Weakly interacting ferromagnetic chains in a field

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The linear renormalization-group transformation is used to study the temperature dependence of the specific heat and susceptibility of the weakly interacting spin chains in a field (h). It has been found that for the coupled Ising chains the shift of the susceptibility maximum under the longitudinal field can be fitted satisfactory to a power-law with exponent (υ) close to 2/3 in a broad range of the field. It is not the case for the Ising spin chains coupled only by four spin interactions where deviation from a single power law is clear and υ changes from 0.39 for small field to 0.68 for higher field. The transition temperature of the uniaxial Heisenberg ferromagnet in the field perpendicular to the easy axis is also found. It is shown that only for very small fields and the anisotropy strong enough this temperature is shifted according to h^2 as predicted within mean-field-approximation.

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I. INTRODUCTION

Quasi-one-dimensional magnets in which the interchain interactions (J') are much weaker than intrachain interactions (J), are usually treated as really one-dimensional spin systems. This is justified at higher temperatures, however, at low temperatures the weak interchain interaction may be responsible for a magnetic ordering and has to be taken into consideration. Among the many quasi-one-dimensional ferromagnets studied so far, one finds systems which can be considered as a $s=1/2$ anisotropic Heisenberg model—$(C_6H_{11}NH_3)CuBr_3$ (CHAB) (Ref. 1); isotropic Heisenberg model—(p-CDTV) (Ref. 2) or Ising model $KEr(MoO_4)_2$.[3] In each of them the magnetic order at sufficiently low temperature is observed. In CHAB the interchain coupling, being of three orders of magnitude lower than the intrachain one, is responsible for the three-dimensional antiferromagnetic ordering below $T_c=1.5$ K.[1] p-CDTV orders ferromagnetically at $T_c=0.67\pm0.02$ K (Ref. 2) and $KEr(MoO_4)_2$ with $J'/J\simeq0.02-0.08$ exhibits magnetic phase transition at $T_c=0.955\pm0.005$ K.[3] The existence of the phase transitions in such systems causes the application of an even small external magnetic field to change drastically the temperature dependence of the thermodynamic quantities. It is obvious that such a behavior can be described neither on the basis of the one-dimensional model nor within a mean-field approximation (MFA). So, it seems interesting to study quasi-one-dimensional systems in the field by using a method that allows one to take into account their really two-dimensional character and to go beyond the MFA.

Recently, we proposed the method based on the linear-perturabtion renormalization-group transformation[4] (LPRG), which can be applied to study a broad class of weakly interacting spin chains. The LPRG method can be used to consider some nonuniversal properties such as location of the critical temperature or temperature dependence of the thermodynamic quantities of the above-mentioned systems. In this paper the LPRG will be applied to find the temperature dependence of the specific heat and susceptibility of the weakly interacting Ising spin chains in a longitudinal external magnetic field and the critical temperature of the quantum uniaxial ferromagnetic chains in a transverse field.

II. ISING CHAINS IN A FIELD

In this section we apply the LPRG method[4] to weakly interacting Ising spins chains in the external longitudinal field. The Hamiltonian of such a system is defined as

$$H=K\sum_{\langle ij\rangle} S_{i,j}S_{i,j+1}+K_1\sum_{\langle ij\rangle} S_{i,j}S_{i+1,j}+\tilde{h}\sum_{l} S_l, \quad (1)$$

where the label i refers to rows and j to columns, the factor $-1/k_BT$ has already been absorbed in the Hamiltonian ($K=J/k_BT$, $K_1=J'/k_BT$, $\tilde{h}=$magnetic field$/k_BT$) and $K_1<K$. As usual, we define the renormalization transformation by

$$\exp[H'(\sigma)]=\mathrm{Tr}_S P(\sigma,S)\exp[H(S)]. \quad (2)$$

The weight operator $P(\sigma,S)$ is chosen in the linear form

$$P(\sigma,S)=\frac{1}{2^{N/2}}\prod_{i,j=0}(1+\sigma_{i+1,j+1}S_{2i+1,2j+1}) \quad (3)$$

and consequently, in each renormalization step, every other spin from every other row survives. Separating the Hamiltonian (1) in a part H_0 containing intrachain interaction (K) and a remainder V containing interchain interaction (K_1), with the notation

