Multivalued dependence of the magnetoresistance on the quantized conductance in nanosize magnetic contacts

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We calculate the quantized conductance of nanosize point contacts between two ferromagnets for different mutual orientations of the magnetic moments. It is found that the magnetoresistance (MR) is a multivalued function of the quantized conductance at the parallel alignment of the magnetizations σ^F. This leads us to the conclusion that experimentally observed large fluctuations of MR versus σ^F are rather due to the conductance quantization than to measurement errors or a poor reproducibility of the results. Using the results of the calculations we are able to understand experimental data obtained by García *et al.* for MR of the magnetic nanocontacts.

I. INTRODUCTION

Recently giant magnetoresistance (GMR) exceeding 200% was discovered by García *et al.* in Ni-Ni (Ref. 1) and Co-Co (Ref. 2) point contacts at room temperature. Somewhat smaller (∼30%) but also very large magnetoresistance was observed in Fe-Fe point contacts.[3] These experiments revealed large fluctuations in the measured values of the magnetoresistance drawn versus the conductance at ferromagnetic alignment of magnetizations in contacts σ^F (*F* conductance). For Ni-Ni and Co-Co contacts, the fluctuations are especially large at σ^F of the order of several elementary conductances e^2/h, which may indicate that the effect observed is related to a conductance quantization.

The quantization of the conductance in magnetic nanosize contacts has been observed experimentally in Refs. 4–6. Costa-Krämer[4] and Oshima and Miyano[5] reported on an odd integer number N of open conductance channels [$\sigma = N(e^2/h)$] in nickel point contacts at room temperature. Ono *et al.*[6] presented evidence of changing the conductance quantum from $2e^2/h$ to e^2/h at room temperature in nickel nanocontacts of another morphology. Imamura *et al.*[7] and Zvezdin and Popkov[8] have calculated the conductance of a point contact between two ferromagnets and demonstrated the e^2/h conductance quantization due to a nonsimultaneous opening of "up" and "down" spin channels. Imamura *et al.*[7] also studied numerically the magnetoresistance as a function of *F* conductance σ^F and came to the conclusion that in the conductance quantization regime, the magnetoresistance oscillated as a function of the conductance.

In this paper, we calculate the conductance and the magnetoresistance of nanosize magnetic contacts in the regime of conductance quantization. We found that at low temperatures, the magnetoresistance is a multivalued function of the conductance at the parallel alignment of magnetizations σ^F. In other words, in the regime of quantization, different samples, having the same *F* conductance σ^F, may have different magnetoresistances. The distribution of the magnetoresistance is extremely broad for the first few open *F*-conductance channels. This leads us to the conclusion that large data fluctuations observed in the experiments by García *et al.*[1–3] may be a direct consequence of the conduction quantization. This means that the data fluctuations are inevitable for the magnetoresistance measurements in the nanosize magnetic contacts and this effect should not be treated as being due to experimental errors or a poor reproducibility of the measurements.

II. BASIC FORMULAS FOR THE CONDUCTANCE AND THE MAGNETORESISTANCE

In a recent paper[9] we applied a quasiclassical (QC) method for calculations of the conductance of point contacts between ferromagnetic metals. We considered a model of two ferromagnetic, single domain half spaces contacting each other through a circular hole of a radius a in an impenetrable membrane, separating the domains. At antiferromagnetically (AF) aligned domains, a domain wall (DW) is created inside the constriction. We argued that the giant magnetoresistance values were determined by peculiarities of the carrier transmission through DW.

If the spin direction does not change when passing through DW, the carriers are strongly reflected by the interface. This effect can easily be understood because, in this situation, the electron moves effectively in a steplike potential. Of course, this reflection is large if the change of the magnetization in the constriction occurs at short distances. The above scenario may be realized provided the DW width is small, $d_W < d_s$, where $d_s = \min(v_F/\omega_Z, v_F T_1)$, T_1 is the longitudinal relaxation rate time of the carriers magnetization, and ω_z is the Zeeman precession frequency.[10] In this limit, the carrier spin does not have enough time to follow the magnetization profile in DW. The strong reflection on DW leads to the magnetoresistance of the order of few hundred percents, if one uses reasonable values of spin polarizations of the conduction band estimated from the experimental data of Refs. 11–14.

