Quantum spin glass with long-range random interactions

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A model of the quantum (transverse) Ising spin glass with a long-range random interaction is proposed and studied here. In this model the random interaction between two spins at a distance r_{ij} apart falls algebraically as $1/r_{ij}^{(d+\sigma)/2}$. Here, apart from the strength of the quantum fluctuations, the interaction range σ is also tunable. We have studied the ground-state properties of the model, extending the "Droplet model" of the short-range quantum spin glass. The model is also studied using a modified form of the effective Landau action describing the transition of the short-range quantum spin glass. The important features which are due to the long-range interaction are clearly mentioned. Field theoretical renormalization group calculations fail to locate any stable weak-coupling fixed point in the non-mean-field region. The simplest conjecture is that beyond the mean-field region, the critical behavior is governed by the infinite randomness fixed point. We extend the phenomenological scaling relation for the short-range quantum spin glass (based on the assumption of the existence of a dangerously irrelevant operator) to the present long-range interacting case. Most of the possible interesting aspects associated with the quantum transition in the present model are elaborately discussed.

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I. INTRODUCTION

Random quantum transitions in the quantum Ising[1,2] and rotor models[1] have attracted a great deal of attention in recent years. Following experimental studies of the insulating, dipolar Ising system $LiHo_xY_{1-x}F_4$, which is ideally modeled in terms of a quantum Ising spin glass,[3,4] an enormous amount of effort has been directed to understand the zero-temperature and finite-temperature transitions in quantum spin glasses (for recent reviews see Refs. 1,5, and 6).

Huse and Miller[7] investigated the transverse Ising spin-glass model with an infinite-range interactions (Sherrington-Kirkpatrick model in a transverse field) and determined exactly the transition from the spin-glass-ordered phase to the paramagnetic phase. Ye, Sachdev, and Read[8] studied the corresponding rotor problem in the limit of infinite-range interactions and infinite spin dimensionality (spherical limit), and derived results similar to the results in Ref. 6. In a subsequent work, they proposed a Landau theory of quantum spin glasses of rotors and Ising spins[9] with a short-range interaction. The zero-temperature transition and the associated critical exponents of the short-range quantum Ising model (Edwards-Anderson model in a transverse field) in two and three dimensions are extensively studied using quantum Monte Carlo techniques.[10,11] A phenomenological droplet theory describing the transitions in short-range quantum Ising systems has also been proposed.[12] The novel features associated with the low-dimensional random quantum transitions are the "activated quantum dynamical scaling" at the quantum critical point and the existence of a Griffiths-McCoy singular region (with continuously varying exponents), where the response function diverges even away from the quantum critical point.[13] This scenario is proved analytically[13] and numerically[14] in the one-dimensional case. Numerical studies on two-dimensional random bond systems corroborate the above conjecture.[15] Further numerical studies on the two-dimensional[16] and the three-dimensional[17] short-range quantum Ising spin glasses clearly indicate the occurrence of the Griffith-McCoy singularities in the disordered phase, although they seem to support a conventional dynamical scaling with a finite value of the dynamical exponent at the quantum critical point. The effect of spatial correlations on quenched disorder in random quantum Ising systems near the quantum critical point has also been studied[18] and it was found that the relevant correlations enhance the critical and off-critical singularities. The random quantum Ising model in $d=2$ has also been studied,[19,20] generalizing the (cluster) renormalization group (RG) method applied to the one-dimensional system.[13] It has been found that at strong randomness the RG flow for the quantum critical point is towards an infinite-randomness fixed point as in one dimension. If quenched randomness grows beyond the limit as the system is coarse grained, the associated fixed point would have infinitely strong randomness and thus be called the infinite-randomness fixed point.[19]

In this communication, we propose a long-range quantum Ising spin glass described by the Hamiltonian

$$H_I=-\sum_{ij}\frac{J_{ij}}{r_{ij}^{(d+\sigma)/2}}\hat{S}_i^z\hat{S}_j^z-\Gamma\sum_i\hat{S}_i^x, \tag{1}$$

where d is the spatial dimension, the $\hat{S}$'s are the noncommuting Pauli matrices, and σ is the tunable range of interaction. The random bonds J_{ij} are chosen from a Gaussian distribution with vanishing mean and variance J given as

$$P(J_{ij})=\frac{1}{(2\pi J^2)^{1/2}}\exp\left(-\frac{J_{ij}^2}{2J^2}\right). \tag{2}$$

Here Γ denotes the noncommuting transverse field that introduces the quantum fluctuations in the model, and therefore tuning Γ appropriately, one can drive the system from the ordered to disordered phase even at $T=0$. Even when the spatial dimensionality of the model is unity, the long-range nature of the random interaction causes frustration and thus the model happens to be an ideal representative of a quantum

Ising spin glass in one dimension. The corresponding short-range system in $d=1$ does not have frustration and can thus be mapped onto the random quantum Ising chain.[13] However, one should note here that in the infinite-randomness limit[20] frustration is irrelevant and the same (infinite-randomness) fixed point should govern both the ferromagnetic and spin-glass quantum critical points. Very recently, the pure quantum Ising system with long-range interactions has also been studied. A Kosterlitz-Thouless transition line and a region of diverging susceptibility in the Γ-T plane for the $d=\sigma=1$ case has also been located.[21]

The classical long-range Ising spin-glass model [$\Gamma=0$ in (1)] was introduced long ago by Kotliar, Anderson, and Stein.[22] The one-dimensional classical Ising spin glass exhibits a nontrivial transition at finite temperature for $\sigma<1$, whereas for $\sigma>1$ thermal fluctuations destroy any type of spin-glass ordering. For $d=1$, the parameter value $\sigma=1$ is especially important. It is the largest value of σ for which the system can sustain long-range spin-glass order at a finite temperature. In this case, the topological defects interact via a logarithmic interaction and thus the chain undergoes a Kosterlitz-Thouless transition to the paramagnetic phase.[23] For arbitrary dimensions, one can evaluate the critical exponents using the perturbative ϵ expansion around the upper critical dimension and range given by the relation $d_u=3\sigma_u$.[22] For a one-dimensional chain, for $\sigma<0.33$, the transition is of mean-field type. This particular classical spin-glass model was also studied in relation to metallic spin glasses[24] and a phase diagram in the (d-σ) plane was proposed.

