Possible origin of the broad peak around 450 cm^{-1} of the *c*-axis optical conductivity of the underdoped $YBa_2Cu_3O_{6+x}$ in the superconducting state

Lou Ping
*Department of Physics, University of Science and Technology of China, Hefei 230026, People's Republic of China and Department of Physics, Anhu University, Hefei 230039, People's Republic of China**
(Received 9 October 2001; revised manuscript received 19 February 2002; published 23 May 2002)

The main features of the *c*-axis infrared optical conductivity of a $d_{x^2-y^2}$-wave superconductor with the interlayer hopping assisted by the spin fluctuations are investigated theoretically. It is surprising to find that the interlayer hopping assisted by the spin fluctuations not only gives rise to the pseudogap structure of the *c*-axis infrared optical conductivity, but also leads to a broad peak anomaly of the *c*-axis infrared optical conductivity in the superconducting state. Based on these results, we propose an interpretation of the broad peak anomaly observed in the *c*-axis infrared optical conductivity of underdoped $YBa_2Cu_3O_{6+x}$ in the superconducting state. Namely, we argue that this broad peak anomaly originates from the mechanism of the interlayer hopping assisted by the spin fluctuations.

DOI: 10.1103/PhysRevB.65.214511 PACS number(s): 74.25.Gz, 74.20.Mn, 72.15.−v, 71.27.+a

I. INTRODUCTION

The *c*-axis optical conductivity of underdoped $YBa_2Cu_3O_{6+x}$ in the superconducting state shows that there is a broad peak around 450 cm^{-1}, a pseudogap starts to emerge well above T_c and does not form a Drude peak at $\omega=0$, and the gap continues to deepen gradually as the temperature is lowered and there is no sign of an anomaly at T_c.[1–4] What is the origin of the broad peak anomaly? Some believe that the origin is due to phonons.[2–4] Others speculate that the origin is due to some kind of resonance taking place between optical phonons and electronic gap excitations $2\Delta_0$.[1] Here we explore an alternative explanation, in which the anomalous behavior of the *c*-axis optical conductivity in the underdoped cuprates arises from the interlayer hopping assisted by the spin fluctuations. It should be noted that neutron-scattering experiments have unambiguously shown the presence of short-range antiferromagnetic spin correlations (fluctuations) in cuprate superconductors such as $La_{2-x}Sr_xCuO_4$ and $YBa_2Cu_3O_{6+x}$ at all doping levels, *x*, and some anomalous physical properties of high-temperature superconductors have been explained in terms of antiferromagnetic spin fluctuations.[5–16] In a recent paper[14] we developed a theory of *c*-axis electronic conductivity due to the competition between interlayer direct hopping and the hopping assisted by the spin fluctuations. On the basis of the theory we analyze the experimental data of the *c*-axis electronic conductivity of $YBa_2Cu_3O_{6+x}$ and conclude that in the underdoped regime the interlayer hopping assisted by the spin fluctuations is dominant. Here we demonstrate that the antiferromagnetic spin fluctuations also have important influence on the form of *c*-axis optical conductivity. In particular, if the interlayer hopping assisted by the spin fluctuations is dominant, then the antiferromagnetic spin fluctuations affect the *c*-axis optical conductivity and lead to the broad peak anomaly of the *c*-axis infrared conductivity of the underdoped cuprates in a superconducting state.

The paper is organized as follows. In Sec. II we develop a theory of the *c*-axis optical conductivity on the basis of the model of the competition between interlayer direct hopping and the hopping assisted by the spin fluctuations. In Sec. III we discuss our results. The paper concludes with a summary in Sec. IV.