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In this paper, we use the model described above for the case when the conductance of the constriction is quantized. The connecting hole is assumed to have a cylindrical shape of arbitrary (but shorter than the mean free path l) length d. In the case of F alignment of the magnetizations, the carriers move effectively in a constant potential. For the AF-aligned domains, the carriers move in a potential corresponding to the magnetization profile of the domain wall (Ref. 15, Fig. 2). The hole connecting the two parts of the space plays the role of a filter selecting only those incidence angles that are allowed by the energy and momentum conservation. As the diameter of the hole is assumed to be very small, we may use the ballistic-limit versions of Eqs. (14), (18), and (19) of our work[9] to calculate the conductance of the constriction:

$$\sigma^F = \sigma_{\uparrow\uparrow} + \sigma_{\downarrow\downarrow} = \frac{e^2}{h} \widetilde{\sum_{m,n}} \{D_{\uparrow\uparrow}(x_{mn}) + D_{\downarrow\downarrow}(x_{mn})\}, \quad (1)$$

$$\sigma^{AF} = \frac{2e^2}{h} \widetilde{\sum_{m,n}} D_{\uparrow\downarrow}(x_{mn}). \quad (2)$$

Similar formulas can also be obtained within the Landauer-Büttiker scattering formalism.[16] In the above expressions, $\sigma^F(\sigma^{AF})$ is the conductance at ferromagnetic (antiferromagnetic) alignment of the domains, $\sigma_{\alpha\alpha}$ is the conductance for the αth spin channel, and $x_{mn} = \cos\theta$ is the cosine of the quasiparticle incidence angle θ measured from the cylinder axis direction, and the allowed values of $\cos\theta$ are defined by Eq. (4). $D_{\alpha\beta}(x)$ is the quantum-mechanical transmission coefficient for the connecting hole. Calculation of this coefficient is straightforward, but lengthy. It is presented in the Appendix, and an explicit expression for $D_{\alpha\beta}(x)$ is given by Eqs. (A9)–(A15).

The conductance quantization is assumed to be due to the quantization of transversal motion in the constriction. In the ballistic regime, when disorder is neglected, the quantization of the transversal motion in the hole imposes the following condition for the component $p_\parallel$ of the quasiparticle momentum parallel to the interface:

$$p_\parallel = p_{F\alpha} \sin\theta = p_{mn} \equiv \hbar a^{-1} Z_{mn}, \quad (3)$$

where $p_{F\alpha}$ is the Fermi momentum for the αth spin channel, Z_{mn} is the nth zero of the Bessel function $\mathbf{J}_m(x)$ (see Appendix), and a is the radius of the hole. The assumption of the ballistic motion is quite reasonable provided the size of the hole is much smaller than the mean free path l. We assume everywhere in this paper that the inequality $a \ll l$ is fulfilled.

Equation (3) is the first basic selection rule. The tilde in Eqs. (1) and (2) means that the summations should be done over the open conduction channels satisfying the condition

$$x_{mn} \equiv \cos\theta = \sqrt{1 - (\hbar Z_{mn}/p_{F\alpha}a)^2} \leq 1. \quad (4)$$

When the alignment of the magnetizations is ferromagnetic, the Fermi momenta on both sides of the contact are equal to each other in the each spin channel. The energy and momentum conservation is already taken into account in Eq.

(1) (both the ingoing and outgoing quasiparticles have the same Fermi energy and the specular character of the scattering follows automatically).

At the antiferromagnetic alignment, the conservation of the momentum parallel to the interface ($p_\parallel \equiv p_{F1\alpha} \sin\theta_1 = p_{F2\alpha} \sin\theta_2$, where the subscript 1 or 2 labels left- or right-hand side of the contact, respectively) introduces the *additional* selection rule into Eq. (3):

$$p_{F\alpha} = \min(p_{Fj\uparrow}, p_{Fj\downarrow}). \quad (5)$$

This selection rule strictly holds when the spin of the electron does not change during the electron flight through the DW. This situation is realized in the model of quantum DW (Ref. 7) and in the model of effectively abrupt DW.[9]

The magnetoresistance is defined as[17]

$$MR = \frac{R^{AF} - R^F}{R^F} = \frac{\sigma^F - \sigma^{AF}}{\sigma^{AF}}. \quad (6)$$

The value of the magnetoresistance is sensitive to the profile of the DW and can become very large for sharp changes of the magnetization. We give an exact solution to a problem for the linear profile of magnetization in DW, which approximates well the behavior of magnetization in a narrow constriction.[15] The limiting case of an infinitely steep slope corresponds effectively to the electron motion in a steplike potential, and gives the maximum possible magnetoresistance. In principle, the solution of the problem for other domain wall profiles can be found by perturbations to our exact solution. However, if the thickness of DW becomes comparable with the Fermi wavelength of the current curriers, then DW becomes effectively sharp even for the classical hyperbolic tangent profile of the magnetization in DW.[18]

III. RESULTS OF MAGNETORESISTANCE CALCULATIONS

In order to find the conductances and the magnetoresistance of the constriction one should take zeros Z_{mn} of the Bessel function $\mathbf{J}_m(Z_{mn}) = 0$, and use the constraint (3). Determining the transmission coefficients $D_{\alpha\beta}(x)$ (see the Appendix) we substitute it into Eqs. (1),(2) and perform summation over the open channels.