The plan of the paper is as follows. In Sec. II we introduce the model and discuss its ground-state properties and the nature of the quantum transition using the droplet picture[25,26] extended to the short-range quantum Ising spin glass.[12] The mathematical framework and the results we discuss are somewhat similar to those in the short-range case (at least at $T=0$) except for necessary modifications due to the long-range nature of the interaction. This study is, however, useful for a complete understanding of the phase diagram of the model. In Sec. III, we use a modified version of the effective Landau functional used in the short-range case[9] and explore the quantum transition using a perturbative renormalization group technique. In Sec. IV, the scaling relations associated with the quantum transition and the finite-temperature transition are discussed. Although our study is mostly restricted to Ising case ($m=1$), it can be easily generalized to the higher spin components ($m>1$, i.e., quantum rotors). We also include an extensive discussion of the different possible critical behaviors of the model in the concluding section.

II. MODEL AND THE DROPLET THEORY

In this section, we shall concentrate on the quantum spin-glass Hamiltonian (1) and explore its ordered state at $T=0$ and the corresponding quantum phase transition at $\Gamma=\Gamma_c$. Note that the classical spin-glass Hamiltonian ($\Gamma=0$) is ordered at $T=0$ for any value of σ and d (see below). Thus, the quantum system described by Eq. (1) has a second-order phase transition from the ordered phase to the paramagnetic phase driven by quantum fluctuations for all values of d and σ. One can visualize the quantum phase transition in the present model (when the transverse field term Γ is nonrandom) as the equivalent of the thermal transition in the $(d+1)$-dimensional classical Ising system with long-range random interaction in d spatial dimensions and short-range ferromagnetic interaction in the $(d+1)$th (imaginary time or Trotter) dimension. Along the Trotter direction the randomness is correlated (or striped).

The classical droplet model[25,26] describing the ordered phase of a classical finite-dimensional spin glass relies on the existence of an effective coupling constant $K(L)$ at a length scale L, which satisfies scaling laws in the ordered phase and near criticality. The scaling theory is based on a renormalization group picture where the zero-temperature fixed point is stable. The droplet theory assumes an ''ergodicity breaking'' of a trivial kind (i.e., assumes the existence of two ground states related by an overall flip), and the picture that emerges out of this is markedly different from the mean-field (Sherrington-Kirkpatrick) spin glass or the conventional ferromagnetic ordered phase. A droplet is defined as a cluster of upturned spins on a particular ground-state configuration.

In the classical droplet picture, at low temperature, the excitation free energies of the droplet at length scale L scale typically as[26]

$$\epsilon_L \sim L^y, \tag{3}$$

where y is the zero-temperature thermal exponent[12] (also called the stiffness exponent since it is related to the stiffness of the ordered phase[25]) of the classical spin glass. The positive (negative) value of the exponent y denotes a finite-(zero-) temperature transition from the paramagnetic to spin-glass phase and $y=0$ at the lower critical dimension. The other zero-temperature exponent d_s is the fractal dimension of the domain walls.[26] The scaling ansatz for the probability distribution $P(\epsilon_L)d\epsilon_L$ of the droplet of free energies ϵ_L at a large scale L is given as

$$P_L(\epsilon_L)d\epsilon_L=\frac{d\epsilon_L}{Y(T)L^y}\mathcal{P}\left(\frac{\epsilon_L}{Y(T)L^y}\right), \quad L\to\infty, \tag{4}$$

where $Y(T)$ is the generalized stiffness modulus that vanishes for $T\geqslant T_c$, and $\mathcal{P}$ here denotes the scaling function.

Let us first recall the known features[26] of a d-dimensional classical Ising spin glass with a long-range algebraic interaction as in Eq. (1), falling as $J_{ij}^2\sim(1/r_{ij}^{d+\sigma})$. The droplet theory for the short-range case is extended to long range (at least at $T=0$) using a restricted definition of droplets (i.e., the weak long-range interaction among the droplets at length scale L is ignored.[26]) For all σ in the range $\sigma_c<\sigma<0$, the corresponding stiffness exponent y is given by $y_\sigma=(d-\sigma)/2$.[26,24] The value of σ_c (defined such that for $\sigma<\sigma_c$ the long-range effect is relevant) is given by condition $y_s=y_\sigma$(at $\sigma=\sigma_c$) which leads to $\sigma_c=d-2y_s$, where y_s is the corresponding y exponent in the short-range case. For $\sigma>\sigma_c$, there is a crossover to the short-range zero-temperature fixed point. This suggests that for a given d, if

$\sigma > d$, the exponent y_σ is negative and so the system does not have a finite-temperature spin-glass transition and the lower critical range (the value of σ for which $y_\sigma = 0$) is $\sigma_l = d$, i.e., $\sigma_l = 1$ for the one-dimensional system, as already discussed.

As in the short-range case,[12] to study the long-range quantum glass, we assume that the lowest-lying excitations above the ground state of the equivalent $(d+1)$-dimensional classical system at a particular imaginary time are the classical d-dimensional droplet excitations. As mentioned already, we shall ignore the weak long-range interaction between the droplets [which is responsible for a topological transition in the classical case of $d = \sigma = 1$ (Ref. 22)] but we have to include the ferromagnetic interaction between the droplets in the $(d+1)$th (Trotter) direction. If one coarse-grains the system along the imaginary time, choosing a value of $\Delta\tau$ (lattice spacing in the Trotter direction), such that droplets with energy $\epsilon_D \sim (\Delta\tau)^{-1}$ are dilute, imaginary-time dynamics of a given droplet is described in terms of a classical one-dimensional Ising model Hamiltonian

$$H_D = \sum_{k=1}^{L_\tau} \left[\frac{1}{2} \Delta\tau \epsilon_D S_D(k\Delta\tau) - K S_D(k\Delta\tau) S_D((k+1)\Delta\tau) \right], \tag{5}$$

where $S_D(\tau) = \pm 1$ represent the state with droplet excitation present and the ground state at the imaginary time τ, respectively. Here k denotes the imaginary-time direction having width L_τ and K stands for the surface free energy having the form $\sim k_0 L^d$ (since the interaction in the Trotter direction is ferromagnetic and short ranged).

