Vortex pinning by large normal particles in high-T_c superconductors

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We present a model of flux pinning on large nonsuperconducting particles. It is assumed that the pins trap different numbers of vortices determined by the Poisson's distribution function. This approach brings an additional factor into the expression for pinning force density that exponentially decays with field. The critical current density thus becomes at low fields nearly pure exponential function of magnetic field, in agreement with numerous experiments on high-T_c superconductors.

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It has been experimentally proved that relatively large (micron-size) RE_2BaCuO_5 (RE-211, secondary phase) particles contribute to flux pinning in (RE)$Ba_2Cu_3O_{7-\delta}$ (RE =rare earth, RE-123) superconductors and enhance superconducting currents, especially at low magnetic fields.[1,2] Vortex lattice interaction with large (volume) pins has been treated in Ref. 3 where the authors used direct summation of elementary pinning forces and found $J_c(B)\propto b^{-1/2}(1-b)$ with $b=B/B_{c2}$. For core pinning by large normal pins Dew Hughes[4] deduced $J_c(B)\propto b^{-1}(1-b)^2$. Murakami *et al.*[5,6] suggested the concept of *surface pinning* assuming volume pinning defects of a cubic shape. Their model predicted $J_c(B)\propto b^{-1/2}$.

All the above models indicated a strong pinning at low magnetic fields, in a qualitative agreement with experiment. A quantitative comparison is not easy. Let us name at least one obvious reason; at low fields the "true" critical current density is not simply proportional to the irreversible magnetic moment[7,8] $M(B)$ and decays with field faster than $M(B)$.[8] Moreover, as reported by Jirsa *et al.*,[9] even the $J_c(B)$ deduced from $M(B)$ using the extended Bean model[7] decays at low fields (central peak region) much faster than with $B^{-1/2}$ or B^{-1}. It was shown that the exponentially decaying function found by Kobayashi *et al.*[10] for $(LaSr)_2CuO_4$ single crystals describes well also the central peak shape in most RE-123 compounds.

In all the above-mentioned theories magnetic flux was supposed to be homogeneously trapped by normal inclusions over the whole sample. Under this assumption, a direct summation of the elementary pinning forces consisted in multiplying the elementary pinning force by the defect concentration. Here we assume that due to magnetic history and irregular distribution of pins and vortices in real samples, the same inclusions can keep under same external conditions different numbers of vortices.

In this work we treat the interaction of an elastic vortex lattice with defects that are large compared to vortex core size ξ. Taking into account stochastic character of the vortex trapping by large inclusions, we calculate and discuss the associated pinning force.

We will suppose a lattice of elastic vortices that can adapt to the structure of large normal defects of the mean radius R, randomly distributed over the sample. The vortices crossing a normal defect lose part of their condensation energy, proportional to the trapped volume $\pi\xi^2 z_1$,

$$W_p=\frac{\mu_0 H_c^2}{2}\pi\xi^2 z_1, \tag{1}$$

where ξ is the core radius and z_1 is the length of the vortex trapped by a normal particle in z direction; H_c is the thermodynamic critical field, and $\mu_0 H_c^2/2$ is the condensation energy density. As $\xi \ll R$, more than one vortex can be trapped by one defect. For the trapped vortex pinning energy can be written as

$$W_{pi}=\mu_0 H_c^2 \pi\xi^2\sqrt{R^2-r_i^2}, \tag{2}$$

where r_i is the distance of the vortex line from the sphere center. The largest pinning energy gain is naturally for the vortex crossing the sphere in its center. However, this is possible only at relatively low fields when the vortex lattice constant is larger than R. In the opposite case, more vortices are trapped by the same defect and due to their repulsion they are more or less regularly distributed over the sphere cross section (top view). The total pinning energy of the vortices trapped by one normal sphere is

$$W_p(x_1)=\mu_0 H_c^2\pi\xi^2\sum_{i=1}^{m}\sqrt{R^2-(x_1+x_i)^2-y_i^2}, \tag{3}$$

here x_i and y_i are the vortex line coordinates in the coordinate system having the origin $\{x_1,0\}$ in the geometrical center of the bundle, and m is the total number of vortices intersecting the sphere. The total energy of the vortex bundle is less by $W_p(x_1)$. On the other hand, the vortices are subject to the Lorentz force that causes an increase in the elastic energy of the bundle.