II. THE THEORY

Following the idea of Ref. 14, the Hamiltonian describing *c*-axis transport properties due to the competition between interlayer direct hopping and hopping assisted by spin fluctuations can be written as

$$H=H^{(1)}+H^{(2)}+H_T, \tag{1}$$

where $H^{(1)}$ is the Hamiltonian for the one-layer carrier of the hopping junction. It contains all many-body interactions and has a $d_{x^2-y^2}$ symmetry superconducting ground state. Similarly, $H^{(2)}$ has all the physics for the two-layer carrier of the hopping junction. These two are considered to be strictly independent. Not only do these two operators commute, $[H^{(1)},H^{(2)}]=0$, but they commute term by term. The interlayer hopping is caused by the term H_T in Eq. (1):

$$\begin{aligned} H_T=&\frac{J}{\sqrt{N}}\sum_{\mathbf{kk}'\mu\mu'}[\mathbf{S}^{(1)}(\mathbf{k}'-\mathbf{k})\boldsymbol{\sigma}_{\mu\mu'}C^{(1)\dagger}_{\mathbf{k}\mu}C^{(2)}_{\mathbf{k}'\mu'}\\ &+\mathbf{S}^{(2)}(\mathbf{k}-\mathbf{k}')\boldsymbol{\sigma}_{\mu'\mu}C^{\mu'(2)\dagger}_{\mathbf{k}'}C^{(1)}_{\mathbf{k}\mu}+\text{H.c.}]\\ &+\frac{1}{\sqrt{N}}\sum_{\mathbf{kk}'\mu\mu'}[D^{\mu\mu'}_{\mathbf{kk}'}C^{(1)\dagger}_{\mathbf{k}\mu}C^{(2)}_{\mathbf{k}'\mu}+D^{\mu'\mu}_{\mathbf{k}'\mathbf{k}}C^{(2)\dagger}_{\mathbf{k}'\mu'}C^{(1)}_{\mathbf{k}\mu}]. \end{aligned} \tag{2}$$

Here, *J* is the constant of the interlayer hopping assisted by the spin fluctuations, $D^{\mu\mu'}_{\mathbf{kk}'}$ is the interlayer direct hopping matrix element, $\boldsymbol{\sigma}_{\mu\mu'}$ is the Pauli matrix element, $\mathbf{S}^{(\mathbf{i})}(\mathbf{q})$ is the *i*-layer spin-fluctuation operator, and $C^{(i)\dagger}_{\mathbf{k}\mu}(C^{(i)}_{\mathbf{k}\mu})$ is the *i*-layer carrier creation (annihilation) operator. Physically, interlayer hopping assisted by spin fluctuations arises from the spin fluctuations scattering [represented by $\mathbf{S}^{(\mathbf{i})}(\mathbf{q})$ which couples to the quasiparticles with strength J], which is analogous to the standard case of phonon-assisted hopping,[17] except that the spin-fluctuation operator replaces the phonon operator. Then the current operator of interlayer direct hopping and hopping assisted by spin fluctuations is given by

$$j_c=\frac{iedJ}{\sqrt{N}}\sum_{\mathbf{k}\mathbf{k}'\mu\mu'}[\mathbf{S}^{(1)}(\mathbf{k}'-\mathbf{k})\boldsymbol{\sigma}_{\mu\mu'}C^{(1)\dagger}_{\mathbf{k}\mu}C^{(2)}_{\mathbf{k}'\mu'}+\mathbf{S}^{(2)}(\mathbf{k}-\mathbf{k}')\boldsymbol{\sigma}_{\mu'\mu}C^{(2)\dagger}_{\mathbf{k}'\mu'}C^{(1)}_{\mathbf{k}\mu}-\text{H.c.}]+\frac{ied}{\sqrt{N}}\sum_{\mathbf{k}\mathbf{k}'\mu\mu'}[D^{\mu\mu'}_{\mathbf{k}\mathbf{k}'}C^{(1)\dagger}_{\mathbf{k}\mu}C^{(2)}_{\mathbf{k}'\mu'}-D^{\mu'\mu}_{\mathbf{k}'\mathbf{k}}C^{(2)\dagger}_{\mathbf{k}'\mu'}C^{(1)}_{\mathbf{k}\mu}]. \tag{3}$$