At this point, we should emphasize some critical aspects of droplet theory applied to the short-range and as well the long-range quantum spin glass. The droplet excitations considered here are purely classical and the exponent y is the stiffness exponent of a d-dimensional classical Ising spin glass. The quantum effect comes into play due to the additional Trotter direction along which the droplets interact with a short-range ferromagnetic interaction K. One thus arrives at an equivalent one-dimensional Ising Hamiltonian given by Eq. (5). Moreover, once we neglect the long-range interaction between the droplets, the mathematical framework is identical to that of the short-range case[12] at least at $T = 0$. The only difference lies in the fact that in the long-range case one has to use the stiffness exponent of the long-range classical Ising spin glass given as $y_\sigma = (d-\sigma)/2$.

The imaginary-time autocorrelation function of the droplets of size L is evaluated by averaging the connected correlation function of the equivalent Ising spins in Eq. (5) (separated by τ along the Trotter direction, with $L_\tau \to \infty$ and Trotter interaction $K \sim \kappa_0 L^d$) over droplet energies ϵ_L, using the distribution given in Eq. (4). The imaginary-time autocorrelation function $\overline{C(\tau)}$ is obtained by summing over all length scales L by the saddle-point method.[12] One finds

$$\overline{C(\tau)} = \overline{\langle S_{\mathcal{D}}(0) S_{\mathcal{D}}(\tau) \rangle_c} \sim \frac{k_0^{y_\sigma/d}}{Y} \frac{\Delta\tau q_{\mathrm{EA}}}{\tau[\ln(\tau/\Delta\tau)]^{1+y_\sigma/d}}. \tag{6}$$

In Eq. (6), the subscript c denotes the cumulants and the overbar denotes averaging over the droplet energies ϵ_L for a given length L, and the Edwards-Anderson order parameter $q_{\mathrm{EA}} = q_{\mathrm{EA}}(T=0,\Gamma)$. The uniform linear static susceptibility is then given as

$$\chi = \int d\tau \overline{C(\tau)}. \tag{7}$$

In the present long-range interacting case, using $y_\sigma = (d-\sigma)/2$, we find that for a particular dimension d, the static linear susceptibility is infinite for $d < \sigma$ and finite for $d > \sigma$. The more general imaginary-time correlation function, obtained through an analogous calculation considering the contribution from droplets of size of the order of or larger than r, is given as

$$\overline{C^2(r,\tau)} \sim \frac{q_{EA}^2 \exp\left(-\kappa_0 r^d - \dfrac{2r}{\Delta\tau} e^{-\kappa_0 r^d} \right)}{\Delta\tau Y r^{y_\sigma}}. \tag{8}$$

The above correlation function falls faster than a simple exponential, a feature of the quantum spin glass which is not seen in the classical case. This is because in the quantum case, the fraction of active droplets is proportional to tunneling rate [or the inverse of the correlation length in the Trotter direction τ in the equivalent Ising chain (5)] which is proportional to $\exp(-\kappa_0 L^d)$.[12]

At finite temperature the Trotter direction L_τ is finite. Thus, the finite-temperature transition in the short-range spin-glass model[12] is explored using a d-dimensional quantum droplet model where we have a two-level system (represented by an Ising spin S^z) with a tunneling term (S^x) necessary to describe the quantum effect. In the present long-range case, however, one should not use the same calculation using the same y_σ to describe the droplet excitations at finite temperature. The droplet picture of the classical long-range spin glass at finite temperature is rather questionable both for $\sigma > \sigma_c$ (where the interplay between the dangerously irrelevant temperature and irrelevant long-range interaction could be important) and $\sigma < \sigma_c$ (where the possibility of having collective droplet excitations cannot be ruled out).[26] Thus, the long-range interaction between the droplets, neglected in the above study, plays an important role. However, if one wishes to use the similar calculation as in Ref. 12 for the long-range case at finite temperature, the results would be quite similar to the short-range case. Of course, the results for $d < \sigma$ ($d > \sigma$) correspond to the results of the short-range systems below (above) their lower critical dimension.

Some crucial remarks are absolutely necessary here. In deriving the above correlation functions,[12] only the contribution from large-scale droplets has been taken into consideration. Actually, in the ordered phase there would always be processes involving small-scale droplets. These more microscopic correlations must dominate over the large-droplet contributions to modify the Ornstein-Zernicke form given in Eq. (8). Cluster renormalization studies[19] suggest that in the ordered phase the short-range quantum random system renormalizes asymptotically to an infinite cluster of active spins (with vanishing transverse field). In the present long-range case as well, it would be justified to expect the formation of such infinite clustersin the ordered phase. The infinite cluster will form only at zero energy below the lower critical dimension of the classical long-range Ising spin glass whereas above the lower critical dimension such a cluster will form at a finite nonzero-energy scale (see Fig. 1 in Sec. V).

III. EFFECTIVE ACTION AND RENORMALIZATION GROUP CALCULATIONS

In this section, we shall extend the renormalization group calculations introduced in Ref. 9 to the present case of long-range interactions.[27] We shall focus on the Ising case ($m=1$), though it should be simple to extend to the rotor case ($m\geqslant 2$). The effective zero-temperature Fourier-transformed Landau functional (in q,ω representation) in the presence of a long-range interaction of the form (1) is modified to (see Refs. 1 and 9 for a rigorous derivation of the action in the short-range case)

