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Interaction corrections at intermediate temperatures: Dephasing time

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We calculate the temperature dependence of the weak-localization correction in a two-dimensional system at arbitrary relation between temperature T and the elastic mean free time. We describe the crossover in the dephasing time $\tau_\varphi(T)$ between the high-temperature, $1/\tau_\varphi \simeq T^2 \ln T$, and the low-temperature $1/\tau_\varphi \simeq T$ behaviors. The prefactors in these dependences are not universal, but are determined by the Fermi-liquid constant characterizing the spin exchange interaction.

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I. INTRODUCTION

The concept of a time scale characteristic of electron-scattering processes in metals has been the focus of intense theoretical research for the last 3 decades.[1–16] It has been established for disordered systems[4] that the time scale, which corresponds to processes that suppress quantum interference (and are thus responsible for the temperature dependence of the weak localization correction, for example), namely the phase relaxation time τ_φ, is quite different from the semiclassical time scales, such as the energy relaxation time τ_E.

Previous work was mostly focused on the universal (independent of interaction strength) contribution of the singlet channel interaction both in the diffusive[4,5] and ballistic[3,5,11,13,14] regimes. The contribution of the triplet channel was only considered in the diffusive regime in Ref. 8. In this paper we fill the gaps by considering both channels at arbitrary relation between temperature and the inverse mean free time and thus describing the crossover between the diffusive and ballistic regimes. We also clarify the relation between thus calculated dephasing time and experimentally observable physical quantities.

A discussion of phase relaxation should begin by defining a physical quantity sensitive to quantum interference (since the phase by itself is not an observable quantity). Therefore, the precise definition of the phase relaxation time depends on the choice of such physical quantity. Consider the weak-localization correction in two dimensions[6,15] in the absence of magnetic field:

$$\delta\sigma_{WL}(T) = -\frac{e^2}{2\pi^2\hbar}\int_{-\infty}^{\infty}\frac{d\epsilon}{4T\cosh^2\dfrac{\epsilon}{2T}}\ln\frac{\tau_\varphi(\epsilon,T)}{\tau}, \quad (1)$$

where τ is the transport elastic mean free time. With the logarithmic accuracy, one can here neglect the dependence of τ_φ on ϵ and estimate[6,17]

$$\delta\sigma_{WL}(T) \approx -\frac{e^2}{2\pi^2\hbar}\ln\frac{\tau_\varphi(0,T)}{\tau}. \quad (2)$$

In the presence of magnetic field H, perpendicular to the plane of the two-dimensional system, the situation is more complicated. The magnetoconductivity can be described[6,17,18] as

$$\sigma(H,T)-\sigma(0,T) = -\frac{e^2}{2\pi^2\hbar}\int_{-\infty}^{\infty}\frac{d\epsilon}{4T\cosh^2\frac{\epsilon}{2T}} \times\left\{ Y[\Omega_H\tau_\varphi(\epsilon,T;H)] + \ln\left(\frac{\tau_\varphi(\epsilon,T;H)}{\tau_\varphi(\epsilon,T)}\right)\right\}, \quad (3)$$

where $\Omega_H = 4DeH/\hbar c$, D is the diffusion constant, and

$$Y[x] = \ln\frac{1}{x} - \psi\left(\frac{1}{2}+\frac{1}{x}\right),$$

with $\psi(x)$ being the digamma function. Notice, that the dephasing time now depends on the magnetic field, see Refs. 6, 18, 15 and Eq. (8) below. We use the notation

$$\tau_\varphi(\epsilon,T) \equiv \tau_\varphi(\epsilon,T;H=0).$$

One can simplify the magnetoconductivity (3) using asymptotic expressions for $Y[x]$ in order to facilitate comparison with experimental data. For strong magnetic fields $\Omega_H\tau_\varphi \gg 1$, one finds ($Y[x\gg1] \approx -\ln x + 2\ln 2 + C$), with $C \approx 0.5772\ldots$ being the Euler constant,