After the standard procedure is applied to the conductivity in the c direction (cf. Ref. 18), the optical conductivity is given by the following formula:

$$\sigma(\omega)=\sigma_c^d(\omega)+\sigma_c^{\text{sf}}(\omega). \tag{4}$$

Here, $\sigma_c^d(\omega)$ is due to interlayer direct hopping and is given by

$$\sigma_c^d(\omega)=\frac{4e^2d}{a^2}\frac{1}{N^2}\int\frac{d\omega'}{2\pi}\sum_{\mathbf{kq}}D^2_{\mathbf{k},\mathbf{q}}[n_F(\omega')-n_F(\omega'+\omega)]\times[A^{(1)}(\mathbf{k},\omega')A^{(2)}(\mathbf{q},\omega'+\omega)+B^{(1)}(\mathbf{k},\omega')B^{(2)}(\mathbf{q},\omega'+\omega)]/\omega, \tag{5}$$

while $\sigma_c^{\text{sf}}(\omega)$ is due to interlayer hopping assisted by spin fluctuations and is given by

$$\sigma_c^{\text{sf}}(\omega)=\frac{6J^2e^2d^2}{VN}\int\int\frac{d\omega_1}{2\pi}\frac{d\omega_2}{2\pi}\sum_{\mathbf{kq}}[n_F(\omega_1)+n_B(\omega+\omega_1-\omega_2)][n_F(\omega_2-\omega)-n_F(\omega_2)]\times\text{Im}\chi^{-+}(\mathbf{k}-\mathbf{q},\omega+\omega_1-\omega_2)[A^{(1)}(\mathbf{q},\omega_1)\times A^{(2)}(\mathbf{k},\omega_2)+B^{(1)}(\mathbf{k},\omega_1)B^{(2)}(\mathbf{q},\omega_2)]/\omega, \tag{6}$$

where $\sigma_c^{\text{sf}}(\omega)$ is the optical conductivity of hopping assisted by spin fluctuations, e is the unit charge, a is the ab-plane lattice constant, and d is the c-axis interlayer distance. $A^{(i)}(\mathbf{k},\omega_i)$ and $B^{(i)}(\mathbf{k},\omega_i)$ are the normal and anomalous spectral functions for the electron in the i layer, $n_F(\omega)$ is the Fermi function, $n_B(\omega)$ is the Bose function, and $\text{Im}\,\chi^{-+}(\mathbf{k},\omega)$ is the spin-fluctuation spectral function (where we only consider the intralayer spin correlation).

Because we primarily study the interlayer hopping of the quasiparticle, we neglect the effects of the in-plane interaction and simply choose the following approximation for $A^{(i)}(\mathbf{k},\omega_i)$ and $B^{(i)}(\mathbf{k},\omega_i)$ (see Ref. 18):

$$A^{(i)}(\mathbf{k},\omega_i)=2\pi[u^2_{\mathbf{k}}\delta(\hbar\omega_i-E^{(i)}_{\mathbf{k}})+v^2_{\mathbf{k}}\delta(\hbar\omega_i+E^{(i)}_{\mathbf{k}})], \tag{7}$$

and

$$B^{(i)}(\mathbf{k},\omega_i)=-\pi\frac{\Delta_{\mathbf{k}}}{E_{\mathbf{k}}}[u^2_{\mathbf{k}}\delta(\hbar\omega_i-E^{(i)}_{\mathbf{k}})-v^2_{\mathbf{k}}\delta(\hbar\omega_i+E^{(i)}_{\mathbf{k}})], \tag{8}$$

where

$$\Delta_{\mathbf{k}}=\frac{1}{2}\Delta_0(T)(\cos k_xa-\cos k_ya) \tag{9}$$

is the order parameter in the superconducting state. $u^2_{\mathbf{k}}$ and $v^2_{\mathbf{k}}$ are the usual superconducting coherence factors. $E^{(i)}_{\mathbf{k}}$ are the excitation energies of the superconductor in the i layer which have the following form:

$$E_{\mathbf{k}}=[\varepsilon^2_{\mathbf{k}}+\Delta^2_{\mathbf{k}}]^{1/2}, \tag{10}$$

with

$$\varepsilon_{\mathbf{k}}=-2t(\cos k_xa+\cos k_ya)-4t'\cos k_xa\cos k_ya-\mu. \tag{11}$$