At the ferromagnetic alignment of the magnetizations the equality $p_{F\alpha} \equiv p_{F\uparrow}$ is fulfilled for the $\sigma_{\uparrow\uparrow}$ contribution to the conductance σ^F, Eq. (1), and $p_{F\alpha} \equiv p_{F\downarrow}$ for the $\sigma_{\downarrow\downarrow}$ contribution. At the antiferromagnetic alignment the minority Fermi momentum should be used instead of $p_{F\alpha}$ in Eqs. (3) and (4) to calculate the conductance σ^{AF}, Eq. (2). The results are displayed on Figs. 1 and 2. The parameter $\delta = p_{F\downarrow}/p_{F\uparrow} \leq 1$ characterizes the conduction band spin polarization and is important for discussion. One can see from the calculations that the results depend on the absolute value of $p_{F\uparrow}$ and, to be specific, we have chosen $p_{F\uparrow} = 1$ Å^{-1}.

Figure 1 displays the results of the calculations for $\delta = 0.7$. The panel (a) shows the dependence of F and AF conductances on the channel radius. The parameters d and $\lambda = d p_{F\uparrow} \hbar^{-1}$ are the length and dimensionless length of the
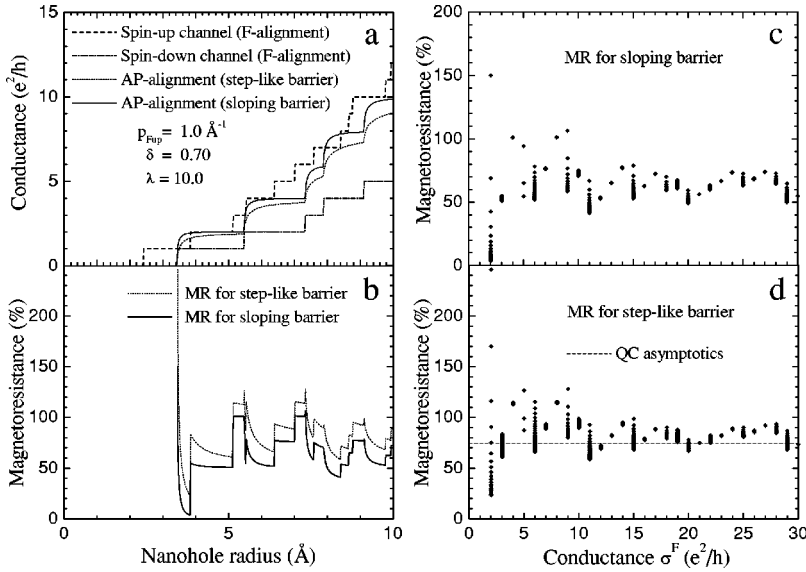
FIG. 1. The dependence of conductance (a), and magnetoresistance (b) on the radius of the hole a. Panels (c) and (d) show dependencies of the magnetoresistance on the number of the open conductance channels at the F alignment of the magnetizations: (c) for the potential described by Eq. (A4), (d) for the steplike potential. $\delta = 0.7$ for all panels.

channel, respectively. The chosen value $\lambda = 10.0$ corresponds to the length $d = 10$ Å. The panel (b) shows the dependence of the magnetoresistance on the radius of the hole. The panels (c) and (d) display the magnetoresistance against F conductance for a potential with a finite slope (c), and for a steplike (d) potential in the hole. Physically, Fig. 1 corresponds to the case when the AF-alignment conduction opens up in the interior part of the first F-conductance plateau. It allows us to make the following conclusions.

(1) The F-alignment conductance is spin dependent and the spin channels open nonsimultaneously [see panel (a)], thus resulting in e^2/h quantization of the conductance.[7,8]

(2) Finite magnetoresistance appears simultaneously with the first spin "down" and AF conductance [panel (b) in correlation with panel (a)].

(3) The magnetoresistance has quasiperiodic oscillations as a function of the hole radius [panel (b)].

(4) Sudden jumps in the magnetoresistance followed by practically flat plateaus appear at points where a new

F-alignment spin "up" conductance channel opens up. They persist until the spin "down" projection opens a new channel [panel (b) in correlation with panel (a)].