$$z_0=\mathrm{Tr}_S P(\sigma,S)\exp[H_0(S)] \quad (4)$$

and

$$\langle A\rangle_0=\frac{1}{z_0}\mathrm{Tr}_S AP(\sigma,S)\exp[H_0(S)], \quad (5)$$

the transformation (2) can be rewritten as

$$H'(\sigma)=\ln[z_0]+\ln[\langle\exp(V)\rangle_0], \quad (6)$$

with the following cumulant expansion for $\langle\exp(V)\rangle_0$:

$$\ln[\langle\exp(V)\rangle_0]=\langle V\rangle_0+\frac{1}{2!}(\langle V^2\rangle_0-\langle V\rangle_0^2)+\ \ldots. \quad (7)$$

To evaluate the cumulants (7) one has to know the averages $\langle S_i,\ldots,S_n\rangle$ in "odd" rows where in the renormalization step every other spin is removed and in ''even'' rows where all spins are removed. Dividing an odd chain into three-spin cells with the following weight operator:

$$P(\sigma,S)=\tfrac{1}{4}(1+\sigma_1 S_1)(1+\sigma_2 S_3), \tag{8}$$

and considering only one cell we can find the chain renormalized interaction

$$z_0^{(odd)}=a_0+a_g(\sigma_1+\sigma_2)+a_x\sigma_1\sigma_2, \tag{9}$$

where

$$a_0\equiv\frac{1}{4}\mathrm{Tr}_S e^{H_0(S)}=\frac{1}{2}[e^{2K}\cosh(2\tilde{h})+e^{-2K}]+\cosh(\tilde{h}),$$

$$a_g\equiv\frac{1}{4}\mathrm{Tr}_S S_1 e^{H_0(S)}=\frac{1}{2}e^{2K}\sinh(2\tilde{h}),$$

$$a_x\equiv\frac{1}{4}\mathrm{Tr}_S S_1 S_3 e^{H_0(S)}=\frac{1}{2}[e^{2K}\cosh(2h)+e^{-2k}]-\cosh(\tilde{h}). \tag{10}$$

The ''odd'' chain spin averages are given by

$$\langle S_1\rangle=\sigma_1,\quad \langle S_3\rangle=\sigma_2,$$

$$\langle S_2\rangle=x_0+x_1(\sigma_1+\sigma_2)+x_x\sigma_1\sigma_2, \tag{11}$$

where

$$x_0=a_3c_0+2a_4c_g+a_5c_x,$$

$$x_1=a_4(c_0+c_x)+c_g(a_3+a_5),$$

$$x_x=a_5c_0+2a_4c_g+a_3c_x. \tag{12}$$

and

$$a_3\equiv\frac{1}{4}\mathrm{Tr}_S S_2 e^{H_0(S)}=\frac{1}{2}e^{2K}\sinh(2\tilde{h})+\sinh(\tilde{h}),$$

$$a_4\equiv\frac{1}{4}\mathrm{Tr}_S S_1 S_2 e^{H_0(S)}=\frac{1}{2}[e^{2K}\cosh(2h)-e^{-2K}],$$

$$a_5\equiv\frac{1}{4}\mathrm{Tr}_S S_1 S_2 S_3 e^{H_0(S)}=\frac{1}{2}e^{2K}\sinh(2\tilde{h})-\sinh(\tilde{h}), \tag{13}$$

$$c_0=(a_0^2-2a_g^2+a_0a_x)/(\lambda_1\lambda_2\lambda_3),$$

$$c_g=-a_g/(\lambda_1\lambda_2),$$

$$c_x=-(a_x^2-2a_g^2+a_0a_x)/(\lambda_1\lambda_2\lambda_3), \tag{14}$$

$$\lambda_{1,2}=a_0\pm 2a_g+a_x,\ \lambda_3=a_0-a_x. \tag{15}$$

By means of the coefficients x_0, x_1, x_x one can express all other averages. For example,

$$\langle S_1S_2\rangle=x_1+x_0\sigma_1+x_x\sigma_2+x_1\sigma_1\sigma_2,$$

$$\langle S_1S_2S_3\rangle=x_x+x_1(\sigma_1+\sigma_2)+x_0\sigma_1\sigma_2. \tag{16}$$