Here t and t' are the nearest- and the next-nearest neighbor hopping amplitudes and μ is the chemical potential. Then from Eqs. (5)–(8) we obtain the c-axis optical conductivity of interlayer hopping of the quasiparticle as follows:

$$\sigma(\omega)=\sigma_c^d(\omega)+\sigma_c^{\text{sf}}(\omega), \tag{12}$$

with

$$\sigma_c^d(\omega)=\frac{e^2d}{\hbar a^2}\frac{1}{N^2}\sum_{\mathbf{kq}}D^2_{\mathbf{k},\mathbf{q}}\Bigg\{\Bigg[\left(1+\frac{\varepsilon_{\mathbf{k}}\varepsilon_{\mathbf{k+q}}+\Delta_{\mathbf{k}}\Delta_{\mathbf{k+q}}}{E_{\mathbf{k}}E_{\mathbf{k+q}}}\right)\delta(\varepsilon_{\mathbf{k}}-\hbar\omega-\varepsilon_{\mathbf{q}})+\left(1-\frac{\varepsilon_{\mathbf{k}}\varepsilon_{\mathbf{k+q}}+\Delta_{\mathbf{k}}\Delta_{\mathbf{k+q}}}{E_{\mathbf{k}}E_{\mathbf{k+q}}}\right)\times\delta(\varepsilon_{\mathbf{k}}-\hbar\omega+\varepsilon_{\mathbf{q}})\Bigg][n_F(\varepsilon_{\mathbf{k}}-\hbar\omega)-n_F(\varepsilon_{\mathbf{k}})]/\hbar\omega-\Bigg[\left(1+\frac{\varepsilon_{\mathbf{k}}\varepsilon_{\mathbf{k+q}}+\Delta_{\mathbf{k}}\Delta_{\mathbf{k+q}}}{E_{\mathbf{k}}E_{\mathbf{k+q}}}\right)\delta(\varepsilon_{\mathbf{k}}+\hbar\omega-\varepsilon_{\mathbf{q}})+\left(1-\frac{\varepsilon_{\mathbf{k}}\varepsilon_{\mathbf{k+q}}+\Delta_{\mathbf{k}}\Delta_{\mathbf{k+q}}}{E_{\mathbf{k}}E_{\mathbf{k+q}}}\right)\delta(\varepsilon_{\mathbf{k}}+\hbar\omega+\varepsilon_{\mathbf{q}})\Bigg]\times[n_F(\varepsilon_{\mathbf{k}}+\hbar\omega)-n_F(\varepsilon_{\mathbf{k}})]/\hbar\omega\Bigg\}, \tag{13}$$