(5) When increasing the hole radius [panel (b)] or the number of open channels [panels (c) and (d)], the amplitude of oscillations and of substeps of the magnetoresistance decreases and its asymptotic value [panel (d)] is given by our quasiclassical theory.[9]

(6) The most intriguing finding is that the magnetoresistance versus the F-alignment conductance is a multivalued function of F conductance σ^F [panels (c) and (d)].

The result (2) demonstrates that the magnetoresistance has a sharp peak when the first conduction channel opens up for the spin "down" electrons at F alignment. If the spin polarization of conduction band is such that the spin "down" conductance channel appears at the first spin "up" conductance plateau, the MR peak should appear at the conductance corresponding to $N^F = 2$ open channels of F conductance $\sigma^F = (e^2/h)N^F$. This is displayed in Fig. 1, and the experi-
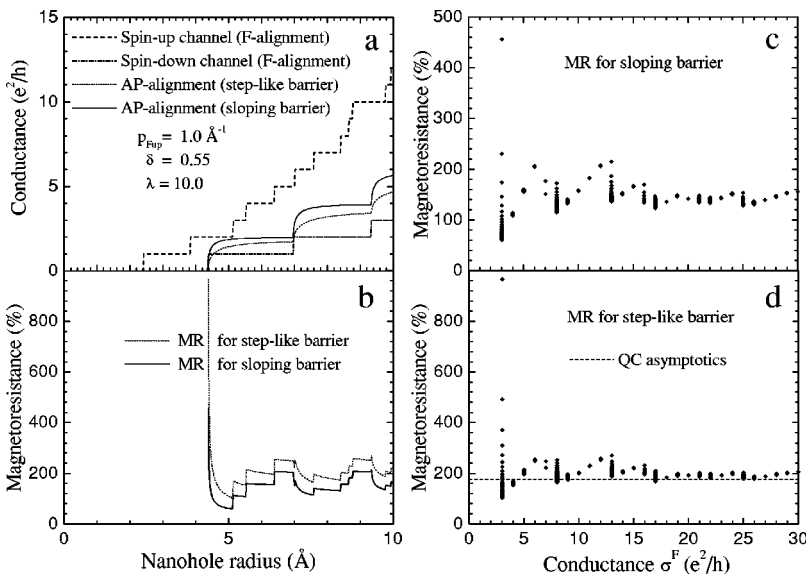

FIG. 2. The same as in Fig. 1, but for $\delta = 0.55$.

mental results by García *et al.*[1–3] for Ni-Ni and Fe-Fe point contacts clearly demonstrate the same tendency.

The result (3) indicates that consecutive maxima in magnetoresistance as a function of the contact radius correspond to opening of the AF-conductance channels. The result (4) leads to a weakly disperse or even nondisperse behavior of magnetoresistance at certain numbers of open *F*-alignment channels $N^F = 3,4,7,8,10,12,13, \ldots$. The nondisperse behavior of MR is due to the fact that the AF conductance is practically independent of the contact radius when a new *F*-conductance channel opens up [see panel (b)]. The result (5) shows that if conductance exceeds the values 10–15 (in the e^2/h units), which corresponds to the values of 8–10 Å for the hole radius, the magnetoresistance fluctuations become relatively small and its mean value converges well to that obtained in the ballistic quasiclassical regime.[9]

The result (6) is crucial for the interpretation of the experimental data. The panel (b) shows a very sharp peak between $a \sim 2.65$ and 3.8 Å. The decay of the peak persists until a new spin "down" conduction channel opens at the *F* alignment. If we draw the peak magnetoresistance versus the number N^F of open channels [panels (c) and (d)] we see that all the points correspond to the single abscissa $N^F = 2$. This means that the magnetoresistance is a *multivalued* function of the number of open conduction channels at the *F* alignment, provided the temperature effects and quenched disorder may be neglected. The magnetoresistance does not oscillate as a function of the conductance σ^F, but there are distributions of MR at fixed values of the *F*-alignment conductance σ^F. The origin of these distributions is clarified by inspection of panel (b) in Fig. 1 and its comparison with the panel (a): in spite of the fact that the actual radius may vary in the range $2.65-3.8$ Å, it gives identical values of the *F* conductance, which are due to the quantization. At the same time, the AF conductance depends on the radius (area) of the connecting channel, and this results in different values of magnetoresistance. The multivalued magnetoresistance as a function of σ^F we predict is simply a consequence of the conductance quantization. This property survives for every reasonable shape of the nanocontact, provided that conductance at the ferromagnetic alignment of magnetizations is quantized (conductance steps exist), and the domain wall in the constriction is effectively sharp. The multivalued behavior leads to extremely large fluctuations in the measured magnetoresistance data at the same F-conductance values. The density of points is considerably larger at small values of the magnetoresistance, than at larger ones. As a consequence of the decreasing density of the points, large values of the magnetoresistance are much less probable than the small ones. When observed experimentally, such a MR distribution should not be interpreted as being due to a poor reliability and reproducibility of experimental data. The giant data fluctuations are inevitable in the magnetoresistance measurements on the quantum magnetic contacts.