$$\begin{aligned}\mathcal{A}=&\frac{1}{\kappa t}\int d\omega\sum_a (r+\omega^2)Q^{aa}(q=0,\omega,-\omega)+\frac{p}{t}\int d^dq d\omega_1 d\omega_2 q^{\sigma}\sum_{a,b} Q^{ab}(q,\omega_1,\omega_2)Q^{ab}(-q,-\omega_1,-\omega_2)\\&+\frac{\tilde{p}}{t}\int d^dq d\omega_1 d\omega_2 q^2\sum_{a,b} Q^{ab}(q,\omega_1,\omega_2)Q^{ab}(-q,-\omega_1,-\omega_2)-\frac{\kappa}{3t}\int d^dq_1 d^dq_2 d^dq_3 d\omega_1 d\omega_2 d\omega_3\sum_{a,b,c} Q^{ab}(q_1,\omega_1,\\&-\omega_2)Q^{bc}(q_2,\omega_2,-\omega_3)Q^{ca}(q_3,\omega_3,-\omega_1)\,\delta^d(q_1+q_2+q_3)+\frac{u}{2t}\int d^dq d\omega_1\cdots d\omega_4\,\delta(\omega_1+\cdots\\&+\omega_4)\sum_a Q^{aa}(q,\omega_1,\omega_2)Q^{aa}(-q,\omega_3,\omega_4)-\frac{1}{t^2}\int d^dq\int d\omega_1 d\omega_2\sum_{a,b} Q^{aa}(q,\omega_1,-\omega_1)Q^{bb}(x,\omega_2,-\omega_2).\end{aligned}\tag{9}$$

In the above action (9), a,b are the replica indices running from 1 to n (with $n\rightarrow 0$ to be taken at the end of the calculation). Here, Q^{ab} is the spin-glass order parameter field and Q^{aa} is the "thermal" operator whose coefficient r drives the quantum transition. In the action $\mathcal{A}$, as in the short-range case,[9] t denotes the "dangerously irrelevant" operator similar to that present in the action of a quantum random field system.[28,29] A dangerously irrelevant operator is an operator, irrelevant at a critical point, that should be retained to get the correct scaling behavior of some field (in the present case free energy). Usually, the field mentioned diverges as some power of the dangerously irrelevant operator as the latter scales down to zero under renormalization. The simplest example is of such irrelevance is the irrelevance of the quartic term in the Landau ϕ^4 theory above the upper critical dimension (i.e., $d>4$).[30]

The presence of the linear term makes the action (9) different from the corresponding classical spin glass. The latter is described in terms of a cubic theory with the mass term coupled to the term quadratic in field operators.[31]

Here the quadratic term is removed by an appropriate choice of the field Q, which generates the linear term in the action and ensures that the random-averaged value of the field Q vanishes at the quantum critical point.[9] The coefficient r in action (9) denotes the departure from the quantum critical point and κ denotes the cubic coupling. The parameter u arises due to the quartic coupling of spins within a single replica and the term with coefficient $1/t^2$ represents the randomness in r or randomness in the transverse field. At finite temperature, the continuous Matsubara frequencies ω are replaced by the corresponding discrete frequencies $\omega_n=2\pi n/\beta$.

Note that due to the long-range interaction, we have both $\tilde{p}q^2$ and pq^{σ} term present in the action. In the presence of such an interaction, the connectivity term K_{ij}^{-1} in Appendix A of Ref. 9 carries an additional multiplicative factor of the form $1/|i-j|^{d+\sigma}$, which when Fourier tranformed to the q representation yields a q^{σ} term in addition to the q^2 term. Note that the algebraic form of the interaction in Eq. (1) is chosen appropriately such that the action (9) carries a simple q^{σ} term. For $\sigma\geqslant 2$, the long-range term q^{σ} is irrelevant in which case the action represents a short-range quantum spin glass. As in the long-range pure classical system,[27] at two-loop order, the coefficient $\tilde{p}$ associated with the short-range term complicates the scenario and one cannot retrieve the short-range results by simply setting $\sigma=2$ in the long-range case. More precisely speaking there is actually a crossover to short-range critical behavior for $\sigma>2-\eta_{\rm SR}$ where $\eta_{\rm SR}$ is the Fisher exponent for the short-range quantum system.[24] Note that the Fisher exponent η determines the the algebraic fall of the (equal time) correlation function at the (quantum) critical point and the subscript "SR" here stands for short-range critical behavior. In short-ranged disordered systems $\eta_{\rm SR}$ may be negative[9] so that the value of σ for which this crossover takes place may in principle be greater than 2.

The saddle-point (mean-field) ansatz is the same as the corresponding short-range case and given as

$$Q^{ab}(q,\omega_1,\omega_2)=(2\pi)^d\delta^{ab}\delta^d(q)\,\delta(\omega_1+\omega_2)D(\omega_1), \tag{10}$$

where $D(\omega)$ is the Fourier transform of $D(\tau_1-\tau_2)$, the effective single-site self-interaction in the infinite-range model.[7] In the saddle-point approximation, therefore, there is no effect of the nonlocal q^σ term present in the action and therefore one retrieves identical results of Ref. 9.

The long-range interaction manifests itself in the tree level. As in the short-range case, we expand Q about its saddle-point value

$$Q^{ab}(q,\omega_1,\omega_2)=\delta^{ab}(2\pi)^d\delta^d(q)\,\delta(\omega_1+\omega_2)D(\omega_1)+\tilde{Q}^{ab}(q,\omega_1,\omega_2). \tag{11}$$

Expanding the action $\mathcal{A}$ up to order $\tilde{Q}^2$, we find the propagator of $\tilde{Q}^{ab}$ to be

$$G(q,\omega_1,\omega_2)=\frac{t}{pq^\sigma+\tilde{p}q^2+\sqrt{\omega_1^2+\tilde{r}}+\sqrt{\omega_2^2+\tilde{r}}}. \tag{12}$$

