Transient electric current through an Aharonov-Bohm ring after switching of a two-level system

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(Received 21 December 2001; published 15 May 2002)

The response of the electronic current through an Aharonov-Bohm ring after a two-level-system is switched on is calculated perturbatively by use of a nonequilibrium Green function. In the ballistic case the amplitude of the Aharonov-Bohm oscillation is shown to decay to a new equilibrium value due to scattering into other electronic states. The relaxation of the Altshuler-Aronov-Spivak oscillation in the diffusive case, due to the dephasing effect, is also calculated. The time scale of the relaxation is determined by characteristic relaxation times of the system and the splitting of a two-level-system. The oscillation phase is not affected. Experimental studies of current response would give us direct information about characteristic times of mesoscopic systems.

DOI: 10.1103/PhysRevB.65.205318 PACS number(s): 73.23.−b

I. INTRODUCTION

Decoherence (or dephasing) caused by external perturbations is an important problem of quantum systems. Within equilibrium statistical mechanics, a convenient formula for estimating the dissipation by the environment was presented by Caldeira and Leggett.[1] There, decoherence was treated as a nonlocal interaction in imaginary time. The formula was shown to be useful in considering macroscopic quantum phenomena,[1] in which the tunneling rate was calculated only as a static quantity.

The effects of decoherence on electron systems were studied in the 1980's in the context of weak localization (e.g., decoherence by phonons and electron-electron interaction).[2,3] Decoherence gives rise to a mass of an electron-electron propagator (a cooperon), which governs magnetoresistance. The decoherence time due to electron-electron interaction was calculated by solving the Cooperon equation[4] and as a mass of the Cooperon.[5] Later it was demonstrated that this dephasing time is equivalent to the time defined in an intuitive way from a decay of the overlap of the wave function.[6,7] One should note, however, that this definition does not always work (see below and in Sec. IV).

Recently decoherence by a quantum two-level system (TLS) has been theoretically studied.[8–10] In these works the temperature dependence of the dephasing time, τ_φ, was calculated, motivated by an experimental finding of the saturation of the dephasing time as $T\to 0$ in disordered metal.[11] The mechanism of saturation appears still controversial.

For studies of decoherence, recent mesoscopic systems are suitable, since decoherence can be detected in a controlled manner. A direct way to study decoherence is to use the interference of two different paths in a small ring. The interference leads to an oscillation of conductance as a function of an external magnetic flux through the ring [Aharonov-Bohm (AB) (Ref. 12) and Altshuler-Aronov-Spivak (AAS) (Ref. 13) oscillations[14]]. The oscillation pattern changes if perturbation causes scattering or dephasing. The first direct measurement of the effect of the phase due to transport through a quantum dot was carried out by use of the AB effect by Yacoby *et al.*[15] Further studies revealed the rigidity of the phase, which is consequence of time-reversal symmetry.[16–18] The amplitude and phase of AB oscillation was calculated in the presence of a dot driven by an ac field in Ref. 19. The effect of a time-varying potential on the conductance of a ring was calculated in Ref. 20. The dynamical properties of quantum dots were studied theoretically intensively in the context of resonant tunneling[21] and the Kondo effect.[22,23]

Recently the AB effect in the ballistic case was experimentally investigated.[24,25] It was argued that the temperature dependence of the AB amplitude indicates a dephasing rate proportional to T^{-1}.[25] This behavior was discussed to be consistent with a theoretical estimate of the dephasing due to charge fluctuation, taking account of the existence of the leads.[26] However, the argument given in Ref. 25 might be too naive, because theoretically the role of dephasing on the AB effect in the ballistic case is not obvious. In fact the dephasing is represented in the calculation by the Cooperon, which exists only in the dirty case (see Sec. IV). A possible dephasing effect on ballistic current may be to change the spectral function.[27]

The aim of this paper is to study the response of the current through a narrow ring with a magnetic flux after a time-dependent environment is switched on. By use of a measurement of electronic properties with a high (THz) time resolution,[28] the observation of such a current response and time-resolved dephasing process is possible. The current response may provide direct information about microscopic relaxation times [elastic (τ) and inelastic (τ_φ) lifetimes) and properties of the perturbation source. As the environment we take a quantum two-level system. The transient current at low temperatures is calculated diagrammatically using a nonequilibrium Green function.[29–31] Coupling to a TLS is included to the second order, and a linear response with respect to the probe electronic field is considered. The AB current is calculated in the ballistic case, treating the arm of the ring as one dimensional. (The response of the AB current to sample-dependent fluctuations in a dirty case would be similar to that of the AAS current.) A generic expression of the AB response is obtained in terms of the correlation functions of the perturbation source. It was shown that only the amplitude of the AB oscillation is affected, consistent with the phase rigidity.[16,17] The reduction of the amplitude is shown to be due simply to the scattering into other electron states, and is not interpreted as dephasing. The overlap of the wave function with the initial state exhibits a decay after the TLS is switched on, but this has nothing to do with dephasing. This is in contrast to the decay caused by electron-electron interaction in a disordered case.[6] Theoretically this distinction is natural, since dephasing in the strict sense cannot be described by one-particle propagation (a Green function with an elastic lifetime). The effect is incorporated only when we take into account the particle-particle ladder (the Cooperon), which represents the interference between a path and the reversed path in the presence of elastic impurity scattering. Physically interaction with a single TLS itself gives a definite phase factor and cannot cause dephasing in the ballistic case. To cause dephasing, some randomness, such as impurities, is needed to give uncertainty to the phase due to the interaction. If there are many TLS's with different energies, dephasing would appear even in the ballistic case.

In the calculation of the AAS current (in Sec. VI), the dephasing time τ_φ is included phenomenologically (we do not care about the origin here). The lowest order contribution we calculate corresponds to the correction to τ_φ by the TLS. The calculation of response of the AAS current is very complicated, and hence we show the leading term only. The effect of oscillating external field is briefly discussed in Sec. V.

II. FORMULATION

The Hamiltonian we consider is $H=H_{\rm e}+H_{\rm TLS}+H'$, where $H_{\rm e}=\Sigma_{\mathbf{k}}\epsilon_{\mathbf{k}}c_{\mathbf{k}}^\dagger c_{\mathbf{k}}+H_{\rm imp}$ is the electron part [$\epsilon_{\mathbf{k}}=\mathbf{k}^2/(2m)-\epsilon_F$, ϵ_F being Fermi energy], and $H_{\rm imp}\equiv v_{\rm i}\Sigma_{\mathbf{kk}'}c_{\mathbf{k}'}^\dagger c_{\mathbf{k}}$ represents the impurity scattering ($v_{\rm i}$ is the coupling constant). $H_{\rm TLS}$ is the Hamiltonian of the TLS, which we describe later. The coupling between the electron and the TLS is

$$H'(t)=\sum_{\mathbf{kQ}} V(t)c_{\mathbf{k}+\mathbf{Q}}^\dagger c_{\mathbf{k}}, \tag{1}$$

where $V(t)$ is an operator of the TLS, which is time dependent. V is treated as independent of the momentum transfer $\mathbf{Q}$, assuming that the TLS is applied to a small area. We consider an electronic field ($\mathbf{E}$) applied on a lead with a frequency of ω. The vector potential $\mathbf{A}$ is then written as $\mathbf{A}(t)=(1/i\omega)\mathbf{E}e^{-i\omega t}$. We consider a limit of $\omega\to 0$ and $\mathbf{E}\parallel z$. The electronic current in linear response is given as $J=J^{(0)}+J^{(\rm A)}$, where

$$J^{(0)}(\mathbf{x},t)=\frac{1}{V}\frac{E_z}{\omega}\left(\frac{e}{2m}\right)^2(\nabla_{\mathbf{x}}-\nabla_{\mathbf{x}'})_z(\nabla_{\mathbf{x}_0}-\nabla_{\mathbf{x}_0'})_z \times Q^<_{\mathbf{xx}',\mathbf{x}_0\mathbf{x}_0'}(t,t',\omega)|_{\mathbf{x}'\to\mathbf{x},\mathbf{x}_0'\to\mathbf{x}_0,t'\to t},$$

$$J^{(\rm A)}(\mathbf{x},t)=-\frac{e^2}{m}A_z(\mathbf{x},t)\langle\langle c^\dagger(\mathbf{x}t')c(\mathbf{x}t)\rangle\rangle|_{t'\to t}, \tag{2}$$

where $\mathbf{x}_0$ and $\mathbf{x}_0'$ represent position in the lead where the electronic field is applied. Double brackets $\langle\langle\ \rangle\rangle$ include the averaging over the electron and impurity. The correlation function $Q_{\mathbf{xx}',\mathbf{x}_0\mathbf{x}_0'}(t,t')$ is defined as

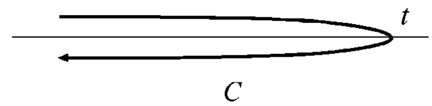

FIG. 1. Path in the complex time plane. t is the time of measurement.

$$Q_{\mathbf{xx}',\mathbf{x}_0\mathbf{x}_0'}(t,t',\omega)\equiv(-i)^2\int_C dt_0 e^{-i\omega t_0}\langle\langle T_C c(\mathbf{x}t)c^\dagger(\mathbf{x}_0't_0)\times c(\mathbf{x}_0t_0)c^\dagger(\mathbf{x}'t')\rangle\rangle. \tag{3}$$

