From Fermi-liquid-like to non-Fermi-liquid behavior of the generalized Anderson impurity model: The Bethe-ansatz solution

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The exact solution for the generalized Anderson impurity model is presented. Here localized electrons interact in the shell via a Hubbard-like repulsion and Hund's rule exchange interaction. An impurity reveals, in general, a mixed-valent behavior. Depending on the relative position of the impurity level and the strengths of Hubbard like and Hund's couplings, an impurity can reveal either a Fermi-liquid-like or non-Fermi-liquid-like behavior. The competition between characteristics of in-shell Coulomb interactions can produce a behavior similar to the channel-anisotropic multichannel Kondo situation, which is responsible for the non-Fermi-liquid physics of the model. Similar conclusions can be made for a two-impurity Kondo model.

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The behavior of hybridization impurities has been theoretically studied in the framework of the Anderson impurity model, which possesses the exact Bethe ansatz solution.[1,2] Usually this pertains to in-shell electrons with only spin internal degrees of freedom, and the Hubbard-like in-shell repulsion between electrons produces the many-body physics with the Kondo effect being the prime example. In the Kondo case the magnetic impurity hybridized with conduction electrons manifests itself in a low-energy Fermi liquid behavior,[3–5] but with the renormalized density of states, which is determined by the crossover scale—the Kondo temperature.[1,2] It is also possible to consider a larger degeneracy of the in-shell electron (large values of the total moment $j \geqslant \frac{1}{2}$), similar Fermi-liquid-like physics results[6,7] for the magnetic localized electron.[1,2] However there exist many materials in which the ground state of ions in the symmetric configuration has an orbital degeneracy in addition to the Kramers (spin) degeneracy.[8] Here the so-called multichannel Kondo situation can appear, whose main feature is the divergency of the density of states for the impurity—the non-Fermi-liquid behavior.[9,10] In this study we propose a Bethe ansatz solution of the generalized Anderson model, in which the Coulomb in-shell coupling reveals itself in the Hubbard-like interaction and Hund's exchange.[11] We show that the competition between those interactions and the relative position of the energy level of the localized electron can produce the effective channel-anisotropic Kondo situation. The latter determines the transition between the Fermi-liquid and non-Fermi-liquid behaviors. The Hamiltonian of the model has the effectively one-dimensional form[1,2]

$$\mathcal{H} = -\int dx \sum_{m,\sigma} \psi^{\dagger}_{m,\sigma}(x)(i\partial/\partial x + \hat{H}_{count})\psi_{m,\sigma}(x) + \left(\sum_{m',\sigma' \neq m,\sigma} [(U/2) f^{\dagger}_{m,\sigma} f^{\dagger}_{m',\sigma'} f_{m',\sigma'} f_{m,\sigma} - (J/2) f^{\dagger}_{m,\sigma} f^{\dagger}_{m',\sigma'} f_{m',\sigma} f_{m,\sigma'}] + \epsilon \sum_{m,\sigma} f^{\dagger}_{m,\sigma} f_{m,\sigma} \right) + V \int dx\, \delta(x) \sum_{m,\sigma} [\psi^{\dagger}_{m,\sigma}(x) f_{m,\sigma} + \text{H.c.}], \quad (1)$$

where $\psi^{\dagger}_{m\sigma}(x)$ ($f^{\dagger}_{m,\sigma}$) creates a conduction electron at site x (in-shell one at $x=0$) with the spin σ and orbital index $m = -l, \ldots, l$, V are hybridization elements (here supposed to be independent on positions, spins, and orbital indices), U and J are the Hubbard-like and Hund's rule (exchange) in-shell interactions, and the Fermi velocity of conduction electrons is equated to unity. The counterterm $\hat{H}_{count} = (1/\Lambda) \times [(\partial^2/\partial x^2) - \delta(x)(x/|x|)[\delta'(x+0) + \delta'(x-0)]$ is necessary to preserve the integrability at the position of the impurity (zero). The parameter Λ measures the curvarture scale of the spectrum. The inclusion of the counterterm does not principally affect physical properties of a single hybridization impurity (cf. Ref. 12).