The region we are interested in here is $\sigma<2-\eta_{\rm SR}(\eta_{\rm SR}=0$ at the tree level), where the coefficient $\tilde{p}$ associated with q^2 (the short-range interaction) is always irrelevant. One can thus set $\tilde{p}=0$ in the propagator when dealing with the critical properties. It is simple to extract the mean-field exponents from the propagator (12). At the quantum critical point ($r=0$), the characteristic Matsubara frequency $|\omega|\sim k^\sigma$, so that the dynamical exponent z associated with the quantum critical point is given by $z=\sigma$. Once again, away from the critical point ($r\neq 0$), there is a length scale $\xi^{-1/2\sigma}$ and one finds the correlation length exponent $\nu=1/2\sigma$. Clearly, we retrieve the short-range mean-field exponents[9] ($z=2$ and $\nu=1/4$) as $\sigma\to 2$. At this point, we use the disconnected spin-glass correlation function[9]

$$\begin{aligned}G^d(x-y,\tau_1-\tau_2,\tau_3-\tau_4)&=\lim_{n\to 0}\sum_{a\neq b}\langle Q^{aa}(x,\tau_1,\tau_2)Q^{bb}(y,\tau_3,\tau_4)\rangle\\&\quad-D(\tau_1-\tau_2)D(\tau_3-\tau_4),\end{aligned} \tag{13}$$

where $\langle\cdots\rangle$ denotes averaging over the n-replicated action (9). The disconnected correlator shows a stronger divergence than the spin-glass correlator (12) at the quantum critical point. One can define the Fisher exponents η and $\bar{\eta}$ through the scaling form of G and G^d at the quantum critical point,

$$G(q,0,0)\sim q^{-2+\eta},\quad G^d(q,0,0)\sim q^{-4+\bar{\eta}}. \tag{14}$$

We shall now study the action $\mathcal{A}$ in the spirit of a renormalization group calculation where one integrates out higher momenta and frequencies. The rescaling transformations we invoke here are

$$q'=bq,\quad \omega'=b^z\omega. \tag{15}$$

As in the short-range case, we shall associate an independent scaling to the dangerously irrelevant operator t (which scales to the fixed point $t=0$) as

$$t'=tb^{-\theta}, \tag{16}$$

where $\theta>0$. Since the long-range term q^σ is relevant, we shall here define the spin-rescaling factor ζ in such a way that the coefficient p of this term is not rescaled. The anomalous dimension exponent η is related to ζ as in the short-range case,

$$\zeta^2=b^{d+2z+2-\eta+\theta}. \tag{17}$$

At the tree level, the leading correction to the coefficient p is given by $b^{-(d+2z+\theta+\sigma)}\times\zeta^2$ which yields

$$\eta=2-\sigma. \tag{18}$$

The interesting feature related to the long-range case[27] comes about because there are no corrections to the coefficient p of the nonlocal term from perturbative elimination of the higher wave-vector modes [i.e., no new $q^\sigma Q(q,\omega)Q(-q,-\omega)$ term is generated in the renormalization process]. This keeps η fixed to $\eta=2-\sigma$ to all orders of ϵ as long as the long-range interaction is stable. Similarly one finds the exponent $\bar{\eta}$ defined in Eq. (14) to be $\bar{\eta}=4-\sigma=2\eta$.

As in the short-range case, the exponents z and θ will be determined from the rescaling of the κ and the $1/t^2$ term in the action $\mathcal{A}$. Therefore the rescaling of the parameters at the tree level is

$$r'=rb^{2z},\quad \kappa'=\kappa b^{(6+\theta-d-3\eta)/2},\quad u'=ub^{2-z-\eta}. \tag{19}$$

The action is invariant under this rescaling of parameters provided we choose

$$\eta=2-\sigma,\quad z=\sigma,\quad \theta=\sigma. \tag{20}$$

The coupling κ scales as

$$\kappa\sim b^{(4\sigma-d)}, \tag{21}$$

so that the coupling is irrelevant for $d>4\sigma$ and relevant otherwise. Thus, we find that the upper critical dimension d_u in the present case is $d_u=4\sigma$ (which goes to $d_u=8$ as $\sigma\to 2$). This relation equivalently yields the upper critical range $\sigma_u=(d/4)$ for a given dimension (such that for $\sigma>\sigma_u$, the coupling term κ is relevant). For the one-dimensional case, $\sigma_u=0.25$ in contrast to the corresponding classical case[22] where $\sigma_u=0.33$. This decrease of the value of σ_u is clearly due to the existence of the dangerously irrelevant operator t. The interaction u is irrelevant for all dimensions above d_u.

One should note here that the exponent θ is determined demanding the invariance of the $1/t^2$ of the action which scales as $b^{2-\eta-\theta}$. As we have discussed already, the exponent η does not renormalize so that one finds that the exponent θ should always remain fixed to its mean-field value $\theta=\sigma$. However, the dynamical exponent z is associated with the correlation length in the Trotter direction with effective short-range interactions and thus will be renormalized as in the corresponding short-range case.[9]

In spirit similar to Ref. 9, one can always study the quantum critical behavior of the present system using an ϵ expansion around the upper critical dimension d_u. The flow equations for the cubic coupling κ and the quantum mechanical interaction u are

$$\frac{d\kappa}{dl}=\frac{4\sigma-d}{2}\kappa+9\kappa^3-\frac{u}{\sqrt{1+r}}, \tag{22}$$

$$\frac{du}{dl}=-2u\kappa^2-\frac{u^2}{\sqrt{1+r}}. \tag{23}$$

We analyze the above equations for $\epsilon=4\sigma-d=0$, $\epsilon>0$, and $\epsilon<0$ in the same spirit as in the short-range case. The results are quite similar. Setting the always relevant operator $r=0$, we find three fixed points for κ and u using the above flow equations. Above the upper critical dimension d_u, the Gaussian fixed point with $u^*=\kappa^*=0$ is attractive for $u>0$. This fixed point corresponds to the mean-field values of the exponents discussed already. The cubic coupling κ and the parameter t are both irrelevant here and there is a logarithmic correction to the exponents for all d above $d_u=4\sigma$, due to the marginality of of the interaction u as in the short-range case.[9] At the upper critical dimension d_u (or range σ_u), there will be an additional logarithmic correction due to the marginality of the coupling κ.

Below the upper critical dimension, as seen in the short-range case,[9] the only fixed point with real coupling κ is the Gaussian fixed point and it is unstable. There is always a flow to the strong coupling. Thus, above the upper critical dimension or range (e.g., for $\sigma>\sigma_u=0.25$ for $d=1$), it is not possible to draw any conclusive inference from an ϵ-expansion analysis.

It is straightforward to generalize the preceding discussion to the n-component rotor system where one has an additional term[9]

$$\frac{v}{2t}\int d^dq\,d\omega_1\cdots d\omega_4\,\delta(\omega_1+\cdots+\omega_4)Q^{aa}_{\mu\nu}(q,\omega_1,\omega_2)\times Q^{aa}_{\mu\nu}(-q,\omega_3,\omega_4),$$

where μ,ν label the m components of the rotor. However, one arrives at similar RG equations with $U=v+mu$.