$$\sigma(H,T)-\sigma(0,T) = \frac{e^2}{2\pi^2\hbar}\ln\alpha\Omega_H\tau_\varphi^H + O\left(\frac{1}{\Omega_H\tau_\varphi^H}\right), \quad (4)$$

where $\alpha = 1/(4e^C) = 0.1403\ldots$, and

$$\tau_\varphi^H = \exp\left(\int_{-\infty}^{\infty}\frac{d\epsilon}{4T\cosh^2\dfrac{\epsilon}{2T}}\ln\tau_\varphi(\epsilon,T)\right). \quad (5)$$

In the opposite limit of the weak field ($Y[x\ll1] \approx -x^2/24$) the magnetoconductivity is quadratic in Ω_H:

$$\sigma(H,T)-\sigma(0,T)=\frac{e^2}{48\pi^2\hbar}(\Omega_H\tau_\varphi^T)^2, \tag{6}$$

where

$$\tau_\varphi^T=\left[\int_{-\infty}^{\infty}\frac{d\epsilon}{4T\cosh^2\dfrac{\epsilon}{2T}}\tau_\varphi^2(\epsilon,T)\right]^{1/2}. \tag{7}$$

As we show below, the two scales τ_φ^T and τ_φ^H may differ by a numerical factor, which is important for a quantitative comparison with experiments.

As a function of ϵ and T, the phase relaxation time is given by[6]

$$\frac{1}{\tau_\varphi(\epsilon,T;H)}=\int_{\max[\Omega_H,\tau_\varphi^{-1}]}^{\infty}d\omega\omega A(\omega)\left[\coth\frac{\omega}{2T}-\frac{1}{2}\left(\tanh\frac{\omega-\epsilon}{2T}+\tanh\frac{\omega+\epsilon}{2T}\right)\right]. \tag{8}$$

Kernel $A(\omega)$ contains all the information about matrix elements of the interaction and wave functions of the disordered system, see, e.g., Refs. 6 and 15, and this is precisely the same kernel that enters into the inelastic collision integral in the Boltzmann equation, see e.g., Refs. 6 and 16:

$$\mathrm{St}_{in}\{f(\epsilon)\}=\int d\omega\int d\epsilon_1 A(\omega)f(\epsilon_1)[1-f(\epsilon_1-\omega)]\times\{-f(\epsilon)[1-f(\epsilon+\omega)]+[1-f(\epsilon)]f(\epsilon-\omega)\}, \tag{9}$$

where f is the distribution function of the quasiparticles.

II. CALCULATION OF THE KERNEL

The kernel $A(\omega)$ can be obtained from the quantum kinetic equation (see Ref. 19), and it is expressed in terms of the interaction propagator $\mathcal{D}^R$ and the propagators $\langle D\rangle$ describing semiclassical dynamics of noninteracting electrons,

$$A(\omega)=\frac{2\nu}{\pi}\int\frac{d^2q}{(2\pi)^2}\{[\mathrm{Re}\langle D\rangle]^2|\mathcal{D}_S^R(\omega,q)|^2+\mathrm{Tr}[\mathrm{Re}\langle\hat{D}\rangle]\hat{\mathcal{D}}_T^R(\omega,q)[\mathrm{Re}\langle\hat{D}\rangle][\hat{\mathcal{D}}_T^R(\omega,q)]^*\}. \tag{10}$$

In the absence of external magnetic field and spin-orbit interaction, the semi-classical propagator is diagonal $\langle\hat{D}\rangle=\delta_{ij}\langle D\rangle$ and

$$\langle D\rangle=\frac{1}{\sqrt{(-i\omega+1/\tau)^2+v_F^2q^2}-1/\tau}. \tag{11}$$

It becomes the usual diffuson[6] in the diffusive limit $\omega, v_Fq\ll 1/\tau$.