Two scattering processes are possible, namely, the scattering of conduction electrons off the localized one, and the scattering between conduction electrons. We can find two-particle scattering matrices (TPSM's) for the first type of scattering. The two-electron wave function for this process can be written as a product of a coordinate wave function referring to the positions (momenta) of electrons, a spin wave function, and an orbital wave function. The global symmetry of the wave function has to be antisymmetric under the exchange of the two electrons. Hence, if the spin and orbital parts have the same symmetry, the coordinate wave function is antisymmetric and vanishes if $x_1 = x_2$, so that the electrons cannot interact. (For instance, for only two orbitals the interacting electrons necessarily form a spin triplet and an orbital singlet or a spin singlet and an orbital triplet. The former situation pertains to an attractive total effective interaction, while for the latter the effective coupling is repulsive.) The TPSM then factorizes (similarly to the two-channel Kondo problem[10])

$$\hat{R}(k_1,k_2) = \frac{[f_s(k_1) - f_s(k_2)]\hat{I}_{\sigma} + i\Gamma \hat{P}_{\sigma}}{f_s(k_1) - f_s(k_2) + i\Gamma} \otimes \frac{[f_m(k_1) - f_m(k_2)]\hat{I}_m + i\Gamma \hat{P}_m}{f_m(k_1) - f_m(k_2) + i\Gamma}, \quad (2)$$

where $k_{1,2}$ are the momenta; $\hat{I}_{m,\sigma}$ and $\hat{P}_{m,\sigma}$ denote the identity matrices and permutation operators in the orbital and spin subspaces, respectively; $\Gamma=\pi N(E_F)V^2$ [$N(E_F)$ is the density of states of conduction electrons at the Fermi level], $f_{s,m}(k)=[p^2-(2\epsilon+U\mp J)p]/(U\mp J)$; and $p_j=k_j/\Lambda$, (cf. Ref. 13; note that this paper only gives the Bethe ansatz equations (BAE's), but not the solution of them). Then we take the scaling limit $\Lambda\to\infty$. For only two orbitals, e.g., when applied to a triplet wave function (either in the spin or the orbital subspace) the corresponding TPSM yields one, while if it acts on a singlet it gives rise to a phase shift. Since the TPSM in each subspace satisfies the Yang-Baxter relations (YBR),[14] their product also does. The TPSM's between conduction electrons have to satisfy the Yang-Baxter relations with $\hat{R}$ (and mutually) and also preserve the integrability. Formally there is no direct coupling between conduction electrons in the model. However, the naive choise of the diagonal scattering matrices for the TPSM's between conduction electrons does not satisfy the YBR. Correlations between conduction electrons are *induced* through the hybridization with in-shell electrons. Hence the hybridization of conduction electrons with (interacting) in-shell electrons *dynamically correlates* the motion of formers. That is why the TPSM between conduction electrons dynamically obtains the form of $\hat{R}(k)$. It turns out that different behaviors of scatterings in spin and orbital subspaces is not novel in the theory of exactly solvable models, and this is similar to the situation in the multichannel channel-asymetric Kondo problem.[12]

Given the elementary TPSM [Eq. (2)] we can derive the set of BAE's for the quantum numbers (*rapidities*), which parametrize the eigenvalues and eigenfunctions of the Schrödinger equation with periodic boundary conditions, in the way similar to that in Refs. 10 and 12. The procedure is standard, and we skip the details. We introduce several sets of rapidities: charge rapidities $\{k_j\}_{j=1}^{N}$ (with N being the number of electrons), spin rapidities $\{\lambda_\alpha\}_{\alpha=1}^{M}$ (with M being the number of down spins), and orbital rapidities $\{\xi_q^{(r)}\}_{q=1}^{M_r}$ (with the number of electrons with the rth orbital index being $N_r=M_{r-1}-M_r$, $r=1,\ldots,2l$). A crystalline electric field (D) can lift the degeneracy of the orbitals, the latters becoming unequally populated. (Similarly, an external magnetic field H lifts the spin degeneracy.) Each eigenstate corresponds to a solution of the BAE, here obtained on a periodic interval of the length L:

$$
\begin{aligned}
e^{-i(k_jL+2\hat{\phi}_j)}&=\prod_{\gamma=1}^{M} g_1(p_j-\lambda_\gamma)\prod_{q=1}^{M_1} e_1(p_j-\xi_q^{(1)}),\\
&\prod_{q=1}^{M_{r-1}} e_1(\xi_f^{(r)}-\xi_q^{(r-1)})\prod_{q=1}^{M_{r+1}} e_1(\xi_f^{(r)}-\xi_q^{(r+1)})\\
&\quad=-\prod_{q=1}^{M_r} e_2(\xi_f^{(r)}-\xi_q^{(r)}),\qquad (3)\\
&\prod_{j=1}^{N} g_1(\lambda_\alpha-p_j)=-\prod_{\delta=1}^{M} g_2(\lambda_\alpha-\lambda_\delta),
\end{aligned}
$$

where $j=1,\ldots,N$, $\alpha=1,\ldots M$, $f=1,\ldots,M_r$, $\xi_j^{(0)}=p_j$, $M_0=N$, $M_{2l+1}=0$, $\hat{\phi}_j=2\tan^{-1}[\Gamma/2(k_j-\epsilon)]$, $e_n(y)=(2y-ic'n)/(2y+ic'n)$, $g_n(y)=(2y-icn)/(2y+icn)$, $c=\Gamma(U-J)/(2\epsilon+U-J)$, and $c'=\Gamma(U+J)/(2\epsilon+U+J)$. The energy and z projections of the total spin and orbital moments are equal to $E=\Sigma_{j=1}^{N}k_j$, $S^z=(N/2)-M$ and $L^z=2(lN-\Sigma_{r=1}^{2l}M_r)$, respectively.

The solutions to BAE (3) in the thermodynamic limit (with $L,N,M,M_r\to\infty$ and finite ratios N/L, M/L, M_r/L) can be classified in the framework of the "string hypothesis"[15] for any values of U and J in the following way. (a) Real charge rapidities—we define the density of rapidities, which belong to the class (a) as $\rho(p)$, the density of their "holes" as $\tilde{\rho}(p)$ and the "dressed" (interactions "dress" "bare" energies of excitations) energy of those excitations $\varepsilon(p)=T\ln(\tilde{\rho}/\rho)$; T is the temperature. (b) Strings of complex spin rapidities—these are bound spin states with $\sigma_n(\lambda)$, $\tilde{\sigma}_n(\lambda)$ and $\phi_n=T\ln(\tilde{\sigma}_n/\sigma_n)$ being the densities of the real parts and dressed energies of those spin excitations, where $n=1,2,\ldots$ denotes the length of the spin string. (c) Strings of complex orbital rapidities—they are bound orbital states with densities of their real parts $\varphi_n^{(r)}(\xi^{(r)})$, their "holes" $\tilde{\varphi}_n^{(r)}(\xi^{(r)})$ and dressed energies $\kappa_n^{(r)}=T\ln(\tilde{\varphi}_n^{(r)}/\varphi_n^{(r)})$ for r-orbital ($r=1,\ldots,2l$) strings of length $n=1,2,\ldots$. (d) Complex spin and charge rapidities, which correspond to bound states of electrons with different spin components—we define their densities and dressed energies as $\sigma(\lambda)$, $\tilde{\sigma}(\lambda)$ and $\Psi(\lambda)=T\ln(\tilde{\sigma}/\sigma)$. (e) Complex orbital and charge rapidities—these pertain to bound states of electrons with different orbital components, with densities of their real parts and dressed energies being defined as $\Sigma_n^{(r)}(\xi^{(r)})$, $\tilde{\Sigma}_n^{(r)}(\xi^{(r)})$ and $\Phi_n^{(r)}=T\ln(\tilde{\Sigma}_n^{(r)}/\Sigma_n^{(r)})$, ($r=1,\ldots,2l$, $n=1,2\ldots$). Note that because $\xi^{(0)}\equiv p_j$, one can identify $\rho=\Sigma_1^{(0)}$ and $\epsilon=\Phi_1^{(0)}$. Which classes are realized in the solution depend on the signs and relative values of the Hubbard-like interaction U, the Hund's rule exchange J, and ϵ. For $c,c'\leqslant 0$ the repulsion exists in both spin and orbital subspaces. Here the solutions of the classes (a), (b) and (c) are valid. For $c\geqslant 0$, $c'\leqslant 0$ one has an effective repulsion in the spin subspace and the effective attraction in the orbital subspace, with the solutions from the classes (a), (b), (c), and (e). For $c\leqslant 0$, $c'\geqslant 0$ the situation is opposite—there is an effective repulsion in the orbital subspace and an effective attraction in the spin subspace [classes (a), (b), (c), and (d)]. Finally, for $c,c'\geqslant 0$ all of the classes are present, because of the effective attraction in both spin and orbital subspaces.