T_C denotes a path order on contour C in a complex time plane (Fig. 1), and superscript $<$ denotes taking the lesser component with respect to $t<t'$ on the path C.[30] The Fourier transform of Q is written as

$$Q_{\mathbf{kk}'}(t,t',\omega)\equiv(-i)^2\int_C dt_0 e^{-i\omega t_0}\langle\langle T_C c_{\mathbf{k}}(t)c_{\mathbf{k}'}^\dagger(t_0)c_{\mathbf{k}'}(t_0)c_{\mathbf{k}}^\dagger(t')\rangle\rangle, \tag{4}$$

and the spatially uniform component of the current $J^{(0)}$ is written as

$$J^{(0)}(t)=\frac{1}{V}\frac{E_z}{\omega}\frac{e^2}{2m}\sum_{\mathbf{kk}'}\frac{k_zk_z'}{m}Q^<_{\mathbf{kk}'}(t,t,\omega). \tag{5}$$

We first consider a case of a simply connected geometry. The second order contribution to Q is the self-energy (SE) type (Fig. 2). [The vertex correction vanishes, since $\mathbf{k}$ and $\mathbf{k}'$ in Eq. (5) are independent of each other (note that V does not depend on the momentum transfer) and $Q^<_{\mathbf{kk}'}$ is an even function of $\mathbf{k}$ and $\mathbf{k}'$.] The SE contribution $Q^{(\rm SE)}_{\mathbf{kk}'}\equiv\delta_{\mathbf{k},\mathbf{k}'}Q^{(\rm SE)}_{\mathbf{k}}$ is written as

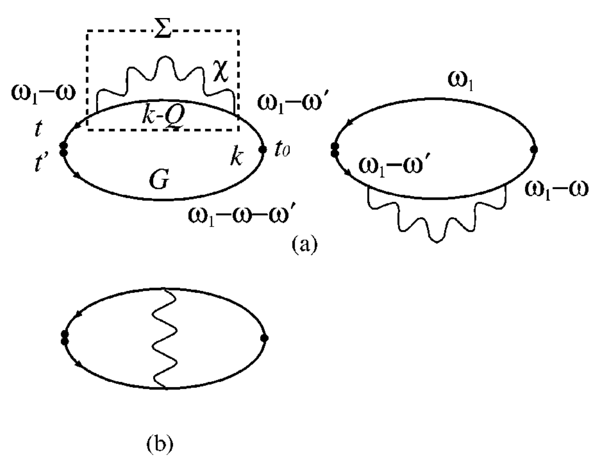

FIG. 2. Second-order contribution to Q. (a) Self-energy type. (b) Vertex correction type, which vanishes since the interaction vertex V does not depend on the momentum transfer.

$$Q_{\mathbf{k}}^{(\mathrm{SE})}(t,t',\omega)=\int_C dt_0 e^{-i\omega t_0}\int_C dt_1\int_C dt_2 \times[G_{\mathbf{k}}(t-t_1)\Sigma(t_1,t_2)G_{\mathbf{k}}(t_2-t_0)G_{\mathbf{k}}(t_0-t') + G_{\mathbf{k}}(t-t_0)G_{\mathbf{k}}(t_0-t_1)\Sigma(t_1,t_2)G_{\mathbf{k}}(t_2-t')]. \tag{6}$$

Here

$$\Sigma(t_1,t_2)\equiv i\Sigma_{\mathbf{Q}}\chi(t_1,t_2)G_{\mathbf{k}-\mathbf{Q}}(t_1-t_2),$$

and

$$\chi(t_1,t_2)\equiv -i\langle T_c V(t_1)V(t_2)\rangle \tag{7}$$

is a correlation function of the TLS. The lesser component ($Q^<$) of Eq. (6) is calculated by use of decomposition rules such as $[\int_C dt_1 A(t-t_1)B(t_1-t')]^< = \int_{-\infty}^{\infty} dt_1[A^r(t-t_1)B^<(t_1-t')+A^<(t-t_1)B^a(t_1-t')]$ and $[A(t-t_1)B(t-t_1)]^< = A(t-t_1)^< B(t-t_1)^<$ (A and B are path-ordered correlation functions).[30] The result is (see Fig. 3)

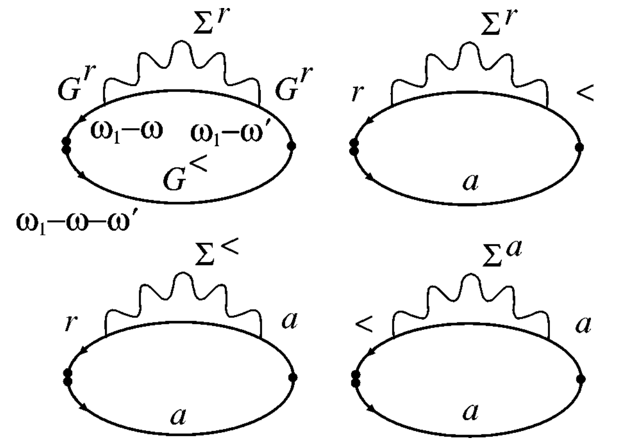

FIG. 3. Decomposition of $Q^<$ into retarded (r), advanced (a), and lesser components ($<$).

$$\begin{aligned}Q_{\mathbf{k}}^{(\mathrm{SE})}(t,t',\omega)=\sum_{\omega'} e^{-i\omega' t}(&G^r_{\mathbf{k},\omega_1-\omega}\Sigma^r_{\omega_1-\omega,\omega_1-\omega'}G^r_{\mathbf{k},\omega_1-\omega'}G^<_{\mathbf{k},\omega_1-\omega-\omega'}\\&+G^r_{\mathbf{k},\omega_1-\omega}\Sigma^r_{\omega_1-\omega,\omega_1-\omega'}G^<_{\mathbf{k},\omega_1-\omega'}G^a_{\mathbf{k},\omega_1-\omega-\omega'}\\&+G^r_{\mathbf{k},\omega_1-\omega}\Sigma^<_{\omega_1-\omega,\omega_1-\omega'}G^a_{\mathbf{k},\omega_1-\omega'}G^a_{\mathbf{k},\omega_1-\omega-\omega'}\\&+G^<_{\mathbf{k},\omega_1-\omega}\Sigma^a_{\omega_1-\omega,\omega_1-\omega'}G^a_{\mathbf{k},\omega_1-\omega'}G^a_{\mathbf{k},\omega_1-\omega-\omega'}+\mathrm{c.c.}),\end{aligned} \tag{8}$$

where

$$\Sigma^r_{\omega_1,\omega_2}=i\sum_{\mathbf{Q}\omega_3}(G^r_{\mathbf{k}-\mathbf{Q},\omega_3}\chi^<_{\omega_1-\omega_3,\omega_2-\omega_3}+G^>_{\mathbf{k}-\mathbf{Q},\omega_3}\chi^r_{\omega_1-\omega_3,\omega_2-\omega_3}), \tag{9}$$

$\Sigma^<_{\omega_1,\omega_2}=i\Sigma_{\mathbf{Q}\omega_3}G^<_{\mathbf{k}-\mathbf{Q},\omega_3}\chi^<_{\omega_1-\omega_3,\omega_2-\omega_3}$, and c.c. denotes conjugate processes. Lesser and greater components of free Green functions are given as $G^<_{\mathbf{k}}(\omega)=f_\omega\Delta G_{\mathbf{k}}(\omega)$ and $G^>_{\mathbf{k}}(\omega)=-(1-f_\omega)\Delta G_{\mathbf{k}}(\omega)$, where $f_\omega\equiv 1/(e^{\beta\omega}+1)$ is the Fermi distribution function and $\Delta G_{\mathbf{k}}(\omega)\equiv G^a_{\mathbf{k}}(\omega)-G^r_{\mathbf{k}}(\omega)$. The expression of $Q^<$ is further simplified if we use

$$\frac{k_z}{m}(G^r_{\mathbf{k}}(\omega))^2=\frac{\partial}{\partial k_z}G^r_{\mathbf{k}}(\omega), \tag{10}$$

and a partial derivative with respect to k_z.

After some calculation the SE contribution is obtained as

$$\begin{aligned}\sum_{\mathbf{k}}\frac{(k_z)^2}{m}Q_{\mathbf{k}}^{(\mathrm{SE})<}(t,t,\omega\to 0)=-i\omega\sum_{\mathbf{k}}\frac{(k_z)^2}{m}\sum_{\omega'}e^{-i\omega' t}\sum_{\omega_1\omega_2}\sum_{\mathbf{q}}&[\Pi^a_{\mathbf{kQ}}(\omega_1,\omega_2,\omega')\partial_{\omega_1}f_{\omega_1}G^r_{\mathbf{k},\omega_1}G^a_{\mathbf{k},\omega_1}G^a_{\mathbf{k},\omega_1-\omega'}\\&+\Pi^r_{\mathbf{kQ}}(\omega_1,\omega_2,\omega')\partial_{\omega_1}f_{\omega_1-\omega'}G^a_{\mathbf{k},\omega_1-\omega'}G^r_{\mathbf{k},\omega_1-\omega'}G^r_{\mathbf{k},\omega_1}]\\&-i\sum_{\mathbf{k}}\sum_{\omega'}e^{-i\omega' t}\sum_{\omega_1\omega_2}\sum_{\mathbf{q}}[\Pi^a_{\mathbf{kQ}}(\omega_1,\omega_2,\omega')f_{\omega_1}\Delta G_{\mathbf{k},\omega_1}G^a_{\mathbf{k},\omega_1-\omega'}\\&+\Pi^r_{\mathbf{kQ}}(\omega_1,\omega_2,\omega')f_{\omega_1-\omega'}G^r_{\mathbf{k},\omega_1}\Delta G_{\mathbf{k},\omega_1-\omega'}\\&+\chi^<_{\omega_2,\omega_2-\omega'}f_{\omega_1-\omega_2}\Delta G_{\mathbf{k}-\mathbf{Q},\omega_1-\omega_2}G^r_{\mathbf{k},\omega_1}G^a_{\mathbf{k},\omega_1-\omega'}],\end{aligned} \tag{11}$$

where

$$\Pi^{\mu}_{\mathbf{kQ}}(\omega_1,\omega_2,\omega') \equiv \chi^{<}_{\omega_2,\omega_2-\omega'} G^{\mu}_{\mathbf{k}-\mathbf{Q},\omega_1-\omega_2} - (1-f_{\omega_1-\omega_2})\Delta G_{\mathbf{k}-\mathbf{Q},\omega_1-\omega_2}\chi^{\mu}_{\omega_2,\omega_2-\omega'}, \tag{12}$$

$(\mu=a,r)$ and $\Sigma_\omega \equiv \int(d\omega/2\pi)$. The current contribution from the SE, $J^{(\mathrm{SE})}$, is defined by Eq. (5), with Q replaced by $Q^{(\mathrm{SE})}$.