The interaction propagator in the singlet channel is given in terms of $\langle D\rangle$ and the Fermi-liquid constant F_0^ρ (Refs. 19 and 20)

$$\mathcal{D}_S^R(\omega,q)=-\frac{1}{\nu}\frac{\nu V_0(q)+F_0^\sigma}{1+[\nu V_0(q)+F_0^\sigma][1+i\omega\langle D\rangle]}. \tag{12}$$

The interaction propagator in the triplet channel is a diagonal matrix in spin indices:

$$\hat{\mathcal{D}}_T^R(\omega,q)=-\frac{\delta_{ij}}{\nu}\frac{F_0^\sigma}{1+F_0^\sigma[1+i\omega\langle D\rangle]}, \tag{13}$$

where F_0^σ is the Fermi-liquid constant in the triplet channel.[19,20] At distances larger than the screening radius, one can take the unitary limit in Eq. (12), $\nu V_0(q)\to\infty$, which coincides with the $F_0^\sigma\to\infty$ limit in Eq. (13) (see Ref. 18 for more details).

We now evaluate the triplet contribution to the kernel $A(\omega)$, separating Eq. (10) into the sum

$$A(\omega)=3A_T(\omega)+A_S(\omega). \tag{14}$$

Using Eqs. (13) and (11) one can rewrite Eq. (10) as

$$A_T(\omega)=\frac{1}{\pi\omega}\mathrm{Im}\int\frac{d^2q}{(2\pi)^2}\mathcal{D}_T^R(\omega,q)[\langle D\rangle+\langle D\rangle^*]. \tag{15}$$

The kernel $A_S(\omega)$ can be obtained from Eq. (15) by taking the limit $F_0^\sigma\to\infty$. The momentum integral in Eq. (15) diverges logarithmically in the ultraviolet. This divergence is typically[5,3,14] cut off at a scale of order k_F (the precise definition of such cutoff is important only for the numerical factor under the logarithm, which we believe is beyond the accuracy of any discussion). In the diffusive limit the divergence does not appear since one is limited by small momenta $v_Fq\ll 1/\tau$. It is therefore convenient to separate $A_T(\omega)$ into two parts, roughly corresponding to the ballistic and diffusive asymptotics, respectively:

$$A_T(\omega)=A_1(\omega)+A_2(\omega). \tag{16}$$

The ''ballistic'' term appears from integrating over momenta larger than inverse elastic mean free time and/or frequency. With logarithmic accuracy we find [here the product of the Fermi momentum and the Fermi velocity (renormalized by interaction) is denoted by $E_F=v_Fk_F/2$]

$$A_1(\omega)=\frac{(F_0^\sigma)^2}{4\pi E_F(1+F_0^\sigma)^2}\ln\frac{E_F^2}{\tau^{-2}+b(F_0^\sigma)\omega^2}, \tag{17a}$$

$$b(x)\approx\frac{1+x^2}{(1+x)^2}.$$

This term has the logarithmic frequency dependence similar to that obtained by several authors for the singlet channel interaction.[5,14]

The ''diffusive'' term, i.e., the term coming from integrating over small momenta, is a generalization of the standard

result.[6] For simplicity, we show the kernel $A_2(\omega)$ in the two limiting cases. For small energy transfers $\omega\tau \ll 1$ we find

$$A_2(\omega\tau \ll 1) = \frac{F_0^\sigma}{(1+F_0^\sigma)(2+F_0^\sigma)} \frac{1}{2g\omega} \times \left[(1+F_0^\sigma) \arctan \frac{1}{\omega\tau} - \arctan \frac{1+F_0^\sigma}{\omega\tau} \right], \tag{17b}$$

where $g = 2\pi\hbar/e^2 R_\square$ is the dimensionless conductance of the system, and $R_\square$ is the sheet resistance.