Let us consider for simplicity the most important physical case $l=\frac{1}{2}$ (only two orbitals). Here only $r=1$ matter in the classes (c) and (e) (and we omit this supraindex in what follows). After some algebra we can write the integral equations, which describe the thermodynamics of our model. The set for densities of rapidities is

$$
\begin{aligned}
&(\Lambda/\pi)+L^{-1}[a_1^{\Gamma+\Lambda c}(\Lambda\lambda-\epsilon)+a_1^{\Gamma-\Lambda c}(\Lambda\lambda-\epsilon)]\\
&\quad=\tilde{\sigma}+\sigma+a_2^{c}*\sigma+a_1^{c}*\rho-b_n^{c'}*\varphi_n,
\end{aligned}
$$

$$
\begin{aligned}
&(\Lambda/\pi)+L^{-1}[a_1^{\Gamma+\Lambda c'}(\Lambda\xi-\epsilon)+a_1^{\Gamma-\Lambda c'}(\Lambda\xi-\epsilon)]\\
&\quad=\tilde{\Sigma}+\Sigma+a_2^{c'}*\Sigma+a_1^{c'}*\rho-b_n^{c}*\sigma_n,\\
&\tilde{\varphi}_n+A_{nm}^{c'}*\varphi_m=-\mathrm{sgn}(c')a_n^{c'}*\rho+b_n^{c'}*\sigma, \qquad (4)\\
&\tilde{\sigma}_n+A_{nm}^{c}*\sigma_m=-\mathrm{sgn}(c)a_n^{c}*\rho+b_n^{c}*\Sigma,\\
&(\Lambda/2\pi)+L^{-1}a_1^{\Gamma}(\Lambda p-\epsilon)=\tilde{\rho}+\rho+a_1^{c}*\sigma+a_1^{c'}*\Sigma\\
&\qquad\qquad+\mathrm{sgn}(c)a_n^{c}*\sigma_n\\
&\qquad\qquad+\mathrm{sgn}(c')a_n^{c'}*\varphi_n.
\end{aligned}
$$