The current $J^{(\mathrm{A})}$ [Eq. (2)] is similarly calculated as

$$\begin{aligned} J^{(\mathrm{A})}(t) &= E_z \frac{e^2}{m}\frac{1}{\omega}\left[\int_C dt_1 \int_C dt_2 G_{\mathbf{k}}(t-t_1)\Sigma(t_1,t_2)G_{\mathbf{k}}(t_2-t')\right]^{<} \\ &= iE_z\frac{e^2}{m}\frac{1}{\omega}\sum_{\mathbf{k}}\sum_{\omega'} e^{-i\omega' t}\sum_{\omega_1\omega_2}\sum_{\mathbf{Q}} \\ &\quad\times[\Pi^{a}_{\mathbf{kQ}}(\omega_1,\omega_2,\omega')f_{\omega_1}\Delta G_{\mathbf{k},\omega_1}G^{a}_{\mathbf{k},\omega_1-\omega'} \\ &\quad+\Pi^{r}_{\mathbf{kQ}}(\omega_1,\omega_2,\omega')f_{\omega_1-\omega'}G^{r}_{\mathbf{k},\omega_1}\Delta G_{\mathbf{k},\omega_1-\omega'} \\ &\quad+\chi^{<}_{\omega_2,\omega_2-\omega'}f_{\omega_1-\omega_2}\Delta G_{\mathbf{k}-\mathbf{Q},\omega_1-\omega_2}G^{r}_{\mathbf{k},\omega_1}G^{a}_{\mathbf{k},\omega_1-\omega'}]. \end{aligned} \tag{13}$$

It is seen that this contribution cancels the second part in Eq. (11). Hence the total current is obtained as

$$\begin{aligned} J(t) &= J_0 + J^{(\mathrm{SE})}(t) + J^{(\mathrm{A})}(t) \\ &= J_0 - i\frac{1}{V}E_z\left(\frac{e}{m}\right)^2\sum_{\mathbf{k}}(k_z)^2\sum_{\omega'}e^{-i\omega' t}\sum_{\omega_1\omega_2}\sum_{\mathbf{Q}} \\ &\quad\times[\Pi^{a}_{\mathbf{kQ}}(\omega_1,\omega_2,\omega')\partial_{\omega_1}f_{\omega_1}G^{r}_{\mathbf{k},\omega_1}G^{a}_{\mathbf{k},\omega_1}G^{a}_{\mathbf{k},\omega_1-\omega'} \\ &\quad+\Pi^{r}_{\mathbf{kQ}}(\omega_1,\omega_2,\omega')\partial_{\omega_1}f_{\omega_1-\omega'}G^{a}_{\mathbf{k},\omega_1-\omega'} \\ &\quad\times G^{r}_{\mathbf{k},\omega_1-\omega'}G^{r}_{\mathbf{k},\omega_1}]. \end{aligned} \tag{14}$$

Here $J_0 \equiv E_z\sigma_0$ is current without the TLS, $\sigma_0 \equiv (e^2/3) \times (k_F/m)^2[N(0)/V]\tau$, and $N(0) \equiv V(mk_F/2\pi^2)$ is the density of states.

Using $\partial_{\omega_1}f_{\omega_1} \simeq -\delta(\omega_1)$ and taking summations over $\mathbf{k}$ and $\mathbf{Q}$, we obtain

$$J(t) = J_0 - 2\pi J_0 N(0)\tau\sum_{\omega'\omega_2}\frac{e^{-i\omega' t}}{1-i\tau\omega'}i[\chi^{<}_{\omega_2,\omega_2-\omega'} - f_{\omega_2}\chi^{a}_{\omega_2,\omega_2-\omega'} + f_{\omega_2-\omega'}\chi^{r}_{\omega_2,\omega_2-\omega'}]. \tag{15}$$

Aharonov-Bohm current

We next consider the case of a ring with a magnetic flux, shown in Fig. 4. For simplicity the perturbation due to the TLS (H') is treated such as to exist only on the upper arm (arm a) and the phase $\phi \equiv 2\pi\Phi/\Phi_0$ ($\Phi_0 \equiv h/2e$ being flux quantum) due to the flux (Φ) affects only the lower arm, b. We consider the case when the ring is slowly varying and the system is ballistic, $L \lesssim l$. The current through the ring is, given by the same expression as Eqs. (2)–(5), but Green functions need to be replaced by those in the ring geometry. The Green function connecting $\mathbf{x}$ and $\mathbf{x}_0$ at the right and left ends of the ring, respectively, is approximated as

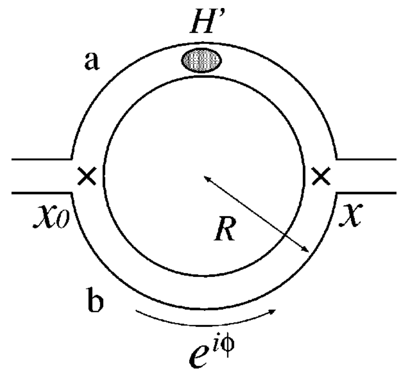

FIG. 4. Ring we consider. The TLS affects only the upper arm (a), and the phase due to magnetic flux (ϕ) is attached only on the lower arm (b).

$$\begin{aligned} G_{\mathrm{ring}}(\mathbf{x}-\mathbf{x}_0) &\simeq G_a(\mathbf{x}-\mathbf{x}_0) + G_b(\mathbf{x}-\mathbf{x}_0) \\ &= [G(\mathbf{x}-\mathbf{x}_0) + (G\Sigma G)(\mathbf{x}-\mathbf{x}_0)] \\ &\quad + e^{i\phi}G(\mathbf{x}-\mathbf{x}_0), \end{aligned} \tag{16}$$

where the first term is the Green function though the arm a ($G_a \equiv G + G\Sigma G$) and $G_b(\mathbf{x}-\mathbf{x}_0) \equiv e^{i\phi}G(\mathbf{x}-\mathbf{x}_0)$ represents propagation through arm b. In Eq. (16) contributions from the multiple circulation through the ring is neglected. The Green function in the opposite direction from $\mathbf{x}$ to $\mathbf{x}_0$ is

$$G(\mathbf{x}_0-\mathbf{x}) = G_a(\mathbf{x}_0-\mathbf{x}) + G_b^{-}(\mathbf{x}_0-\mathbf{x}), \tag{17}$$

where $G_b^{-}(\mathbf{x}_0-\mathbf{x}) \equiv e^{-i\phi}G(\mathbf{x}_0-\mathbf{x})$ carries the opposite phase as G_b. The current through the ring is calculated from Eq. (2) as

$$J_{\mathrm{ring}}(t) \equiv J_a + J_b + J_{ab} \tag{18}$$

where J_a and J_b are currents through arms a and b, which are given as ($\alpha=a,b$)

$$\begin{aligned} J_\alpha &= \frac{1}{V}\frac{E_z}{\omega}\left(\frac{e}{m}\right)^2\sum_{\mathbf{k}}k_z^2\sum_{\omega'}e^{-i\omega' t}\sum_{\omega_1\omega_2}[G^{r}_{\alpha\mathbf{k}}(\omega_1+\omega',\omega_2+\omega) \\ &\quad\times G^{-<}_{\alpha\mathbf{k}}(\omega_2,\omega_1) + G^{<}_{\alpha\mathbf{k}}(\omega_2,\omega_1)G^{-a}_{\alpha\mathbf{k}}(\omega_1-\omega,\omega_2-\omega')] \\ &\quad+J^{(\mathrm{A})}_\alpha, \end{aligned} \tag{19}$$