Note that this result reduces to that of the diffusive theory for $\omega\tau \ll 1+F_0^\sigma$ (see Refs. 19 and 21 for a more detailed discussion of the crossover to the ballistic regime). For the singlet channel (which can be obtained by setting $F_0^\sigma \to \infty$) Eq. (17b) coincides with the standard result.[6]

For large energy transfers $\omega\tau \gg 1$ integration over small moments yields a correction to Eq. (17a):

$$A_2(\omega\tau \gg 1) = \frac{1}{1+F_0^\sigma} \frac{1}{2g\omega} \left[\arctan \frac{1+F_0^\sigma}{\omega\tau(1-F_0^\sigma)} + (1+F_0^\sigma) \arctan \frac{1}{\omega\tau} \right]. \tag{17c}$$

For numerical reasons, contributions of Eq. (17c) to the final results are small compared to that of Eq. (17a) for all temperature regimes.

III. RESULTS FOR THE DEPHASING TIME

We now use the explicit form of the kernel Eq. (17) to find the dephasing time from the self-consistency equation (8). According to Eqs. (14) and (16), it takes the form

$$\frac{1}{\tau_\varphi(\epsilon,T;H)} = I_1(\epsilon,T) + I_2(T, \max[\Omega_H, \tau_\varphi^{-1}]). \tag{18}$$

The kernel (17a) is not diverging at small energy transfers, and the calculation of term I_1 can be performed by setting the lower limit of integration in Eq. (8) to zero. We find with logarithmical accuracy

$$I_1 = \frac{\tau^2 T^2 + \epsilon^2}{8\pi E_F} \times \left[\frac{3(F_0^\sigma)^2}{(1+F_0^\sigma)^2} \ln \left(\frac{E_F^2}{b(F_0^\sigma)T^2 + \tau^{-2}} \right) + \ln \left(\frac{E_F^2}{T^2+\tau^{-2}} \right) \right], \tag{19a}$$

where function $b(x)$ is defined in Eq. (17a).

The diffusive term I_2 is logarithmically divergent at the lower limit. The divergent contribution is independent of ϵ and the divergence is cut according to Eq. (8),

$$I_2(T,\Omega) = \left(1 + \frac{3(F_0^\sigma)^2}{(1+F_0^\sigma)(2+F_0^\sigma)} \right) \frac{T}{g} \ln \left(\frac{T}{\Omega} \right). \tag{19b}$$

A. Diffusive limit ($T\tau \ll 1+F_0^\sigma$)

In this case the dephasing time follows from the self-consistency Eq. (8) where one has to include the singlet channel contribution. The time scales τ_φ^T, see Eq. (7), and τ_φ^H, see Eq. (5), are given by

$$\frac{1}{\tau_\varphi^H} = \frac{1}{\tau_\varphi^T} = \left(1 + \frac{3(F_0^\sigma)^2}{(1+F_0^\sigma)(2+F_0^\sigma)} \right) \frac{T}{g} \ln[g(1+F_0^\sigma)] + \frac{\pi}{4} \left(1 + \frac{3(F_0^\sigma)^2}{(1+F_0^\sigma)^2} \right) \frac{T^2}{E_F} \ln(E_F\tau). \tag{20}$$

At stronger fields $\Omega_H > T$ Eq. (20) is not applicable, since in that regime the interaction correction in the Cooper channel becomes of the same order as the weak-localization correction.[6] We will not dwell on this issue here.

The results (20) are valid while the condition $g(1+F_0^\sigma) \gg 1$ holds. The same condition guarantees the exponential smallness of temperature-independent dephasing induced by spontaneously spin-polarized regions.[22]

Comparing the dominant, diffusive term [the first term in Eq. (20), which came from integrating Eq. (17c)] to the second, ballistic term, we find that the two become of the same order when $T\tau \sim 1+F_0^\sigma$, which sets the limit of applicability to the purely diffusive theory.