which is due to interlayer direct hopping of the quasiparticle, and

$$\sigma_c^{\mathrm{sf}}(\omega)=\frac{3\sigma_0 t^2}{2N^2}\sum_{\mathbf{kq}}\left\{\left[\mathrm{Im}\,\chi^{-+}(\mathbf{k},\varepsilon_{\mathbf{q}}+\hbar\omega-\varepsilon_{\mathbf{k+q}})[n_F(\varepsilon_{\mathbf{q}})+n_B(\varepsilon_{\mathbf{q}}+\hbar\omega-\varepsilon_{\mathbf{k+q}})]\left(1+\frac{\varepsilon_{\mathbf{k}}\varepsilon_{\mathbf{k+q}}+\Delta_{\mathbf{k}}\Delta_{\mathbf{k+q}}}{E_{\mathbf{k}}E_{\mathbf{k+q}}}\right)\right.\right.$$
$$\times+\mathrm{Im}\,\chi^{-+}(\mathbf{k},\varepsilon_{\mathbf{q}}-\hbar\omega+\varepsilon_{\mathbf{k+q}})[n_F(\varepsilon_{\mathbf{q}})+n_B(\varepsilon_{\mathbf{q}}-\hbar\omega+\varepsilon_{\mathbf{k+q}})]\left.\left(1-\frac{\varepsilon_{\mathbf{k}}\varepsilon_{\mathbf{k+q}}+\Delta_{\mathbf{k}}\Delta_{\mathbf{k+q}}}{E_{\mathbf{k}}E_{\mathbf{k+q}}}\right)\right]$$
$$\times[n_F(\varepsilon_{\mathbf{k+q}}-\hbar\omega)-n_F(\varepsilon_{\mathbf{k+q}})]/\hbar\omega-\left[\mathrm{Im}\,\chi^{-+}(\mathbf{k},\varepsilon_{\mathbf{q}}-\hbar\omega-\varepsilon_{\mathbf{k+q}})[n_F(\varepsilon_{\mathbf{q}})+n_B(\varepsilon_{\mathbf{q}}-\hbar\omega-\varepsilon_{\mathbf{k+q}})]\right.$$
$$\times\left(1+\frac{\varepsilon_{\mathbf{k}}\varepsilon_{\mathbf{k+q}}+\Delta_{\mathbf{k}}\Delta_{\mathbf{k+q}}}{E_{\mathbf{k}}E_{\mathbf{k+q}}}\right)+\mathrm{Im}\,\chi^{-+}(\mathbf{k},\varepsilon_{\mathbf{q}}+\hbar\omega+\varepsilon_{\mathbf{k+q}})[n_F(\varepsilon_{\mathbf{q}})+n_B(\varepsilon_{\mathbf{q}}+\hbar\omega+\varepsilon_{\mathbf{k+q}})]$$
$$\left.\left.\times\left(1-\frac{\varepsilon_{\mathbf{k}}\varepsilon_{\mathbf{k+q}}+\Delta_{\mathbf{k}}\Delta_{\mathbf{k+q}}}{E_{\mathbf{k}}E_{\mathbf{k+q}}}\right)\right]\times[n_F(\varepsilon_{\mathbf{k+q}}+\hbar\omega)-n_F(\varepsilon_{\mathbf{k+q}})]/\hbar\omega\right\}, \qquad (14)$$

which is due to interlayer hopping assisted by spin fluctuations. $\sigma_0=J^2e^2d/\hbar a^2t^2$ is a characteristic c-axis conductivity scale. Above T_c the gap $\Delta_{\mathbf{k}}$ goes to zero, and Eq. (13) andEq. (14) reduce to the normal-state expressions. Our model contains two limiting cases. In the limit $J=0$ and $D_{\mathbf{k,q}}\neq 0$, only interlayer direct hopping is present. The opposite limiting case is that $D_{\mathbf{k,q}}=0$ and $J\neq 0$ (i.e., only interlayer hopping assisted by spin fluctuations is present). Because in the underdoped cuprate interlayer hopping assisted by spin fluctuations is dominant,[14] interlayer direct hopping can be omitted. Thus in the next section we will calculate the c-axis infrared conductivity of a $d_{x^2-y^2}$-wave superconductor with hopping assisted by spin fluctuations only for the underdoped cuprate (i.e., $D_{\mathbf{k,q}}=0$ and $J\neq 0$).

III. RESULTS

In this section, we first introduce the spin-fluctuation spectral function. Then we present and discuss the results computed by using Eq. (14), which was derived above.

A recent paper[19] investigated in-plane infrared conductivity caused by mobile charge carriers coupling to spin fluctuations based on the following phenomenological form of the spin-fluctuation spectral function:

$$\chi^{-+}(\mathbf{q},\hbar\omega)=\frac{1}{1+\xi^2(\mathbf{q}-\mathbf{Q})^2}\,\frac{F}{\omega_0^2-\omega^2-i\omega\gamma}, \qquad (15)$$

where ξ is the magnetic correlation length, and ω_0 and γ are the frequency and the broadening of the resonance peak, respectively. F is an overall temperature-independent constant and $\mathbf{Q}$ is the position of the peak in momentum space which is assumed to be commensurate, i.e., $\mathbf{Q}=(1,1)\pi/a$. Here, for the sake of simplifying computation, we will use it to investigate the c-axis infrared conductivity arising from interlayer hopping assisted by spin fluctuations.