$J^{(A)}_\alpha$ being the the contribution from $J^{(\mathrm{A})}$ on arm α and $G_a^{-} \equiv G_a$. Current J_a is equal to Eq. (15) and $J_b = J_0$, since $H'=0$ on arm b. The Fourier transform of the Green functions in Eq. (19) is defined as $G^{\mu}_{\alpha\mathbf{k}}(\omega_1,\omega_2) \equiv \int_{-\infty}^{\infty}dt_1\int_{-\infty}^{\infty}dt_2 e^{-i\omega_1 t_1}e^{i\omega_2 t_2}G^{\mu}_{\alpha\mathbf{k}}(t_1,t_2)$. ($\omega_1$ is not necessarily equal to ω_2, since G_α includes the self-energy due to the TLS, which is not energy conserving). In Eq. (18), the interference effect is included in J_{ab}, which reads

$$J_{ab}=\frac{1}{V}\frac{E_z}{\omega}\left(\frac{e}{m}\right)^2\sum_{\mathbf{k}} k_z^2\sum_{\omega'} e^{-i\omega' t}\sum_{\omega_1}[G^r_{a\mathbf{k}}(\omega_1-\omega,\omega_1-\omega')$$
$$\times G^{<}_{\bar{b}\bar{\mathbf{k}}}(\omega_1)+G^{<}_{a\mathbf{k}}(\omega_1-\omega,\omega_1-\omega')G^{a}_{\bar{b}\bar{\mathbf{k}}}(\omega_1-\omega-\omega')$$
$$+G^r_{b\mathbf{k}}(\omega_1)G^{<}_{a\mathbf{k}}(\omega_1-\omega,\omega_1-\omega')$$
$$+G^{<}_{b\mathbf{k}}(\omega_1)G^{a}_{a\mathbf{k}}(\omega_1-\omega,\omega_1-\omega')]+J^{(A)}_{ab}$$
$$\equiv J^{(0)}_{ab}+J^{(2)}_{ab}. \tag{20}$$

Here $J^{(0)}_{ab}=2J_0\cos\phi$ and [by use of Eqs. (16) and (17)]

$$J^{(2)}_{ab}=\frac{1}{V}\frac{E_z}{\omega}\left(\frac{e}{m}\right)^2\sum_{\mathbf{k}} k_z^2\sum_{\omega'} e^{-i\omega t}[e^{i\phi}Q^{+<}_{\mathbf{k}}(\omega,\omega')$$
$$+e^{-i\phi}Q^{-<}_{\mathbf{k}}(\omega,\omega')]+J^{(A)^{(2)}}_{ab}, \tag{21}$$

where $J^{(A)^{(2)}}_{ab}$ is the contribution from $J^{(A)}$ and

$$Q^{+<}_{\mathbf{k}}(\omega,\omega')\equiv\sum_{\omega_1}[G^r_{\mathbf{k},\omega_1}(G\Sigma G)^{<}_{\omega_1-\omega,\omega_1-\omega'}$$
$$+G^{<}_{\mathbf{k},\omega_1}(G\Sigma G)^{a}_{\omega_1-\omega,\omega_1-\omega'}],$$

$$Q^{-<}_{\mathbf{k}}(\omega,\omega')\equiv\sum_{\omega_1}[(G\Sigma G)^{r}_{\omega_1-\omega,\omega_1-\omega'}G^{<}_{\mathbf{k},\omega_1-\omega-\omega'}$$
$$+(G\Sigma G)^{<}_{\omega_1-\omega,\omega_1-\omega'}G^{a}_{\mathbf{k},\omega_1-\omega-\omega'}]. \tag{22}$$

These are calculated similarly to the derivation of Eq. (11) as

$$Q^{+<}_{\mathbf{k}}(\omega\to0,\omega')=-i\omega\sum_{\omega_1\omega_2}\partial_{\omega_1}f_{\omega_1}G^r_{\mathbf{k},\omega_1}G^a_{\mathbf{k},\omega_1}G^a_{\mathbf{k},\omega_1-\omega'}$$
$$\times\Pi^a_{\mathbf{kQ}}(\omega_1,\omega_2,\omega')+Q^{+\prime},$$

$$Q^{-<}_{\mathbf{k}}(\omega\to0,\omega')=-i\omega\sum_{\omega_1\omega_2}\partial_{\omega_1}f_{\omega_1-\omega'}G^r_{\mathbf{k},\omega_1}G^r_{\mathbf{k},\omega_1-\omega'}$$
$$\times G^a_{\mathbf{k},\omega_1-\omega'}\Pi^r_{\mathbf{kQ}}(\omega_1,\omega_2,\omega')+Q^{-\prime}, \tag{23}$$

where $Q^{\pm\prime}$ are terms which cancel with $J^{(A)^{(2)}}_{ab}$. The final result of $J^{(2)}_{ab}$ is

$$J^{(2)}_{ab}=-\frac{2\pi^2}{3}\frac{1}{V}E_z\left(\frac{e}{m}\right)^2[N(0)k_F\tau]^2\sum_{\omega'}\frac{e^{-i\omega' t}}{1-i\omega'\tau}$$
$$\times\sum_{\omega_2} i\frac{1}{2}[e^{i\phi}(\chi^{<}_{\omega_2,\omega_2-\omega'}-2f_{\omega_2}\chi^{a}_{\omega_2,\omega_2-\omega'})$$
$$+e^{-i\phi}(\chi^{<}_{\omega_2,\omega_2-\omega'}+2f_{\omega_2-\omega'}\chi^{r}_{\omega_2,\omega_2-\omega'})]. \tag{24}$$

From Eqs. (19) and (24), we obtain the total current through the ring as

$$J_{\rm ring}(t)=2(1+\cos\phi)J_0$$
$$-2\pi J_0N(0)\tau\sum_{\omega'}\frac{e^{-i\omega' t}}{1-i\omega'\tau}\sum_{\omega_2}\frac{i}{2}$$
$$\times[(1+e^{i\phi})(\chi^{<}_{\omega_2,\omega_2-\omega'}-2f_{\omega_2}\chi^{a}_{\omega_2,\omega_2-\omega'})$$
$$+(1+e^{-i\phi})(\chi^{<}_{\omega_2,\omega_2-\omega'}+2f_{\omega_2-\omega'}\chi^{r}_{\omega_2,\omega_2-\omega'})]. \tag{25}$$

III. CORRELATION FUNCTIONS OF A TWO-LEVEL SYSTEM

The Hamiltonian of the TLS we consider is

$$H_{\rm TLS}=\frac{\Omega}{2}\sigma_z, \tag{26}$$

where the two levels are represented by the Pauli matrix σ_z. The interaction H' is switched on at $t=0$ till $t=T_0$ (T_0 is later set equal to the time of measurement), and is written as

$$V(t)=(u\sigma_z+v\sigma_x)\theta(t)\theta(T_0-t), \tag{27}$$

where u and v are coupling constants and $\theta(t)$ is a step function. Here we consider the case in which the TLS is initially at $|\sigma_z=m\rangle$ ($m=\pm1$) at $t=0$. The correlation functions are given as

$$\chi^{<}(t_1,t_2)=-i\langle m|V(t_2)V(t_1)|m\rangle,$$
$$=-i(u^2+v^2e^{-im\Omega(t_1-t_2)})$$
$$\times\theta(t_1)\theta(t_2)\theta(T_0-t_1)\theta(T_0-t_2),$$
$$\chi^{>}(t_1,t_2)=-i(u^2+v^2e^{im\Omega(t_1-t_2)})\theta(t_1)$$
$$\times\theta(t_2)\theta(T_0-t_1)\theta(T_0-t_2), \tag{28}$$
$$\chi^{r}(t_1,t_2)=\theta(t-t')(\chi^{>}-\chi^{<})(t_1,t_2),$$
$$\chi^{a}(t_1,t_2)=-\theta(t'-t)(\chi^{>}-\chi^{<})(t_1,t_2).$$

The Fourier transform is defined as ($\mu=\langle,\rangle,r,a$)

$$\chi^{\mu}_{\omega,\omega'}\equiv\int_{-\infty}^{\infty}dt_1\int_{-\infty}^{\infty}dt_2e^{i\omega t_1}e^{-i\omega' t_2}\chi^{\mu}(t_1,t_2), \tag{29}$$

where we note that ω and ω' are not nesessarily equal. These are calculated as

$$\chi^{<}_{\omega_2,\omega_2-\omega'}=-i[u^2\Gamma_{\omega_2}\Gamma_{\omega'-\omega_2}+v^2\Gamma_{\omega_2-m\Omega}\Gamma_{\omega'-\omega_2+m\Omega}],$$

$$\chi^{a}_{\omega_2,\omega_2-\omega'}=v^2\sum_{\pm}\frac{\pm}{\omega_2\pm m\Omega}(\Gamma_{\omega'}-\Gamma_{\omega'-\omega_2\mp m\Omega}), \tag{30}$$

$$\chi^{r}_{\omega_2,\omega_2-\omega'}=v^2\sum_{\pm}\frac{\pm}{\omega_2-\omega'\pm m\Omega}(\Gamma_{\omega'}-\Gamma_{\omega_2\pm m\Omega}),$$

where

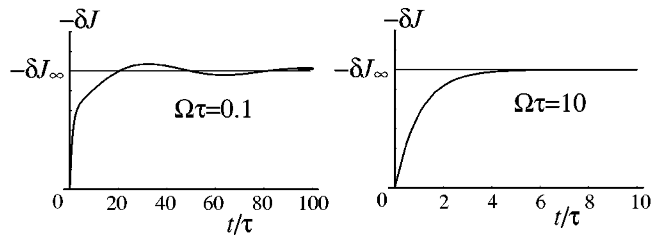

FIG. 5. Behavior of the current [Eq. (32)] for the two cases $\tilde{\Omega}=0.1$ and $\tilde{\Omega}=10$. u and v are chosen as 1.

$$\Gamma_\omega(T_0)\equiv\int_0^{T_0}dt' e^{i\omega t'}=\frac{e^{i\omega T_0}-1}{i(\omega+i0)} \tag{31}$$

(note that Γ_ω depends on T_0).