Increasing the spin polarization of the conduction band (decreasing the parameter δ) we see from our model that opening of the spin "down" conductance channel moves towards the second step in *F* conductance. Then, weakly disperse MR distributions, which originate from the flat magne-

toresistance graph sections [panels (b) of Figs. 1 and 2], become strongly disperse moving closer to the steps of the spin "down" F conductance. Figure 2 is drawn using the parameter $\delta = 0.55$, so that the spin "down" conductance appears now at the second plateau of the spin "up" conductance at *F* alignment [panel (a)]. Obviously, the MR points appear now at $N^F = 3$ open conductance channels of *F* alignment. The figure reveals the seventh finding: the minimal number of open channels N^F at which the magnetoresistance points appear allows us to estimate the lower bound for the conduction band spin polarization parameter δ.

A further increase of the conduction band polarization leads to the following interesting behavior. (a) MR points appear at $N^F = 4$ and larger numbers of the open *F*-conductance channels. (b) The theory predicts a huge enhancement of MR at high conduction band polarizations (small δ). From our calculations we conclude that if nanosize point contacts made of highly spin polarized metals [$\delta < 0.4$: NiMnSb, LMSO, CrO$_2$ (Ref. 12)] with the *F* conductance in the range of 5–10 channels were available experimentally, they would show MR of 1000% and higher.

Our calculations, panels (c) of Figs. 1 and 2, show that the finite length of the constriction does not influence qualitatively the results, which can be deduced from the calculations for the steplike potential barrier corresponding to DW. All the above conclusions hold, but the magnitude and the overall width of MR distributions decrease as compared to the results of calculations for the model of the steplike potential describing DW in the constriction [panels (d) of Figs. 1 and 2].

IV. DISCUSSION OF THE EXPERIMENTS

Our calculations show that in the quantized conductance regime, the minimal number of open *F*-conductance channels, at which the values of the magnetoresistance appear, is determined by the conduction band polarization δ. In Fig. 1 the AP-alignment conductance channel opens up at the first plateau of the spin "up" conductance. Corresponding magnetoresistance points appear at $N^F = 2$ open *F*-conductance channels. Our analysis shows that the threshold of the magnetoresistance rise moves from $N^F = 2$ to $N^F = 3$ open *F*-conductance channels at $\delta \simeq 0.63$. The experiments by García *et al.* on Ni-Ni contacts [Fig. 2(b) of Ref. 1, Fig. 1(a) of Ref. 2, and Fig. 2 of Ref. 3] and on Fe-Fe contacts [Fig. 1(a) of Ref. 3] clearly indicate that the MR points appear close to $N^F = 2$ for the both materials. This means that δ for both Ni and Fe is *larger* than 0.63 for our choice of the parameter $p_{F\uparrow} = 1$ Å^{-1}.

In contrast, the experimental data for Co-Co contacts, Fig. 2(a) of Ref. 2, and Fig. 2 of Ref. 3, indicate that MR appears at $N^F \simeq 3$ open channels, that is at the second plateau of the spin "up" *F* conductance [see panels (a),(b) of Fig. 2]. This suggests that polarization of the conduction band in Co is higher (and δ is smaller) as compared with Ni and Fe, and allows us to estimate the lower bound as $\delta(\text{Co}) \simeq 0.47 - 0.63$.

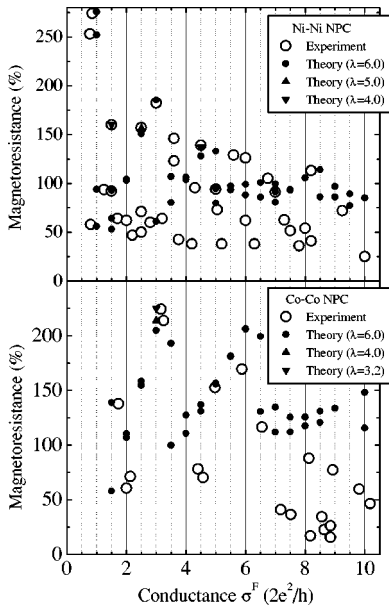Additional information can be extracted from the distributions of the MR points at small numbers of open

FIG. 3. Comparison between the theoretical and experimental values of the magnetoresistance for Ni ($\delta = 0.64$) and Co ($\delta = 0.57$) nanosize point contacts. The experimental data are taken from Ref. 3. For a discussion of the calculated MR values see the text.