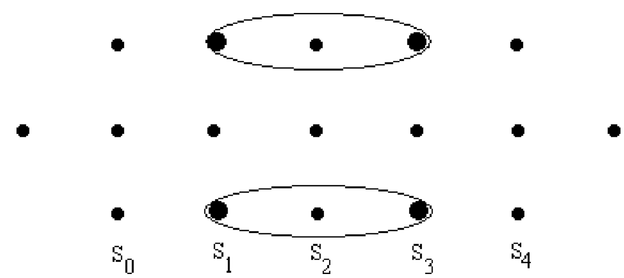

FIG. 1. The cluster used to get renormalized Hamiltonian (6). Small dots denote decimated spins.

The averages of the spins from even, removed rows could be calculated using a chain of an arbitrary length, however, in our procedure[4] already in the lowest nontrivial order of the cumulant expansion (7) all possible bilinear couplings between several spins from the adjoining renormalized rows come into play. Thus, in order to carry out effectively the LPRG transformation we have to confine ourselves to a cluster of a reasonable size. In the second order in the cumulant expansion we have to take into account three chains and we chose the cluster consisting of five spins from odd and seven spins from even rows (see Fig. 1).

In order to find the contribution to the interactions between the effective spins (σ) one should take into account the averages of all possible products of spins from the considered cluster. For example, this contribution from the average $\langle S_1S_3S_4\rangle$ is

$$x_1\sigma_1+x_0\sigma_1\sigma_2, \tag{17}$$

and from $\langle S_0S_1S_3S_4\rangle$ is

$$x_1^2+x_0x_1(\sigma_1+\sigma_2)+x_0^2\sigma_1\sigma_2, \tag{18}$$

and so on. For the cluster presented in Fig. 1, to the second order in the cumulant expansion, three new interactions (K_2,K_3,K_4) are generated:

$$K_2S_{i,j}S_{i+1,j+1}+K_3S_{i,j}S_{i+1,j}S_{i+1,j+1}+K_4S_{i,j}S_{i+1,j}S_{i,j+1}S_{i+1,j+1}. \tag{19}$$

Now, we can find the recursion relations for six parameters ($K',K_1',K_2',K_3',K_4',\tilde{h}'$), which in our case define the renormalization group-transformation (6). As usual in each step of this transformation a constant (independent of effective spins σ) term $G(K_i,\tilde{h})$ appears which can be used to calculate the free energy per site according to the formula[5]

$$f=\sum_{n=1}^{\infty}\frac{G(K_i^{(n)},\tilde{h}^{(n)})}{3^n}. \tag{20}$$

Using the recursion relations for the interaction parameters and the formula (20) one can find numerically the thermodynamic quantities of the models described by the Hamiltonian (1) with the additional interactions (19). We will calculate the specific heat (C) and the magnetic susceptibility (χ) for two special cases: (1). $K_2=K_3=K_4=0$—the Ising chains coupled by standard bilinear interaction K_1 with the

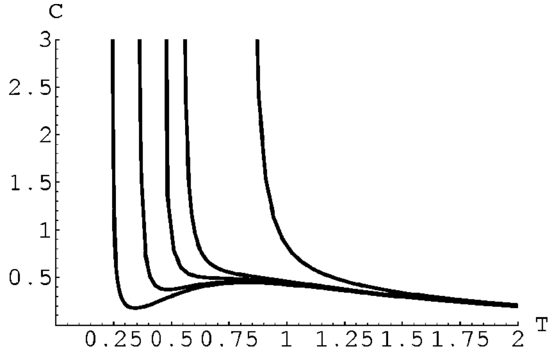

FIG. 2. Temperature dependence of the zero-field specific heat for Ising chains with $K_1/K=0.02,\ 0.03,\ 0.04,\ 0.05$, and 0.1 from left to the right.

finite-temperature phase transition in the field-free case; (2). $K_1=K_2=K_3=0$—the Ising chains interacting via four-spin interactions K_4 with interacting spins located on a 2×2 plaquette, where no finite-temperature long-range order is observed in the absence of the external field.