Furthermore, the goal of this paper is not to quantificationally compare the theoretical calculation result with experiment, but to investigate the main features of the c-axis infrared conductivity arising from interlayer hopping assisted by spin fluctuations and to speculate on the origin of the broad peak anomaly observed in the c-axis infrared optical conductivity of underdoped $YBa_2Cu_3O_{6+x}$ in the superconducting state. Therefore, by substituting Eq. (15), the phenomenological form of the spin-fluctuation spectral function; Eq. (9), the order parameter in the superconducting state;Eq. (10), the excitation energy of the superconductor; and Eq. (11), the single-particle energy spectrum into Eq. (14), the c-axis infrared conductivity arising from interlayer hopping assisted by spin fluctuations (i.e., neglecting direct hopping and only considering hopping assisted by spin fluctuations), we have computed $\sigma_c^{\mathrm{sf}}(\omega)/\sigma_0$ [instead of $\sigma_c^{\mathrm{sf}}(\omega)$] versus $\hbar\omega/t$. Note that here and hereafter, σ_0 has been redefined, namely, $\sigma_0=FJ^2e^2d/\hbar a^2t^2$. In the computation for $YBa_2Cu_3O_{6.6}$(YBCO), we choose $\hbar\omega_0=34$ (meV), $\hbar\gamma=10$ (meV), and $\xi=12$ (Å) for the values of the parameter of the spin-fluctuation spectral function in the superconducting state and the normal state near the superconducting transition temperature T_c. It should be pointed out that the values of these parameters were estimated in Ref. 19 from the neutron-scattering experiments in the superconducting state. The values of the parameters of the single-particle energy spectrum used are $t=200$ (meV), $t'/t=-0.3$, and $\mu/t=-1.1$.[20] The temperature-dependent part of the order parameter of the superconducting state is given by

$$\Delta_0(T)=\Delta_0\tanh(\alpha\sqrt{T_c/T-1}). \qquad (16)$$

$\alpha=3$, $T_c=63$ (K), and $\Delta_0/\kappa_B T_c=2.14$ have been used. Numerical results for $\sigma_c^{\mathrm{sf}}(\omega)/\sigma_0$ versus $\hbar\omega/t$ at different temperatures T are shown in Fig. 1. It shows that there is no Drude peak at $\omega=0$ anda pseudogap starts to emerge for $\omega<\omega^*$ ($\omega^*\simeq 0.3t$) for $T>T_c$, and persists into the superconducting state, with no sign of an anomaly near T_c. After going into the superconducting state, we find that a broad peak around ω^* starts to emerge and this unusual broad peak increases in strength as the temperature is lowered. It is obvious that the c-axis infrared conductivity calculated on the basis of the model of interlayer hopping assisted by spin fluctuations, shown in Fig. 1, captures the main features of experiments,[1–4] which we mentioned in the Introduction.

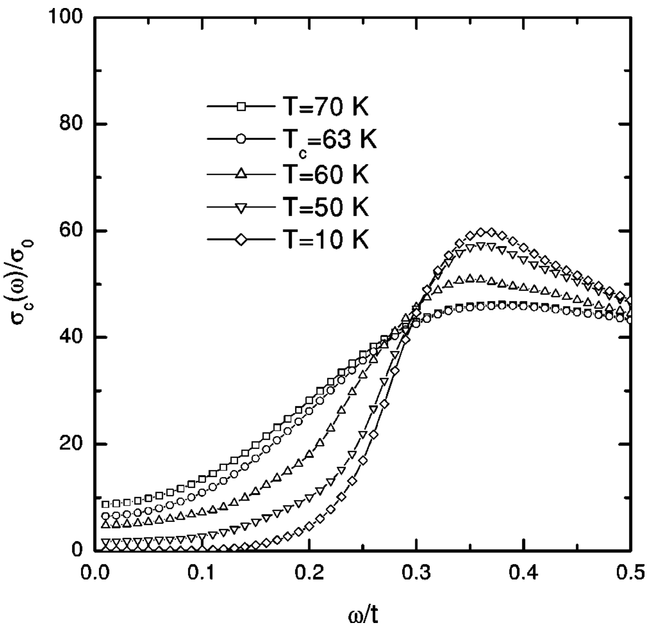

FIG. 1. $\sigma_c(\omega)/\sigma_0$ versus $\hbar\omega/t$ at several T for YBCO in the underdoped regime. The parameters used are $\hbar\omega_0=34$ (meV), $\hbar\gamma=10$ (meV), $\xi=12$ (Å), $t=200$ (meV), $t'/t=-0.3$, and $\mu/t=-1.1$.