As discussed above, the RG calculations in the short-range case fail to locate a stable nontrivial fixed point for all $d<8$.[9] The simplest conjecture is that for any $d<8$, the critical fixed point, at least for strong randomness, is the infinite-randomness fixed point (IRFP) where frustration is irrelevant.[13,19,20] The quantum critical point is a novel percolation point where the annihilation and aggregation of clusters compete at all energy scales. For a cluster at energy scale Ω,Ω is related to the diameter L of the cluster through the relation

$$\ln\left(\frac{\Omega_0}{\Omega}\right)\sim L^{\psi},$$

where ψ is a new exponent and Ω_0 is the basic energy scale.[19] The magnetic moment of the cluster scales as $L^{\phi\psi}$. The exponents ψ,ϕ and the correlation length exponent ν are involved in this phenomenological scaling theory.[19]

In the present long-range case as well, RG calculations cannot find a stable nontrivial fixed point for any value of σ and d beyond the mean-field region. One can similarly argue that even in the present case, the critical fixed point beyond the mean-field region happens to be the infinite-randomness fixed point as discussed above. However, due to the existing long-range interaction, the situation seems to be further complicated here. For small values of σ, when the long-range interaction is dominant, we expect the critical fixed point to be effectively of long-range nature with the associated exponents ψ,ϕ, and ν depending on σ. On the other hand, for larger value of σ, when the interaction between the distant spins is very feeble (nearly absent), the critical fixed point should be the short-range IRFP with short-range exponents.[20] This crossover is observed in the one-dimensional random quantum Ising chain with algebraic disorder correlation.[18] In the short-range case, if we consider a typical "rare" ferromagnetic cluster (with correlated randomness in the Trotter direction) of linear size L, the relaxation time (correlation length in the Trotter direction) is typically given as $\ln\xi_\tau\sim L^d$. In the present case, with algebraic interaction of the form $1/r_{ij}^{(d+\sigma)/2}$ between spins, a similar consideration yields $\ln\xi_\tau\sim L^{(3d-\sigma+2)/2}$. Comparison with the short-range case naively indicates a possibility of crossover to the short-range IRFP as $\sigma\to(d+2)$. We must mention here that this is just a heuristic estimation (obtained comparing the value of ξ_τ) of the crossover value of σ.

IV. SCALING RELATIONS

In this section, we shall discuss the scaling relations that are expected to hold near the quantum transition point in the present model. These scaling properties are quite general and do not depend on the replica picture. We assume here a conventional quantum dynamical scaling and a finite value of the dynamical exponent z at the quantum critical point. This should be a valid description for quantum rotors ($m\geqslant 2$) but appears to be inappropriate for the Ising case with higher values of σ due to the possible activated quantum dynamical scaling.[13] Following Ref. 9, we shall proceed with the assumption of the presence of a single dangerously irrelevant variable t. The key points where the long-range nature of the interaction causes a marked difference are discussed. Clearly, these scaling relations should be valid in the region $\sigma<2-\eta_{\rm SR}$ where the long-range interaction always dominates over the corresponding short-range one.

Using the scaling properties of the disconnected propagator, we propose the exponent $\overline{\eta}$ (at the quantum critical point) as[9]

$$G^d(x,\tau,\tau)\sim x^{-(d+2z-4+\overline{\eta})}, \tag{24}$$

for a fixed τ/x^z. Note that we have $2z$ term in the above since an operator bilocal in time is involved. The present problem is unique as we have discussed already, in the sense that the exponent $\bar{\eta}$ has the fixed value $\bar{\eta}=4-2\sigma$ as long as the long-range interaction is relevant. The spin-glass correlator, as defined earlier, will be given by t times the scaling dimension of G^d and thus scales as

$$G(x,0,0)\sim x^{-(d+2z-4+\bar{\eta}+\theta)}. \quad (25)$$

Once again, using the definition of the exponent η [see Eq. (14)], we have

$$G(x,0,0)\sim x^{-(d+2z-2+\eta)}. \quad (26)$$

The exponent η is similarly fixed to the value $\eta=2-\sigma$ so that the above two equations lead us to the relation

$$\theta=2+\eta-\bar{\eta}=\sigma. \quad (27)$$

Interestingly, as we have discussed already, the exponent θ describing the dangerous irrelevance of the quantum fluctuations does not vary from its mean-field value for all $\sigma<2-\eta_{\mathrm{SR}}$. In short, as in the short-range case,[9] the usual scaling relations are satisfied but in the hyperscaling relation d is replaced by $d+2z-\theta$, with $2z$ coming from the bilocal (in τ) field and θ due to the dangerous irrelevance of t. The free energy density, on the other hand, is local in time. Considering a correlation volume ξ^{d+z} and with the assumption that the singular part of the free energy scales as ξ^{θ} [where ξ is the spatial correlation length scaling as $(r-r_c)^{-\nu}$], one finds the scaling form of the free-energy density given as $\xi^{-(d+z-\theta)}\sim(r-r_c)^{\nu(d+z-\theta)}$ near the quantum critical point $r=r_c$ and $T=0$.