A. Bilinear interactions

The specific heat as a function of the temperature ($t=K^{-1}=k_BT/J$) in case (1) for several values of the ratio $K_1/K=J'/J$ in the field-free case is presented in Fig. 2. According to the exact result,[13,14] the specific heat has a singularity characterizing a critical point for any value of the ratio K_1/K. It should be emphasized that in contrast to the numerical method where a critical point is found by using some fitting procedure in LPRG, the critical point is found from the flow diagram analysis.

As shown in Fig. 3 according to our expectation the application of an even small longitudinal external field destroys the phase transition and the specific heat has a maximum instead of a singularity. This maximum shifts toward higher temperatures as the field is increased.

In Fig. 4 the temperature dependence of the longitudinal magnetic susceptibility χ is presented. The "universal" character of this dependence for the ferromagnetic systems—the existence of the maximum at $t=t_m>t_c$ (where t_c is the critical temperature at $h\equiv\tilde{h}/K=0$) which decreases and shifts

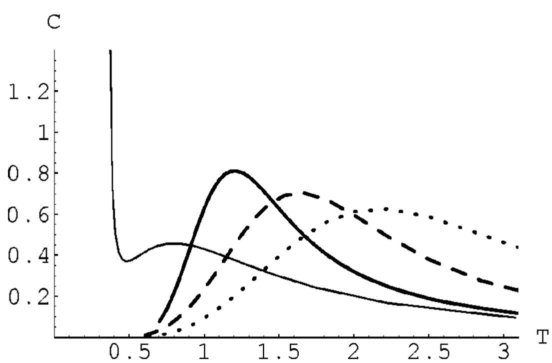

FIG. 3. Temperature dependence of the specific heat for Ising chains with $K_1=0.03$ K for $h=0$ (thin line), $h=0.2$ (solid line), $h=0.5$ (dashed line), and $h=1$ (dotted line).

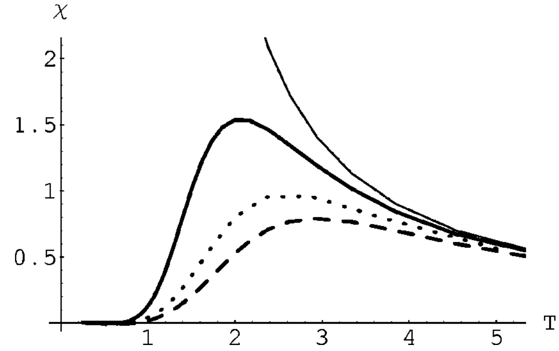

FIG. 4. Temperature dependence of the magnetic susceptibility for Ising chains with $K_1=0.02$ K for $h=0$ (thin line), $h=0.5$ (solid line), $h=0.8$ (dotted line), and $h=1$ (dashed line).

according to $h^{2/3}$ with increasing field—has been studied so far, mainly by using the MFA.[6–9] Recently, the shift of the maximum of the field-dependent susceptibility in one-dimensional anisotropic ferromagnetic materials has been studied by using the linear RG transformation.[10] It was shown that only for the transverse susceptibility the shift of the maximum can be satisfactorily described by the power law for a broad range of the field with the exponent υ close to 2/3 for the anisotropy small enough.

In Fig. 5 the log-log plot of the susceptibility maximum shift for weakly interacting Ising spin chains in longitudinal field is presented. The whole curve can be reasonablly fitted to the formula $y=0.94+0.58$, which means

$$t_m-t_c\sim h^{\upsilon},\quad \upsilon\approx0.58. \tag{21}$$

However, as it is shown in Fig. 5 the fit is much better for small fields if $\upsilon\approx0.65$, and for higher fields if $\upsilon\approx0.71$. Anyway, one can conclude that with reasonable approximation in a rather broad range of the field the longitudinal susceptibility maximum of the weakly interacting Ising chains is shifted according to the 2/3 law.