F-conductance channels. We interpret these distributions as a manifestation of the multivalued behavior of the magnetoresistance as a function of F conductance. In Fig. 3, the calculated values of MR are compared with the results of the experiments on Ni and Co point contacts. Solid circles at every quantized value of the conductance show the range of the magnetoresistance distributions calculated at $\lambda = 6.0$. In addition, some points close to the experimentally measured ones are shown inside the regions. The maximum theoretically available MR values for this length of the channel are $\sim 500\%$ for Ni (at $N^F = 2$) and $\sim 1600\%$ for Co (at $N^F = 3$).

Surprisingly, our simple model reproduces the MR fluctuations and the extreme MR values at $\delta(\text{Ni}) \simeq 0.64$ and $\delta(\text{Co}) \simeq 0.57$ and small numbers of open conduction channels well. A deviation of the calculated MR from the experimental ones at $N^F \geqslant 6-8$ (in e^2/h units) may be because of various reasons.

(1) When the diameter of the constriction becomes large the domain wall is no longer effectively abrupt (independent of the actual shape), and the magnetoresistance begins to drop very fast down to the values 2–11 % given by the Levy-Zhang mechanism of scattering enhancement in the domain wall.[19]

(2) One or several impurities or lattice defects may be located just at the constriction causing additional random deviations of conductance values from integer numbers of e^2/h (see, for example, Refs. 4, 20, 21, and references therein).

(3) The shape of the constriction may deviate substantially from the cylindrical one. According to calculations by Torres *et al.*[22] for variable cross-section constrictions with the hyperbolic geometry, the conductance quantization steps

survive at the opening solid angles up to 90°, at least for the small number of open conductance channels. This also leads to deviations of conductance values from integer numbers of e^2/h.

(4) The noncylindrical cross section of the connecting channel (it can be verified for the elliptic and rectangular cross sections) influences the quantization conditions and, hence, the sequence of openings of the spin channels.

The latter reason may change the assignment of some open conduction channels from the spin "up" quantization to the spin "down" one and *vice versa*. It may influence the structure and the width of the MR distributions at fixed values of the F conductance σ^F in Fig. 3, but does not destroy the overall consistency of the theory with the experiment.

Thus, in addition to the conduction band polarization parameter δ, the contact size, shape, and length of the channel determine the values of the magnetoresistance. The real nanocontacts by García *et al.* have been made by pressing a sharpened ferromagnetic tip into another piece of a ferromagnet. Every MR point has been measured for a particular contact with individual shape, size, and length of the constriction. That is why we believe that overall agreement of the theory with the experiment is fairly good.

Thus, our analysis of García *et al.* measurements[1–3] suggests that the conduction band polarization parameters δ for different materials obey the following inequalities: $\delta(\text{Fe}) \geqslant \delta(\text{Ni}) > \delta(\text{Co})$. This means that cobalt has the most polarized conduction band. On the other hand, one can extract the information about the spin polarization of a ferromagnet's conduction band from the ferromagnet-insulator-superconductor (FIS) tunneling spectroscopy and the FS Andreev reflection spectroscopy. The FIS tunneling data[11] provide the following estimates for the mean values of the conduction band polarization parameter δ: 0.63 for Ni, 0.48 for Co, and 0.43 for Fe. The Andreev reflection spectroscopy[12–14] gives the mean values of δ: 0.62 for Ni, 0.64 for Co, 0.62 for Fe—from Ref. 12, 0.72 for Ni, and 0.68 for Co—from Ref. 13, and 0.6 for Ni and 0.62 for Fe—from Ref. 14. So, our estimated values for δ, $\delta(\text{Ni}) \simeq 0.64$ for Ni and $\delta(\text{Co}) \simeq 0.57$ for Co, are rather close to those obtained from the tunnel and Andreev spectroscopies. At the same time, these spectroscopies indicate that iron probably has the highest polarization of the conduction band. This does not agree with the conclusions obtained from measurements of the magnetic point contact magnetoresistance.