The set of integral equations for dressed energies can be written as

$$
\begin{aligned}
&\Psi-\Psi_0+2\mu+2D-Ta_2^{c}*\ln[1+e^{-(\Psi/T)}]\\
&\quad=Ta_1^{c}*\ln[1+e^{-(\varepsilon/T)}]-Tb_n^{c'}*\ln[1+e^{-(\kappa_n/T)}],\\
&\Phi-\Phi_0+2\mu+H-Ta_2^{c'}*\ln[1+e^{-(\Phi/T)}]\\
&\quad=Ta_1^{c'}*\ln[1+e^{-(\varepsilon/T)}]-Tb_n^{c}*\ln[1+e^{-(\phi_n/T)}],\\
&2nD-T\ln[1+e^{(\kappa_n/T)}]+TA_{nm}^{c'}*\ln[1+e^{-(\kappa_m/T)}]\\
&\quad=-T\mathrm{sgn}(c')a_n^{c'}*\ln[1+e^{-(\varepsilon/T)}]\\
&\qquad+Tb_n^{c'}*\ln[1+e^{-(\Psi/T)}], \qquad (5)\\
&nH-T\ln[1+e^{(\phi_n/T)}]+TA_{nm}^{c}*\ln[1+e^{(\phi_m/T)}]\\
&\quad=-T\mathrm{sgn}(c)a_n^{c}*\ln[1+e^{-(\varepsilon/T)}]+Tb_n^{c}*\ln[1+e^{-(\Phi/T)}],\\
&\varepsilon-\varepsilon_0+\mu+(H/2)+D=Ta_1^{c}*\ln[1+e^{-(\Psi/T)}]+Ta_1^{c'}*\ln[1\\
&\qquad\qquad+e^{-(\Phi/T)}]+T\mathrm{sgn}(c)a_n^{c}*\ln[1\\
&\qquad\qquad+e^{-(\phi_n/T)}]+T\mathrm{sgn}(c')a_n^{c'}*\ln[1\\
&\qquad\qquad+e^{-(\kappa_n/T)}],
\end{aligned}
$$

where μ is the chemical potential, the symbol $*$ denotes the convolution, and the Fourier transforms of the kernels are

$$
\begin{aligned}
&A_{n,m}^{z}(\omega)=\coth\left(\frac{|z\omega|}{2}\right)[a_{|n-m|}^{z}(\omega)-a_{n+m}^{z}(\omega)],\\
&b_n^{c,c'}(\omega)=2a_n^{c,c'}(\omega)\cosh\left(c',\frac{c\omega}{2}\right)-\delta_{n,|c'|/|c|}(\delta_{n,|c|/|c'|}), \qquad (6)\\
&a_n^{z}(\omega)=\exp\left(-\frac{n|z\omega|}{2}\right);
\end{aligned}
$$

$\varepsilon_0\approx\Lambda p$, $\Psi_0\approx 2\Lambda\lambda$, and $\Phi_0=2\Lambda\xi$. The summation is meant over repeated indices. The free energy is equal to

$$
\begin{aligned}
F=-(T\Lambda/2\pi)\Bigg(&\int dp\,\ln[1+e^{-(\varepsilon/T)}]+2\int d\lambda\\
&\times\ln[1+e^{-(\Psi/T)}]+2\int d\xi\,\ln[1+e^{-(\Phi/T)}]\Bigg). \qquad (7)
\end{aligned}
$$