Numerical results for the $\sigma_c^{sf}(\omega)/\sigma_0$ versus $\hbar\omega/t$ curve in the superconducting state with different superconducting transition temperatures T_c are displayed in Fig. 2. In this figure, we find that as the superconducting transition temperature T_c decreases, the broad peak around ω^* gradually disappears. It is worthwhile to present the experimental result of the Zn-doping effect on the c-axis optical conductivity of the underdoped cuprates,[1] because this experimental result may be indirect experimental evidence for the theoretical prediction. In Ref. 1 Fukuzumi, Mizuhashi, and Uchida reported that when T_c is suppressed by Zn, the normal-state pseudogap structure seen in c-axis optical conductivity is robust. But in the superconducting state, as Zn doping radically suppresses T_c, the broad peak around 450 cm^{-1} gradually disappears. It should be noted that in Ref. 21 Mizuhashi, Takenaka, Fukuzumi, and Uchida interpreted the experimental result found that Zn doping does not affect the basic T dependence of in-plane resistivity of $YBa_2Cu_3O_{6+x}$ on the basis of the model of mobile charge carriers coupling to spin fluctuations. They concluded that although Zn doping radically affects the parts of the spin-fluctuation spectrum which are responsible for superconducting pair formation, the global feature of spin fluctuations is that they are not sensitive to Zn doping. Based on this suggestion and the present theoretical scenario, we not only can understand that Zn doping does not affect the basic T dependence of in-plane resistivity arising from mobile charge carriers coupling to spin fluctuations,[21] but also that Zn doping does not affect both the basic T dependence of c-axis resistivity and the pseudogap structure of c-axis optical conductivity, which originate from interlayer hopping assisted by spin fluctuations.[1] Keeping this analysis in mind, we conclude that the theoretical prediction, displayed in Fig. 2, is qualitatively in agreement with experiment.[1] Thus, it is reasonable to speculate that the main origin of the broad peak anomaly of c-axis infrared conductivity of underdoped cuprates in the superconducting state is due to the mechanism of interlayer hopping assisted by spin fluctuations.

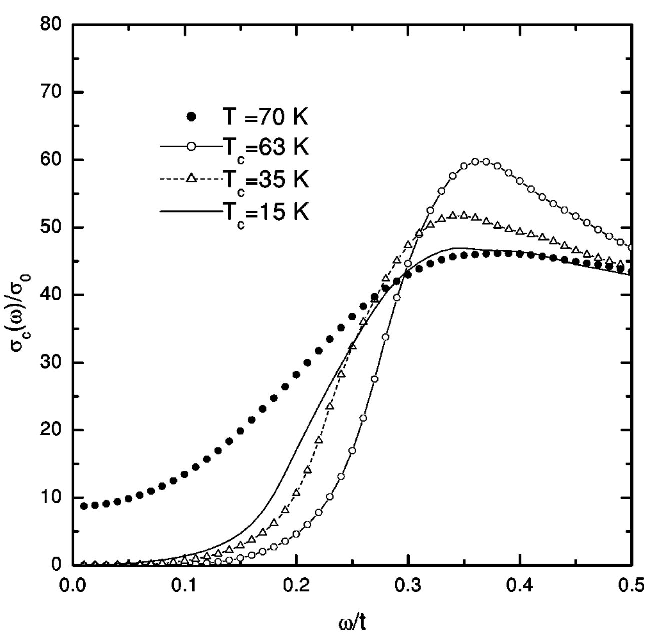

FIG. 2. $\sigma_c(\omega)/\sigma_0$ versus $\hbar\omega/t$ at $T=10$ (K) for YBCO in the underdoped regime. Except at $T=70$ (K) they are all in the superconducting state. The parameters used are the same as in Fig. 1.