We assume that near the sphere the vortex lines tilt to adopt to the pinning landscape, as illustrated in Fig. 1. This tilt occurs in the distortion volume V_d and is accompanied by the increase in the elastic energy

$$W_{el}=\frac{c_{44}\epsilon_{xz}^2}{2}V_d, \tag{4}$$

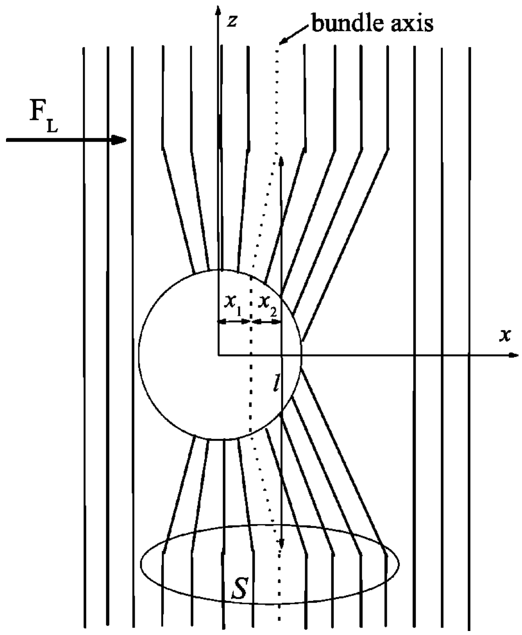

FIG. 1. Vortex displacements inside and out a large spherical defect.

where $\epsilon_{xz} = \sin\gamma$ and γ is the tilt angle. The distortion volume is $V_d = Sl\cos\gamma$, where S is the top cross section of the undistorted bundle (in the distance $l/2$ from the defect center) and l is the length of the tilted vortex bundle as indicated in Fig. 1. We denote the elastic displacement of the central vortex as x_2 and assume $x_2/l \ll 1$, so that $\epsilon_{xz} \approx x_2/l$. The elastic energy can be now expressed as

$$W_{el} = \frac{2c_{44}Sx_2^2}{l}. \tag{5}$$

The energy associated with the Lorentz force is

$$W_L = -BJSl\left(x_1 + \frac{x_2}{2}\right). \tag{6}$$

Both energies (5) and (6) increase the total energy of the pinned vortex bundle.

The equilibrium position of the vortex bundle (coordinates x_1 and x_2) is obtained by minimizing the total energy

$$W = W_p(x_1) + W_{el}(x_2) + W_L(x_1, x_2) \tag{7}$$

with respect to x_1 and x_2,

$$\sum_{i=1}^{m} \frac{(\eta_1 + \eta_i)}{\sqrt{1 - (\eta_1 + \eta_i)^2 - \varsigma_i^2}} = \frac{BJSl}{\mu_0 H_c^2 \pi \xi^2}, \tag{8}$$

$$\eta_2 = \frac{BJl^2}{8c_{44}R}, \tag{9}$$

where $\eta = x/R$ and $\zeta = y/R$.

In the single-vortex pinning regime $m=1$, $\eta_1 = \zeta_1 = 0$. The critical current density can be calculated from Eq. (8) taking into account that the maximum pinning force is reached at $x_1 = x_c = \pm|R - x_i|$.

The left side of Eq. (8) represents the pinning force acting on all vortices trapped by the sphere, normalized to $\mu_0 \pi H_c^2 \xi^2$. The total pinning force is the sum of forces caused by individual spheres and depends on the statistical distribution of vortices captured by these spheres. Thus, for a sample with N_0 normal spheres one can rewrite Eq. (8) as

$$J_c B = F(B) = \mu_0 H_c^2 \pi \xi^2 \sum_{m=0}^{m_{\max}} \varphi(m) n(m), \tag{10}$$

$$\varphi(m) = \sum_{i=1}^{m} \frac{(\eta_c + \eta_i)}{\sqrt{1 - (\eta_c + \eta_i)^2 - \varsigma_i^2}}, \tag{11}$$

where $n(m)$ is the concentration of spheres keeping m vortices, and $m_{\max}$ is the maximum possible number of vortices captured by a sphere of the given radius. Herein we suppose the maximum possible shift of the bundle center $x_1 = x_c = |R - R_b - \xi|$, where R_b is the bundle radius. As was explained above, this shift corresponds to the maximum value of the pinning force.

For a given value of magnetic field one can calculate the probability P_{m} for m vortices to be captured by one sphere. We can consider the vortices to independently occupy defects if the vortex-pin interaction is significantly larger than that of the vortex-vortex interaction. In low and moderate fields the probabilities should thus obey the Poisson's distribution

$$P_m = \frac{a^m}{m!} e^{-a}, \tag{12}$$

where a is the parameter of the distribution. For $B||c$ axis and perpendicular to the slab surface this parameter is $a = S_d \rho = S_d B/\Phi_0$, where S_d is the defect cross section perpendicular to vortex line, $\rho = 2/(a_0^2 3^{1/2})$ is the local vortex lattice density, and a_0 is the vortex lattice constant. For a spherical shape, $S_d = \pi R^2$. With these options, the probability to find m vortices trapped by the sphere of radius R at the internal field B is

$$P_m(B) = \frac{1}{m!} \left(\frac{\pi R^2 B}{\Phi_0}\right)^m \exp\left(-\frac{\pi R^2 B}{\Phi_0}\right). \tag{13}$$