B. Four-spin interaction

The two-dimensional Ising model made using spin chains coupled only by four-spin interaction K_4 (19) does not exhibit any finite-temperature phase transition in the field-free

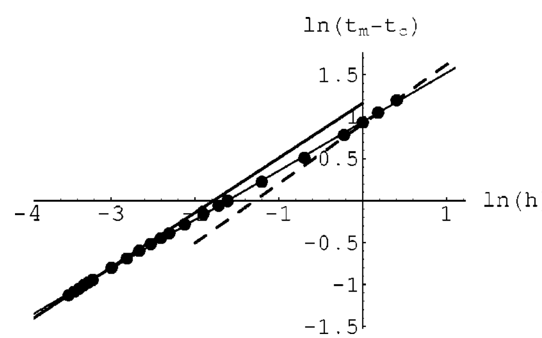

FIG. 5. Log-log plot of the field dependence of the susceptibility maximum location for Ising chains with $K_1=0.05$ K. The thin, solid and dashed lines denote the curves $y=0.94+0.58x$, $y=1.16+0.65x$, and $y=0.91+0.71x$, respectively.

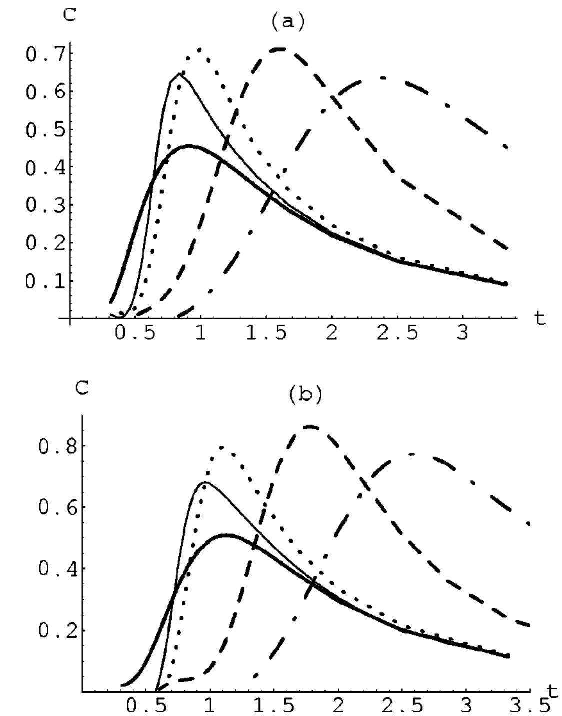

FIG. 6. Temperature dependence of the specific heat for Ising chains coupled by four-spin interaction $K_4 = 0.05$ K (a) and $K_4 = 0.2$ K (b) for $h = 0$ (solid line), $h = 0.05$ (thin line), $h = 0.1$ (dotted line), $h = 0.5$ (dashed line), and $h = 1.2$ (dashed-dotted line).

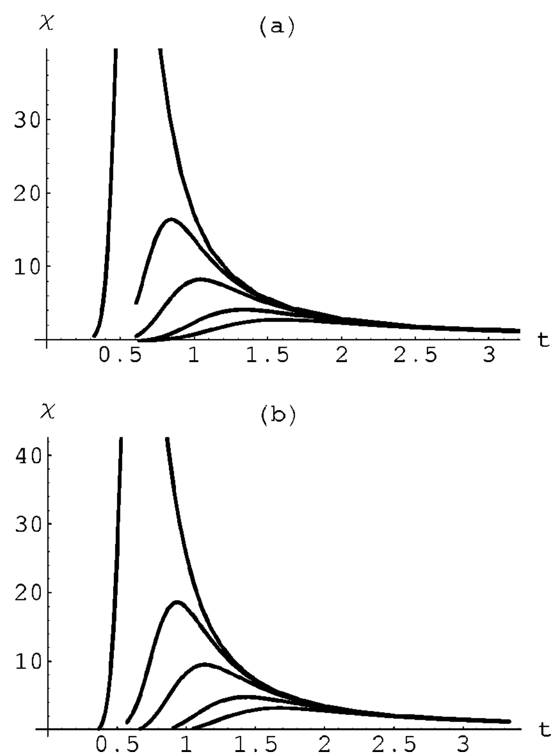

FIG. 7. Temperature dependence of the susceptibility for Ising chains coupled by four-spin interaction $K_4 = 0.05$ K (a) and $K_4 = 0.2$ K (b) for $h = 0.01$, 0.05, 0.1, 0.2, and 0.3 from top to the bottom.