Of course, complex band structures of the contacting metals in the Andreev and the point contact measurements may affect the values of the conduction band spin polarization obtained from these experiments. However, we would like to stress here that this discrepancy may also be due to the character of the electron transmission through the contact. In the point contact of two ferromagnetic metals, an electron traverses the domain wall at the AF alignment of magnetizations, which contrasts the tunneling in tunnel and Andreev spectroscopies. We believe that the regime of the spin conservation during the flight through the constriction could not been satisfied in Fe. Then, the effective domain width is large, $d_w(\text{Fe}) \sim l_s(\text{Fe})$, and the electron spin partially follows the domain wall profile. This results in the appearance of the

AF conductance and magnetoresistance at smaller numbers of open F-conductance channels ($N^F = 2$), rather than at $N^F = 3$ or 4 as one might expect from our theory. This may be also the reason why the magnitude of magnetoresistance is reduced in iron to $\sim 30\%$.

Let us say a few words about the influence of disorder in the area of contact and of temperature effects on the magnetoresistance. Strong disorder should be avoided in experiments because it destroys quantization (see, for example, Ref. 21, and citations therein), and hence the huge enhancement of magnetoresistance that we predict. One may estimate from Ref. 21 that typical range for fluctuations of the disorder potential energy should be below 10% of the Fermi energy. The experiments by García *et al.* have been made on pure metals at room temperature, so we do not expect any effects that would result in weak localization.[23] If there is a single impurity just in the constriction, the transmission coefficient changes,[20,24] and this leads to deviations of the F-conduction values from the integer numbers of e^2/h. However, provided the condition $V_i/\varepsilon_F \ll 1$ is fulfilled, where V_i is the impurity potential, the effect of the impurity scattering on the conductance is small, and our analysis for the ballistic transmission should be qualitatively valid.

We expect that temperature effects are relatively small if the temperature does not exceed the Fermi energy ε_F and the Curie temperature T_{Curie}, $k_B T \ll \varepsilon_F, k_B T_{\text{Curie}}$. Moreover, phonon and magnon assisted relaxation processes are quenched because of a large, $1-3$ eV, exchange splitting of the conduction band. The experimental observation of sharp conduction quantization steps in the nickel nanosize contacts at room temperature[4-6] confirms the above expectation.

From the above analysis we conclude that our theory is consistent with the experimental data.[1-3] Therefore, it is reasonable to think that the origin of large fluctuations of the magnetoresistance as a function of conductance σ^F at the ferromagnetic alignment is the quantization of conductance, but not measurement errors or poor reproducibility of the results. The smallest number of open F-conductance channels, at which the magnetoresistance data appear, allowed us to estimate the low bound of the spin polarization of the conduction band of a ferromagnet. For a more detailed comparison of our theory with experiments more experimental data points for the magnetoresistance, as well as more experimentally determined or controlled parameters such as d, T_1, ω_Z, and more information about the shape of the constriction are needed. In addition to the experiments by García *et al.*, our theory has obvious implications to future experiments with a nanocontact between two ferromagnetic islands made of a short nanowire.

APPENDIX: THE TRANSMISSION COEFFICIENT FOR THE CYLINDRICAL CHANNEL

In this appendix we solve the quantum mechanical problem of a particle motion in the cylindrical channel of the length d and the radius a and find the exact coefficient of transmission through this channel. The solution of the Schrödinger equation in the cylindrical coordinates is sought in the form

$$\Psi(\rho,x) = \Phi(x)\mathbf{J}_m\left(\frac{Z_{mn}\rho}{a}\right)e^{im\varphi}, \qquad (A1)$$

where Z_{mn} is the discrete set of zeros of the Bessel function $\mathbf{J}_m(Z_{mn}) = 0$ and the variable x is chosen along the axis of the cylinder. The boundary condition is chosen as

$$\Psi(a,x) = 0. \qquad (A2)$$

It results in the quantization of the transverse (with respect to the x axis) motion by the zeros of the Bessel function. The longitudinal motion is described by the equation

$$\hbar^2\frac{\partial^2\Phi}{\partial x^2} + [p_{F0}^2 - a^{-2}Z_{mn}^2\hbar^2 + 2mU(x)]\Phi = 0. \qquad (A3)$$

In order to simplify the calculations we model the dependence of the magnetization of the domain wall (Ref. 15, Fig. 2) on the coordinate x perpendicular to the membrane by the following function:

$$U(x) = \begin{cases} I, & x > d/2, \\ 2Ix/d, & -d/2 < x < d/2, \\ -I, & x < -d/2. \end{cases} \qquad (A4)$$

A general solution to Eq. (A3) can be written as

$$\Phi(x) = C_1\text{Ai}(\xi) + C_2\text{Bi}(\xi), \qquad (A5)$$

where $\text{Ai}(\xi)$ and $\text{Bi}(\xi)$ are the Airy functions[25]

$$\xi(x) = \left(\frac{4mI\hbar}{d}\right)^{-2/3}\left[\frac{4mI}{d}x - (p_{F0}^2 - a^{-2}Z_{mn}^2)\right]. \qquad (A6)$$