The total number of electrons and z projections of the total spin and orbital momenta (for conduction electrons and the impurity) are $N=\int dp\rho(p)+2\int d\lambda\sigma+2\int d\xi\Sigma$, $S^z=(1/2)\int dp\rho(p)+\int d\xi\Sigma-\Sigma_{n=1}^{\infty}\int d\lambda\sigma_n$, and $L^z=\int dp\rho(p)+2\int d\xi\sigma-2\Sigma_{n=1}^{\infty}\int d\xi\varphi_n$, respectively. The solution of integral equations (4) and (5) yields the thermodynamic properties of the model as a function of U, J, ϵ, T, μ, H, and D. The terms in Eqs. (4), which do not depend on ρ, σ, Σ, σ_n, and φ_n (driving terms) of order of L^{-1}, determine the behavior of the impurity, while the remaining driving terms in those equations determine the behavior of conduction electrons. Note that the set of integral equations for dressed energies does not depend on impurity terms explicitly. It turns out, however, that the behavior of the impurity follows the properties of the host. Because of the division of driving terms in Eqs. (4) into ones of order of unity and the ones of order of L^{-1}, and because those equations are linear, we can separate each of densities of rapidities, respectively, into contributions from the host and from the impurity, like $\rho=\rho_h+L^{-1}\rho_{imp}$; etc.[1,2] (for each class of densities). Hence we can write equations, similar in structure to Eqs. (4), for host densities and for impurity densities of rapidities. In a similar way we can divide the energy, spin, orbital moment, and number of electrons (valence for the impurity in-shell state) into the host part and the impurity contribution. It is not difficult to show that the behavior of conduction electrons in the model [determined by those equations for the host densities of rapidities and by Eqs. (5)] is *the same* as for a free-electron gas (as it must be). Hence from now on we will concentrate on the behavior of an impurity, omitting the index *imp* in what follows.

For high energies ($T\gg V$) the model describes the high-T behavior of a single noninteracting impurity shell, whose properties are well known.[1,11] The most interesting properties are revealed in the ground state and at low temperatures. Dressed energies can be separated into their positive and negative parts. According to the Fermi statistics obeyed by the rapidities[15] (all quasiparticles are hard-core ones), positive parts of dressed energies pertain to empty states in the ground state (holes), while negative parts correspond to the occupied states (filling up of Dirac seas). Note that the symmetry of eigenfunctions is *not* fermionic but rather of an anyonic nature[16,17] due to interactions.

The case $c,c'\geqslant 0$, as we discussed above, pertains to all classes of excitations (all possible bound states) being the solutions of the BAE. This case is similar to the case of the degenerate Anderson model,[2] but with *different* values of effective interactions for orbital and spin degrees of freedom. In the ground state the classes (a), (d), and (e) matter. In general the valence of the impurity reveals either nonmagnetic, mixed-valence, and magnetic behavior,[1,2] depending on the relative position of ϵ. The behavior of the impurity in

the magnetic regime with the valence of the in-shell electron being close to 1 is the Fermi-liquid like.[3–5]

The most interesting case is, e.g., with $c' \geqslant 0$, $c \leqslant 0$ (i.e., $U+J \geqslant 0$, $U+J+2\epsilon \geqslant 0$, and $U \geqslant J$, $U+2\epsilon \leqslant J$ or $U \leqslant J$, $U+2\epsilon \geqslant J$). Here one can show after some straightforward calculations that only solutions of classes (a) unbound electron excitations, (c) one can call these chargeless and spinless excitations "orbitons" and their bound states (for $n=1$ and in the commensurate case for integer $n=|c'|/|c|$: let $|c'| \geqslant |c|$), and (d) those low-lying excitations are spin-singlet orbital-triplet Cooper-like pairs (they carry charge $-2e$, spin zero and orbital moment 1) and can have negative parts for any μ, H, and D. The Fermi points for those excitations are obtained from the conditions $\varepsilon(\pm B)=0$, $\kappa_{1,n}(\pm A_{1,n})=0$ and $\Psi(\pm Q)=0$. [Generally speaking[18] Eqs. (3) imply symmetric distributions for rapidities; however, for the low-energy behavior (with the characteristic energies being much less than than the Fermi energy) one can consider the lower limits being infinite.] Then the ground-state integral equations for densities become