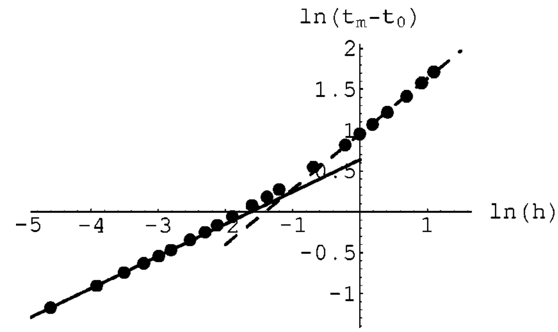

FIG. 8. Log-log plot of the field dependence of the susceptibility maximum location for Ising chains coupled by four-spin interaction with $K_4 = 0.05$ K. The solid and dashed lines denote the curves $y = 0.64 + 0.39x$ and $y = 0.95 + 0.68x$, respectively.

case.[4] It means that even in the zero field the specific heat has no a singularity but only a maximum. In Fig. 6 the temperature dependence of the specific heat for several values of the field is shown. As the field increases the specific heat maximum, at first, grows and moves toward lower temperatures, then changes the shift direction and finally decreases.

The temperature dependence of the susceptibility of the Ising chains coupled by four-spin interactions with $K_4 = 0.05$ K is presented in Fig. 7. Except for the field-free behavior where in this case the sharp maximum instead of the singularity is observed the shift of the susceptibility maximum resembles that for the standard Ising model (Fig. 4). However, it is easy to see from Fig. 8 that in case (2) the field dependence of the maximum susceptibility shift cannot be described by a single power law. $t_m - t_0 \sim h^{\upsilon}$, where t_0 is the location of the susceptibility maximum at zero field. For $K_4 = 0.05$ we have found $t_0 \approx 0.26$. Surprisingly, for the higher fields the fit $\upsilon \approx 0.68$ is closer to the 2/3 law than the fit for small field $\upsilon \approx 0.39$

III. QUANTUM SPIN CHAINS IN A TRANSVERSE FIELD

In this section, we consider weakly interacting quantum spin chains described by the Hamiltonian

$$H = \sum_{\alpha = x,y,z} K^{\alpha} \sum_{\langle ij \rangle} S^{\alpha}_{i,j} S^{\alpha}_{i,j+1} + \sum_{\alpha = x,y,z} K^{\alpha}_1 \sum_{\langle ij \rangle} S^{\alpha}_{i,j} S^{\alpha}_{i+1,j} + \tilde{h} \sum_{l} S^{x}_{l}. \tag{22}$$

As in the preceding section the label i refers to rows and j to columns and the factor $-1/k_B T$ has already been absorbed in the Hamiltonian. For $K^z > K^x = K^y > 0$ this Hamiltonian describes the uniaxial ferromagnet in the external field directed perpendicular to the easy axis (z). In such a system the external field does not destroy the phase transition from ferromagnetic phase (with magnetization making some angle with the external field) to the paramagnetic phase (with magnetization along the field).[11] In the MFA the temperature of this transition is given by[15,11]

$$t_c - t_c(h) = b h^{\omega}, \quad \omega = 2, \tag{23}$$

where $t_c=(K_c^z)^{-1}=k_BT_c/J^z$ denotes the critical temperature in zero field, $h=\tilde{h}/K^z$ and b is some constant. Within the MFA the exponent ω neither depends on the system dimensionality nor on the interactions and anisotropy values. Here, to find the field dependence of the critical temperature of the model described by the Hamiltonian (22) we apply the LPRG method presented for the quantum spins in the previous paper.[4] Due to the noncommutativity of several terms of the Hamiltonian (22), and also because of the three components of the spin operators, the application of the LPRG method to the quantum spin model is much more complicated. For example, instead of the relations (9) and (11) for the chain renormalized interaction and averages of the Ising model, now we have

$$\mathbf{z}_0^{odd}=a_0+a_g(\sigma_1^x+\sigma_2^x)+\sum_{\alpha=x,y,z}a_\alpha\sigma_1^\alpha\sigma_2^\alpha \tag{24}$$

and

$$\langle S_1^x\rangle=x_1\sigma_1^x+x_3\sigma_2^x+x_y(\sigma_1^y\sigma_2^y-\sigma_1^z\sigma_2^z), \tag{25}$$