For a real sample in critical state, both the local magnetic field and current density depend on coordinates. Further, we will operate with these quantities averaged over the sample volume. Figure 2(a) shows the field dependence of probability (13) for $m=1$, 4, and 8 and two different values of R. With increasing m the probability maximum decreases and shifts to higher fields. In Fig. 2(b) the probability to trap 50 vortices is plotted as a function of the defect size for two values of magnetic induction, 0.5 and 2 T. In low fields, the larger spheres are evidently more effective than smaller ones.

By definition, the probability (12) can be rewritten as $P_m = n(m)/n_0$, where $n_0 = N_0/V$ is the total concentration of

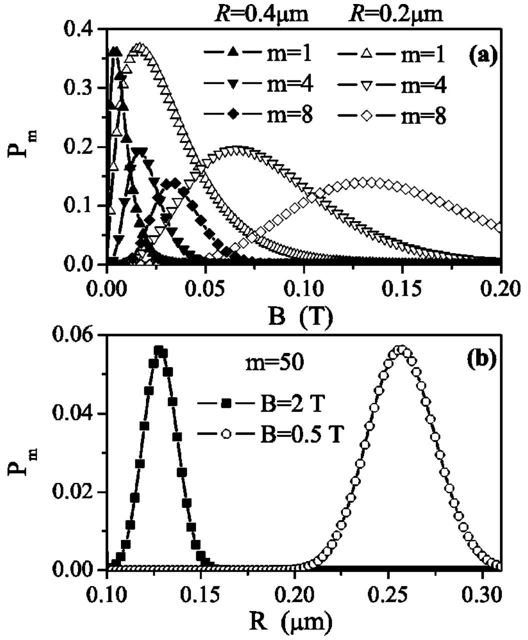

FIG. 2. (a) Field dependence of the probability to catch m vortices in a normal sphere of the radius $R=0.2$ and 0.4 μm for $m=1$, $m=4$, and $m=8$; (b) Probability to catch $m=50$ vortices at $B=2$ T and $B=0.5$ T as a function of the sphere radius.

the sample defects. It allows us to write the volume density of the total pinning force in the form

$$F(B)=F_c n_0 \sum_{m=0}^{m_{\max}} \varphi(m) P_m(B), \tag{14}$$

where $\varphi(m)$ is determined by Eq. (11), $\varphi(0)=0$, and $F_c=\mu_0 H_c^2 \pi \xi^2$. From Eqs. (13) and (14) we obtain a general expression for the pinning force density,

$$F(B)=F_c n_0 \frac{\pi R^2 B}{\Phi_0} \exp\left(-\frac{\pi R^2 B}{\Phi_0}\right) \times \sum_{m=1}^{m_{\max}} \varphi(m) \frac{1}{m!} \left(\frac{\pi R^2 B}{\Phi_0}\right)^{m-1}. \tag{15}$$

Equation (15) represents the total pinning force density for a random distribution of normal particles of radius R. Evidently, $\Phi_0/\pi R^2$ is a field scaling factor here and we denote it B_1. For a general shape of the particle, one can use $B_1=\Phi_0/S_d$. Equation (15) then simplifies to

$$F(B)=F_c n_0 b \exp(-b)\left(\varphi(1)+\frac{\varphi(2)}{2!} b+\frac{\varphi(3)}{3!} b^2+\cdots\right), \tag{16}$$

where $b=B/B_1$. We see that the total pinning force (15) or (16) is a power series in b the coefficients of which are determined by Eq. (11).

In Ref. 11 the authors successfully fitted the experimental $J_c(B)$ dependences by the expression

$$J_c(B)=J_{c1} \exp(-\gamma B)+J_{c2}(B/B_p)\exp[\{1-(B/B_p)^n\}/n], \tag{17}$$

where J_{c1}, J_{c2} were magnitudes of the central and second peak, respectively, n and γ were coefficients, and B_p was the second peak position. It is obvious that for low fields expressions (16) and (17) become equivalent.