$$
\begin{aligned}
&a_1^{\Gamma+\Lambda c}(\Lambda\lambda-\epsilon)+a_1^{\Gamma-\Lambda c}(\Lambda\lambda-\epsilon)\\
&\quad=\sigma+\int_{-\infty}^{Q}d\lambda' a_2^{c'}\sigma+\tilde{\sigma}+\int_{-\infty}^{B}dk a_1'^{c}\rho\\
&\qquad-\int_{-\infty}^{A_1}d\xi b_1^c\varphi_1-\int_{-\infty}^{A_n}d\xi b_n^c\varphi_n,\\
&\tilde{\varphi}_1+\varphi_1+\int_{-\infty}^{A_1}d\xi' a_2^c\varphi_1=\int_{-\infty}^{B}dk a_1^c\rho+\int_{-\infty}^{Q}d\lambda b_1^c\sigma\\
&\qquad\qquad-\int_{-\infty}^{A_n}d\xi' A_{1n}^c\varphi_n,\\
&a_1^{\Gamma}(\Lambda p-\epsilon)-\tilde{\rho}-\rho=\int_{-\infty}^{Q}d\lambda a_1^{c'}\sigma-\int_{-\infty}^{A_1}d\xi a_1^c\varphi_1\\
&\qquad\qquad-\int_{-\infty}^{A_n}d\xi a_n^c\varphi_n,\\
&\tilde{\varphi}_n+\varphi_n+\int_{-\infty}^{A_n}d\xi' A_{nn}^c\phi_n=\int_{-\infty}^{B}dk a_n^c\rho+\int_{-\infty}^{Q}d\lambda b_n^c\sigma\\
&\qquad\qquad-\int_{-\infty}^{A_1}d\xi' A_{1n}^c\varphi_1.
\end{aligned}
\tag{8}
$$

[For the incommensurate situation one has to remove the last equation from Eqs. (8), and in the remaining equations to remove the last terms in their right-hand sides.] For $A_{1,n}=\infty$ ($L^z=0$) one can obtain the solutions for $\varphi_{1,n}$ as functions of σ and ρ. In a zero magnetic field one can eliminate ρ, and find the valence of the impurity (the average number of localized in-shell electrons), which varies depending on the relative position of ϵ, as it must. To find the relative dependence of the characteristics in the magnetic regime, charge degrees of freedom can be eliminated,[10] and one obtains the effective equation for ρ only. The alternative way of solving the BAE is the *fusion* procedure.[12] The latter is the search for a solution to BAE's for charge rapidities within the class of orbital bound states. Those which have maximal spin are only important for the low-energy physics.[12] The number of orbitons bound in those states is determined by the number of orbitals, i.e., for our case it is 2. One conduction electron, however, is bound at the impurity orbital (i.e., its charge rapidity is real, $k_q=\epsilon$, with a fixed ratio ϵ/Λ). In the limit of $\Lambda\to\infty$ all real parts of those string solutions can be neglected[12] (except for the rapidity of the conduction electron bound at the impurity site). In fact the fusion is a way of eliminating orbital rapidities from the BAE. After this procedure host rapidities become present in the fused BAE only with the effective spins $(1/2)(c'\pm c)$. In the limit $J\to\infty$ only effective spins 1 are relevant in the low-energy physics of the model, which is characteristic of the two-channel Kondo problem.[10] In the opposite case of vanishing J one recovers the single-channel Kondo case.[1] For the behavior of an impurity two low-energy scales are important: $T_K=(N/L)\exp(\pi\epsilon/|c|)$ and $T_a=(N/L)\cos(\pi|c|/2|c'|)\times\exp(\pi\epsilon/|c'|)$. The solution of the BAE reveals that in the ground state (for small enough ϵ) the mixed valence of the impurity increases with the growth of the band filling of conduction electrons (i.e., the valence of the impurity depends explicitly on the *total number of electrons* in the system.) The ground-state magnetization of the localized electron for $H \ll T_a \ll T_K$ is proportional to H/T_a with standard Kondo logarithmic corrections, i.e., $M_{loc}^z \sim H/T_a(1+|\ln H/aT_a|^{-1}-\cdots)$ (a is some nonuniversal constant). The latter are characteristic for the asymptotically free behavior of an impurity spin. It is typical of a simple one-channel Kondo problem[1] with a finite magnetic susceptibility. However for $T_a \ll H \ll T_K$ the magnetization of the impurity reveals a logarithmic behavior $M_{loc}^z \sim -(H/T_a)\ln(H/aT_K)$ (with divergent susceptibility), typical for the two-channel Kondo behavior. For nonzero low temperatures $T \ll T_a \ll T_K$ we obtain the low-T (Sommerfeld) coefficient of the specific heat for the localized electron $\gamma_{loc} \sim T_a^{-1}[1-(3T_a/\pi T_K)\ln(T_a/T_K)](1+|\ln T/T_a|^{-1}-\cdots)$ and the finite ground-state susceptibility $\chi_{loc} \sim T_K^{-1}\ln(T_a/T_K)(1+|\ln T/T_K|^{-1}-\cdots)$ (both with typical logarithmic corrections of an asymptotically free spin). This case pertains to the single-channel Kondo physics, though two different energy scales for χ_{loc} and γ_{loc} mean that the Wilson ratio differs from the Fermi-liquid one.[5] For $T_a \ll T \ll T_K$ we have $\gamma_{loc} \propto \chi_{loc} \sim -(T_K)^{-1}\ln(T/T_K)$ (i.e., logarithmically divergent) and with the remnant entropy of the impurity $\mathcal{S}_{loc}=\ln\sqrt{2}$. For higher temperatures the magnetic susceptibility of the impurity manifests the Curie-like behavior with typical logarithmic corrections. The temperature dependence of the resistivity is determined by the scattering of conduction electrons off the spins of localized electrons (magnetic impurities).[6] We calculate it in a standard way, taking into account subleading irrelevant perturbations in the renormalization-group sense. It can be approximated (for a small hybridization anisotropy) at low temperatures by $\Delta\rho(T) \sim A(T/T_a)^2+\cdots$ for $T \ll T_a \ll T_K$ and $\Delta\rho(T) \sim B(T/T_K)^{1/2}+\cdots$ for $T_a \ll T \ll T_K$.