B. Ballistic limit

At temperatures $T\tau \gg 1+F_0^\sigma$ the leading asymptotics is controlled by Eq. (19a). In this regime we can no longer neglect the ϵ dependence in $\tau_\varphi(\epsilon,T)$. As a result, the time scales τ_φ^T, see Eq. (7), and τ_φ^H, see Eq. (5), are different by a numerical factor:

$$\tau_\varphi^{H,T} = \tau_\varphi(0,T) B^{H,T}, \tag{21a}$$

where

$$\frac{1}{\tau_\varphi(0,T)} = \frac{\pi}{4} \frac{T^2}{E_F} \left[\ln \left(\frac{2E_F}{T} \right) + \frac{3(F_0^\sigma)^2}{(1+F_0^\sigma)^2} \ln \left(\frac{E_F}{T\sqrt{b(F_0^\sigma)}} \right) \right], \tag{21b}$$

where $b(x)$ is defined in Eq. (17a), and the numerical factors are

$$B^T = \left[\int_0^\infty \frac{dz}{\cosh^2 z} \left(1 + \frac{4z^2}{\pi^2} \right)^{-2} \right]^{1/2} \approx 0.8437\ldots;$$

$$B^H = \exp\left[-\int_0^\infty \frac{dz}{\cosh^2 z} \ln \left(1 + \frac{4z^2}{\pi^2} \right) \right] \approx 0.7931\ldots.$$

The observable dephasing times are different from $\tau_\varphi(0,T)$ by the above numerical constants. The temperature dependence of $\tau_\varphi(0,T)$, Eq. (21b), coincides with that found earlier in Refs. 3, 5, 11, 13, and 14 for the case $F_0^\sigma=0$.[23,24] The numerical factor $\pi/4$ is the same as in Refs. 11, 13, and 14, while Ref. 5 reports the factor $\pi/2$ and Ref. 3 reports the factor of $1/2\pi$. The correct numerical factor $\pi/4$ was also obtained in Ref. 12, however it was claimed that this result should be further renormalized (reduced by a factor of 4) by taking into account higher-order forward-scattering processes. We believe such renormalization is erroneous and is a result of misidentification of the Fermi-liquid constant in the singlet channel.[25]

IV. SUMMARY

We have calculated the temperature dependence of the weak-localization correction at arbitrary relation between T and elastic mean free time τ. The prefactors in the temperature dependences of the dephasing rate are not universal and are determined by the single Fermi-liquid constant F_0^σ. The very same constant determines the temperature dependence of the longitudinal resistivity,[19] the Hall coefficient,[21] and magnetoresistance in the parallel magnetic field.[26] Since the number of different observable quantities exceeds the number of input parameters this theory posesses the predictive power.

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[23] We note that the expression of the relaxation rate in terms of the Fermi-liquid constant is somewhat similar to the taking into account of so-called ''local field corrections'' of Ref. 13. We believe, that the deviation of our results from Eq. (24) of this reference is caused by the fact that the singlet (charge) and triplet, $L_z=0$, contributions were not separated properly [see Eq. (19) of this reference].

[24] Reference 14 argues that the contribution of other ''nongolden rule'' diagrams, see Fig. 1 of Ref. 14 decreases the coefficient by a factor of 2. We think that this statement is not correct—this diagram, in fact, describes the renormalization of F_0^ρ and the effect of interaction in the Cooper channel on inelastic processes. The latter is suppresed by the factor of $1/\ln^2(E_F/T)$ for the repulsive interaction.

[25] The correct form of the propagator in the singlet channel is given by Eq. (12), which at distances greater than the screening radius reduces to $\mathcal{D}_S^R(q=0)=1/\nu$. We believe that the resummation of Ref. 12 results in the incorrect form of the singlet propagator, $\mathcal{D}_S^R(q)=F_0^\rho/\nu+V(q)/[1+\nu V(q)]$, where for forwardscattering $F_0^\rho\to-1/2$, hence the extra factor of 1/4 in the final result of Ref. 12. Opinion of B. Laikhtman (private communication) is that disagreement by a factor of 4 is caused by algebraic mistakes in Eqs. (24) and (25) of Ref. 12.

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