In the limiting case $b \to 0$ we obtain

$$J_c(0)=\mu_0 H_c^2 \pi^2 \xi^{3/2} n_0 \frac{R^{5/2}}{\Phi_0 \sqrt{2}}, \tag{18}$$

where $\varphi(1) \approx [R/(2\xi)]^{1/2}$, cf. Eq. (11). Substituting for H_c $[=\Phi_0/(\sqrt{8}\pi \xi \mu_0 \lambda)]$, we get

$$J_c(0)=\frac{\Phi_0 n_0 R^{5/2}}{8 \mu_0 \lambda^2 \sqrt{2\xi}}=\frac{3 \Phi_0 V_{\mathrm{f}}}{32 \pi \mu_0 \lambda^2 \sqrt{2 R \xi}}, \tag{19}$$

where $V_{\mathrm{f}}=n_0(4/3)\pi R^3$ is the pinning phase volume fraction. The present model thus gives a finite value of critical current density for $B \to 0$ [unlike the previous models where $J_c(B)$ diverged]. We can see that $J_c(0)$ is proportional to $V_{\mathrm{f}}/\sqrt{R}$, instead of V_{f}/R.[3–5] This is a direct consequence of the spherical shape of the pins. For different defect geometries the dependence will differ.

Setting $\lambda=150$ nm, $\xi=1.2$ nm, the mean particle radius $R=0.3$ μm, and $n_0=(0.44\text{–}3.5)10^{18}$ m^{-3} (corresponding to $V_{\mathrm{f}}=0.05\text{–}0.4$), we get from Eq. (19) $J_c(0)$ in the range of $10^9\text{–}10^{10}$ A/m^2, in accord with the low-temperature values observed in melt-textured RE-123 samples[12] [Eq. (19) applies for zero temperature]. We should bear in mind that Eq. (19) is not exactly remanent critical current density as $b \neq 0$ at the zero *applied* field. The above comparison is therefore only approximate.

It would be useful to estimate maximum number of vortices trapped by one particle at a given internal field. As a criterion one can take the limit when the local mean field around the defect becomes less than the field inside the trapped vortex bundle. The mean field inside the bundle of m trapped vortices is $B_d=m\Phi_0/S_d$. In the framework of Bean model the local internal field in the remanent state varies linearly between zero and the full penetration field, B_p. Thus, the maximum number of pinned vortices is defined by $B_d \leqslant B$ or $m \leqslant B S_d/\Phi_0$. For $B_p=1$ T and defect size, e.g., $R=0.4$ μm the inequality provides maximum values of m at a remanant state to vary between $m_{max}=0$ (for $B=0$) and 240 (for B_p). In general, the relation between local internal field and applied field is affected by the sample magnetic history, sample and experiment geometries, surface barriers, pinning parameters, and other circumstances. Therefore, the estimation of m_{max} as a function of the applied field is not simple.

To simplify Eq. (16), we introduce the pinning force $\varphi(\langle m \rangle)$ of a sphere that pins the average number of vortices, $\langle m \rangle$. For the Poisson's distribution $\langle m \rangle = a = \pi R^2 B/\Phi_0$, which allows one to change the argument of the φ function so that

$$\varphi(B)=\varphi(\langle m\rangle)=\sum_{i=1}^{\langle m\rangle}\frac{(\eta_c+\eta_i)}{\sqrt{1-(\eta_c+\eta_i)^2-\varsigma_i^2}}. \qquad (20)$$

Expression (20) can be associated with the sum in the brackets of Eq. (16). In this case, the prefactor $n_0 b\exp(-b)$ in Eq. (16) should be replaced by function $n(B)$, representing the mean concentration of spheres containing $\langle m\rangle$ vortices. Thus the total pinning force density will be proportional to $n(B)\varphi(B)$,

$$F(B)=\mu_0 H_c^2 \pi \xi^2 n(B)\varphi(B) \qquad (21)$$

with $n(B)\propto n_0 b^k \exp(-b)$, where $k\geqslant 1$ is a numerical parameter.

We found that for spherical defects $\varphi(B)$ can be very well approximated by $\varphi(B)=c_1 R\sqrt[4]{3}[B/(2\Phi_0)]^{1/2}$ ($c_1=2.01736$). It is worth noting that for $n(B)=\text{const}$, as supposed in previous theories, Eq. (21) gives[5] $J_c(B)\propto 1/\sqrt{B}$.

In general, the $n(B)$ function reflects the magnetic and thermal history of the sample. Several different types of $n(B)$ functions were suggested for pinning by columnar defects in Refs. 13–15. For another defect geometry one should use a properly modified $\varphi(m)$ function.

In summary, it was shown that the varying number of vortices trapped at low fields by normal particles brings into the expression for total pinning force density a new factor, exponentially decaying with field. The $F(B)$ dependence can be expressed as a product of this exponential term and a power series in B. For low fields the present model justifies the previously found empirical exponentially decaying $J_c(B)$ dependence.[9,11] The critical current density for $B\rightarrow 0$ is in very good accord with low-temperature experiments on melt-textured RE-123 materials doped by secondary phase particles.

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