0)k\sqrt{\int_\tau^t\zeta^2(\tau)\exp(-2\tau)d\tau}-\frac{k\sqrt{r}}{\sqrt{2}}\sqrt{t-\tau}\right),$$

$$c(t,\tau)=\left(1+\frac{\sqrt{2}n(0)\zeta^2(\tau)\exp(-2\tau)\sqrt{t-\tau}}{\sqrt{r}\sqrt{\int_\tau^t\zeta^2(x)\exp(-2x)dx}}\right)^{-1}. \tag{9}$$

For $k\sqrt{r}\gg 1$ function $\partial G(t,\tau)/\partial\tau\to 0$ for all points except $\tau\to t$. Hence, the main contribution to the integral in Eq. (9) comes from the vicinity of $t-\tau=0$, and we can replace $c(t,\tau)$ with the first terms of its expansion at that point,

$$c(t,\tau)\approx 1-\sqrt{2/r}n(0)\zeta(\tau)\exp(-\tau). \tag{10}$$

Using Eq. (10) and the following notation for the Laplace transforms:

$$\tilde{\zeta}(p)=\int_0^\infty\zeta(t)\exp(-pt)dt,$$

$$\tilde{G}(p)=\int_0^\infty\exp\left(-n(0)k\sqrt{\int_0^t\zeta^2(\tau)\exp(-2\tau)d\tau}-\frac{k\sqrt{r}}{\sqrt{2}}\sqrt{t}\right)\exp(-pt)dt,$$

$$\tilde{\Psi}(p)=\int_0^\infty\zeta^2(t)\exp[-(p+1)t]dt, \tag{11}$$

$$D\zeta(p)=\int_0^\infty\frac{\partial\zeta(t)}{\partial t}\exp(-pt)dt,$$

$$\tilde{F}_0(p)=\int_0^\infty F_0(t)\exp(-pt)dt,$$

we find

$$D\zeta(p)=-\frac{\tilde{F}_0(p)}{\tilde{G}(p)}+1-\frac{\sqrt{2}n_0}{\sqrt{r}}\frac{1-p\tilde{G}(p)}{\tilde{G}(p)}\tilde{\Psi}(p), \tag{12}$$

where

$$\tilde{F}_0(p)=\tilde{G}(p)-\left(\frac{\alpha}{n(0)}-1-\alpha\beta\right)\times\frac{F(k\sqrt{r/[8p]})/p-F(k\sqrt{r/[8(1+\alpha\beta)]})/(1+\alpha\beta)}{(p-1-\alpha\beta)}. \tag{13}$$

The asymptotic behavior ($t\to\infty$) of a function is determined by the pole of its Laplace image with a largest real part.[8] For $t\to\infty$ the number of excited ions decreases, so the upconversion is slowing down and $\zeta(t)\to$const. As it follows from Eq. (11), $\tilde{\Psi}(p)$ has simple pole $p_0=-1$. Thus for the asymptotic behavior of $D\tilde{\xi}(p)/\tilde{\Psi}(p)$, we have, for $k\sqrt{r}\gg 1$,

$$\lim_{p\to -1} \frac{D\tilde{\xi}(p)}{\tilde{\Psi}(p)} \to \frac{\sqrt{2}n_0}{\sqrt{r}} \left[\frac{1-pG(p)}{G(p)} \right] \Bigg|_{p=-1} \propto \frac{n_0 k^2 \sqrt{r}}{2\sqrt{2}}. \tag{14}$$

Thus, we find from Eqs. (11) and (14) the following equation in the time domain:

$$\frac{d\zeta(t)}{dt} = -\frac{n_0 k^2 \sqrt{r}}{2\sqrt{2}} \zeta^2(t) \exp(-t). \tag{15}$$

From Eq. (15) we finally obtain the asymptotic form of the upconversion rate $W_{\rm up}$ for $t\to\infty$ and high Er^{3+} concentration $(k\sqrt{r} \gg 1)$,

$$W_{\rm up}^{\rm CWES} = -\frac{1}{n(t)} \frac{dn(t)}{dt} - 1 = -\frac{\partial\zeta(t)}{\partial t} \frac{1}{\zeta(t)} = \frac{k^2\sqrt{r}}{2\sqrt{2}} n(t). \tag{16}$$

Comparison with the results of Ref. 5 obtained for the limit $n\to 0$ shows that the asymptotic behavior of the upconversion rate for vanishing population inversion ($n\to 0$ or $t\to\infty$) is the same for both the CW and CWES regimes.

Based on a numerical solution of Eqs. (5) we find upconversion rate as function of population inversion for CWES regime. Numerical data, as well as data for CW (Ref. 5) and experimental results for CWES,[2] are shown in Fig. 1.