IV. RESPONSE OF AHARONOV-BOHM CURRENT TO A TLS

The expression of Eq. (15) is estimated by use of Eq. (30) with $T_0\to t$ as

$$J(t)=J_0-2\pi J_0 N(0)\tau\left[\left\{u^2+v^2\left(1-\frac{2}{\pi}\tan^{-1}\tilde{\Omega}\right)\right\}(1-e^{-\tilde{t}}) + \frac{2}{\pi}v^2\int_0^{\tilde{\Omega}}\frac{dx}{1+x^2}\left(1-\cos x\tilde{t}+\frac{\sin x\tilde{t}}{x}\right)\right], \tag{32}$$

where $\tilde{\Omega}\equiv m\Omega\tau$, $\tilde{t}\equiv t/\tau$. In the case of low frequency, $|\tilde{\Omega}|\ll 1$, integration over x is carried out to be

$$J(t)=J_0\left[1-2\pi N(0)\tau\left\{(u^2+v^2)(1-e^{-\tilde{t}})+v^2\frac{2}{\pi}\times\left(\mathrm{Si}(\tilde{\Omega}\tilde{t})-\frac{\sin\tilde{\Omega}\tilde{t}}{\tilde{t}}+\tilde{\Omega}e^{-\tilde{t}}\right)\right\}\right] \quad (|\tilde{\Omega}|\ll 1), \tag{33}$$

where $\mathrm{Si}(x)\equiv\int_0^x(dy/y)\sin y$. After the TLS ($H'$) is switched on, the current relaxes to a new equilibrium value ($J_0[1-2\pi N(0)\tau(u^2+2v^2)]\equiv J_0+\delta J_\infty$) in the time scale of Ω^{-1} (Fig. 5). In the opposite case, $|\tilde{\Omega}|\gg 1$, the scale becomes τ:

$$J(t)=J_0[1-2\pi N(0)\tau(u^2+2v^2)(1-e^{-\tilde{t}})] \quad (|\tilde{\Omega}|\gg 1). \tag{34}$$

The result for the ring [Eq. (25)], is similarly calculated as

$$J_{\mathrm{ring}}(t)=2(1+\cos\phi)J_0-(1+\cos\phi)J_0 2\pi N(0)\tau\times\left[\left\{u^2+v^2\left(1-\frac{2}{\pi}\tan^{-1}\tilde{\Omega}\right)\right\}(1-e^{-\tilde{t}})+\frac{2}{\pi}v^2\int_0^{\tilde{\Omega}}\frac{dx}{1+x^2}\left(1-\cos x\tilde{t}+\frac{\sin x\tilde{t}}{x}\right)\right]. \tag{35}$$

It is seen that the amplitude of the AB oscillation is reduced by the TLS, but the reduction is due to the reflection of the electron by the TLS in the same way as in a wire [Eq. (32)]. This indicates that the TLS does not affect the coherence of electrons in the ballistic transport. This is clear in the static limit $\Omega\to 0$, where dephasing cannot occur. In fact the decay rate in this case, $2\pi N(0)(u^2+v^2)$, is simply equal to the transition probability calculated from the self-energy, $\Gamma=2i\Sigma_{\mathbf{k}}\mathrm{tr}[(u\sigma_z+v\sigma_x)G^r_{\mathbf{k}}(\omega=0)(u\sigma_z+v\sigma_x)]$. The effect of the TLS surviving in the high-frequency limit, $\Omega\to\infty$, also excludes the possibility of the dephasing mechanism. We will see that these limiting behaviors are different in case of the AAS current [Eq. (57)]. The phase of the oscillation ($\cos\phi$) is not modified, similarly to the equilibrium case, in which case $\sin\phi$ term is forbidden since it violates the time-reversal symmetry.[16]

The behavior at $t\sim 0$ of the current [Eq. (35)] is given as

$$\begin{aligned}J_{\mathrm{ring}}(t)&\simeq(1+\cos\phi)J_0[2-2\pi N(0)(u^2+v^2)t]\\&\simeq(1+\cos\phi)2J_0e^{-\Gamma t/2}\end{aligned} \tag{36}$$

where $\Gamma\equiv 2\pi N(0)(u^2+v^2)$ and a factor of 1/2 is to account for the TLS applied only on one of the two arms. This decay rate Γ is nothing but the rate obtained by Fermi's golden rule. In fact the transition probability of the electron from momentum $\mathbf{k}$ to $\mathbf{k}'$ is given by

$$\begin{aligned}|A_{\mathbf{k}'m'\mathbf{k}m}(t)|^2&=\left|-i\int_0^t dt_1\langle\mathbf{k}'m'|H'(t_1)|\mathbf{k}m\rangle\right|^2\\&=u^2\delta_{m'm}\left(\frac{\sin[(\epsilon_{\mathbf{k}'}-\epsilon_{\mathbf{k}})t/2]}{(\epsilon_{\mathbf{k}'}-\epsilon_{\mathbf{k}})/2}\right)^2\\&\quad+v^2\delta_{m',-m}\left(\frac{\sin[(\epsilon_{\mathbf{k}'}-\epsilon_{\mathbf{k}}-m\Omega)t/2]}{(\epsilon_{\mathbf{k}'}-\epsilon_{\mathbf{k}}-m\Omega)/2}\right)^2\end{aligned} \tag{37}$$

where m and $m'(=\pm)$ are the initial and final state of the TLS. By use of

$$\left(\frac{\sin[\epsilon t/2]}{\epsilon/2}\right)^2\to 2\pi\delta(\epsilon)t$$

for $t\to\infty$, we obtain $\Sigma_{\mathbf{k}'mm'}|A_{\mathbf{k}'m'\mathbf{k}m}(t)|^2\to 2\pi t\Sigma_{\mathbf{k}'}[u^2\delta(\epsilon_{\mathbf{k}'}-\epsilon_{\mathbf{k}})+v^2\delta(\epsilon_{\mathbf{k}'}-\epsilon_{\mathbf{k}}-m\Omega)]=\Gamma t$.

This rate Γ is also evaluated from the overlap of the state at t and $t=0$,

$$\begin{aligned}\langle 0|Te^{-i\int_0^t dt_1 H'(t_1)}|0\rangle&\simeq 1-\frac{i}{2}\int_0^t dt_2\int_0^t dt_2\,\theta(t_1-t_2)\\&\quad\times[\chi^>(t_1,t_2)+\chi^<(t_2,t_1)]\\&\quad\times\sum_{\mathbf{k}\mathbf{k}'}G^<_{\mathbf{k}'}(t_2-t_1)G^>_{\mathbf{k}}(t_1-t_2),\end{aligned} \tag{38}$$

which results in $\simeq e^{-\Gamma t}$ for $\Gamma t\ll 1$.

In the case of electron-electron interaction, the decay rate of the overlap integral was shown to be equivalent to dephasing time.[6,7] In the present case of ballistic transport, the de-

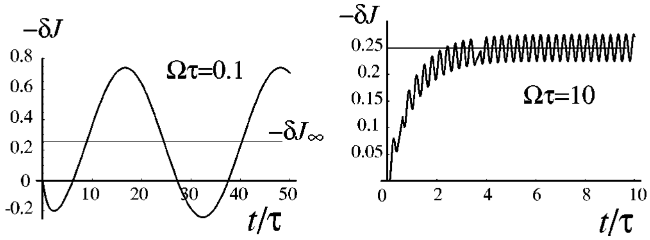

FIG. 6. Behavior of $|\delta J| \equiv |J(t)-J_0|$ for the oscillating external field [Eq. (40)], plotted in units of $2\pi J_0 N(0)\tau v^2$. At $t\to\infty$ the oscillation is around $|\delta J| = 1/4\times 2\pi J_0 N(0)\tau v^2$.

cay of the amplitude of the AB oscillation [Eq. (35)] as well as the overlap integral are not related to dephasing, but are due simply to the scattering into other states. What is crucial here is the lack of randomness needed to put an uncertain phase on the wave function. Dephasing is taken into account when the effect of the Cooperon is considered in the presence of random disorder (Sec. VI).

V. EFFECT OF AN OSCILLATING EXTERNAL FIELD

Our ballistic results, Eqs. (15) and (25), are general and can be applied to other perturbation sources. We here consider current (15) with an oscillating external field, $V(t) = v \sin \omega t$. In this case $\chi^a = \chi^r = 0$ and

$$\chi^<_{\omega_2,\omega_2-\omega'} = \frac{i}{4} v^2 \sum_{\pm} [\Gamma_{\omega_2 \pm \Omega}(\Gamma_{-\omega_2+\omega' \pm \Omega} - \Gamma_{-\omega_2+\omega' \mp \Omega})]. \tag{39}$$

Current (15) is obtained as

$$J(t) = J_0 - 2\pi J_0 N(0)\tau \frac{v^2}{4} \frac{1}{1+4\tilde{\Omega}^2}[2(1-\cos 2\Omega t - 2\tilde{\Omega}\sin 2\Omega t) - (1-4\tilde{\Omega}^2)(1-e^{-\tilde{t}})]. \tag{40}$$

As seen in Fig. 6, the current oscillates around new equilibrium value $[J_\infty = J_0 - 2\pi J_0 N(0)\tau v^2/4]$ if the external field is slowly varying ($\tilde{\Omega} \ll 1$), but oscillation is not dominant if the perturbation is too fast for the electron to accommodate ($\tilde{\Omega} \gg 1$).