where

$$x_1=2a_gc_g+a_0c_0+a_xc_x,$$

$$x_3=2a_gc_g+a_xc_0+a_0c_x,$$

$$x_y=a_g(c_y-c_z) \tag{26}$$

a_i are expectation values defined analogous to the definitions (10) and c_i are coefficients of the $\mathbf{z}_0^{odd}$ inverse operator

$$[\mathbf{z}_0^{odd}]^{-1}=c_0+c_g(\sigma_1^x+\sigma_2^x)+\sum_{\alpha=x,y,z}c_\alpha\sigma_1^\alpha\sigma_2^\alpha \tag{27}$$

and, for example,

$$c_g=\frac{a_g}{a_z^2-2a_ya_z+a_y^2-a_x^2-2a_0a_x+4a_g^2-a_0^2}. \tag{28}$$

To find the renormalized Hamitlonian we apply the cluster presented in Fig. 1. However, in the quantum case to the second order in the cumulant expansion the RG transformation generates 22 new two-, three-, and four-spin interactions. So, together with the initial Hamiltonian couplings $(K^\alpha,K_1^\alpha,\tilde{h})$ one has to consider 29 interaction parameter space. The evaluation of the contribution from all new interactions is straightforward but rather tedious. So, we confine ourselves to consider the contribution to the effective interaction from two-spin interactions up to the second order and from three- and four-spin interactions to the first order. We have evaluated numerically the renormalization transformation from the original set of 29 coupling parameters to the set of renormalized parameters and have found two stable fixed points describing the behavior of the system at $T=0$ and $T=\infty$, and the critical surface in the 29-dimensional space of the parameters. The critical temperature was found for two cases: (i) $K_x=K_y=0.5K_z$, $K_1^\alpha=0.2K_\alpha$; (ii) $K_x=K_y=0.2K_z$, $K_1^\alpha=0.3K_\alpha$. The zero-field critical temperatures t_c are (i) 1.04 and (ii) 1.37, respectively (for the standard Ising model $t_c\approx2.269$).[13]

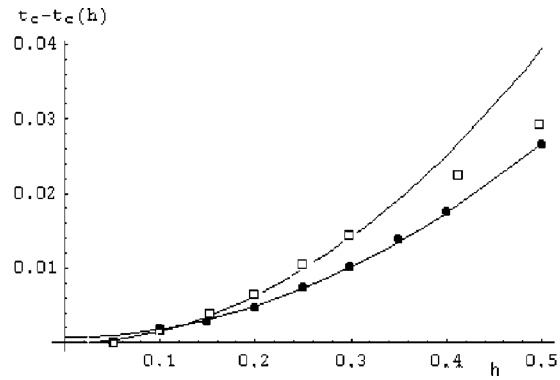

FIG. 9. The critical temperature of the uniaxial ferromagnets in the field perpendicular to the easy axis. Squares: $K_x=K_y=0.5K_z$, $K_{1\alpha}=0.2K_\alpha$; Points: $K_x=K_y=0.2K_z$, $K_{1\alpha}=0.3K_\alpha$.

The shifts of the critical temperatures under external field are presented in Fig. 9. In both cases the results are fitted to the formula (23), with $b=0.16$ and 0.1 for the cases (i) and (ii), respectively. As shown in Fig. 9, in both cases the fit is quite good for a field small enough, although in the case (i) for $h>0.3$ the deviation from the single power-law (23) is very clear.

IV. CONCLUSION

The LPRG method has been applied to study the temperature dependence of the thermodynamic quantities and the location of the transition temperature in weakly interacting ferromagnetic chains at an external magnetic field. It should be emphasized that even in the field-free Ising spin model the renormalization-group transformation has been obtained by using two approximations. First, connected with the perturbation with regard to interchain interaction, which is valid if $K_1<K$ (and $K_1\ll1$) and second, related to the truncation of the interaction generated in the LPRG procedure. For the quantum case, because of the noncommutativity of several terms of the Hamiltonian, additional approximations are necessary. Namely, the used procedure takes quantum effect into account within a single linear cell and neglects the effects of noncommutativity of several cells.[4,12] We have also neglected higher than second-order contributions to the effective interactions from three- and four-spin interactions. All these approximations are high-order approximations. On the other hand, the smaller the ratio K_1/K, the lower critical temperature. So, the proposed approach can be used to evaluate the temperature dependence of the thermodynamic values in the higher temperature and to find the critical point location if a transition does not take place in a very low temperature, i.e., if K_1/K is not too small. Of course, the approximation could be improved by taking into account the higher orders in the cumulant expansion and by increasing the used cluster. Unfortunately, for the quantum spins, especially in the presence of the external field, the higher-order calculations become labor and time consuming. However, we be-