IV. CONCLUDING REMARKS

In this paper, we have computed the c-axis infrared conductivity of a $d_{x^2-y^2}$-wave superconductor with hopping assisted by spin fluctuations based on the oversimplified form of the spin-fluctuation spectral function (i.e., neglecting the detailed structure of the spin-fluctuations). However, the present form of the spin fluctuation spectral function is not only analytical, but can also incorporate well the narrow magnetic peak observed at low temperatures by neutron scattering.[19] In addition, the aim of the present paper was to explore the main features of c-axis infrared conductivity arising from interlayer hopping assisted by spin fluctuations, but not to investigate the detailed properties of the spin system itself. Keeping this fact in mind and the conclusion that charge transport has an intimate connection with the global feature of spin fluctuations,[21] we expect that the mechanism of interlayer hopping assisted by spin fluctuations does not depend on the detail of the spin-fluctuation spectral function, and accordingly the intrinsic conclusions obtained in this paper are not altered by the more improved form of the spin-fluctuation spectral function (for example, the spin-fermion model[22]) at least qualitatively.

Although the focus of this paper was to argue that the mechanism of interlayer hopping assisted by spin fluctuations is the main origin of the broad peak anomaly of c-axis infrared optical conductivity of underdoped $YBa_2Cu_3O_{6+x}$ in the superconducting state, it is worthwhile to simply comment on the pseudogap phenomena. The pseudogap phenomena are universal phenomena observed in various compounds of underdoped cuprates,[23,24] which not only appear in

ab-plane properties, but also in c-axis properties. As regards the origin of the pseudogap, for instance, the pseudogap seen in ab-plane properties, a number of scenarios such as pair formation well above T_c,[25–27] spin-charge separation,[28,29] spin-density wave (SDW) or antiferromagnetic fluctuations,[20,30,31] and a hidden order parameter[32–34] have been proposed. However, no consensus has been reached so far, as to the correct microscopic theory. In addition, it is still uncertain whether the pseudogaps seen in both the ab-plane properties and the c-axis properties arise from the same origin, or whether there is an intimate connection between them. For example, some researchers have suggested that the pseudogap structure seen in c-axis optical conductivity may be caused mainly by non-Fermi ''Luttinger'' liquid in-plane behavior,[35] but others argued that the pseudogap structure seen in c-axis optical conductivity is main due to the result of the phase fluctuation of the superconducting order parameter.[36] It should be noted that although their views are divergent on the origin of the pseudogap seen in ab-plane properties (i.e., the former researcher[35] believes that it arises from ''Luttinger'' liquid behavior, while the latter[36] believe that it is caused by pairing without long-range phase coherence), they all believe that the pseudogap structure seen in c-axis optical conductivity is due to the effect of the pseudogap seen in ab-plane properties. In this paper, we suggest that the pseudogap structure seen in c-axis optical conductivity is due instead to the mechanism of interlayer hopping assisted by spin fluctuations. It should be pointed out that the present paper does not rule out the possible existence of other effects (for example, the effect caused by the non-Fermi ''Luttinger'' liquid in-plane behavior[35] or superconducting pairing without long-range order[36]).

In summary, the formulas of c-axis infrared conductivity of a $d_{x^2-y^2}$-wave superconductor with competition between interlayer direct hopping and hopping assisted by spin fluctuations have been derived. On basis of these formulas, we have computed c-axis infrared conductivity arising from interlayer hopping assisted by spin fluctuations (i.e., neglecting direct hopping and only considering hopping assisted by spin fluctuations). Although there still needs some simplification to improve, for example, the calculation based on the more improved form of the spin-fluctuation spectral function (for example, the spin-fermion model[22]), the essence of c-axis infrared optical conductivity of underdoped cuprates has been mimicked by theoretical predictions. Namely, the theoretical predictions capture the main characteristics of experiment. Based on these results, we argue that the mechanism of interlayer hopping assisted by spin fluctuations is not only the main origin of the broad peak anomaly of c-axis infrared optical conductivity of underdoped cuprates in the superconducting state, but also the main origin of the pseudogap structure seen in c-axis optical conductivity.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation under Grant No. 19974001 and the Education Commission of Anhui.

*Address for correspondence.

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