This result has the possibility of various applications. One example is a ballistic transport through a nanoscale metallic magnetic contacts. In magnetic contacts a large magnetoresistance is observed due to a strong scattering by a domain wall trapped in the contact region.[32,33] Recently a nonlinear *I-V* characteristic was observed in half-metallic oxide contacts, which is argued to be due to deformation of the wall.[34] In these small contacts, the application of a small oscillating magnetic field might drive a slow oscillation of the wall position and shape. This causes a time-varying scattering potential of the electron, and hence would be detectable by measuring time-resolved current through the contact. A current measurement may be useful to observe mesoscopic dynamics.

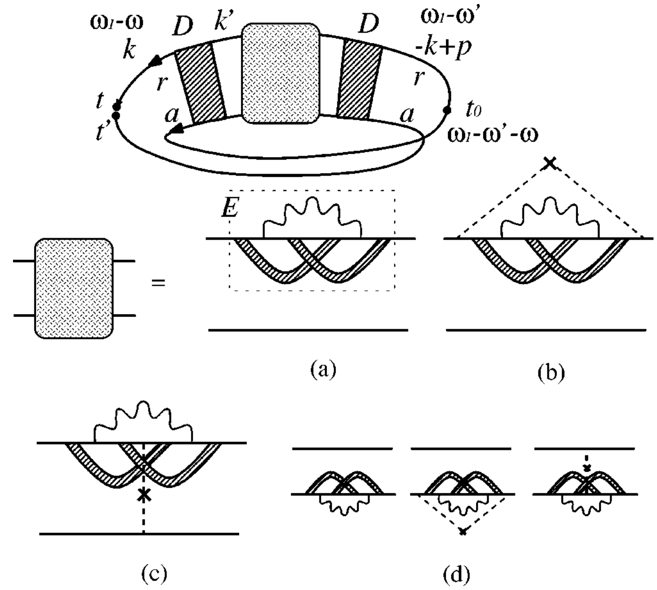

FIG. 7. Corrections by the TLS to the AAS oscillation. The shaded thick line denotes a Cooperon. Scattering by a normal impurity is indicated by a dotted line.

VI. RESPONSE OF ALTSHULER-ARONOV-SPIVAK OSCILLATION

In this section we study the effect of switching of the TLS on the Altshuler-Aronov-Spivak (AAS) oscillation.[13] This oscillation is due to the interference of a particle-particle propagator (Cooperon) induced by successive elastic scattering. The oscillation is $\cos(2\phi)$, reflecting the charge of $2e$ carried by a Cooperon. The AAS contribution is calculated from Eq. (2) with the Cooperon taken into account. In the absence of a TLS, the Cooperon contribution to the current is calculated as[14]

$$J^{(0)}_{AAS} = \frac{E_z}{V}\left(\frac{e}{m}\right)^2 n_{\rm i} v_{\rm i}^2 \sum_{\mathbf{k}} k_z(-k_z) G^r_{\mathbf{k}} G^r_{-\mathbf{k}} G^a_{\mathbf{k}} G^a_{-\mathbf{k}} C(0), \tag{41}$$

where

$$C(0) \equiv \sum_{\mathbf{p}} \sum_{n=0}^{\infty} \left(n_{\rm i} v_{\rm i}^2 \sum_{\mathbf{k}} G^r_{\mathbf{k}} G^a_{\mathbf{p}-\mathbf{k}} \right)^n \simeq \sum_{\mathbf{p}} \frac{1}{(Dp^2 + 1/\tau_\varphi)\tau} \tag{42}$$

is a Cooperon. $n_{\rm i}$ and $v_{\rm i}$ are the density and strength of impurity scattering, respectively, which are related to τ as $1/\tau = 2\pi n_{\rm i} v_{\rm i}^2 N(0)$. We have phenomenologically added an inelastic lifetime $\tau_\varphi{}^2$, which is assumed to arise from other mechanisms than TLS's. For $L \gtrsim l_\varphi$ [$l_\varphi \equiv \sqrt{D\tau_\varphi}$ is the inelastic mean free path (dephasing length)], $C(0)$ is calculated as [we assume that the width of the ring is smaller than inelastic mean free path ($L_\perp \lesssim l_\varphi$) and carry out summation over p as in one dimension]

$$C(0) \simeq \frac{3 L l_\varphi}{8\pi^2 l^2}(1 + 2e^{-L/l_\varphi}\cos 2\phi). \tag{43}$$

(Higher-order contributions $\propto e^{-nL/l_\varphi}$, $n\geqslant 2$ are neglected.) The AAS current in the absence of a TLS is thus

$$J_{AAS}^{(0)}=-E_z\sigma_0\frac{3}{2\pi k_F^2 ab}\frac{l_\varphi}{l}e^{-L/l_\varphi}\cos 2\phi, \quad (44)$$

a and b being the width and thickness of the ring, respectively.

Now we calculate the effect by the TLS. This is done by considering a correction to the Cooperon. Most important processes are shown in Figs. 7(a)–7(c). Process (a) is calculated as

$$\begin{aligned}Q_{\mathbf{k}}^{(a)<}(t,t,\omega)=(n_{\rm i}v_{\rm i}^2)^2\sum_{\omega'\omega_1}e^{-i\omega' t}\sum_{nn'=0}^{\infty}\sum_{\mathbf{k}'\mathbf{k}_i\mathbf{k}_i'}\sum_{\mathbf{p}}&\\ \times[G_{\mathbf{k},\omega_1-\omega}D^{(n)}_{\{\mathbf{k}_i\},\omega_1-\omega}G_{\mathbf{k}',\omega_1-\omega}&\\ \times E_{\omega_1-\omega,\omega_1-\omega'}G_{\mathbf{k}',\omega_1-\omega'}D^{(n')}_{\{\mathbf{k}_i'\},\omega_1-\omega'}&\\ \times G_{-\mathbf{k}+\mathbf{p},\omega_1-\omega'}G_{-\mathbf{k}+\mathbf{p},\omega_1-\omega-\omega'}&\\ \times D^{(n)}_{\{-\mathbf{k}_i+\mathbf{p}\},\omega_1-\omega-\omega'}G_{-\mathbf{k}'+\mathbf{p},\omega_1-\omega-\omega'}&\\ \times D^{(n')}_{\{-\mathbf{k}_i'+\mathbf{p}\},\omega_1-\omega-\omega'}G_{\mathbf{k},\omega_1-\omega-\omega'}]^<,&\end{aligned} \quad (45)$$

where

$$D^{(n)}_{\{\mathbf{k}_i\},\omega_1}\equiv\Pi_{i=1}^{n}(\sqrt{n_{\rm i}v_{\rm i}^2}G_{\mathbf{k}_i,\omega_1}) \quad (46)$$

is a Green function connected by successive impurity scattering,

$$\begin{aligned}E(t,t')\equiv(n_{\rm i}v_{\rm i}^2)^2\sum_{nn'=0}^{\infty}\int_C dt_1\int_C dt_2[D^{(n)}G]&\\ \times(t-t_1)i\chi(t_1,t_2)F^{nn'}(t_1-t_2)[GD^{(n')}](t_2-t'),&\end{aligned} \quad (47)$$

and $F^{nn'}(t_1-t_2)\equiv[GD^{(n')}GD^{(n)}G](t_1-t_2)$ (We write $[AB](t-t')\equiv\int_C dt'' A(t-t'')B(t''-t')$ and subscripts are partially suppressed). An important Cooperon behavior [Eq. (42)] arises in $Q_{\mathbf{k}}^{(a)<}$ only when all $G_{\mathbf{k}_i}$'s in $D^{(n)}_{\{\mathbf{k}_i\},\omega_1-\omega}$ and $D^{(n')}_{\{\mathbf{k}_i'\},\omega_1-\omega'}$ are retarded Green functions and $G_{-\mathbf{k}_i+\mathbf{p}}$'s in $D^{(n)}_{\{-\mathbf{k}_i+\mathbf{p}\},\omega_1-\omega-\omega'}$ and $D^{(n')}_{\{-\mathbf{k}_i'+\mathbf{p}\},\omega_1-\omega-\omega'}$ are advanced Green functions, and for $p\sim 0$. By use of

$$\sum_{n=0}^{\infty}\sum_{\mathbf{k}_i}D^{(n)r}_{\{\mathbf{k}_i\},\omega_1-\omega}D^{(n)a}_{\{-\mathbf{k}_i+\mathbf{p}\},\omega_1-\omega-\omega'}\simeq C_{\mathbf{p},\omega'}\quad(p\sim 0), \quad (48)$$

where $C_{p\omega}\equiv 1/[(Dp^2+1/\tau_\varphi-i\omega)\tau]$, the dominant contribution of Eq. (45) is calculated as

$$\begin{aligned}Q_{\mathbf{k}}^{(a)<}(t,t,\omega)&=\sum_{\omega'\omega_1}e^{-i\omega' t}(n_{\rm i}v_{\rm i}^2)^2(f_{\omega_1-\omega-\omega'}-f_{\omega_1-\omega'})[GD^{(n)}GEGD^{(n')}G]^r_{\omega_1-\omega'}[GD^{(n)}GD^{(n')}G]^a_{\omega_1-\omega-\omega'}\\ &=\sum_{\omega'\omega_1}e^{-i\omega' t}(n_{\rm i}v_{\rm i}^2)^2(f_{\omega_1-\omega-\omega'}-f_{\omega_1-\omega'})\sum_{\mathbf{p}}C_{\mathbf{p}\omega}C_{\mathbf{p}\omega'}\sum_{\mathbf{k}'}G^r_{\mathbf{k},\omega_1-\omega}G^r_{\mathbf{k}',\omega_1-\omega}E^r_{\omega_1-\omega,\omega_1-\omega'}G^r_{\mathbf{k}',\omega_1-\omega'}\\ &\quad\times G^r_{-\mathbf{k}+\mathbf{p},\omega_1-\omega'}G^a_{-\mathbf{k}+\mathbf{p},\omega_1-\omega-\omega'}G^a_{-\mathbf{k}'+\mathbf{p},\omega_1-\omega-\omega'}G^a_{\mathbf{k},\omega_1-\omega-\omega'}.\end{aligned} \quad (49)$$