At finite temperature, the length scale ξ scales with temperature as $\xi\sim T^{-(1/z)}$, and hence the free-energy density scales as $\xi^{-(d+z-\theta)}\sim T^{(d+z-\theta)/z}$, so that the specific heat scales as $T^{(d-\theta)/z}$. At the upper critical dimension ($d_u=4\sigma$) with $z=\theta=\sigma$, one retrieves expected T^3 behavior of the infinite-range system.[8]

At this point one can introduce the local variable $\psi(x,\tau)$ which couples to the mass term r that drives the zero-temperature transition and introduce the corresponding correlation function exponents η_ψ and $\bar{\eta}_\psi$ as in the short-range case.[9] Identical considerations yield the scaling form of the nonlinear susceptibility χ_{nl} with the associated length scale ξ,

$$\chi_{nl}\sim\xi^{z+2-\eta}\sim\xi^{z+\sigma}, \quad (28)$$

where we have used $\eta=2-\sigma$. The divergence of the nonlinear susceptibility χ_{nl} as $T\to 0$ is given by $\chi_{nl}\sim T^{-(z+\sigma)/z}$. Studies of the experimental sample $\mathrm{LiHo}_x\mathrm{Y}_{1-x}\mathrm{F}_4$ (Ref. 3 and 4) seem to indicate that this divergence decreases with decreasing temperature and there may well be no divergence of χ_{nl} at the quantum critical point. None of the previous analytical studies of the short-range quantum spin-glass systems reproduces this experimental observation. It has been suggested[6] that an appropriate choice of long-range interaction present in the experimental system may be important here. The present work, however, at least in the framework of the scaling theory we propose, seems to indicate a divergence of χ_{nl} as $T\to 0$. Of course, the Griffiths singularities present in the quantum systems at $T=0$ might well have influenced the experimental results.[6]

V. CONCLUSIONS

To conclude, we report studies of a quantum spin-glass model with long-range random interactions. We have used the existing droplet theory[12] and the Landau theory[9] for the short-range quantum spin glass (with the necessary extension for the long-ranged interacting case) to understand qualitatively the phases of the model.

We have already discussed in the Introduction that the quantum transition in the random quantum Ising system is associated with a "Griffiths-McCoy" singular region and an "activated quantum dynamical scaling"[13] at the quantum critical point. In the mean-field region and for quantum rotors with spin components $m\geqslant 2$,[7–9] the dynamical scaling at the quantum critical point is expected to be conventional. Our study shows that for any given dimension mean-field theory results are valid for a range of the parameter $\sigma\leqslant\sigma_u(=d/4)$. Thus, even in the one-dimensional quantum Ising model with long-range random interactions, the dynamical scaling at the quantum critical point should be conventional for $\sigma<0.25$.

Let us now address the question of quantum Griffiths-McCoy (GM) effects associated with the quantum transition in the present quantum spin-glass model. In the zero-temperature disordered phase of the short-range system,[19,20] under renormalization the system renormalizes to a collection of asymptotically uncopuled clusters with a broad distribution of effective fields. In the quantum-disordered phase, a clusters much larger than the correlation length are exponentially "rare." These rare regions typically have an exponetially large correlation length ξ_τ in the Trotter direction (relaxation time). These exponentially rare regions with an exponentially large ξ_τ result in a the divergence of the response and a continuously varying dynamical exponent in the disordered phase. In the present long-range case, one expects similar GM effects to occur in the quantum-disordered phase for any d and σ where the exponent ψ, as discussed earlier, may depend on the range of interactions σ.

At this point it would be useful to recall the droplet theory for the classical long-range Ising spin glass. As discussed already, for $\sigma>\sigma_c=d-2y_s$, there is a crossover from the long-range zero-temperature (droplet) fixed point to the corresponding short-range zero-temperature fixed point. Thus one finds that for the $d=1$ case, the long-range interactions are relevant in determining the ground-state properties for all $\sigma<3$ (using $y_s=-1$ for one-dimensional random chain.[25]) For $\sigma\geqslant 3$, the zero-temperature transition in the classical model is effectively of short-range nature. In a similar spirit, one can estimate the values of σ_c using the numerically obtained values of y_s for higher-dimensional classical systems.

Using these above information, we indicate the ordered phase of the present quantum spin glass in Fig. 1. Region **A** of Fig. 1 corresponds to the zero-temperature droplet fixed point (of the classical long-range Ising spin-glass) of long-range nature whereas region **B** refers to the short-range droplet fixed point. As discussed above the boundary between

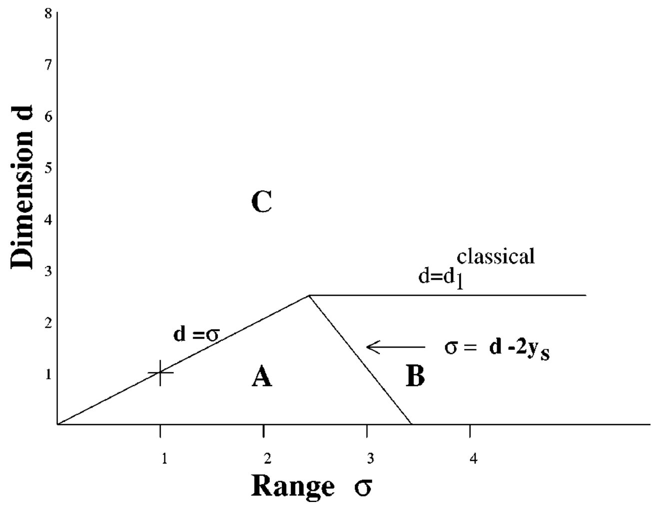

FIG. 1. Schematic diagram showing the ordered phases of the quantum Ising spin glass [in the (d-σ) plane] with long-range random interactions. We have used here the exact value $y_s=-1$ and the numerically obtained value $y_s\sim-0.3$ for one-dimensional and two-dimensional short-range classical spin glasses. It is also assumed that the lower critical dimension ($d_1^{\text{classical}}$) for a short-range classical spin glass lies in between $d=2$ and 3. The different regions of this phase diagram—the boundaries and the special point ($d=\sigma=1$)—are explained in the text.