For $c' \leqslant 0$, $c \geqslant 0$ the situation is opposite to the above. The low-energy physics is determined by unbound electron

excitations, spinons, and spin-triplet orbital-singlet Cooper-like pairs. One has a formal similarity to the previous case with the interchange $c \leftrightarrow c'$, $H/2 \leftrightarrow D$ and $L^z \leftrightarrow 2S^z$. Finally, for $c, c' \leqslant 0$ only unbound electron excitations, spinons, and orbitons can have their Dirac seas (states with negative energies). In this case the situation is reminiscent to the Anderson impurity model with the in-shell attraction of electrons.[1]

Summarizing, we have presented an exact solution for the generalized Anderson impurity model. The impurity in-shell electron is hybridized with conduction ones. Localized electrons interact via the Hubbard-like repulsion and Hund's rule exchange interactions. Depending on the relative position of the impurity's energy level and strengths of the Hubbard-like and Hund's exchange couplings, the impurity can manifest either a Fermi-liquid-like behavior or a transition to a non-Fermi-liquid-like behavior (with the divergent magnetic susceptibility and Sommerfeld coefficient). This is why the competition between characteristics of the in-shell Coulomb interactions can produce a behavior similar to the channel-anisotropic multichannel Kondo behavior, which is responsible for the transition to the non-Fermi-liquid-like physics in the model. Finally, we can point out that in Ref. 13 the authors reported a similarity in the Bethe ansatz equations, which determine the behaviors of the generalized Anderson model and that of the two-impurity Kondo model[19] and the two-impurity Anderson model.[20] This means that the two-impurity Kondo model, according to our results, also has to reveal the properties characteristic of the channel-anisotropic multichannel Kondo model, i.e., the transition between the Fermi-liquid-like and non-Fermi-liquid-like behaviors. The results of our calculations can be applied to the non-Fermi-liquid-like behaviors of some rare-earth and actinide compounds,[21,22] and also to the behaviors of a split-gate quantum dot[23] or double-dot configurations.[24]

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