The retarded part of $E(t,t')$ here is given as

$$\begin{aligned}E^r(t,t')=(n_{\rm i}v_{\rm i}^2)^2\sum_{nn'=0}^{\infty}\int_{-\infty}^{\infty}dt_1\int_{-\infty}^{\infty}dt_2[D^{(n)r}G^r](t-t_1)i[\chi^<(t_1,t_2)F^{nn'r}(t_1-t_2)&\\ +\chi^r(t_1,t_2)F^{nn'>}(t_1-t_2)][G^rD^{(n')r}](t_2-t').&\end{aligned} \quad (50)$$

In terms of the Fourier transform (Fig. 8),

$$\begin{aligned}E^r_{\omega_1-\omega,\omega_1-\omega'}&=\sum_{\omega_4}[D^{(n)r}G^r]_{\omega_1-\omega}i[\chi^r_{\omega_1-\omega_4-\omega,\omega_1-\omega_4-\omega'}F^{nn'>}_{\omega_4}+\chi^<F^{nn'r}][G^rD^{(n')r}]_{\omega_1-\omega'}\\ &\simeq -i(n_{\rm i}v_{\rm i}^2)^2\sum_{\omega_4}(1-f_{\omega_4})\chi^r_{\omega_1-\omega_4-\omega,\omega_1-\omega_4-\omega'}[D^{(n)r}G^r]_{\omega_1-\omega}[GD^{(n')}GD^{(n)}G]^a_{\omega_4}[G^rD^{(n')r}]_{\omega_1-\omega'}\\ &=-i(n_{\rm i}v_{\rm i}^2)^2\sum_{\omega_4}(1-f_{\omega_4})\chi^r_{\omega_1-\omega_4-\omega,\omega_1-\omega_4-\omega'}\sum_{\mathbf{p}'}C_{\mathbf{p}',\omega_1-\omega_4-\omega}C_{\mathbf{p}',\omega_1-\omega_4-\omega'}\\ &\quad\times\sum_{\mathbf{k}_1\mathbf{k}_2}G^r_{\mathbf{k}_1,\omega_1-\omega}G^a_{\mathbf{k}_1-\mathbf{Q},\omega_4}G^a_{-\mathbf{k}'+\mathbf{p}',\omega_4}G^a_{\mathbf{k}_2-\mathbf{Q},\omega_4}G^r_{\mathbf{k}_2,\omega_1-\omega'}.\end{aligned} \quad (51)$$

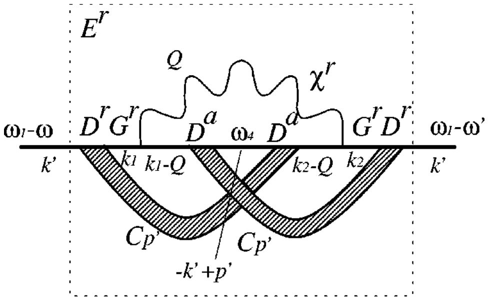

FIG. 8. Diagrammatic representation of E^r.

We thus obtain

$$
\begin{aligned}
Q_{\mathbf{k}}^{(a)<}(t,t,\omega)=&-in_{\mathrm{i}}v_{\mathrm{i}}^2\tau^2\sum_{\omega'\omega_1\omega_4}e^{-i\omega' t}\sum_{\mathbf{p}}G^r_{\mathbf{k},\omega_1-\omega}\\
&\times G^r_{-\mathbf{k}+\mathbf{p},\omega_1-\omega'}G^a_{-\mathbf{k}+\mathbf{p},\omega_1-\omega-\omega'}G^a_{\mathbf{k},\omega_1-\omega-\omega'}\\
&\times(f_{\omega_1-\omega-\omega'}-f_{\omega_1-\omega'})(1-f_{\omega_4})\\
&\times\chi^r_{\omega_1-\omega_4-\omega,\omega_1-\omega_4-\omega'}C_{\mathbf{p}\omega}C_{\mathbf{p}\omega'}\\
&\times\sum_{\mathbf{p}'}C_{\mathbf{p}',\omega_1-\omega_4-\omega}C_{\mathbf{p}',\omega_1-\omega_4-\omega'}\\
&\times[2+3i\tau(\omega_1-\omega_4)-4p^2D\tau]. \qquad (52)
\end{aligned}
$$

Other processes in Figs. 7(b) and 7(c) are similarly calculated as

$$
\begin{aligned}
Q_{\mathbf{k}}^{(\mathrm{b+c})<}(t,t,\omega)=&-in_{\mathrm{i}}v_{\mathrm{i}}^2\tau^2\sum_{\omega'\omega_1\omega_4}e^{-i\omega' t}\sum_{\mathbf{p}}G^r_{\mathbf{k},\omega_1-\omega}\\
&\times G^r_{-\mathbf{k}+\mathbf{p},\omega_1-\omega'}G^a_{-\mathbf{k}+\mathbf{p},\omega_1-\omega-\omega'}\\
&\times G^a_{\mathbf{k},\omega_1-\omega-\omega'}(f_{\omega_1-\omega-\omega'}-f_{\omega_1-\omega'})\\
&\times(1-f_{\omega_4})\chi^r_{\omega_1-\omega_4-\omega,\omega_1-\omega_4-\omega'}C_{\mathbf{p}\omega}C_{\mathbf{p}\omega'}\\
&\times\sum_{\mathbf{p}'}C_{\mathbf{p}',\omega_1-\omega_4-\omega}C_{\mathbf{p}',\omega_1-\omega_4-\omega'}\\
&\times[-2-i\tau(4\omega_1-4\omega_4-\omega-\omega')+5p^2D\tau]. \qquad (53)
\end{aligned}
$$

It is seen that one of the four Cooperons is canceled after summation of the three processes (a)–(c),[35] and we obtain $Q_{\mathbf{k}}^{AAS<}\equiv Q_{\mathbf{k}}^{(\mathrm{a})<}+Q_{\mathbf{k}}^{(\mathrm{b+c})<}$ as (noting $p,p'\ll k$ and $\omega'\tau\ll 1$)

$$
\begin{aligned}
Q_{\mathbf{k}}^{AAS<}(t,\omega\to 0)=&-i\omega n_{\mathrm{i}}v_{\mathrm{i}}^2\tau^2\\
&\times\sum_{\omega'\omega_1}e^{-i\omega' t}(G^r_{\mathbf{k}}G^a_{\mathbf{k}})^2\\
&\times\sum_{pp'}C_{p\omega}C_{p\omega'}C_{p'\omega_1}\\
&\times(f_{\omega_1-\omega'}\chi^r_{\omega_1,\omega_1-\omega'}-f_{\omega_1}\chi^a_{\omega_1,\omega_1-\omega'}), \qquad (54)
\end{aligned}
$$

where χ^a term is due to the complex processes [Fig. 7(d)] and $G_{\mathbf{k}}\equiv G_{\mathbf{k},\omega=0}$.

The current at low temperature is obtained by use of Eq. (30) [and Eq. (5)] as

$$
\begin{aligned}
\delta J^{AAS}(t)=&J_0\frac{\pi v^2}{mk_FV}\sum_{\mathbf{pp}'}\frac{1}{A_p}\int_{-\infty}^{0}d\omega\sum_{\pm}(\pm)\\
&\times\mathrm{Im}\Bigg[-\left(1-e^{-A_pt}-iA_pe^{-A_pt}\frac{e^{-i(\omega\pm\Omega)t}-1}{\omega\pm\Omega}\right)\\
&\times\frac{3\omega\pm\Omega+i(2A_{p'}-A_p)}{[(\omega\pm\Omega)^2+A_p^2](\omega+iA_{p'})[\omega+i(A_{p'}-A_p)]}\\
&+\left(\frac{1-e^{-(A_{p'}-i\omega)t}}{\omega+iA_{p'}}-e^{-(A_{p'}-i\omega)t}\frac{e^{-i(\omega\pm\Omega)t}-1}{\omega\pm\Omega}\right)\\
&\times\frac{1}{[\omega+i(A_{p'}-A_p)](\pm\Omega-iA_{p'})}\Bigg], \qquad (55)
\end{aligned}
$$

where $A_p\equiv Dp^2+1/\tau_\varphi$. The slowest relaxation is governed by the contribution from $p=p'=0$ of the square bracket part. The oscillation part of this contribution is obtained as

$$
\delta J^{AAS}(t)\simeq J_0\cos(2\phi)\frac{3}{4\pi}\frac{l_\varphi}{l}\frac{v^2}{k_F^2ab}\tau_\varphi^3F, \qquad (56)
$$

where

$$
\begin{aligned}
F\equiv\mathrm{Im}\int_{-\infty}^{0}d\omega\sum_{\pm}\frac{\pm}{\tau_\varphi^3}\Bigg[&-\left(1-e^{-t/\tau_\varphi}-i\frac{e^{-t/\tau_\varphi}}{\tau_\varphi}\frac{e^{-i(\omega\pm\Omega)t}-1}{\omega\pm\Omega}\right)\\
&\times\frac{3\omega\pm\Omega+i/\tau_\varphi}{[(\omega\pm\Omega)^2+\tau_\varphi^{-2}](\omega+i/\tau_\varphi)\omega}+\Bigg(\frac{1-e^{-(1/\tau_\varphi-i\omega)t}}{\omega+i/\tau_\varphi}\\
&-e^{-(1/\tau_\varphi-i\omega)t}\frac{e^{-i(\omega\pm\Omega)t}-1}{\omega\pm\Omega}\Bigg)\frac{1}{\omega(\pm\Omega-i/\tau_\varphi)}\Bigg]. \qquad (57)
\end{aligned}
$$