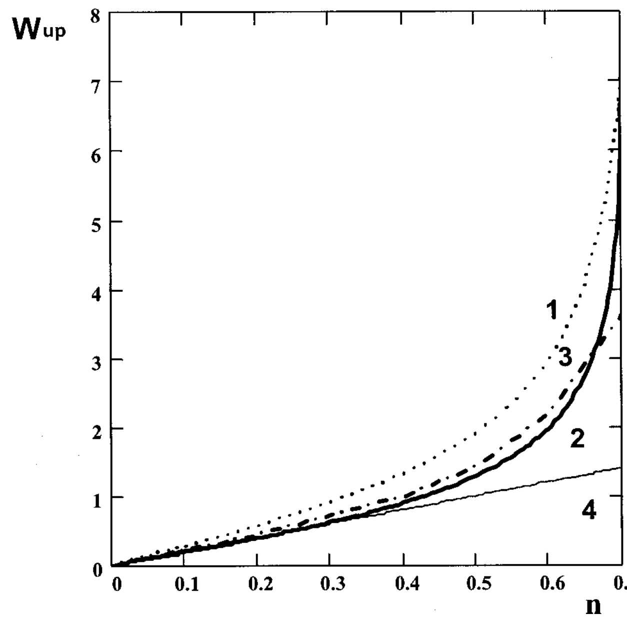

FIG. 1. Normalized upconversion rate $W_{\rm up}$ as a function of the population inversion n for (1) CW regime (calculated in Ref. 5); (2) CW excitation switch-off regime [calculated from Eqs. (5)]; (3) measured in Ref. 2; (4) a linear asymptote for $n\to 0$ [Eq. (16)]. $R_{\rm up}=11.2$ Å, $r=60$ (1,2,4), $c_{\rm Er}=8.6\times 10^{25}$ ions/m^3 (1–4), $\tau = 11$ ms. $W_{\rm up}$, n are dimensionless values.

III. DISCUSSION

The main result of this paper is the generalization of our model[5] to the case of CW excitation switch-off regime. As it follows from Fig. 1, numerically and analytically obtained data are in good correspondence with experimental data from Ref. 2.

The obtained results can be interpreted as follows. Upconversion is strongly dependent on the distance between the excited ions. Hence, for high population inversion each of the excited ions is upconverted by a different ensemble of surrounding excited ions in the following way: the denser the surrounding is, the higher the rate of upconversion will be. As a result, the rate of upconversion after averaging over possible ensemble configuration is not proportional to the population inversion. Upconversion process burns holes in the excitation distribution where the ions are densest spaced. For the CW operation both the pump and migration tend to refill the holes, thus enhancing the overall upconversion rate. For the switch-off case only the migration effect is left and hence the upconversion is weaker (Fig. 1, curves 1 and 2). For small population inversion, however, the influence of the pump on the upconversion is weak for both CW and CWES regimes. Hence, for both regimes, upconversion is accelerated mainly by migration. The last one leads to equalization of different ensembles, and the averaged value of upconversion would be proportional to the concentration of excited ions (compare curves 1, 2, and 4). Based on fitting results of experiments from Ref. 2 we have found the critical distance of upconversion of $R_{\rm up}=11.2$ Å. The deviation of the theoretical curve from experimental one for the region of population inversion $n>0.65$ can be explained by the low accuracy in the measurement of population inversion and upconversion rate for the saturation case. In experiments from Ref. 2, decays were sampled by the oscilloscope with a sampling time of 50 μs. For high erbium concentrations, upconversion rate decreases substantially during first 50 μs after pump power blocking by chopper. As a result, the rates of upconversion for population inversions close to the maximum value of 0.7 are underestimated.

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*Present address: Ericsson Telecom AB. Optical Networks Research Lab. SE-12625, Stockholm, Sweden.

[1] R. S. Quimby, W. J. Miniscalo, and B. Thompson, J. Appl. Phys. **76**, 4472 (1994).

[2] J. L. Philipsen, J. Broeng, A. Bjarklev, S. Helmfrid, D. Bremberg, B. Jaskorzynska, and B. Palsdonir, IEEE J. Quantum Electron. **35**, 1741 (1999).

[3] J. Nilsson, P. Blixt, B. Jaskorzynska, and J. Babonas, J. Light-

wave Technol. **13**, 341 (1995).
[4] J. L. Philipsen and A. Bjarklev, IEEE J. Quantum Electron. **33**, 845 (1997).
[5] S. V. Sergeyev and B. Jaskorzynska, Phys. Rev. B **62**, 15 628 (2000).
[6] M. Hemstead, J. E. Roman, C. C. Ye, J. S. Wilkinson, P. Camy, P. Laborde, and C. Lerminiaux, in *Proceedings of the 7th European Conference on Integrated Optics* (Delft University Press, Delft, 1995), p. 233.
[7] J. E. Roman, C. Ye, and M. Hemstead, Appl. Phys. Lett. **67**, 470 (1995).
[8] H. S. Carslaw, J. C. Jaeger, *Operational Methods in Applied Mathematics* (Oxford University Press, Cambridge, 1953) p. 279.