Introducing the spin "up" ($p_{F\uparrow}$) and spin "down" ($p_{F\uparrow}$) Fermi momenta and using the relation

$$\frac{p_{F\uparrow}^2}{2m} - \frac{p_{F\downarrow}^2}{2m} = 2I, \qquad (A7)$$

we obtain

$$\xi\left(-\frac{d}{2}\right) = -\left(\frac{p_{F\uparrow}^2 - p_{F\downarrow}^2}{d\hbar^{-1}}\right)^{-2/3}[p_{F\uparrow}^2 - p_{\parallel}^2],$$

$$\xi\left(\frac{d}{2}\right) = -\left(\frac{p_{F\uparrow}^2 - p_{F\downarrow}^2}{d\hbar^{-1}}\right)^{-2/3}[p_{F\downarrow}^2 - p_{\parallel}^2], \qquad (A8)$$

where $p_{\parallel} = a^{-1}Z_{mn}\hbar$, is the parallel to the interface (but transverse with respect to the x axis of the cylinder) projection of the momentum allowed by the quantization condition.

Combining Eqs. (A5) and (A1) we find the general solution of the Schrödinger equation for the particle moving in a cylinder.

To find the transmission coefficient through the cylinder, connecting two bulk ferromagnetic metals, we match the wave functions and their derivatives at the interfaces and relate the outgoing probability flux to the ingoing one. As a result, the exact expression for the transmission coefficient reads

$$D = \frac{\text{Num}}{\text{Denum}}, \qquad (A9)$$

where

$$\text{Num} = 4 p_{x1} p_{x2} [|\gamma^2| \phi_{1+}^2 + \phi_{2+}^2 + (\gamma + \gamma^*) \phi_{1+} \phi_{2+}], \qquad (A10)$$

$$\begin{aligned}\text{Denum} &= |\gamma^2| [p_{x1}^2 \phi_{1-}^2 + (\phi_{2-}')^2] + p_{x1}^2 \phi_{2-}^2 + (\phi_{2-}')^2 \\ &\quad + (\gamma + \gamma^*) [p_{x1}^2 \phi_{1-} \phi_{2-} + (\phi_{1-}')^2 (\phi_{2-}')^2] \\ &\quad + i(\gamma - \gamma^*) p_{x1} [\phi_{1-} \phi_{2-}' - \phi_{2-} \phi_{1-}'].\end{aligned} \qquad (A11)$$

In the above expressions

$$\phi_1 = \mathbf{J}_m\!\left(\frac{Z_{mn}\rho}{a}\right) \text{Ai}(\xi) \cos(m\varphi + \gamma), \qquad (A12)$$

$$\phi_2 = \mathbf{J}_m\!\left(\frac{Z_{mn}\rho}{a}\right) \text{Bi}(\xi) \cos(m\varphi + \gamma), \qquad (A13)$$

$$\phi_i' = \frac{\partial \phi_i}{\partial x}, \qquad \phi_{i\pm} \equiv \phi_i\!\left(x = \pm\frac{d}{2}\right), \qquad (A14)$$

$$\gamma = -\frac{\phi_{2+}' - i p_{x2} \phi_{2+}}{\phi_{1+}' - i p_{x2} \phi_{1+}}. \qquad (A15)$$

The projection of the momentum of incident particles on the x axis is

$$p_{xi} = p_{Fi} \cos \theta_i. \qquad (A16)$$

In the limit $d \to 0$ the expression for D (A9) considerably simplifies and reduces to a familiar expression for the transmission coefficient for scattering on the potential step

$$D_{\text{step}} = \frac{4 p_{x1} p_{x2}}{(p_{x1} + p_{x2})^2}, \qquad (A17)$$

which has been used for checking the numerical calculations with D (A9).

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[17] Formally, the magnetoresistance defined by Eq. (6) may diverge if the AF conductance σ^{AF} approaches zero, which would mean that the nanocontact with one open channel of the F conductance serves as an ideal quantum spin valve. However, this might be possible only if the spin of conduction electrons conserves when the electrons traverse the domain wall. The spin-flip process leads to a finite AF conduction and introduces an overall cutoff preventing the magnetoresistance to increase infinitely. In this situation, there should be also a peak of the magnetoresistance at one open channel of F conductance. The reason why no finite magnetoresistance has been observed in the experiment at one open conduction channel for the F alignment is unclear. Hereafter we will imply that the conductance σ^{AF} in Eq. (6) is finite.

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