It is easy to check that $\delta J^{AAS}>0$. This enhancement of the AAS current is explained as due to the dephasing effect of the TLS, which suppresses localization. The phase of the oscillation is not modified (i.e., $\delta J^{AAS}\propto\cos 2\phi$), and only the amplitude relaxes after the TLS is switched. If $\Omega\ll\tau_\varphi$ the time scale of the relaxation is $\sim\Omega^{-1}$. In the opposite case of $\Omega\gg\tau_\varphi$, there first appears a rise in the time scale of τ_φ followed by a rapid decay with small oscillation of frequency of $\sim\Omega$ (Fig. 9). The effect of the TLS vanishes both in the low- and high-frequency limits; $\propto\Omega$ for $\Omega\ll 1$ and $\propto 1/\Omega$ for $\Omega\gg 1$. The vanishing of the effect in these limits, which is distinct from the ballistic case [Eq. (35)], is consistent with the explanation by dephasing effect.

VII. SUMMARY AND DISCUSSION

We have calculated the electronic current through an Aharonov-Bohm (AB) ring after a quantum two-level-system (TLS) is switched on. The TLS affects the amplitude of AB and AAS oscillations, which relaxes to a new equilibrium value. Phases of both oscillations are not affected. If the

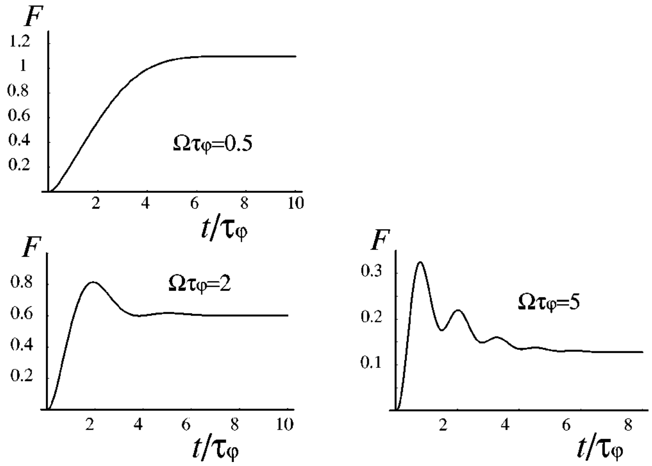

FIG. 9. Relaxation of the amplitude of the AAS oscillation [F of Eq. (57)] after a switching of the TLS for $\Omega\tau_\varphi = 0.5$, 2, and 5. For $\Omega\tau_\varphi \ll 1$ the behavior is monotonic, but for $\Omega\tau_\varphi \gtrsim 1$ a bump appears in the time scale of $\sim\tau_\varphi$ and then a decay.

energy splitting of the TLS, Ω, is large, the time scale of the amplitude relaxation is given by the characteristic time of the system, which is the elastic lifetime τ in the ballistic case and the inelastic lifetime τ_φ in the diffusive case. In the opposite case, $\Omega \ll \tau^{-1}, \tau_\varphi{}^{-1}$, the time scale becomes Ω^{-1}. Although the relaxation of the current appears similar in both ballistic and diffusive cases, the physics behind the relaxation is different. In the ballistic case the relaxation is due to a scattering of the states into other states, which is not dephasing. In the diffusive case the relaxation is interpreted as due to dephasing. The crucial difference between the two is that in the diffusive case, on the one hand, the phase produced by the TLS is randomly accumulated because of the contribution from the random paths the electron travels; in the ballistic case, on the other hand, there is no randomness. The dephasing effect would appear in the ballistic case if the energy of the TLS is distributed.

The effect of an oscillating external field is also calculated. The amplitude of the current oscillates if the external oscillation is slow enough for the electron to accommodate, but the current oscillation becomes unclear in the fast varying limit.

Recent high (THz) time-resolved measurements of electronic properties[28] make it possible to observe the current response and time-resolved dephasing processes. The current response may provide us with direct information about microscopic relaxation times [elastic (τ) and inelastic (τ_φ) lifetimes] and properties of the perturbation source.

In nanoscale magnetic contacts,[32,34] a motion such as a slow oscillation of a magnetic domain wall may be detectable as an oscillation of electronic current through the contact. Time-resolved transport measurement may become a powerful method in studying mesoscopic dynamics.

ACKNOWLEDGEMENTS

G.T. thanks H. Matsukawa and H. Kohno for valuable discussion. He is grateful to The Mitsubishi Foundation for financial support.

[1] A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. **46**, 211 (1981); A. O. Caldeira and A. J. Leggett, Ann. Phys. (N.Y.) **149**, 374 (1983).
[2] G. Bergmann, Phys. Rep. **107**, 1 (1984).
[3] P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).
[4] B. L. Altshuler, A. G. Aronov, and D. E. Khmelnitsky, J. Phys. C **15**, 7367 (1982).
[5] H. Fukuyama and E. Abrahams, Phys. Rev. B **27**, 5976 (1983).
[6] A. Stern, Y. Aharonov, and Y. Imry, Phys. Rev. A **41**, 3436 (1990).
[7] Y. Imry, *Introduction to Mesoscopic Physics* (Oxford University Press, Oxford, 1997).
[8] Y. Imry, H. Fukuyama, and P. Schwab, Europhys. Lett. **47**, 608 (1999).
[9] K.-H. Ahn and P. Mohanty, Phys. Rev. B **63**, 195301 (2001).
[10] I. L. Aleiner, B. L. Altshuler, and Y. M. Galperin, Phys. Rev. B **63**, 201401(R) (2001).
[11] P. Mohanty, E. M. Q. Jariwala, and R. A. Webb, Phys. Rev. Lett. **78**, 3366 (1997).
[12] Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959).
[13] B. L. Altshuler, A. G. Aronov, and B. Z. Spivak, Pis'ma Zh. Éksp. Teor. Fiz. **33**, 101 (1981) [JETP Lett. **33**, 94 (1981)].
[14] A. G. Aronov and Yu. V. Sharvin, Rev. Mod. Phys. **59**, 755 (1987).
[15] A. Yacoby, M. Heiblum, D. Mahalu, and H. Shtrikman, Phys. Rev. Lett. **74**, 4047 (1995).
[16] A. Yacoby, R. Schuster, and M. Heiblum, Phys. Rev. B **53**, 9583 (1996).
[17] G. Hackenbroich and H. A. Weidenmüller, Phys. Rev. Lett. **76**, 110 (1996).
[18] R. Schuster, E. Buks, M. Heiblum, D. Mahalu, V. Umansky, and H. Shtrikman, Nature (London) **385**, 417 (1997).
[19] A. P. Jauho and N. S. Wingreen, Phys. Rev. B **58**, 9619 (1998).
[20] K. Yakubo and J. Ohe, J. Phys. Soc. Jpn. **69**, 2170 (2000).
[21] A. -P. Jauho, N. S. Wingreen, and Y. Meir, Phys. Rev. B **50**, 5528 (1994).
[22] A. Kaminski, Y. V. Nazarov, and L. I. Glazman, Phys. Rev. Lett. **83**, 384 (1999).
[23] P. Nordlander, M. Pustilnik, Y. Meir, N. S. Wingreen, and D. C. Langreth, Phys. Rev. Lett. **83**, 808 (1999).
[24] S. Pedersen, A. E. Hansen, A. Krisensen, C. B. Sorensen, and P. E. Lindelof, Phys. Rev. B **61**, 5457 (2000).
[25] A. E. Hansen, A. Krisensen, S. Pedersen, C. B. Sorensen, and P. E. Lindelof, Phys. Rev. B **64**, 245313 (2001).
[26] G. Seelig and M. Büttiker, Phys. Rev. B **64**, 045327 (2001).
[27] S. Q. Murphy, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **52**, 14 825 (1995).
[28] M. C. Beard, G. M. Turner, and C. A. Schmuttenmaer, Phys. Rev. B **62**, 15764 (2000).
[29] L. V. Keldish, Zh. Éksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys. JETP **20**, 1018 (1965)].

[30] H. Haug and A. -P. Jauho, *Quantum Kinetics in Transport and Optics of Semi-conductors* (Springer-Verlag, Berlin, 1998).
[31] J. Rammer and H. Smith, Rev. Mod. Phys. **58**, 323 (1986).
[32] N. Garcia, M. Munoz, and Y. W. Zhao, Phys. Rev. Lett. **82**, 2923 (1999).
[33] G. Tatara, Y.-W. Zhao, M. Muñoz, and N. García, Phys. Rev. Lett. **83**, 2030 (1999).
[34] J. I. Versluijs, M. A. Bari, and J. M. D. Coey, Phys. Rev. Lett. **87**, 026601 (2001).
[35] H. Fukuyama, Suppl. Prog. Theor. Phys. **84**, 47 (1985).