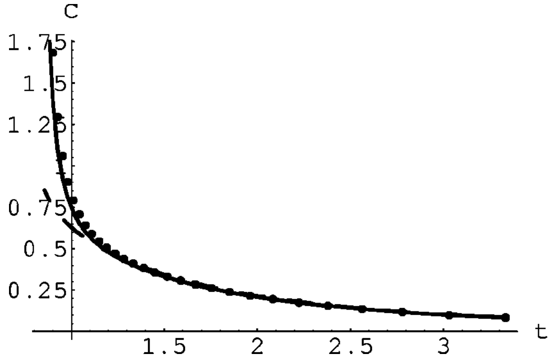

FIG. 10. Temperature dependence of the specific heat for Ising chains with $K_1/K=0.1$ found by using clusters: 3-5-3 (dashed line), 5-7-5 (dotted line), and 7-9-7 (solid line).

lieve that the cluster presented in Fig. 1 allows one to describe reasonably the basic features of the considered models especially at high temperatures and small fields. The basis for this belief is the analysis of the results found by using three different clusters 3-5-3 (three spins in odd and five spins in even rows, respectively), 5-7-5, and 7-9-7 for Ising chains at zero field. As an example the temperature dependence of the specific heat based on these three clusters is shown in Fig. 10. It is seen that the results for the two bigger clusters are very close to each other. Therefore we would expect that further increasing of the cluster does not change much qualitatively the results. The quantitative discrepancy in the low-temperature region ($t\langle 1.1$) for the smallest cluster arises from neglecting the nearest-neighbor interactions in the odd rows.

The good point of the LPRG is that, contrary to the nonlinear renormalization-group transformation, it does not require the choice of the weight operator. In fact this choice is not obvious for weakly interacting spin chains especially in the presence of the external field. Next, opposite to the linear Migdal-Kadanoff transformation the LPRG method does not have to be connected with the bond-moving mechanism that gives rather poor results even for the standard two-dimensional Ising model.[4,12] Finally, as opposed to numerical methods, one has not to use any fitting procedure to estimate the critical temperature that can be found from the analysis whose fixed point is approached by the Hamiltonian as the renormalization transformation goes on. So, using the LPRG one can expect to get reasonable results for quite complicated classical and quantum systems made of weakly interacting chains at sufficiently high temperature.

In this paper we found the temperature dependence of the specific heat and susceptibility for the two classical spin systems in the longitudinal field: the standard Ising model that exhibits finite-temperature phase transitions in the field-free case, and the Ising spin chains coupled only by the four-spin interactions. In the latter case there is no phase transition at any finite temperature even at $h=0$.[4] The result of our approach concerning the existence of the critical point for an arbitrary small bilinear interchain interaction K_1 is qualitatively correct although, as mentioned above, the location of this point for small ratio (K_1/K) is not so good in comparison with the exact results. As one expects the application of an external field destroys the phase transition and both the specific heat and magnetic susceptibility have maxima instead of singularities. In the case of the susceptibility this maximum is shifted toward higher temperature according to the 2/3 power law for the broad range of the fields. For the Ising with only four-spin interaction according to the exact result[4] there are no singularities in specific heat and susceptibility even at $h=0$, although χ exhibits a very sharp peak. In this case also the magnetic field shifts the susceptibility maximum but this shift cannot be fitted to the single power law in the broad range of fields. For small fields the exponent that characterized the shift is $\upsilon\approx 0.39$.

The LPRG has been also applied to quantum uniaxial Heisenberg model in the field perpendicular to the easy axis. It has been shown that, according to the MFA results, such a field does not destroy the continuous phase transition and shifts the critical point towards a lower temperature. For the small enough field, this shift is described by the power law with exponent $\omega=2$. For higher fields and smaller anisotropy the deviation from the single power law seems to be clear. However, it should be noted that our approximation can be worse for higher fields.

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