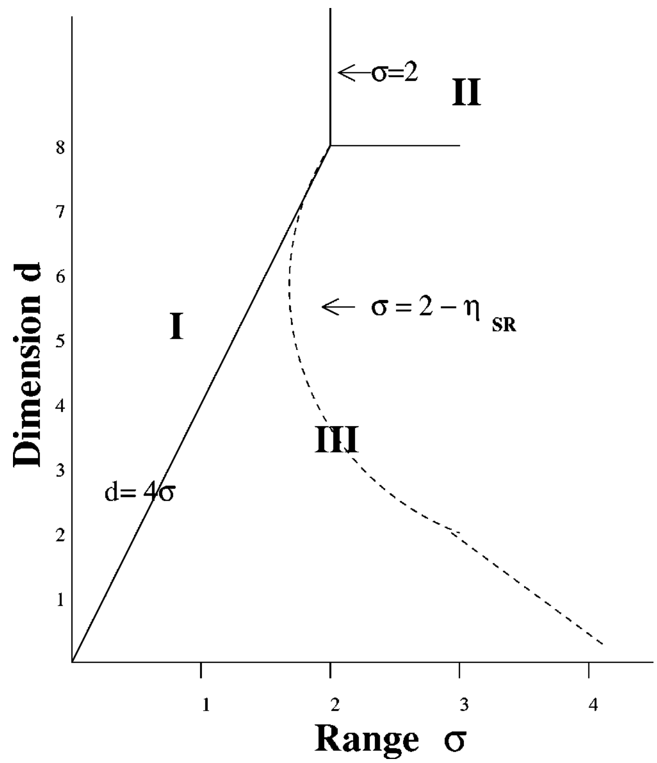

FIG. 2. Schematic diagram of the quantum critical behavior of the quantum Ising spin glass [in the (d-σ) plane] with long-range random interactions. We have used numerically obtained values of the exponent η_{SR}. In the present notation, $\eta_{\text{SR}}=-1$ for $d=2$ (Ref. 11) and $\eta_{\text{SR}}=-0.2$ for $d=3$ (Refs. 9 and 10). The three regions of this phase diagram and the boundaries are explained in the text.

these two regions is given by $\sigma=d-2y_s$. In region **C** the quantum system undergoes both the zero-temperature quantum transition and finite-temperature classical transition while in regions **A** and **B** there is a quantum transition but there is no long-range order at any finite temperature. As already discussed, the droplet theory for the long-range classical spin glass is really questionable for finite temperature. The boundary between regions **A** and **C** is $d=\sigma$ (as obtained from the droplet theory) whereas the boundary between regions **B** and **C** is given by $d=d_1^{\text{classical}}$ where $d_1^{\text{classical}}$ is the lower critical dimension of the classical short-range Ising spin glass. In the language of cluster renornalization group arguments, proposed in the short-range case,[19] in the ordered phase an asymptotic infinite cluster only forms at zero-energy in regions **A** and **B** whereas in region **C**, an infinite cluster is formed at a finite energy scale Ω_∞ (which is of the order of the classical transition temperature). The point of special interest happens to be $d=\sigma=1$ where we have a zero-temperaure transition and the classical transition at any finite temperature is a topological transition.[22]

Let us now concentrate on the $T=0$ quantum transition of the present long-range quantum spin glass and discuss the associated critical fixed points. Depending on the nature of the quantum critical behavior, we typically denote three regions I–III in Fig. 2. In region I, the mean-field theory with relevant long-range interactions (exponents depending on the value of σ) holds true (see Sec. III). The quantum critical fixed point is the long-range Gaussian fixed point. In region II, the mean-field theory for the short-range interacting quantum spin glass[9] is valid and here the critical fixed point is the short-range Gaussian fixed point. As already explained the crossover from the long-range Gaussian fixed point (region I) to the short-range one (region II) occurs at $\sigma=2$. In region III, the system has a nontrivial (not mean-field) quantum transition. The ϵ-expansion calculations fail to locate any nontrivial fixed point here. This is true even for quantum spin glasses with purely short-range interactions.[9] As discussed in Sec. III, in this region the critical fixed point should be the infinite-randomness fixed point. The boundary between region I and region III is given by the condition $d_u=4\sigma$ which yields $d=8$ for $\sigma=2$. We expect a crossover in region III from the IRFP with long-range exponents to the short-range one for sufficiently large values of σ. The ϵ-expansion calculations, however, indicate a crossover from long-range quantum critical behavior to the short-range one at $\sigma=2-\eta_{\text{SR}}$ where, as mentioned already, η_{SR} is the Fisher exponent for the short-range quantum system. This crossover is indicated with a dotted line in Fig. 2. As mentioned above, the ϵ expansion, on the other hand, fails to grasp the occurrence of the IRFP. It would be interesting to probe such a crossover behavior using numerical techniques.

We should specifically mention once again the different types of fixed points that are involved in the above discussion: (i) the quantum-disordered phase, where the system renormalizes to uncoupled clusters, and (ii) the ordered phase fixed point (see Fig. 1) of the quantum system where the system asymptotically renormalizes to an infinite cluster. Moreover, in describing the ordered phase, we refer to the zero-temperature droplet fixed points of the classical long-range Ising spin glass (regions **A** and **B** of the Fig. 1). In describing the quantum critical behavior, we come across two quantum critical fixed points: (a) Gaussian (long-range Gaussian for region I and short-range Gaussian for region II of Fig. 2) and (b) the infinite-randomness fixed point (region III of Fig. 2).

We have generalized the ϵ-expansion calculations used in the Ref. 9 to the present long-range problem. This method fails to find any stable weak-coupling fixed point for $d > d_u$ for a given σ and $\sigma > \sigma_u$ for a given d (i.e., in regions III and IV of Fig. 1). This is not surprising keeping in mind that even in the short-range case no stable weak-coupling fixed point is located for $d > 8$, but rather a flow to strong coupling is found. The simplest scenario here is that for both the short-range and the long-range case beyond the mean-field region the critical fixed point should be the infinite-randomness fixed point.[13,19] However, ϵ-expansion calculations provide a nice mean-field description even in the present long-range interacting cases. We find that the upper critical range and dimension are related through the expression $d_u = 4\sigma_u$, so that the upper critical range for $d = 1$ is $\sigma_u = 0.25$, in contrast to the classical case where $\sigma_u = 0.33$ for $d = 1$.[22] This is clearly due to the existence of a dangerously irrelevant operator t in the quantum problem. Interestingly, due to the long-range nature of interaction, we have a wide region of physically relevant parameter space (i.e., $\sigma_u = 3/4$ for $d = 3$) where mean-field theory is valid and the mean-field results hold good. We present as well a phenomenological scaling description based on the assumption of the existence of a dangerously irrelevant operator t,[9] which should be the valid near the quantum critical point. The scaling relations are derived under the assumption of a conventional dynamical scaling at the quantum critical point. As discussed already, this may not be a valid assumption for the Ising system with a general value of σ.

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